

Article

Theoretical Investigation on the Impact of Two HDR Dampers on First Modal Damping Ratio of Stay Cable

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Abstract: Stay cables are one of the vital components of a cable-stayed bridge. Due to their flexible nature, stay cables are vulnerable to external excitation and often vibrate with large amplitude under wind action which leads to the fatigue failure of the cables. To suppress such kind of large amplitude vibration by improving the damping ratio of the cable various dampers such as magnetorheological damper, friction damper; oil damper; or high damping rubber (HDR) damper are utilized and gained popularity over time. This paper focuses on improving the damping ratio of stay cables using a combination of two HDR dampers. First, the theoretical model is formulated considering cable bending stiffness to evaluate the damping effect of cable-HDR dampers system. Then, the impact of various design parameters of HDR dampers on cable damping considering the cable stiffness is performed. The comparative analysis of results shows that the considered parameters such as loss factor, spring factor, and installation location of dampers have much effect on the stay cables damping ratio. Finally, the optimal parameters of the two HDR dampers are proposed for damper design.



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Keywords: two HDR dampers; stay cable; damping ratio; spring factor; loss factor; cable bending stiffness

1. Introduction

Stay cables have low mass, low frequencies, and low damping ratio, so they are vulnerable to external excitation. The vibration of stay cable under ambient excitation (such as wind or earthquake loading) and live load can cause fatigue in cable attachment, which might reduce the bridge safety. There are mainly two methods to control cable vibration. The first method is to modify the cable surface to improve its aerodynamic performance [1–3] and the other one is to install the energy dissipating devices such as magnetorheological damper, friction damper; oil damper; or HDR damper [4–27]. The installation of an external damper can increase the cable damping ratio. Consequently, its vibration was mitigated.

Studies into the cable vibration with an external damper including an HDR damper have been investigated by many researchers [4–27]. Single HDR damper device has widely been applied for stayed cables with short lengths as it can efficiently dissipate the vibration energy of the cable. Nakamura et al. [6] optimized the parameters of single HDR damper for obtaining the maximum damping of stay cable. In addition, Fujino Y., and Nam H. [7] proposed the formulas for calculating the cable damping ratios of stay cable with a single HDR-damper. Cu et al., further investigated on stay cable with HDR damper [11]. They found that the damping ratio has strong dependency on loss factor and damper location. However, their study did not consider the bending stiffness of taut cable for calculating the cable damping. Recently, Le et al. (2020) also proposed an approximated formula for estimating the damping ratio of a stay cable with single HDR-damper considering the uncertainties of boundaries conditions [12]. They concluded that the efficiency of the damper can be increased by adding more rotational restraint

stiffness. According to previous studies [4,9–11], the maximum modal damping of stay cable is directly proportional to the distance of the damper from its end. The effect of single HDR damper and other damper types on stay cable damping has attracted many further studies [4,6,8,9,11–18]. However, due to the bridge aesthetic, the damper is often installed near the cable end within a few percentages of cable length that lessens the efficiency of the damper. Therefore, it is difficult to obtain an expected damping level using a single damper for very long-stay cables [4]. In this regard, the use of two dampers at different locations can provide a possible countermeasure to suppress the cable vibration by increasing the total damping [10]. Caracoglia et al. [19] investigated the impact of two viscous dampers on taut cable damping. They figured out that the dampers located at opposite ends of taut cable can enhance the damping as compared to the case with dampers installed on one side near the bridge deck. Nam H. and Fujino Y. [10] also confirmed that the total damping ratio of cable with two dampers is equal to the summation of two single damper effects. Xu and Yu [20] presented a numerical example of combining two oil dampers to increase the stay cable damping ratio. This study showed that damper stiffness, damper installation location, and oil-damper direction influence its effectiveness. Furthermore, according to Tabatabai and Mehrabi [21], bending stiffness affects noticeably the damping ratio of stay cable. These authors have developed a numerical framework to calculate vibration frequencies and damping ratios of sagging cables with non-negligible bending stiffness equipped with a viscous damper. Vo and Nguyen proposed the theoretical model for evaluating damping of stay cable with two friction dampers to reduce cable galloping considering cable bending stiffness [22]. Fangdian Di et al., investigated the effect of HDR and viscous dampers to increase cable damping by theoretical analysis and field experiments [23]. In addition, Haijun Z. et al., reported the field measurement for stay cable with different type of dampers. They figured out that the damping of field measurement is lower than the theoretical value [24]. Other studies on viscous dampers combined with internal devices to enhance cable damping were also proposed [25–27]. As discussed above, most of the previous studies mainly focused on the effects of single damper, two viscous dampers or viscous damper combined with another type. The model for estimating the cable damping ratio of two HDR dampers considering the cable bending stiffness has been not studied yet and their effect is also not fully understood.

In this study, the performance of stay cable with two HDR dampers is investigated to obtain higher damping ratio. Particularly, the theoretical model is proposed to evaluate damping effect of the cable with two HDR dampers considering its bending stiffness. The impact of loss factor, spring factor, dampers installation positions, and bending stiffness (EI) of stay cable on the first modal damping ratio is also elucidated. Finally, an optimization study for two HDR dampers is carried out and discussed in detail.

