applied sciences

MDPI

Article

Reliability Modeling and Analysis of Multi-Degradation of Momentum Wheel Based on Copula Function

Yan-Feng Li [1], Ming Huang [1], Song Bai [1], Yuan Chen [2] and Hong-Zhong Huang [1,*]

[1] School of Mechanical and Electrical Engineering, University of Electronic Science and Technology of China, Chengdu 611731, China; yanfengli@uestc.edu.cn (Y.-F.L.); huangming@std.uestc.edu.cn (M.H.); baisong@std.uestc.edu.cn (S.B.)
[2] Science and Technology on Reliability Physics and Application of Electronic Component Laboratory, The Fifth Electronics Research Institute of Ministry of Industry and Information Technology, Guangzhou 511370, China; chenyuan@ceprei.com
* Correspondence: hzhuang@uestc.edu.cn

Abstract: The momentum wheel is a key component of the satellite attitude control system and has a direct impact on the reliability and overall life of the satellite. The momentum wheel has the characteristics of a high reliability, long life, and complex failure mechanics, which leads to expensive maintenance and a low reliability of the test sample. Therefore, it is challenge to implement an accelerated life test. The traditional life data statistical method has great difficulty in solving the reliability analysis of the momentum wheel. A reliability calculation method based on copula function for multi-degradation is proposed. Firstly, the key factors affecting the reliability of the momentum wheel are analyzed, and the lubricant residual quantity and current are selected as the degradation quantity. Secondly, the wiener process is used to model the degradation of a single degradation quantity, and the edge distribution function of the momentum wheel reliability is obtained. Considering that the correlation between multiple degradation quantities has a non-negligible influence on the reliability analysis result, the copula function is introduced to describe the correlation, and the edge distributions are fused to obtain the joint distribution function of the momentum wheel reliability.

Keywords: reliability; degradation; copula function; wiener process; momentum wheel

Citation: Li, Y.-F.; Huang, M.; Bai, S.; Chen, Y.; Huang, H.-Z. Reliability Modeling and Analysis of Multi-Degradation of Momentum Wheel Based on Copula Function. *Appl. Sci.* **2021**, *11*, 11563. https://doi.org/10.3390/app112311563

Academic Editor: José A.F.O. Correia

Received: 2 November 2021
Accepted: 29 November 2021
Published: 6 December 2021

1. Introduction

With the development of modern equipment to complexity, reliability research is facing more and more challenges, for example the multi-state [1], common cause failures [2,3], survival signature [4], epistemic uncertainty [5,6] and multi-degradation of complex systems should be taken into consideration when conducting reliability modeling and analysis. In the satellite orbiting process, due to gravity gradient torque, solar radiation moment, aerodynamic torque, and geomagnetic torque, attitude deviation will occur. To this end, the attitude control system is required to play a controlling role to overcome the interference of the ambient torque and eliminate the attitude measurement. The deviation between the actual attitude and the desired attitude is given. The momentum wheel is the main execution part of the satellite three-axis stability control system, which generally includes four parts: bearing assembly, motor assembly, housing assembly and wheel assembly. The life cycle of the momentum wheel directly affects the service life of the satellite. Accurate calculation of the reliability of the momentum wheel is important to improve the reliability of the satellite and to extend the service life of the satellite.

Because the mechanism of momentum wheel failure is complex and expensive, it is difficult to carry out a large-scale accelerated life test. Therefore, there is no mature momentum wheel failure mechanism model. For this type of object, using the equipment degradation data to establish its performance, the degradation model is a widely used life prediction method [7]. Existing research shows that in satellite attitude control systems, momentum wheel failure is related to many factors [8]. Seasonal changes in the external environment, bearing temperature, failure of the lubricant system, micro-vibration, current, speed, and other factors may affect the life of the momentum wheel [9]. The existing data-based residual life prediction methods mostly consider only the effect of a single degradation amount on the remaining life. In reality, the life of the momentum wheel is influenced by multiple factors. For example, [10] studied the physical properties of lubricant failure, established a lubricant wear model to predict the life of the JB-3 momentum wheel bearing assembly and [11] took the momentum wheel bearing temperature as the life characteristic of the momentum wheel and used it. Empirical mode decomposition extracts time trends, establishes the degradation model of momentum wheel, and performs on-orbit momentum wheel life prediction; [12] uses the RVM-PF-based prediction method and takas momentum wheel bearing temperature and other indicators as life cycle characteristics of momentum wheel, completing the establishment of the momentum wheel degradation model and its life prediction; [13] analyzed the data of a model of five momentum wheel bearing temperature telemetry data, its shaft temperature performance degradation trend modeling, used to predict the momentum wheel remaining life. Moreover, [14] pointed out that the key to the failure of the momentum wheel is the remaining oil quantity of the oil supply system. The failure mechanism of the momentum wheel lubrication is studied to obtain the life reliability curve of the satellite momentum wheel. In addition, [15] studies the momentum wheel shaft. As a result of the test data, it was found that the slow rising trend of the current data was caused by the increase in bearing friction. A time series model of momentum wheel currents was established. Based on its performance degradation characteristics, momentum wheel life prediction curves were deduced. Ref. [16] was based on two similar models of momentum wheel product data, assuming that their lifetimes obey Weibull distribution, and the Weibull distribution was estimated. The shape parameters describe the degree of similarity in the life distribution between similar products of different models; [17] gives test results of momentum wheel life tests under conditions of high-temperature edge lubrication, seasonal changes, temperature, and micro-vibration; [18] established the network topology between the shaft temperature, current and momentum wheel component failure modes, and used the Bayesian network to model the reliability of the momentum wheel. Refs. [19,20] used the multi-source information fusion to estimate the residual life and useful life of a momentum wheel in a satellite, respectively. Ref. [21] collected 450 original frictional torque data of satellite momentum wheel bearings by experiment to predict its dynamic reliability. Ref. [22] conducted the finite element analysis under multiple coupling operating conditions and frictional heat on momentum wheel bearings and established the reliability model of bearings based on stress-strength interference. Ref. [23] proposed a custom detector based on the requirements specific to attitude control, which can autonomously monitor motor health to detect degradation and failure for real-time on-orbit recovery to maximize attitude control lifetime.

