

Article

# A Method for Calculating the Velocity of Corner-to-Corner Rear-End Collisions of Vehicles Based on Collision Deformation Analysis

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**Abstract:** To improve the efficiency of solving vehicle collision velocity and provide sufficient evidence for the identification of accident responsibility, we proposed a method combining the momentum equation and finite element simulation. We built a finite element simulation model of a vehicle where multiple collision simulation experiments were carried out, and studied the calculation method of collision deformation. After fitting and analyzing the simulated deformation data of an accident vehicle under different velocities through collision simulation experiments, a relationship model between collision velocity and deformation was established, and a method to solve the collision velocity was proposed by combining the existing two-dimensional collision momentum equation of the vehicle. For actual collision cases, the proposed velocity solution method and the simulation software were used for reconstruction analyses, respectively, and the results of the instantaneous contact velocity of vehicle collision were compared. It was found that the velocity calculation results obtained using the two methods above were in good agreement; the shape and depth of the simulated deformation were consistent with the actual deformation of the vehicle.

**Keywords:** rear-end collision; collision deformation; collision velocity; finite element simulation experiment; accident reconstruction



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## 1. Introduction

With the rapid development of the global economy and the rapid growth of vehicle ownership, vehicles have become an important means of transportation for people, but at the same time, road traffic accidents have also become a global public hazard. Once a vehicle crash has happened, it is impossible to reappear. This irreversible characteristic makes it difficult to accurately determine a vehicle's velocity before collision, causing a great deal of inconvenience in identifying accident liability, especially if the parties involved have different opinions on the circumstances of the accident. In numerous vehicle collision accidents, corner-to-corner rear-end collisions of vehicles are more common, especially in urban road systems. Therefore, it is necessary to use relevant technical means to find a solving method to study the collision velocity of corner-to-corner rear-end vehicle collision accidents.

At present, scholarly research is mainly focused on dynamic calculations and finite element analyses. In terms of dynamic calculations, the National Highway Traffic Safety Administration (NHTSA) has successfully developed the world's first vehicle crash simulation software, SMAC [1], which is mainly used for simulation analyses of collisions between two vehicles, and in which the theoretical model is mainly based on numerical solution of classical Newton's laws. According to the relationship between the average impact force of a vehicle and residual deformation, a model was built, and CRASH analysis software was developed [2]. Dr. Hermann Steffan from Austria constructed the model based on the momentum theory and PC-Crash simulation software based on the Kudlich-Slibar

model [3]. Zhang and Huang et al. studied a dynamic model of vehicle-to-vehicle collision and concluded that vehicle collision parameters can be optimized using the wheel imprint and the stationary position of the vehicle. They also put forward the error factors of a stationary vehicle, in the middle position, and on a direction angle, and established the objective function of collision parameter optimization. Based on actual accident cases, PC-Crash software was used to simulate the accident and verify the effectiveness of the vehicle-to-vehicle collision dynamics model [4,5].

On this basis, Inhwon et al. proposed an analysis method based on qualitative vehicle collision mechanics using the law of conservation of momentum [6]. Zou et al. analyzed the uncertainty of accident reconstruction using mathematical theory, transformed it into a problem of extreme value, and proposed a simplified algorithm for accident reconstruction. The algorithm was applied to cases of actual accidents, and error analysis showed that the algorithm could be used in road accident reconstruction within an acceptable range [7].

Based on momentum conservation, information regarding body deformation was used. According to the theory of the relationship between deformation and energy, Guo obtained a reconstruction method for traffic accidents and found that the collision deformation was consistent with that of the accident vehicle through calculations of actual accident cases [8]. Zhiqiang et al., using PC-Crash simulation software and the principle of energy and momentum conservation, analyzed the relationship between total deformation energy and velocity before collision, and proposed an energy-based model for calculating the side impact velocity of a vehicle, of which the validity was verified using numerical simulations [9]. Voevodin et al. proposed a calculation model of vehicle driving dynamics during collision by studying the destruction of vehicle elements after collision [10].

Vangi has performed a great deal of research in reducing errors and improving efficiency. In order to reduce errors in the process of vehicle collision, a computational model of linear vehicle velocity and angular velocity was proposed to analyze the phase of a vehicle after collision [11]. In order to quickly calculate the energy loss during collision, a “triangle method” for defining body damage was proposed [12]. In addition, in order to reduce the calculation time, a reduced order model for accident reconstruction was proposed, which accurately calculated kinematic data and deformation of the vehicle [13]. Ji et al. used energy loss-based vehicular injury severity (ELVIS) to describe the impact of energy absorption during vehicle collisions, and a regression model was used to study the relationship between injury and collision mechanisms [14].