2. Governing Equation of Stay Cable with Two HDR Dampers

Figure 1 shows the considered taut cable model with two HDR dampers. The x -axis coincides with the longitudinal axis of the stay cable. The combination of two HDR dampers is located at l_1 and l_2 distances from each cable end. The mass per unit length of taut cable, cable length, bending stiffness and tension force in the stay cable is denoted as m , L , EI , and S , respectively. Since the rubber pads in the HDR dampers have the hysteretic characteristic, so the damping force does not depend on the frequency [14] and can be displayed as:

$$f_j(x, t) = K_j \cdot (1 + i \cdot \lambda_j) \cdot u(l_j, t), \quad (j = 1, 2) \quad (1)$$

where K_j is the spring factor of j th damper; λ_j the material loss factor of j th damper; $i^2 = -1$; $u(l_j, t)$ —the displacement of stayed-cable in location attached j th damper at time t .

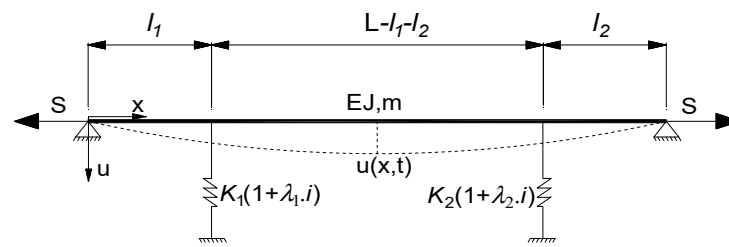


Figure 1. A stay cable with two HDR dampers.

It is assumed that the stay cable tension is much greater than its self-weight and the frictional force between the air and stay cable is negligible. The governing equation of in-plane cable motion can be expressed by:

$$m \cdot \frac{\partial^2 u}{\partial t^2} + EJ \cdot \frac{\partial^4 u}{\partial x^4} - S \cdot \frac{\partial^2 u}{\partial x^2} = f_j(x, t) \cdot \delta(x - l_j) \quad (2)$$

where $\delta(\cdot)$: the Dirac delta function is to simulate the concentrated damping force $f_j(t)$ ($j = 1, 2$) located at $x = l_j$.

By using the variables separation of Fourier, the result of Equation (2) can be described by:

$$u(x, t) = X(x)T(t) \quad (3)$$

The frequency equation of a stay cable with two HDR dampers can be derived by applying the boundary conditions and assuming equilibrium in the vertical direction at the positions where the dampers are attached, as shown in Equation (4). In which, η is the stay cable frequency at j th mode; setting k as a cable eigenvalue with complex form, k can be defined as Equation (5).

$$\begin{aligned} & \left(\frac{EJ \cdot m}{S^2} \eta^2 - 1 \right)^2 \left[\cot(\eta \sqrt{\frac{m}{S}} l_1) \cdot \cot[\eta \sqrt{\frac{m}{S}} (L - l_1 - l_2)] + \cot(\eta \sqrt{\frac{m}{S}} l_1) \cdot \right. \\ & \left. \cot(\eta \sqrt{\frac{m}{S}} l_2) + \cot(\eta \sqrt{\frac{m}{S}} l_2) \cdot \cot[\eta \sqrt{\frac{m}{S}} (L - l_1 - l_2)] - 1 \right] \\ & - \left(\frac{EJ \cdot m}{S^2} \eta^2 - 1 \right) \left[\left(\cot[\eta \sqrt{\frac{m}{S}} (L - l_1 - l_2)] + \cot(\eta \sqrt{\frac{m}{S}} l_2) \right) \frac{K_1 \cdot (1 + i \cdot \lambda_1)}{\eta \sqrt{mS}} \right. \\ & \left. + \left(\cot[\eta \sqrt{\frac{m}{S}} (L - l_1 - l_2)] + \cot(\eta \sqrt{\frac{m}{S}} l_1) \right) \frac{K_2 \cdot (1 + i \cdot \lambda_2)}{\eta \sqrt{mS}} \right] \\ & = - \frac{K_1 \cdot (1 + i \cdot \lambda_1)}{\eta \sqrt{mS}} \cdot \frac{K_2 \cdot (1 + i \cdot \lambda_2)}{\eta \sqrt{mS}} \end{aligned} \quad (4)$$

$$k_j = \sigma_j + \phi_j \cdot i = \left(-\xi_j + i \cdot \sqrt{1 - \xi_j^2} \right) \frac{\eta_j}{\omega_1^0} \quad (5)$$

$$\xi_j = \sqrt{1 + \frac{\phi_j^2}{\sigma_j^2}} \quad (6)$$

where ω_1^0 is the cable natural frequency without damper devices; ξ_j , is the cable damping ratio at j th mode.

By substituting Equation (5) into Equation (4), the Equations (7) and (8) can be obtained by expanding and separating the real and imaginary parts.