To date, the reliability modeling of momentum wheel has been mainly based on physics of failure (PoF). However, because the current momentum wheel failure mechanism model is not yet clear, the traditional PoF-based reliability model is not accurate enough to some extent. With the rapid development of prognostics health management (PHM) in recent years, system reliability modeling and evaluation are often combined with PHM. For a momentum wheel, it is more convincing and closer to engineering practice to conduct a reliability analysis through monitoring data such as degradation data. Therefore, based on performance degradation data of the momentum wheel, and considering that there is more than one performance degradation inside the system, this research proposes a reliability modeling and analysis of multi-degradation of a momentum wheel. Moreover, according to actual experience and failure analysis of the momentum wheel, it can be confirmed that there is a correlation between different performance degradations; therefore, it is necessary to quantity the correlation to accurately conduct reliability research on momentum wheels.

The structure of this paper is organized as follow. In Section 2, key factors affecting the life of the momentum wheel are analyzed, and the correlation between different performance degradations is explained. In Section 3, the modeling method based on Wiener process is introduced. In Section 4, the Copula-based multi-degradation reliability modeling method is presented. In Section 5, a case study on reliability modeling and analysis of a momentum wheel is conducted. In Section 6, some conclusions are obtained.

2. Key Factors Affecting the Life of the Momentum Wheel

Analysis of engineering practices and a large number of tests have shown that the main cause of failure in the rail momentum wheel is the failure of its lubrication system and bearing failure. There are three major influencing factors for lubrication system failure: metal wear, cage wear, and lack of lubricant. For bearing, bearing geometry, material selection, environmental factors, etc., are some causes of bearing failure [24]. The failure mode of the bearing and the main failure mode of the momentum wheel are given in [12,25]. In the case of excessive oil supply to the bearing and excessive lean oil, the shaft temperature will continue to be high, the current telemetering value will increase, and the speed will decrease. When the motor drive current is short-circuited or open-circuited, the current telemetry value fluctuates, the speed drops, and the relationship between the control command and the current value is disordered. After analysis, the shaft temperature, current, speed, and other performance data have complex relationships with the momentum wheel failure modes. Refs. [17,26] give the results of monitoring the rotational speed, current, and temperature, and found that there is a close relationship between the three. When one of the variables changes, the other two quantities also change. (1) The existing literature has conducted in-depth studies on the relationship between the remaining amount of lubricant and the remaining life of the momentum wheel and (2) there is a clear correspondence between the remaining amount of lubricant and the shaft temperature, current and speed. However, there are no research results on how to describe the relationship between them. In this paper, lubricant residual quantity and current are selected as degradation variables, and momentum life wheel residual life prediction model based on multi-degradation quantity is established.

As the main indexes describing the performance degradation of the momentum wheel, lubricant residual quantity and current have common impression factors such as the temperature inside the system. It can be further concluded that there is a correlation between different performance degradations, which would cause a huge error in reliability modeling and analysis if it is ignored. To take the correlation into consideration during the analysis process, the Copula function, a professional solution to problems of failure correlation, is introduced in this research. The Copula functions and reliability calculation procedure are illustrated in Section 4 in detail.

3. Establishing a Degenerate Model

3.1. Modeling Ideas

Let $X(t)$ denote the amount of degradation of the long-lived product. The growth law of product degradation can be represented by the average function $E[X(t)]$ of the degradation amount. Due to changes in the environment and individual characteristics, the actual degradation process and growth law of the product is a random process. The process is well described by the Wiener process. If $X(t)$ is satisfied:

$$X(t) = \mu t + \sigma B(t) \tag{1}$$

where μ is the drift coefficient, σ is the diffusion coefficient, and $B(t)$ is the Brownian motion, then the product degradation process obeys the linear drift Wiener process. At this point, $X(t)$ satisfies the following properties:

(1) The incremental amount of degradation at any time obeys the normal distribution, that is $\Delta X = X(t + \Delta t) - X(t) \sim N(\mu \Delta t, \sigma^2 \Delta t)$

(2) The increments in any two disjoint periods are independent of each other. The corresponding mathematical description is as follows: for any $0 < t_1 < t_2 < \ldots < t_n$, then $X(t_1) - X(t_0), X(t_2) - X(t_1), \ldots, X(t_n) - X(t_{n-1})$ are independent;

(3) $X(0) = 0$, and $X(0)$ is continuous at $t = 0$.