From the aspect of finite element simulations, Day and York improved the finite element collision method for automobiles and increased the efficiency of finite element simulation analysis [15]. For vehicle-pier collision accidents, El-Tawil et al. used the inelastic transient finite element method to reconstruct an accident and obtained deformation data under different conditions by changing the velocity. Through analysis of the experimental results, some suggestions for collision design specification were put forward [16]. Fahlstedt et al. built a finite element collision model of vehicles and carried out simulation analyses to study the effects of the initial impact velocity and angle on the results of accident reconstruction [17]. Macurová et al. discussed the possibility of studying the value of energy-equivalent velocity using PC-Crash software and provided a theoretical basis [18]. Based on finite element simulation, Evtiukova proposed a method to calculate vehicle velocity through vehicle collision deformation, but it is only applicable for calculating the velocity of frontal vehicle collisions with non-deformable obstacles [19].

In order to improve the simulation efficiency of finite element simulation, Zhang et al. used the method of combining neural networks and finite element analysis to simulate vehicle collision accidents and found that the application of neural networks can improve the simulation efficiency; thus obtaining the relationship between initial vehicle collision parameters and deformation. Used to verify cases of typical accidents in which the pre-collision velocities were obtained, the method is noted as being applicable for vehicle accidents without tire tracks [20]. Chen et al. developed an artificial neural network method using virtual collision data generated by finite element simulation software, and

proved that plastic deformation characteristics can effectively invert the data before vehicle collision [21].

To sum up, research in the field of vehicle collision is based on a single theory and lacks mutual combination. A calculation method based on the dynamics principle fails to take advantage of the important information of vehicle collision deformation, and the reliability of velocity results is insufficient. The finite element reconstruction-based method requires repeated simulation experiments to determine the velocity at the moment of vehicle collision contact, which is inefficient; moreover, the vehicle finite element model has a strong directivity and it is difficult to apply it to other accidents.

In view of the above, with our focus on corner-to-corner rear-end vehicle collision accident patterns, we constructed a vehicle collision model, obtained collision deformation results at different velocities with the help of a large number of finite element collision simulation experiments, and established a relationship model between impact velocity and deformation through data fitting. A method for calculating the impact velocity based on classical mechanics and the finite element method is proposed using the existing vehicle collision momentum equations. The empirical results have important theoretical significance in revealing the formation mechanism of vehicle collision deformations and developing a theory of accident-reconstruction analysis, and also have important value for practical applications in improving the level of accident reconstruction analysis and accident handling as well as the determination of responsibility.

## 2. Collision Simulation Experiment

### 2.1. Design of the Experiment

Two vehicles drove in the same direction from front to back (the rear vehicle is defined as vehicle 1 and the front vehicle is defined as vehicle 2). Vehicle 2 slowed down and changed lanes, and vehicle 1 cannot avoid it, resulting in a diagonal rear-end collision. During the collision, the included angle between vehicle 1 and vehicle 2 was 15°.

The vehicle collision model was built using Ls-Prepost and the model and mesh were optimized using HyperMesh software. The areas that had a great impact on the results were finely divided, and the mesh size of the other areas was increased appropriately. The initial velocity of vehicle 1 and vehicle 2 was 25 km/h and 20 km/h, respectively, with an additional gradient of 5 km/h. The ending velocity of the experiment was 65 km/h and 60 km/h, respectively. The simulation experiment of 30 km/h–20 km/h was introduced as an example. LS-DYNA was used to simulate the angular and diagonal rear end collision under different combinations of velocities. After verifying the reliability of the vehicle model, the vehicle deformation data were obtained using post-processing software, and then follow-up data analysis was carried out. The simulation experiment process is shown in Figure 1.

### 2.2. Finite Element Model of Vehicle

#### (1) Material Property

According to materials of actual vehicle bodies, the material properties of the vehicle model were set [22]. The material parameters of the main body parts are shown in Table 1.

#### (2) Contact Way

The possible contact of the vehicle was set using Ls-Prepost software. Contact among the parts of the vehicle was set using the keyword \*CONTACT \*AUTOMATIC\_SINGLE\_SURFACE. The contact settings among the vehicle bodies were set using \*AUTOMATIC\_SINGLE\_SURFACE\_TO\_SURFACE in the \*CONTACT keyword. Because the instantaneous automobile collision process is generally 70–120 ms, the friction between the vehicle's tires and the ground was ignored.