$$\begin{aligned}
 & [\sigma(A^2 - 2A - 4B^2 + 1) - 4\phi B(A - 1)] \left[\begin{aligned} & \frac{\sin 2e \sin 2a - \sinh 2f \sinh 2b}{M_1 M_3} \\ & + \frac{\sin 2c \sin 2a - \sinh 2d \sinh 2b}{M_1 M_2} \\ & + \frac{\sin 2e \sin 2c - \sinh 2f \sinh 2d}{M_2 M_3} - 4 \end{aligned} \right] \\
 & + [\phi(A^2 - 2A - 4B^2 + 1) + 4\sigma B(A - 1)] \left[\begin{aligned} & \frac{\sin 2e \sinh 2b + \sin 2a \sinh 2f}{M_1 M_3} \\ & + \frac{\sin 2e \sinh 2d + \sin 2c \sinh 2f}{M_2 M_3} \\ & + \frac{\sin 2c \sinh 2b + \sin 2a \sinh 2d}{M_1 M_2} \end{aligned} \right] \\
 & + 4B \left[\begin{aligned} & \left(\frac{\sin 2a}{M_1} + \frac{\sin 2e}{M_3} \right) \Phi_2 \lambda_2 + \left(\frac{\sin 2a}{M_1} + \frac{\sin 2c}{M_2} \right) \Phi_1 \lambda_1 \\ & - \left(\frac{\sinh 2b}{M_1} + \frac{\sinh 2f}{M_3} \right) \Phi_2 - \left(\frac{\sinh 2b}{M_1} + \frac{\sinh 2d}{M_2} \right) \Phi_1 \end{aligned} \right] \\
 & - 2(A - 1) \left[\begin{aligned} & \left(\frac{\sinh 2b}{M_1} + \frac{\sinh 2f}{M_3} \right) \Phi_2 \lambda_2 + \left(\frac{\sinh 2b}{M_1} + \frac{\sinh 2d}{M_2} \right) \Phi_1 \lambda_1 \\ & + \left(\frac{\sin 2a}{M_1} + \frac{\sin 2e}{M_3} \right) \Phi_2 + \left(\frac{\sin 2a}{M_1} + \frac{\sin 2c}{M_2} \right) \Phi_1 \end{aligned} \right] \\
 & - 4\Phi_1 \Phi_2 \lambda_1 \lambda_2 \frac{\sigma}{(\sigma^2 + \phi^2)} + 4\Phi_1 \Phi_2 (\lambda_1 + \lambda_2) \frac{\phi}{(\sigma^2 + \phi^2)} + 4\Phi_1 \Phi_2 \frac{\sigma}{(\sigma^2 + \phi^2)} = 0
 \end{aligned} \tag{7}$$

$$\begin{aligned}
 & [\phi(A^2 - 2A - 4B^2 + 1) + 4\sigma B(A - 1)] \left[\begin{aligned} & \frac{\sin 2e \sin 2a - \sinh 2f \sinh 2b}{M_1 M_3} \\ & + \frac{\sin 2c \sin 2a - \sinh 2d \sinh 2b}{M_1 M_2} \\ & + \frac{\sin 2e \sin 2c - \sinh 2f \sinh 2d}{M_2 M_3} - 4 \end{aligned} \right] \\
 & - [\sigma(A^2 - 2A - 4B^2 + 1) - 4\phi B(A - 1)] \left[\begin{aligned} & \frac{\sin 2e \sinh 2b + \sin 2a \sinh 2f}{M_1 M_3} \\ & + \frac{\sin 2e \sinh 2d + \sin 2c \sinh 2f}{M_2 M_3} \\ & + \frac{\sin 2c \sinh 2b + \sin 2a \sinh 2d}{M_1 M_2} \end{aligned} \right] \\
 & - 2(A - 1) \left[\begin{aligned} & \left(\frac{\sin 2a}{M_1} + \frac{\sin 2e}{M_3} \right) \Phi_2 \lambda_2 + \left(\frac{\sin 2a}{M_1} + \frac{\sin 2c}{M_2} \right) \Phi_1 \lambda_1 \\ & - \left(\frac{\sinh 2b}{M_1} + \frac{\sinh 2f}{M_3} \right) \Phi_2 - \left(\frac{\sinh 2b}{M_1} + \frac{\sinh 2d}{M_2} \right) \Phi_1 \end{aligned} \right] \\
 & - 4B \left[\begin{aligned} & \left(\frac{\sinh 2b}{M_1} + \frac{\sinh 2f}{M_3} \right) \Phi_2 \lambda_2 + \left(\frac{\sinh 2b}{M_1} + \frac{\sinh 2d}{M_2} \right) \Phi_1 \lambda_1 \\ & + \left(\frac{\sin 2a}{M_1} + \frac{\sin 2e}{M_3} \right) \Phi_2 + \left(\frac{\sin 2a}{M_1} + \frac{\sin 2c}{M_2} \right) \Phi_1 \end{aligned} \right] \\
 & + 4\Phi_1 \Phi_2 \lambda_1 \lambda_2 \frac{\phi}{(\sigma^2 + \phi^2)} + 4\Phi_1 \Phi_2 (\lambda_1 + \lambda_2) \frac{\sigma}{(\sigma^2 + \phi^2)} - 4\Phi_1 \Phi_2 \frac{\phi}{(\sigma^2 + \phi^2)} = 0
 \end{aligned} \tag{8}$$

where $A = \Delta \cdot (\sigma^2 - \phi^2)$; $B = \Delta \cdot \sigma \cdot \phi$; $a = \pi \cdot \left(1 - \frac{l_1}{L} - \frac{l_2}{L}\right) \cdot \sigma$; $b = \pi \cdot \left(1 - \frac{l_1}{L} - \frac{l_2}{L}\right) \cdot \phi$; $c = \pi \cdot \frac{l_2}{L} \cdot \sigma$; $d = \pi \cdot \frac{l_2}{L} \cdot \phi$; $e = \pi \cdot \frac{l_1}{L} \cdot \sigma$; $f = \pi \cdot \frac{l_1}{L} \cdot \phi$; $M_1 = \cosh^2 b - \cos^2 a$; $M_2 = \cosh^2 d - \cos^2 c$; $M_3 = \cosh^2 f - \cos^2 e$. In which Δ , Φ_1 and Φ_2 are non-dimensional parameters, shown as following $\Delta = \pi^2 \cdot \frac{EJ}{SL^2}$; $\Phi_1 = \frac{K_1 \cdot L}{\pi S}$; $\Phi_2 = \frac{K_2 \cdot L}{\pi S}$.