3.2. Failure Distribution

According to the definition of degraded failure, the failure distribution of long-life products is the time distribution of degraded performance degradation to failure threshold l for the first time, i.e.,

$$T = \inf\{t | X(t) \geq l, t \geq 0\} \tag{2}$$

which is

$$P\{T \geq t\} = P\{X(\tau) < l(0 \leq \tau < t), X(t) \leq l, X(0) = 0\} \tag{3}$$

Let the distribution density of $X(t)$ at time t be $g_x(x,t)$.

$$P(X(\tau) < l(0 < \tau < t), X(t) \leq x) = \int_{-\infty}^{x} g_x(\xi,t)d\xi \tag{4}$$

Let the probability density function of the product failure probability distribution be $f(t)$, according to the definition:

$$\begin{aligned} f(t) &= -\tfrac{\partial}{\partial t}P(T \geq t) \\ &= -\tfrac{\partial}{\partial t}P(X(\tau) < l\ (0 \leq \tau < t), X(t) \leq l) \\ &= -\tfrac{\partial}{\partial t}\int_{-\infty}^{l} g_x(\xi,t)d\xi;\ t > 0 \end{aligned} \tag{5}$$

Find $g_x(x,t)$ in the formula to find $f(t)$. When studying the distribution of the Wiener process at the time of first arrival [27], by defining an absorption column when the degradation amount reaches the failure threshold, the form of $g_x(x,t)$ is given using the Kolmogorov forward equation. $g_x(x,t)$ is

$$g_x(x,t) = \frac{1}{\sigma\sqrt{2\pi t}}\left\{\exp\left[-\frac{(x-\mu t)^2}{2\sigma^2 t}\right] - \exp\left[\frac{2\mu l}{\sigma^2} - \frac{(x-2l-\mu t)^2}{2\sigma^2 t}\right]\right\} \tag{6}$$

Then,

$$\begin{aligned} f(t) &= -\tfrac{d}{dt}\int_{-\infty}^{l} g_x(\xi,t) \\ &= -\tfrac{d}{dt}\left[\Phi\left(\tfrac{l-\mu t}{\sigma\sqrt{t}}\right) - \exp\left(\tfrac{2\mu l}{\sigma^2}\right)\Phi\left(\tfrac{-l-\mu t}{\sigma\sqrt{t}}\right)\right] \\ &= \Phi\left(\tfrac{\mu t-l}{\sigma\sqrt{t}}\right) + \exp\left(\tfrac{2\mu l}{\sigma^2}\right)\Phi\left(\tfrac{-l-\mu t}{\sigma\sqrt{t}}\right);\ t > 0 \end{aligned} \tag{7}$$

where $\Phi(x) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{\xi^2}{2}}d\xi$ is the standard normal distribution function, and $\Phi(x)$ is its density function.

The failure distribution function for the degeneration model is

$$F(t) = \Phi\left(\frac{\mu t - l}{\sigma\sqrt{t}}\right) + \exp\left(\frac{2\mu l}{\sigma^2}\right)\Phi\left(\frac{-l-\mu t}{\sigma\sqrt{t}}\right);\ t > 0 \tag{8}$$

The unknown parameters of the failure distribution function are the drift parameter μ and the variance coefficient σ. The degradation failure threshold l can generally be given based on engineering experience combined with the failure mechanism of long-life products. Now, we only need to find the estimated value of unknown parameters to get the product's failure distribution function, and then obtain more reliability information.

3.3. Parameter Estimation

Assume the data form $\{X_{ij}; i = 1, 2, \ldots, m; j = 1, 2, \ldots, n\}$, where X_{ij} represents the performance degradation value obtained from the j-th sample of the i-th measurement and the initial performance degradation is 0.

For the model parameters, the maximum likelihood estimation method can be used to obtain the estimation values $\hat{\mu}$ and $\hat{\sigma}$ of the parameters μ and σ. The increment of the degradation amount follows the normal distribution $N(\mu\Delta t, \sigma^2\Delta t)$, so using the long-life product degradation data $\{X_{ij}; i = 1, 2, \ldots; j = 1, 2, \ldots\}$, the performance degradation increment between the measurement times of the j-th product is

$$\Delta z_{ij} = X_{i+1,j} - X_{i,j}(i = 1, 2, \ldots, m; j = 1, 2, \ldots, n) \tag{9}$$

From the incremental data of product degradation, the likelihood function of the unknown parameters μ and σ can be obtained.

$$L(\mu, \sigma) = \prod_{j=1}^{n}\prod_{i=1}^{m} \frac{1}{\sqrt{2\pi\Delta t\sigma}} \exp\left[-\frac{(\Delta z_{ij} - \mu\Delta t)^2}{2\sigma^2\Delta t}\right] \tag{10}$$