#### (3) Basic Parameter Setting

The velocity of vehicle 1 was set to an X-axis component of 8333.33 mm/s, and a Y-axis component of 0 mm/s; the velocity of vehicle 2 was set to an X-axis component of 5366.28 mm/s and a Y-axis component of 1437.89 mm/s, with a pre-collision horizontal

distance of 7.3 mm and an angle of 15°. To ensure that the model collision was fully effective in the simulation experiment, the termination time of the experiment was 130 ms and the hourglass coefficient was a default value of 0.1.

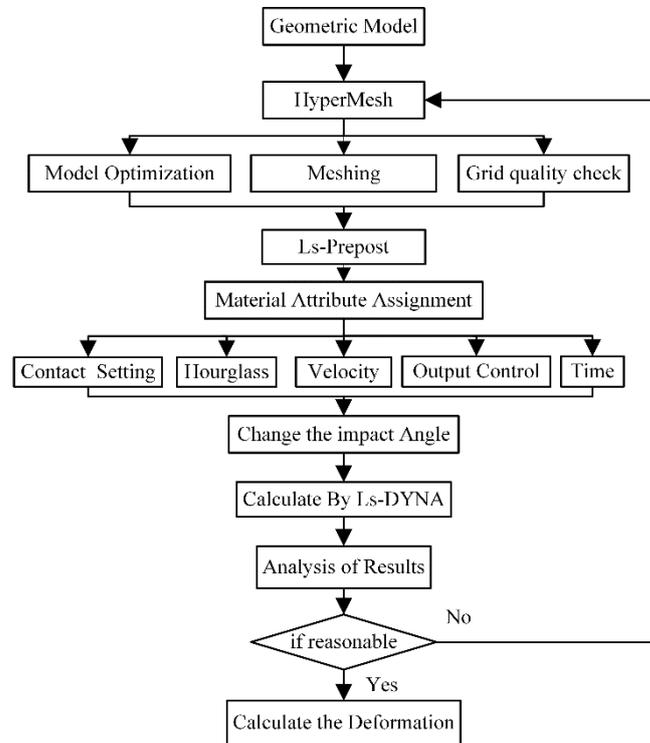


Figure 1. Simulation experiment construction process.

Table 1. Materials and properties of major automotive components.

Number	Parts	Material Number	Density (t/mm <sup>3</sup> )	Young's Modulus (Mpa)	Poisson's Ratio	Yield Stress (Mpa)
1	Front fender	024	1.415 × 10 <sup>-9</sup>	1.00 × 10 <sup>3</sup>	0.3	20
2	Bumper	024	7.890 × 10 <sup>-9</sup>	2.00 × 10 <sup>5</sup>	0.3	800
3	Door and Hood	024	7.890 × 10 <sup>-9</sup>	2.00 × 10 <sup>5</sup>	0.3	271
4	Engine	001	1.582 × 10 <sup>-9</sup>	2.00 × 10 <sup>4</sup>	0.3	\
5	Engine Cover	001	7.890 × 10 <sup>-9</sup>	2.00 × 10 <sup>5</sup>	0.3	\
6	Tailgate	024	1.005 × 10 <sup>-9</sup>	1.00 × 10 <sup>3</sup>	0.3	20
7	Roof	024	7.890 × 10 <sup>-9</sup>	2.00 × 10 <sup>5</sup>	0.3	220
8	Window Glass	123	2.500 × 10 <sup>-9</sup>	7.00 × 10 <sup>4</sup>	0.22	30
9	Tire	001	1.750 × 10 <sup>-9</sup>	3.00 × 10 <sup>2</sup>	0.3	\