Due to the nonlinear nature of Equations (7) and (8), the iteration method is used to solve these equations. The algorithm to deal with Equations (7) and (8) illustrates in Figure 2.

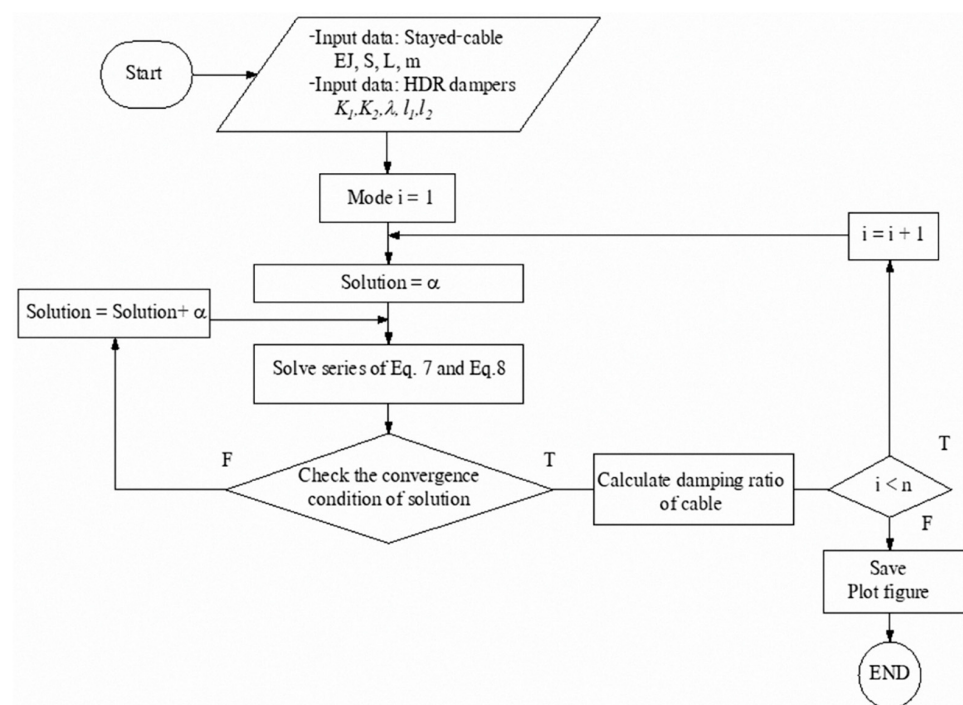


Figure 2. Model algorithm.

3. Results and Discussion

3.1. Model Validation

Due to the scarcity of experimental data of two HDR dampers, the model validation will be carried out using single HDR damper results of Cu and Han [11]. In this case, the system will be considered without a second HDR damper ($K_2 = 0$, $l_2 = 0$, $\lambda_2 = 0$) whereas the other input parameters of the remained HDR damper is same to the work of [11]. Particularly, the bending stiffness of stay cable is considered in the present model compared to [11]. The non-dimensional spring factor is redefined by $K^* = K_1 \cdot l_1 / S$ with a loss factor equal to 0.4 and the tension force, $S = 2900$ kN. The nonlinear equations 7–8 are solved using the algorithm shown in Figure 2 to obtain the damping ratio of HDR damper utilizing spring factor (K^*) and installation position (l_1/L) values varying from 0.5 to 1.5 and from 0 to 0.5, respectively [11]. Figure 3 presents the damping ratio computed by present model and [11] for different damper locations. In general, the damping ratio calculated by the present model shows high agreement with findings of Cu and Han [11]. The validation metrics of present model and [11] are shown in Table 1. The predicted value of the present model and [11] exhibits a high correlation. The correlation coefficient (C_C) is approximate 0.999 for three different cases. In addition, the mean absolute gross error (E_{MAGE}) and root mean square error (E_{RMSE}) are very small, less than 1% for all cases (Table 1). The results obtained with the current model agree very well with [11] in terms of damping ratio value as well as the trend in the results for three different cases. In short, present model can be successfully utilized for approximate estimation of damping ratio for stay cable.

3.2. Effect of Spring Factor of HDR Dampers on First Modal Damping Ratio

To investigate the influence of two HDR dampers on improving cable damping, various properties of damping devices such as K_j -spring stiffness factor, λ_j -the loss factors, and the attached locations of dampers have been analyzed. For each HDR- damper parameter, the stay cable damping ratio is solved from the system of Equations (7) and (8). According to previous studies, the modal damping ratios do not depend much on the vibration modes for the stay cable installed HDR damper without rotational constraint at cable ends [7,10]. Therefore, the following sections present and discuss the effects of two HDR damper's parameters on only the first modal damping ratio of stay cable.