Using equations

$$\begin{cases} \frac{\partial \ln(L(\mu,\sigma))}{\partial \mu} = 0 \\ \frac{\partial \ln(L(\mu,\sigma))}{\partial \sigma} = 0 \end{cases} \tag{11}$$

can obtain the estimation of $\hat{\mu}$ and $\hat{\sigma}$:

$$\begin{cases} \hat{\mu} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n} \Delta z_{ij}}{n\sum_{i=1}^{m} \Delta t_i} \\ \hat{\sigma} = \sqrt{\sum_{i=1}^{m}\sum_{j=1}^{n} \frac{(\Delta z_{ij} - \mu\Delta t_i)^2}{\Delta t_{ij}}} \end{cases} \tag{12}$$

Substituting the obtained parameter estimates into the formula, the corresponding failure distribution function can be obtained.

4. Momentum Wheel Reliability Calculation Based on Copula Function

4.1. Several Copula Functions

The single degenerate quantities of the equipment are independent of each other. According to the mutually independent random variable theory in probability theory, the residual distribution function of the equipment remaining life obtained under the single degenerate quantity is multiplied, and the joint distribution function of the remaining life of the equipment can be obtained. If the correlation of the single-degenerate quantities of the equipment is known, that is, the correlation coefficient and the covariance are known, the joint distribution of the remaining life of the equipment can be obtained through the covariance matrix in the mathematical statistics. If the correlation of the single-degenerate quantities of the equipment is unknown, neither of the above two methods are feasible [12]. In engineering practice, the correlation between degenerate variables is often unknown. Given the marginal distribution of individual degenerative quantities, how to determine their joint distribution becomes a very important issue. There is a correlation between the residual amount of lubricant in the satellite's momentum wheel and the current, but the correlation coefficient is unknown. In view of this situation, this paper chooses Copula function to derive the joint distribution function of the remaining life of the equipment.

Table 1 shows three widely used binary copula functions, where u and v, are the residual lifetime marginal distribution functions of the momentum wheel corresponding to the remaining lubricant quantity and current, i.e., $u = F_1(t)$, $v = F_2(t)$; γ is the correlation coefficient, when γ at the lower boundary value, the random variable u, v tends to be independent. When γ takes the upper boundary value, the random variable u, v tends to be completely correlated.

Table 1. The three most common binary copula functions.

Model	Parameter γ Range	$C_\gamma(u,v)$
Gumble	$[1,\infty)$	$\exp\left\{-\left[(-\ln u)^\gamma + (-\ln v)^\gamma\right]^{\frac{1}{\gamma}}\right\}$
Clayton	$[-1,\infty)/\{0\}$	$\max\left[\left(u^{-\gamma} + v^{-\gamma} - 1\right)^{-\frac{1}{\gamma}}, 0\right]$
Frank	$(-\infty,\infty)/\{0\}$	$-\frac{1}{\gamma}\ln\left[1 + \frac{(e^{-\gamma u}-1)(e^{-\gamma v}-1)}{e^{-\gamma}-1}\right]$

The selection of Different Copula functions significantly influences the description of the correlation between variables. The probability density of the Frank Copula function has symmetry and can be used to describe the correlation between variables with symmetric structures. It is not appropriate to describe the correlation between asymmetric structures of variables; the Gumbel Copula function mainly describes the features with strong tail-related features. The correlation between the variables cannot capture the correlation between the tails of the variables; the Clayton Copula function emphasizes that the variables have stronger characteristics of the lower tail, and the change of the correlation of the variables at the upper tail is not sensitive.

4.2. Reliability Calculation Based on Copula Function

After determining the marginal distribution of the remaining life of the momentum wheel under each single degeneration, the Copula function is used to calculate the joint distribution function of the remaining life of the momentum wheel. The procedure is described as follows:

(1) The lifetime marginal distribution $F_1(t)$ of the known residual quantity of the lubricant and the lifetime failure margin distribution $F_2(t)$ of the current, let $U = F_1(t)$, $V = F_2(t)$.

(2) The Copula function contains an unknown parameter γ, so parameter estimation is needed. Since the margins of lubricant residuals and currents are known, the Canonical maximum likelihood method (CML) is used to estimate the parameters γ.

$$\hat{\gamma} = \operatorname{argmax} \sum_{i=1}^{n} \ln c(u_i, v_i; \gamma) \tag{13}$$

(3) Calculate Copula density function $c_\gamma(u,v)$ and Copula distribution function $C_\gamma(u,v)$.

(4) Calculate the momentum wheel residual life probability function $R(t)$ based on the Copula distribution function according to the joint probability distribution formula in probability theory.

$$\begin{aligned} R(t) &= P(y_1(t) \geq D_1, y_2(t) \leq D_2) \\ &= 1 - P(y_1(t) < D_1) - P(y_2(t) > D_2) \\ &+P(y_1(t) < D_1, y_2(t) > D_2) \\ &= 1 - F_1(t) - F_2(t) + C_\gamma(u,v) \\ &= R_1(t) + R_2(t) + C_\gamma(u,v) - 1 \end{aligned} \tag{14}$$

where $F_1(t)$ and $F_2(t)$ are the momentum wheel failure distribution functions based on the remaining quantity of lubricant and the current; $R_1(t)$ and $R_2(t)$ are the corresponding momentum wheel reliability functions; $C_\gamma(u,v)$ is the Copula joint distribution function.