### 2.3. Model Checking

#### (1) System Energy Analysis

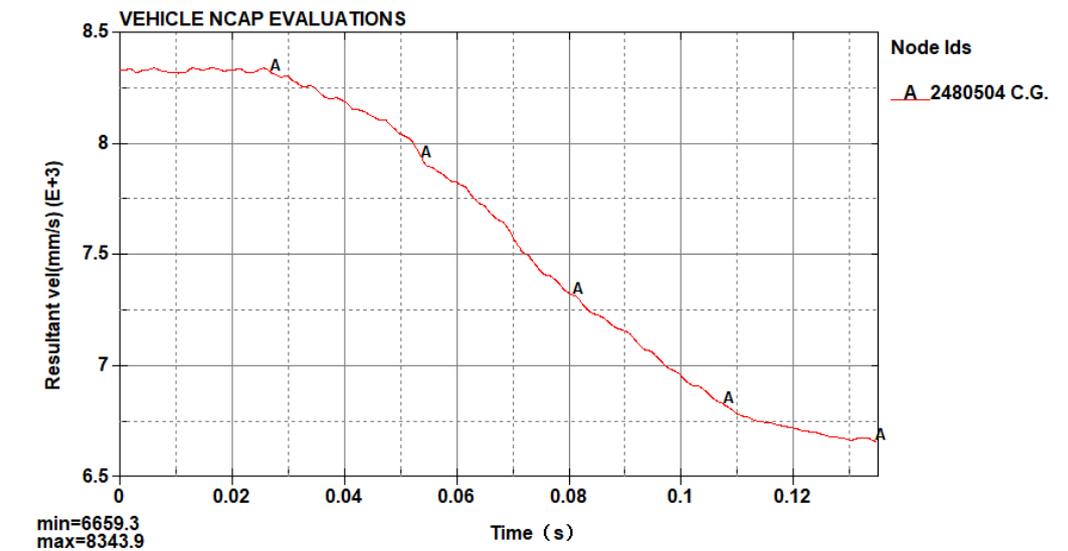
The energy conversion curves of the collision process were obtained through a hypergraph, from which can be seen that at t = 0.02–0.04 s, the kinetic energy and internal energy curves change significantly at the moment of vehicle collision and the curve of total energy basically remains stable; after t = 0.08 s, the kinetic energy and internal energy curves remain horizontal, indicating that the energy conversion at the moment of vehicle collision is over. Overall, the kinetic energy of the system is converted into internal energy in the collision process, and the total energy remains the same.

The post-processing software calculated that the percentage of the hourglass energy to the total energy of the system was 0.8%, and the mass increase was 0.99%, both of which

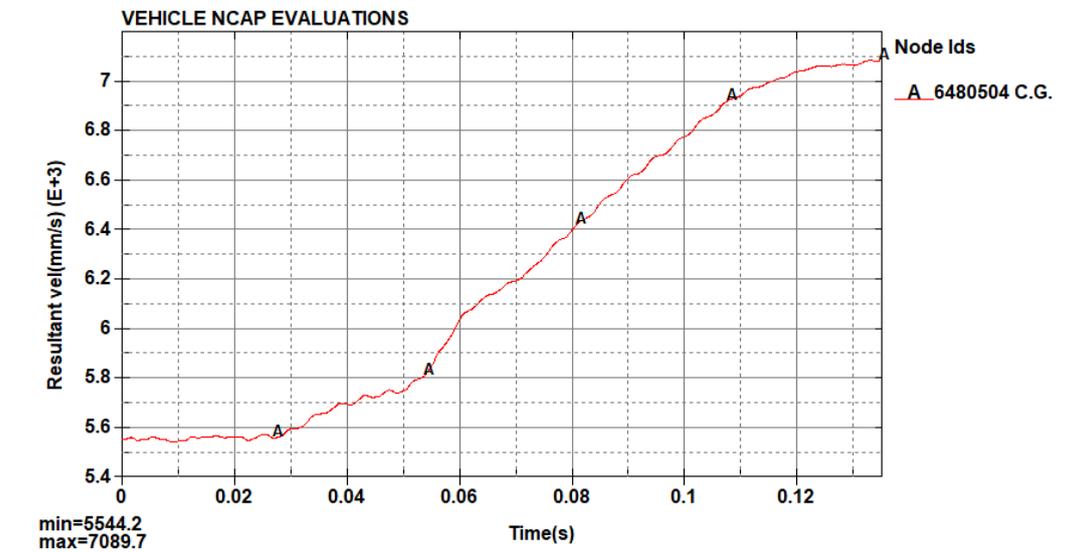
were far less than 5%, making the impact negligible [23]. Therefore, the finite element model is reasonable.

(2) Vehicle Velocity Analysis

By observing the change curve in the vehicle velocity, it could be found that the velocity of vehicle 1 decreased significantly, while that of vehicle 2 increased. After  $t = 0.12$  s, the velocity of vehicle 1 dropped to about  $6.67 \times 10^3$  mm/s, and the velocity of vehicle 2 increased to  $7.1 \times 10^3$  mm/s, as shown in Figure 2. By comparing the velocity differences between the two vehicles, it can be inferred that the two vehicles were in a collision separation state. The change of velocity conforms to the law of real vehicle collision velocity.