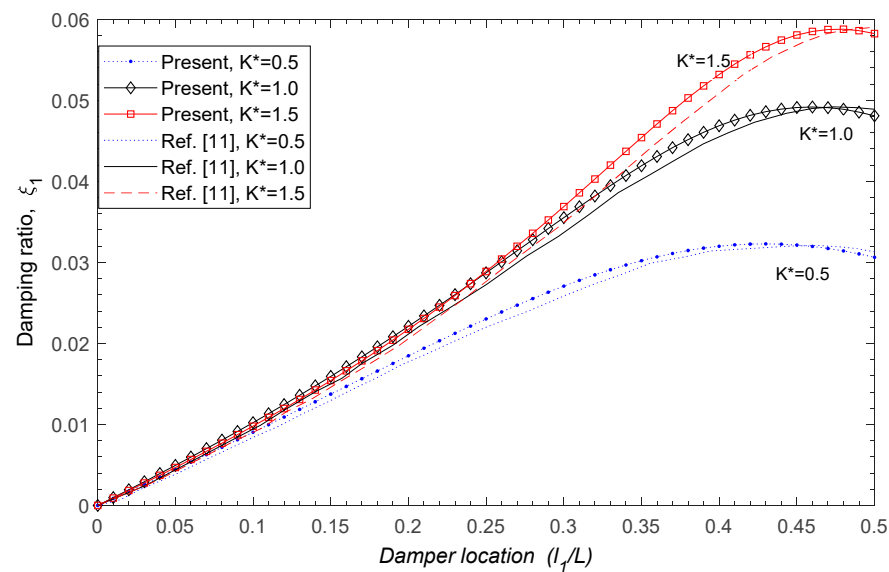


Figure 3. Comparison of present model and [11].

Table 1. Validation metrics between present study and [11].

Validation Metrics	$K^* = 1.5$	$K^* = 1.0$	$K^* = 0.5$
C_C	0.9997	0.9994	0.9992
R_{MSE}	0.0014	0.0011	0.0007
E_{MAGE}	0.0002	0.0002	0.0001

As discussed in Section 2, the spring factor of the damping device can be characterized by non-dimension coefficients $\Phi_i = K_i L / \pi S$ [13]. Where, L , is cable length and S , is cable tension, those are fixed for each stay cable. To figure out the relationship between Φ_i and K_i , a stay cable with tension force (S) of 3200 KN and L of 100 m are considered in this investigation. Figures 4 and 5 illustrate the changes of 1st modal damping ratio (ξ_1) with respect to the Φ_1 and Φ_2 corresponding to l_1/L and l_2/L equal to 0.03. In Figure 4, the Φ_1 varies from 0 to 60, while the Φ_2 values are fixed at 1, 2, 5, 10, 40, respectively. For each non-dimensional coefficient Φ_2 , the 1st modal damping ratio (ξ_1) increases with the increase of Φ_1 until reaching the maximum value. Then, the damping ratio (ξ_1) decreases gradually, and the curve becomes flat. Therefore, for each value of Φ_2 , we can find an optimum value of Φ_1 having the highest damping ratio. The similar results are illustrated in Figure 5. Consequently, the optimum parameters for two HDR dampers in the current study are $\Phi_1 = 1$ and $\Phi_2 = 1$ corresponding to $K_1 = 100$ and $K_2 = 100$, respectively. The maximum 1st modal damping ratio (ξ_{1max}) may reach up to 0.0284. The detail of the results is summarized in Tables 2 and 3.

3.3. Impact of Loss Factor (λ) of HDR Dampers on First Modal Damping Ratio

This section clarifies the effect of loss factor (λ_i) for different values of K_1 and K_2 on the first modal damping ratio (ξ_1). The spring stiffness factors (K_1 and K_2) are varied for 100, 200, 500, 1000, and 1500. Figure 6 presents the relationship between damping ratio and loss factor for different values of K_1 . For each value of K_1 , there is an optimum value of loss factor (λ_1) for which the damping ratio reaches the maximum value. For $K_1 = 100$, the highest damping ratio 0.0284 is obtained at $\lambda_1 = 12$. As the K_1 value increases, the optimum value of the loss factor (λ_1) decreases, and the magnitude of the highest damping ratio reduces from 0.0284 to 0.0207. The K_2 and λ_2 have a similar effect on cable damping ratio as shown in Figure 7. It is found that the 1st modal damping ratio is in direct proportion with the loss factor before reaching the optimum value. After that, the 1st modal damping

ratio decreases as the loss factor increases. The optimum values of the 1st modal damping ratio are presented in Tables 3 and 4.

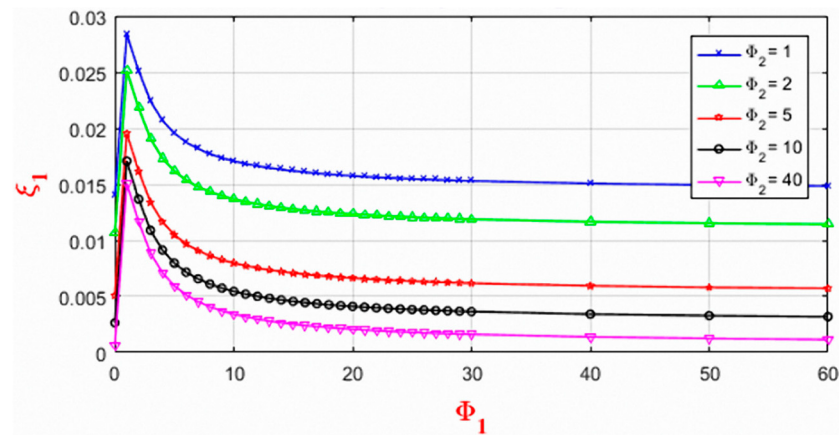


Figure 4. Relationship of Φ_1 and ζ_1 .