5. Analysis of Examples

5.1. Reliability Function Based on Lubricant Residual Quantity

In this paper, the data of the remaining lubricant in [10] are used to establish the degradation trajectory model. The remaining amount of lubricant data is simulated by the ground test to simulate satellite airborne conditions and is obtained after an interval of time by dissecting momentum wheels, infrared spectroscopy, and other methods. Table 2 shows the quality change data of the bearing lubricant system, including the initial value of lubricant quality, the measured value of the eighth month, and the measured value of the 11th month.

Table 2. Change in weight of bearing lubricant system.

No.	Suttle	Initial Weight	The Weight in the First Anatonmic Analysis	The Weight in the Second Anatonmic Analysis
1	36.334	41.560	41.521	41.497
2	36.356	41.727	40.526	40.456
3	36.350	41.730	41.681	41.306
4	36.344	41.525	41.305	41.227
5	36.345	41.344	41.179	41.076
Mean	36.346	41.577	41.242	41.112

Dealing with test data, the data of the bearing lubrication system's degradation is input into Equation (12), and the estimated results of the corresponding parameters are then obtained.

$$\begin{cases} \hat{u} = 0.04225 \\ \hat{\sigma} = 0.46730 \end{cases}$$

Substituting the parameter estimates into the equation, the reliability of the mission time t is estimated by $\hat{u}$, $\hat{\sigma}^2$ as

$$\begin{aligned} R(t) &= F(t) = 1 - F\left(t;\hat{\mu};\hat{\sigma}^2\right) \\ &= \Phi\left(\frac{l-\hat{\mu}t}{\hat{\sigma}\sqrt{t}}\right) - \exp\left(\frac{2\hat{\mu}l}{\hat{\sigma}^2}\right)\Phi\left(\frac{-l-\hat{\mu}t}{\hat{\sigma}\sqrt{t}}\right) \end{aligned} \quad (15)$$

According to expert analysis, failures can be considered only when the weight of the oil supply system is close to dry weight. At this point, the failure threshold of the remaining lubricant is determined. By substituting the relevant parameters and the failure threshold estimated above into Equation (15), the reliability curve of the remaining amount of lubricant can be obtained and is shown in Figure 1.

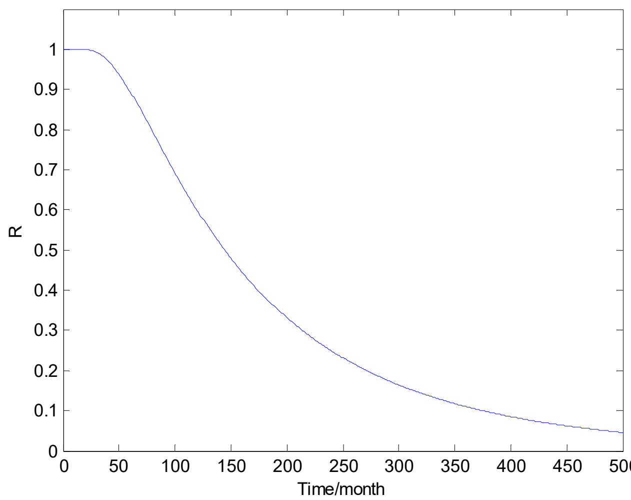

Figure 1. Momentum wheel reliability curve based on lubricant residual amount.

5.2. Reliability Function Based on Current Data

In this paper, the current data in [28] is used to establish the degradation trajectory model. Table 3 shows the current degradation data.

Table 3. Degradation data of momentum wheel current.

Time/Month	1	2	3	4	5
0	0.000	0.000	0.000	0.000	0.000
4	0.077	0.083	0.182	0.100	0.182
8	0.154	0.167	0.182	0.200	0.273
12	0.385	0.250	0.364	0.400	0.546
16	0.615	0.667	0.727	0.700	0.818

Similarly to the lubricant residual quantity, the corresponding parameter can be estimated through Equation (12)

$$\begin{cases} \hat{u} = 0.0441 \\ \hat{\sigma} = 0.2392 \end{cases}$$

The reliability functions for currents are shown in Figure 2.