(a) vehicle 1



(b) vehicle 2

Figure 2. Change curve of vehicle velocity.

The finite element model of the whole vehicle was reasonably established through comprehensive analyses, and the collision simulation experiments also met the objective laws and relevant requirements.

### 3. Methodology

#### 3.1. Calculation Method of Deformation

Calculation steps of the deformation are: ① determine the deformation range; ② determine key points; and ③ calculate the deformation with the help of a formula. Taking vehicle 1 as an example, in the upper, middle, and lower horizontal positions of the front deformation area, an average of 4 key points was selected for each row. The key points of each row and column were required to be on the same straight line with the deformation range divided into six regions by key points, and the overall deformation of the deformation area was calculated by the deformation of these key points. The selection of the key points is shown in Figure 3.

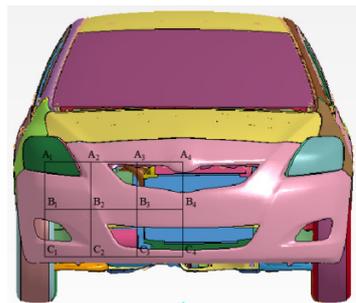


Figure 3. Diagram of selection of vehicle 1 deformation range and key points.

The overall deformation (C) calculation formula is:

$$C = \sqrt{Cx^2 + Cy^2 + Cz^2} \tag{1}$$

In the formula,  $C_x$ ,  $C_y$ , and  $C_z$  are the deformation amount of the deformation area in the X, Y, and Z directions, respectively, which is obtained by calculating the average value of the key points in these directions.

Deformation data of the key points of vehicle 1 are shown in Table 2.

Table 2. Deformation data of vehicle 1 (30 km/h).

Key Point	Deformation (mm)			Key Point	Deformation (mm)		
	X	Y	Z		X	Y	Z
A1	4	−18	49	B3	61	−34	39
A2	15	−25	49	B4	24	−24	43
A3	23	−27	54	C1	19	−19	39
A4	11	−23	56	C2	19	−20	41
B1	23	−21	41	C3	15	−19	43
B2	71	−37	40	C4	10	−18	50

According to Formula (1),  $C_x$ ,  $C_y$ , and  $C_z$  are 25 mm, −24 mm, and 45 mm, respectively, and the overall deformation (C) of vehicle 1 is 57 mm.

#### 3.2. Data Collection

In the same way, the deformations of the two vehicles after experiments under different velocities were calculated, and the statistics are shown in Table 3.

**Table 3.** Deformation of vehicle 1 under collision at different velocities (mm).

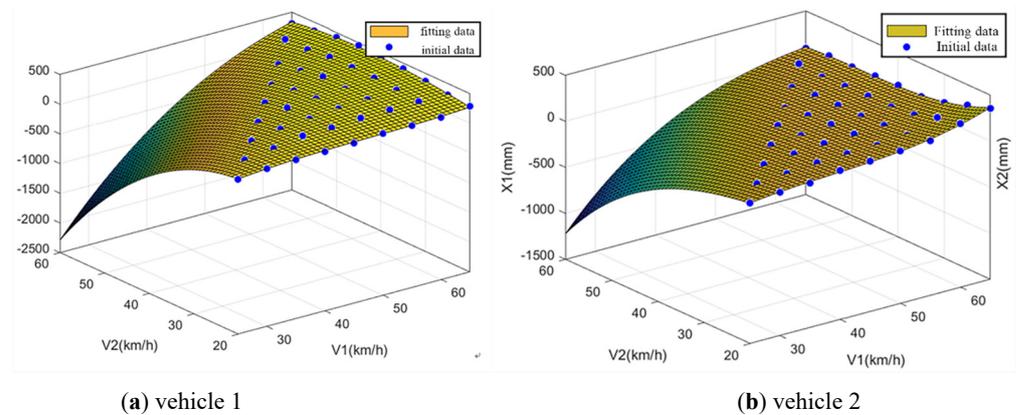
		Overall Deformation of the Vehicle 1: C1									
V		20 km/h	25 km/h	30 km/h	35 km/h	40 km/h	45 km/h	50 km/h	55 km/h	60 km/h	65 km/h
Overall deformation of the vehicle 2: C2	20 km/h	-	29	57	70	119	132	155	178	277	354
	25 km/h	108	-	36	58	88	98	101	123	229	276
	30 km/h	156	134	-	43	69	96	110	116	143	189
	35 km/h	174	196	150	-	53	67	102	103	114	139
	40 km/h	179	275	217	164	-	63	85	108	120	148
	45 km/h	191	278	255	273	186	-	50	101	111	135
	50 km/h	221	311	294	292	257	182	-	71	96	119
	55 km/h	222	317	320	325	362	285	224	-	126	125
	60 km/h	228	324	346	379	387	388	342	343	-	92
	65 km/h	286	305	365	403	410	452	415	354	306	-