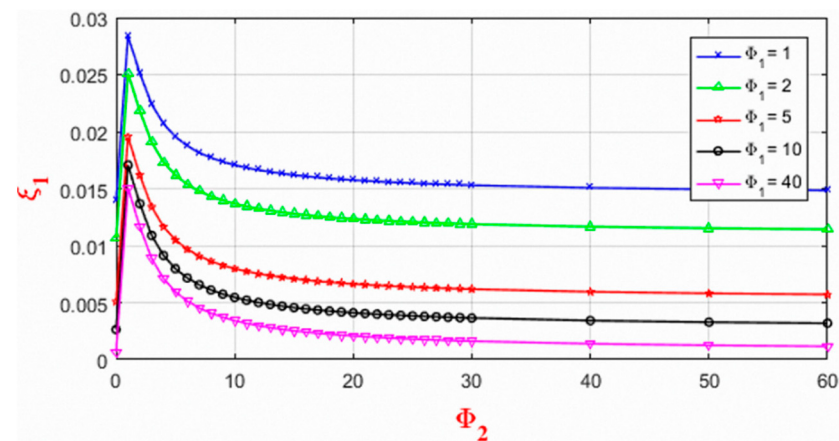


Figure 5. Relationship between Φ_1 , Φ_2 and ζ_1 .

Table 2. Maximum damping ratio $\zeta_{1\max}$ for each Φ_1 .

Φ_2	1	2	5	10	40
$\zeta_{1\max}$	0.0284	0.0251	0.0195	0.0171	0.0151
Φ_1	1	1	1	1	1

Table 3. Maximum damping ratio $\zeta_{1\max}$ for each Φ_2 .

Φ_1	1	2	5	10	40
$\zeta_{1\max}$	0.0284	0.0251	0.0195	0.0171	0.0151
Φ_2	1	1	1	1	1

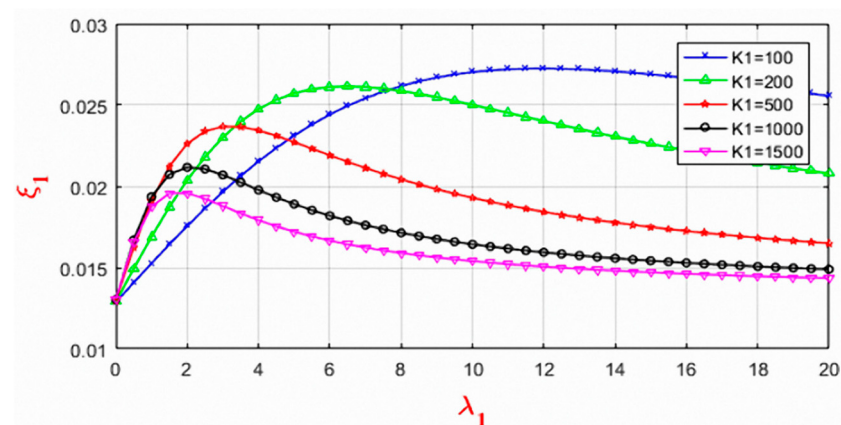


Figure 6. Relationship of $(\lambda_1; \xi_1)$ and K_1 .

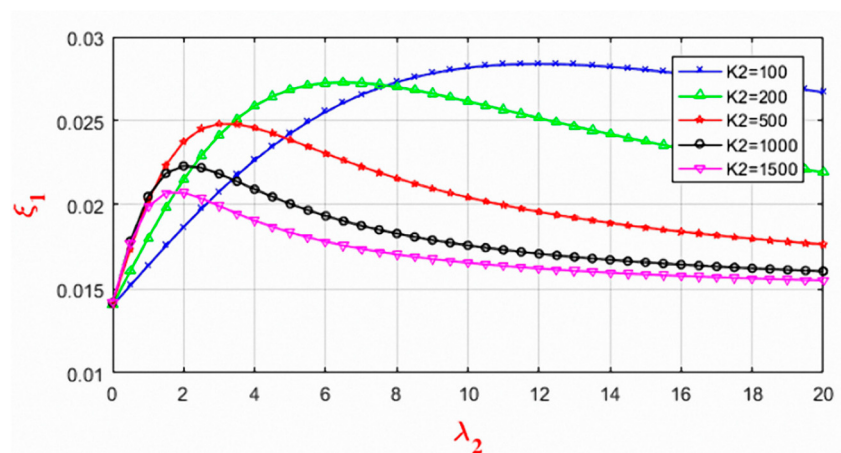


Figure 7. Relationship between λ_2 and ξ_2 for different values of K_2 .

Table 4. Damping ratio $\xi_{1\max}$ with λ_1 .

K_1	100	200	500	1000	1500
$\xi_{1\max}$	0.0284	0.0273	0.0248	0.0223	0.0207
λ_1	12	6	3	2	1.5

3.4. Effect of Installation Position of Two HDR Dampers ($l_1/L, l_2/L$)

To figure out the impact of installation positions of two HDR dampers ($l_1/L, l_2/L$) on cable damping ratio, the installation position (l_1/L) of dampers are varied from 0.01 to 0.2 for different values of K_1 and K_2 . Figures 8 and 9 show the relationship of ξ_1 and l_1/L for different values of K_1 and K_2 . The damping ratio is directly proportional to the installation positions of HDR dampers. For any value of K_1 , the damping ratio increases with the increase of l_1/L and l_2/L . Furthermore, the values of damping ratio decrease with the increase of spring stiffness factors for an installation position of HDR dampers. The highest value the damping ratio 0.05 is found when l_1/L equals to 0.2. However, if the installation position is too far from cable ends, it is hard to fabricate, install and maintain the dampers. Moreover, it affects the aesthetic aspect of the bridge as well. Usually, the HDR dampers are normally installed around a normalized value of 0.02 to 0.05 from the cable ends [13].