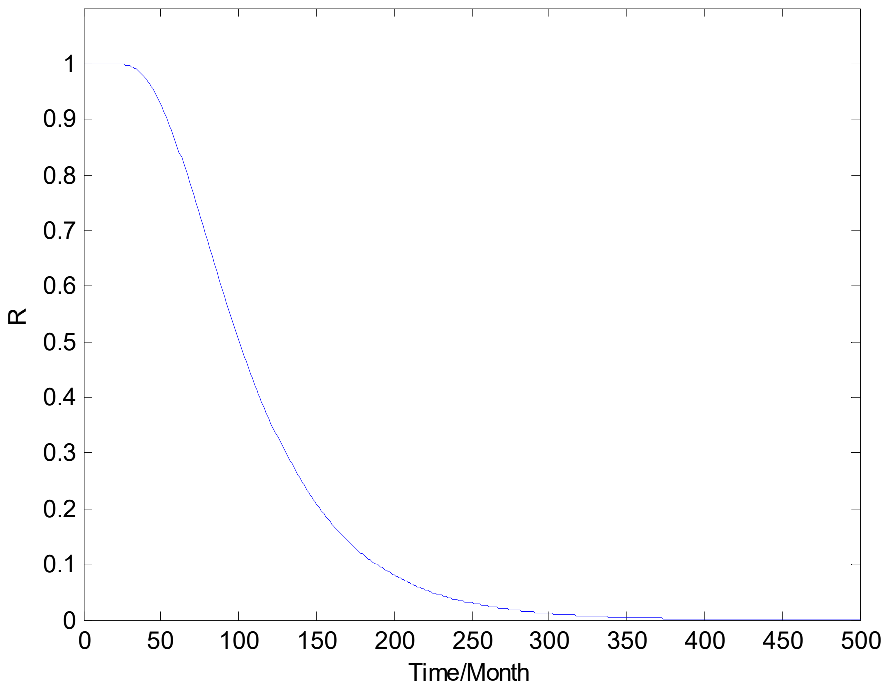

Figure 2. Momentum wheel reliability curve based on current degradation.

5.3. Copula-Based Multi-Degradation Reliability Calculation

In this paper, the Frank Copula function is used to analyze the correlation between the residual quantity of lubricant and the edge distribution function of the current. The data obtained above are substituted into the formula, and the reliability function based on Copula function can be obtained and shown in Figure 3.

The Frank Copula joint distribution curve and the two edge distribution curves in the comparison show that for the same research object momentum wheel, the lubricant-based momentum wheel life prediction curve conforms to the life degradation of mechanical components, while the current-based momentum wheel life prediction conforms to the life degradation of electronic components. The curve corresponds to the life degradation of the electronic device. However, there is a certain difference in the momentum wheel life predicted by the two. Therefore, it is necessary to establish a multi-degenerate joint distribution function for the momentum wheel. In this figure, the momentum wheel reliability curve based on the Frank Copula function is consistent with the lubricant-based momentum wheel life prediction curve before 55 months, and then is similar to the current-based momentum wheel life prediction curve. This situation shows that in the early stage of the operation of the momentum wheel, the life degradation has a lot to do with the lubricant. With the passage of time, the degradation of the momentum wheel life is more related to the degradation of the life of electronic components. Compared with single degradation reliability curve, the Copula-based multi-degradation reliability curve is more consistent with the actual operation of the momentum wheel.

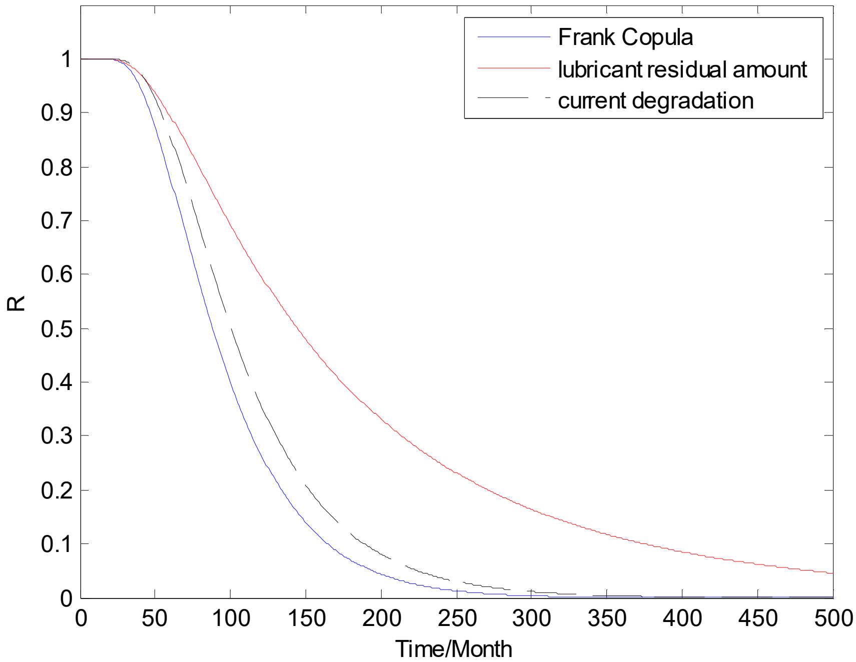

Figure 3. Momentum wheel reliability function curve based on frank copula function.

If ignoring the correlation between two performance degradations of the momentum wheel, $R_1(t)$ and $R_2(t)$ can be considered independent, and the reliability of the momentum wheel can be calculated by Equation (16).

$$R'(t) = R_1(t) \cdot R_2(t) \tag{16}$$

The comparison between Couple-based reliability function and independent reliability function is shown in Figure 4. In this figure, the Couple-based reliability curve is consistent with the independent reliability curve before 65 months, but after 65 months, it is obvious that the reliability analysis results without the consideration of correlation between these 2 performance degradations are more conservative. This conclusion of the comparison fully conforms with the general rule of reliability research considering multiple failure correlation, which further confirms the importance of considering the multi-degradation correlation in reliability modeling and analysis of complex electromechanical coupling systems, for example, the momentum wheel.