3.3. Data Fitting

The experimental data were fitted many times to obtain a three-dimensional surface diagram of the relationship between the deformation and driving velocity (as shown in Figure 4), and the corresponding relations (2) and (3) were established.

$$X_1 = A_{1i}v_1^3 + A_{1i}v_2^3 + A_{1i}v_1^2 + A_{1i}v_2^2 + A_{1i}v_1^2v_2 + A_{1i}v_1v_2^2 + A_{1i}v_1 + A_{1i}v_2 + A_{1i}v_1v_2 + b_1 \tag{2}$$

$$X_2 = A_{2i}v_1^3 + A_{2i}v_2^3 + A_{2i}v_1^2 + A_{2i}v_2^2 + A_{2i}v_1^2v_2 + A_{2i}v_1v_2^2 + A_{2i}v_1 + A_{2i}v_2 + A_{2i}v_1v_2 + b_2 \tag{3}$$



**Figure 4.** A 3D curved surface diagram of the relationship between deformations  $X_1$  and  $X_2$  and the collision velocity ( $v_1$  and  $v_2$ ).

In Formulas (2) and (3),  $b_1 = -62.94$ ,  $b_2 = -28.49$ . The values of  $A_{1i}$  and  $A_{2i}$  are shown in Table 4.

**Table 4.** Values of coefficients  $A_{1i}$  and  $A_{2i}$ .

$i$	1	2	3	4	5	6	7	8	9
$A_{1i}$	0.00967	-0.01029	-0.6034	-0.2137	-0.02575	0.02614	19.21	-17.93	0.8444
$A_{2i}$	0.00585	-0.00231	-0.2844	-1.818	-0.02741	0.02769	-5.814	14.45	1.79

For research on the solution of the vehicle collision velocity, the basic equation for two-dimensional eccentric dynamics was determined according to the momentum equation and the momentum conservation law [24]. For the establishment of the equations, some scholars used the elastic coefficient of restitution and some introduced a relative friction coefficient and other methods, but the selection of coefficients inevitably involves personal subjective

thinking, which will reduce the credibility of the results. Therefore, the relationship between the amount of deformation and the driving velocity ((2) and (3)) are used as equations, which are combined with the dynamics equations to build the calculation model of vehicle collision dynamics parameters.

#### 4. Case Study

##### 4.1. Brief Case

In December 2009, near the intersection of Xinyang Road and Anguang Street in Daoli District, Harbin, China, a black Audi and a white Jetta were driving in the same direction from east to west when a rear-end collision occurred, causing damage to both vehicles. It is known that the accident occurred in winter, on an icy, snowy road and at a gradient of 2.8%. The total mass of the Audi was 1200 kg, and the total mass of the Jetta was 2025 kg. When the two vehicles were stationary, the base of the left rear wheel of the Jetta and Audi front right wheel were 1.6 m apart; tire brake tracks were 8 m and 8.5 m, respectively; and the collision angle of the two vehicles was 15°.

##### 4.2. Case Solving

###### (1) Model Solving

Calculated according to the vehicle's trace inspection report: the overall deformation of the Audi was 30 mm and the overall deformation of the Jetta was 122 mm.

$X_1 = 30$  mm,  $X_2 = 122$  mm are brought into Formulas (2) and (3), the  $v_1$  of Audi and  $v_2$  of Jetta before collision are obtained:

$$v_1 = 35.42 \text{ (km/h)}$$

$$v_2 = 32.20 \text{ (km/h)}$$

By using the two-dimensional dynamic equation and the slip trajectory equation, the instantaneous velocities after collision are obtained:  $v_{1'} = 32.86$  (km/h),  $v_{2'} = 33.77$  (km/h); angular velocity:  $w_1 = -0.45$  rad/s,  $w_2 = 0.86$  rad/s.