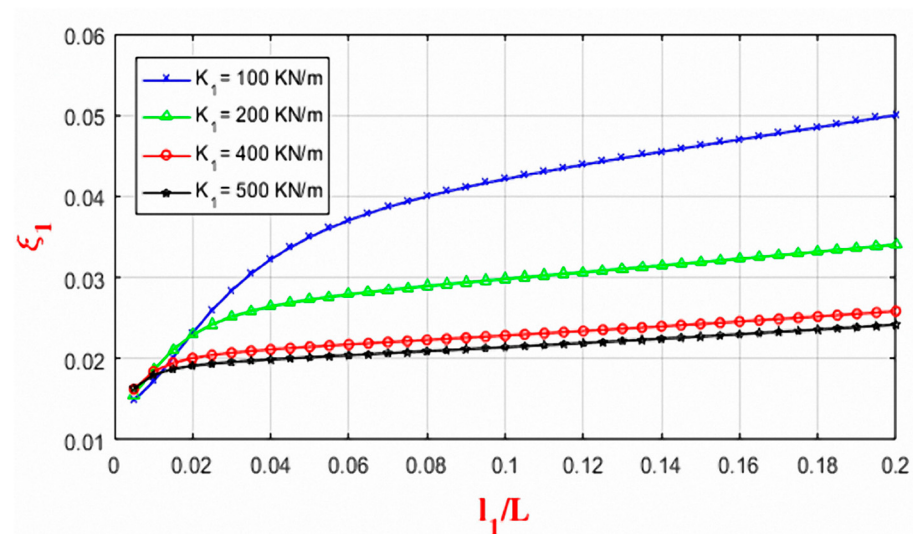


Figure 8. Relationship of ζ_1 and l_1/L with different K_1 .

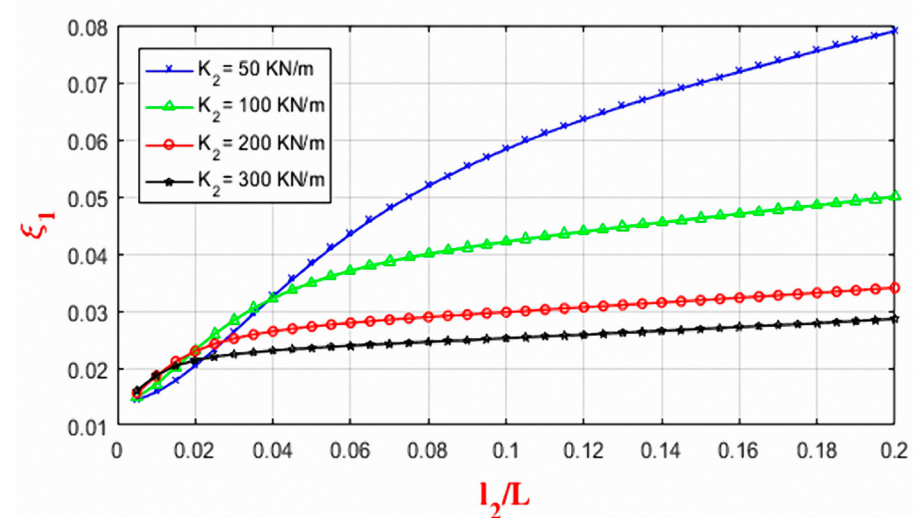


Figure 9. Relationship of ζ_1 and l_2/L with different K_2 .

3.5. Effect of Bending Stiffness (EJ) of Stay Cable

The stay cables for long-span bridges have quite a big diameter. Especially, they are always covered by mortar or epoxy. As a result, the bending stiffness of the cable increases significantly. Therefore, cable bending stiffness should be considered during the analysis of cable vibration. Generally, the damping ratio of the stay cable depends on not only the spring factor, loss factor, installation position but also its bending stiffness, cable tension, and cable length.

The non-dimensional parameter $\Delta = \pi^2 \cdot \frac{EJ}{SL^2}$ will be used for investigating the effect of bending stiffness. Parameter Δ reflects the cable stiffness and its sectional properties. The variation of the 1st modal damping ratio with non-dimensional parameter Δ can be seen in Figure 10. In this Figure 10, the value of Δ changes from 0 to 0.15 and the installation positions of HDR dampers varies from 0.02 to 0.05. Obviously, an increase in the damping ratio is directly proportional to Δ . The highest damping ratio is around 3.13% when the bending stiffness is taken into account in the current model.

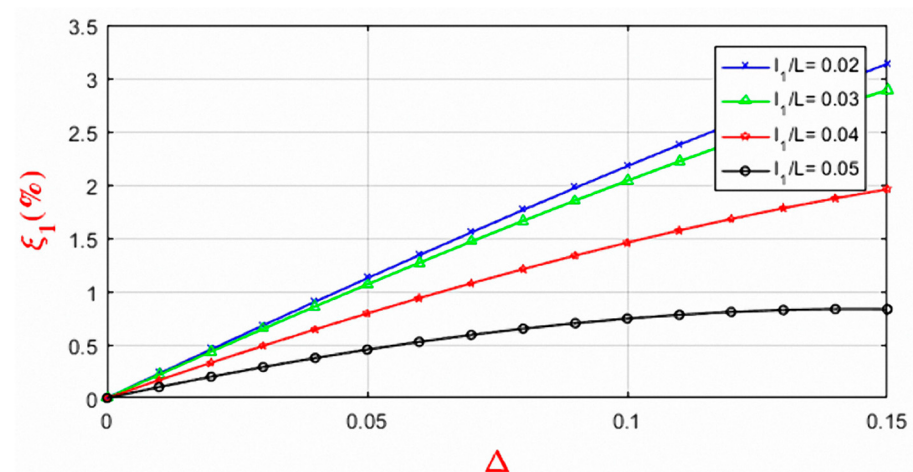


Figure 10. Relationship between Δ and $\xi_1\%$ with different l_1/L .