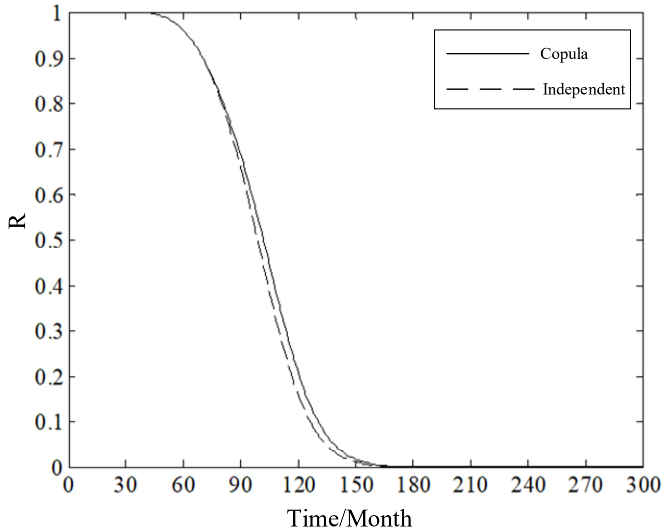

Figure 4. Comparison between Couple-based reliability function and independent reliability function.

6. Conclusions

In this paper, the reliability of the momentum wheel is studied using the degenerate modeling based on the wiener stochastic process. The main factors affecting the reliability of the momentum wheel are analyzed and the momentum wheel degeneracy model with single degradation and multiple degradation is established. Compared with the existing methods, the main innovation of this paper is its proposition of a momentum wheel degradation model based on the multi-degenerate variables of the Wiener stochastic process. When the correlation between the two degradation quantities is unknown, the Copula function is used to fuse the two. The edge distribution gives a joint distribution of momentum wheel reliability. The results show that the edge distribution of lubricants and currents is obtained through the Wiener random process degradation modeling method, and then the Copula function is used to calculate the joint distribution. The resulting momentum wheel degradation model is consistent with practical engineering experience. It would be a good reference for the reliability calculation of equipment with multiple degeneracy and unknown correlation between variables.

Due to the lack of momentum wheel degradation data under the same working conditions of the same model, an empirical study of the prediction method cannot be completed yet. In the process of single degradation modeling, the common degenerative process of the Wiener stochastic process is used to construct the degradation trajectory function of the two residuals of lubricant residual quantity and current. However, in practical applications, there are other factors that have a significant impact on the life of the momentum wheel. For example, we would further study how to use the modeling and analysis of the performance degradation process when considering the impact of environmental, use, and manufacturing factors on the life and reliability of moving parts (for example, analysis of the cyclical rise mechanism of bearing temperature). In addition, further theoretical and experimental research is needed.

In summary, this research provides a new idea for the reliability modeling and analysis of the momentum wheel, and the proposed method can be further applied in complex electromechanical coupling systems with a high reliability, long life and monitorable degradation data.

Author Contributions: Conceptualization, Y.-F.L.; methodology, H.-Z.H.; investigation, M.H.; resources, Y.C.; writing—original draft preparation, S.B. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the Project of Science and Technology on Reliability Physics and Application Technology of Electronic Component Laboratory under grant number 61428060305.

Acknowledgments: Special thanks to Li, H. (University of Electronic Science and Technology of China) for his instructive advice and useful suggestions on the thesis and methodology.

Conflicts of Interest: The authors declare no conflict of interest.