According to the parameters of the accident vehicles, finite element models of the Audi and Jetta were established. The obtained values for the velocities were taken as the driving value of velocity, and the instantaneous motion state of the vehicles was reproduced by LS-DYNA.

The collision between two vehicles led to the conversion of system energy. It can be seen in Figure 5 that, during the collision, the vehicle running velocities decreased suddenly, and the corresponding kinetic energy also changed, with the overall kinetic energy of the system decreasing from 64,244,600 MJ to 60,928,400 MJ. Due to the collision friction, the internal energy increased from 0 MJ to 2,795,150 MJ, and the vehicle collision experienced a conversion from kinetic energy to internal energy. Owing to the limitations of the experimental setup, the hourglass energy of 521,050 MJ generated during the simulation was much less than 5% of the total energy. The experiment accords with the objective facts and the law of energy conservation; therefore, the obtained data have a certain credibility.

###### (2) Simulation reconstruction.

PC-Crash software is a mature simulation software with a long history of use and high authority in the field of automobile collisions. The above-mentioned traffic accident was reconstructed with the help of PC-Crash.

After constant adjustment of the experimental parameters, it was finally determined that when the velocity of the Audi was 33.91 km/h and the velocity of the Jetta was 33.53 km/h, the trajectory of the vehicle after the collision, the stopping location, and the collision position were closest to those of the actual scene. The stopping location of the accident vehicle, the collision point, and the slip trajectory of the simulated vehicles are shown in Figure 6.

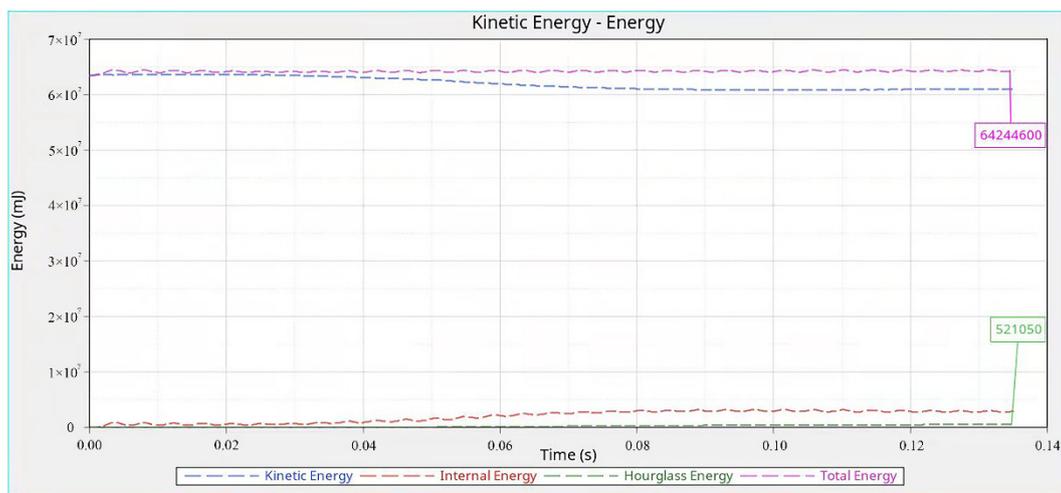
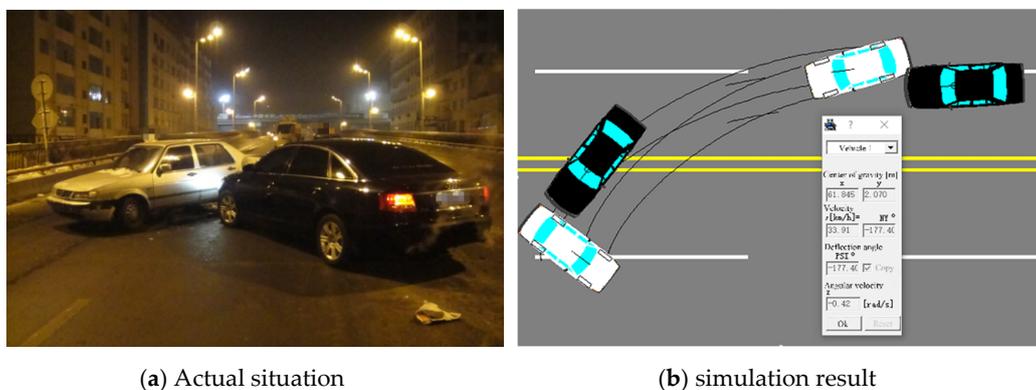


Figure 5. Energy variation curves.