3.6. Optimization Parameters for Two HDR Dampers

To evaluate the damping efficiency of two HDR dampers, the three-dimension relationship of the first modal damping ratio, loss factor, and installation position of dampers are explored in this section. Figure 11 presents the relationship between loss factors of two HDR dampers with the damping ratio. In this analysis, the installation position of two HDR dampers is fixed at 0.03. From the figure, it is evident that loss factors of dampers have a significant influence on the damping ratio of the dampers. As the loss factors of the dampers increases, the damping ratio increases as well. The highest value of damping ratio is obtained for a loss factor value of 12. On the other hand, the relationship between the installation positions of two HDR dampers with the damping ratio is presented in Figure 12. As the magnitude of installation position increases from the cable ends, the higher damping ratio of the cable increases as well. Furthermore, the peak value of the damping ratio is found for an installation position (l/L) of 0.1 from the cable ends.

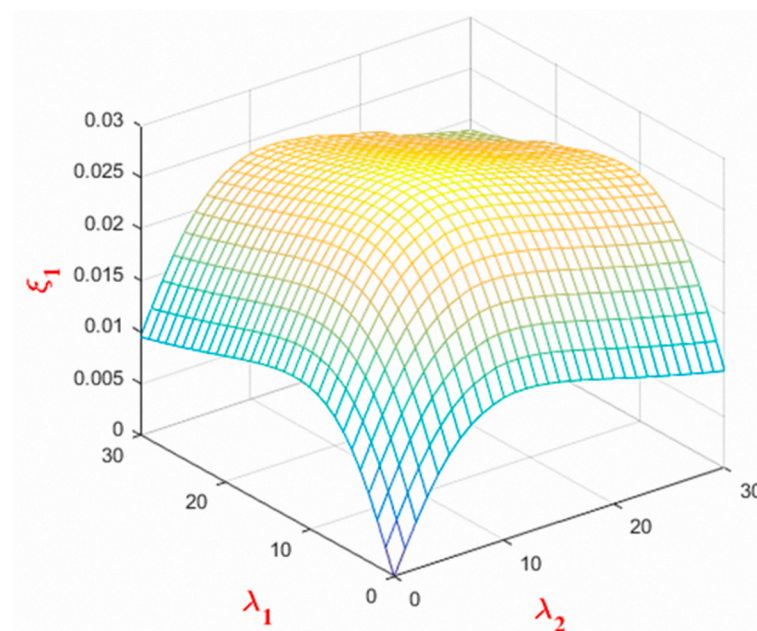


Figure 11. Loss factors optimization for two HDR dampers.

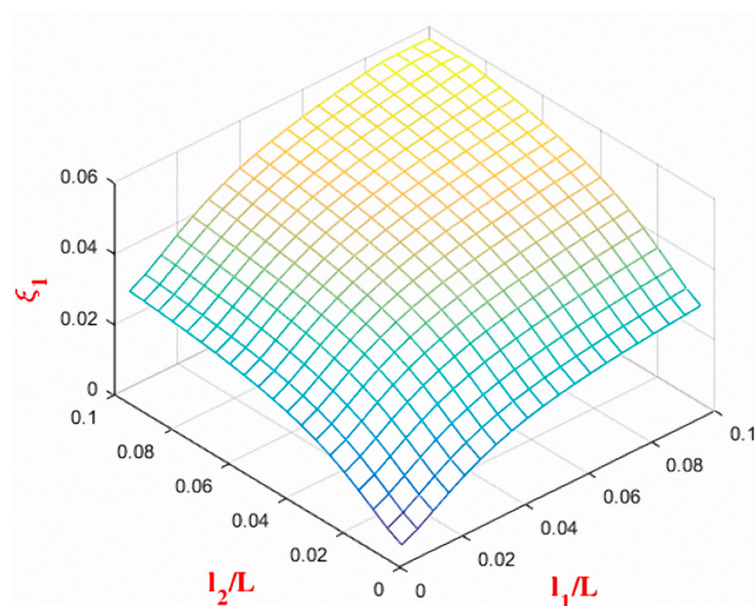


Figure 12. Installation positions optimization for two HDR dampers.

4. Conclusions

This paper already investigated the effect of combined two HDR dampers on 1st modal damping ratio of stay cable under considering cable bending stiffness. The theoretical model for evaluating cable damping with two HDR dampers has been proposed. The comparative analysis of results elucidated that the parameters such as loss factor, spring factor, and installation positions of dampers have much effect on the cable damping ratio. The conclusions obtained from this study are as follow:

- Theoretical formulation for evaluating the modal damping ratio of stay cable with two HDR dampers has been proposed and validated.
- For a stay cable with fixed values of tension (S) and cable length (L), the optimum spring factor value of HDR dampers can be obtained when the parameter $\Phi_i (K_i L / \pi S)$ becomes 1.
- Loss factors play a significant role in the damping efficiency of HDR dampers. There is always an optimum value for the loss factor. By varying the spring factor, the optimum loss factor value can be figured out.
- Based on the present analysis, it is found that the damping ratio is directly proportional to the installation positions of HDR dampers. The larger the value of installation distance from the cable ends, the damping ratio also becomes higher. In addition, cable damping is also directly proportional to its bending stiffness.
- Present model can be an effective tool for evaluating the stay cable damping with two HDR-dampers. Designer can choose appropriate values among various design parameters to obtain the highest damping ratio.

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