References

1. Li, Y.F.; Huang, H.Z.; Mi, J.; Peng, W.; Han, X. Reliability analysis of multi-state systems with common cause failures based on Bayesian network and fuzzy probability. *Ann. Oper. Res.* **2019**, *276*, 1–15. [CrossRef]
2. Li, Y.F.; Liu, Y.; Huang, T.; Huang, H.Z.; Mi, J. Reliability assessment for systems suffering common cause failure based on Bayesian networks and proportional hazards model. *Qual. Reliab. Eng. Int.* **2020**, *36*, 2509–2520. [CrossRef]
3. Mi, J.; Li, Y.F.; Peng, W.; Huang, H.Z. Reliability analysis of complex multi-state system with common cause failure based on evidential networks. *Reliab. Eng. Syst. Saf.* **2018**, *174*, 71–81. [CrossRef]
4. Mi, J.; Beer, M.; Li, Y.F.; Broggi, M.; Cheng, Y. Reliability and importance analysis of uncertain system with common cause failures based on survival signature. *Reliab. Eng. Syst. Saf.* **2020**, *201*, 106988. [CrossRef]
5. Mi, J.; Li, Y.F.; Yang, Y.J.; Peng, W.; Huang, H.Z. Reliability assessment of complex electromechanical systems under epistemic uncertainty. *Reliab. Eng. Syst. Saf.* **2016**, *152*, 1–15. [CrossRef]
6. Mi, J.; Li, Y.F.; Liu, Y.; Yang, Y.J.; Huang, H.Z. Belief universal generating function analysis of multi-state systems under epistemic uncertainty and common cause failures. *IEEE Trans. Reliab.* **2015**, *64*, 1300–1309. [CrossRef]
7. Ramasso, E.; Gouoriveau, R. Remaining useful life estimation by classification of predictions based on a neuro-fuzzy system and theory of belief functions. *IEEE Trans. Reliab.* **2014**, *63*, 555–566. [CrossRef]
8. Qi, H.M.; Cheng, Y.H.; Jiang, B. Remaining lifetime prediction based on multiple fault states for satellite attitude control system. *J. Nanjing Univ. Aeronaut. Astronaut.* **2015**, *47*, 29–36.
9. Kang, G.H.; Xia, Q.; Cheng, J. Design of attitude and orbit control computer of micro satellite based on SoPC. *J. Nanjing Univ. Aeronaut. Astronaut.* **2013**, *45*, 763–768.
10. Jin, G.; Liu, Q.; Zhou, J. RePofe: Reliability physics of failure estimation based on stochastic performance degradation for the momentum wheel. *Eng. Fail. Anal.* **2012**, *22*, 50–63. [CrossRef]
11. Jin, G.; Matthews, D.; Fan, Y. Physics of failure-based degradation modeling and lifetime prediction of the momentum wheel in a dynamic covariate environment. *Eng. Fail. Anal.* **2013**, *28*, 222–240. [CrossRef]
12. Pecht, M.; Gu, J. Physics-of-failure-based prognostics for electronic products. *Trans. Inst. Meas. Control* **2009**, *31*, 309–322. [CrossRef]
13. Li, H.T.; Jin, G. Momentum wheel wiener process degradation modeling and life prediction. *J. Aerosp. Power* **2011**, *26*, 622–628.
14. Liu, Q.; Zhou, J.L.; Jin, G. Based on random threshold Gauss\rown fure physical model of reliability assessment of momentum wheel. *J. Astronaut.* **2009**, *30*, 2109–2115.
15. Liu, Q. Reliability Modeling and Evaluation Method of Satellite Momentum Wheel's Performance. Ph.D. Thesis, National University of Defense Technology, Changsha, China, 2006.
16. Jin, G.; Feng, J. Bayes-Weibull reliability assessment method for long life satellite moving component. *Syst. Eng. Electron.* **2009**, *31*, 2020–2083.
17. McMahon, P.; Laven, R. Results from 10 Years of Reaction/Momentum Wheel Life Testing. In Proceedings of the 11th European Space Mechanisms and Tribology Symposium, Lucerne, Switzerland, 21–23 September 2005; pp. 299–305.
18. Li, H.T. Reliability Modeling and Analysis of Momentum Wheel Based on Baysian Network. Ph.D. Thesis, National University of Defense Technology, Changsha, China, 2007.
19. Zhao, Q.; Jia, X.; Cheng, Z.J.; Guo, B. Bayesian estimation of residual life for Weibull-distributed components of on-orbit satellites based on multi-source information fusion. *Appl. Sci.* **2019**, *9*, 3017. [CrossRef]
20. Yu, K. Reliability Estimation for Momentum Wheels Based on Information Fusion. Master's Thesis, University of Electronic and Science of Technology of China, Chengdu, China, 2019.
21. Chen, X.F.; Xia, X.T. Dynamic Prediction of Friction Torque Reliability of Super-Precision Rolling Bearings. In Proceedings of the 2020 Prognostics and Health Management Conference (PHM-Besançon), Besançon, France, 4–7 May 2020; pp. 345–348.
22. Huang, H.Z.; Yu, K.; Huang, T.D.; Li, H.; Qian, H.M. Reliability estimation for momentum wheel bearings considering frictional heat. *Eksploat. Niezawodn. Maint. Reliab.* **2020**, *22*, 6–14. [CrossRef]
23. Mohr, H.D. Real-Time on-Orbit Momentum Wheel Health Monitoring for Robust Satellite Attitude Control. In Proceedings of the 2021 IEEE Aerospace Conference, Now Virtual, 6–13 March 2021; pp. 1–7.
24. Liu, L.Y. Study on Wear Life of Momentum Wheel Bearing. Ph.D. Thesis, Henan University of Science and Technology, Luoyang, China, 2011.
25. Liu, W.J.; Liu, C.R. The diagnosed evaluation of momentum wheel failure combination of quantitative and qualitative. *Chin. Space Sci. Technol.* **2011**, *8*, 55–63.
26. Zhang, J.X.; Hu, C.H. Multiple degradation variables modeling for remaining useful life estimation of gyro based on Copula function. *Acta Aeronaut. Astonanytica Sin.* **2014**, *35*, 1111–1121.
27. Cox, D.R.; Miller, H.D. *The Theory of Stochastic Processes*; Methuen & Co. Ltd.: London, UK, 1965.
28. Liu, S.N.; Lu, N.Y.; Cheng, Y.H. Residual life prediction method based on multi-degradation quantity of momentum wheel. *J. Nanjing Univ. Aeronaut. Astronaut.* **2015**, *47*, 360–366.