(a) Actual situation

(b) simulation result

Figure 6. Vehicle collision track.

### 4.3. Result Analysis

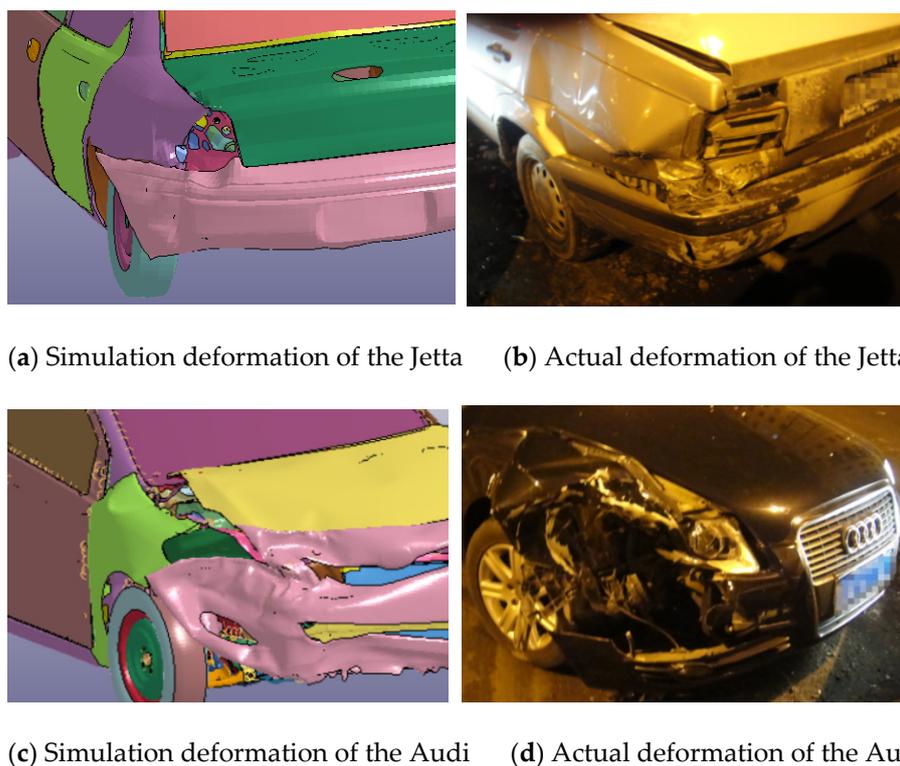
Table 5 shows the results obtained from the two calculation methods and compares the relative errors.

Table 5. Summary of results calculated by the two methods.

Calculation Results	Method of Calculation		Comparative Relative Error
	PC-Crash	Mathematical Model Calculation	
Driving Velocity: $v_1$ (km/h)	33.91	35.42	4.45%
Driving Velocity: $v_2$ (km/h)	33.53	32.20	3.97%

The results show that the relative error of the two methods is less than 5%, which indicates that the calculation results of the mathematical model are close to those of the PC-Crash simulation.

Through the finite element simulation experiment, it is found that the simulation experiment is consistent with the actual deformation parts and deformation range of the vehicle. The overall deformation of the Audi and the Jetta are 34 mm and 136 mm respectively, with errors of less than 15% compared with the actual vehicle deformation. The LS-DYNA simulation results are compared with the actual vehicle deformation, as shown in Figure 7.



**Figure 7.** Deformation comparisons between the simulation model and the actual vehicles.

## 5. Conclusions

- (1) Based on the principles of classical mechanics and the finite element method, a method for solving the velocity of rear-end vehicle collisions is proposed. The case study shows that the results obtained using this method are in good agreement with those calculated with PC-Crash software, and the error is within 5%; reconstruction results of collision deformation are basically consistent with actual deformations, and the error is within 15%.
- (2) This method utilizes information from vehicle collision deformations and improves the reliability of analysis results of corner to corner rear-end collision of vehicles. A relationship model between collision velocity and deformation is established that overcomes the limitations of the traditional finite element method, such as low reconstruction efficiency and the model has strong pertinence.
- (3) By combining the finite element simulation with the dynamic model method, the vehicle velocity can be quickly calculated according to the vehicle deformation, which breaks through the traditional algorithm of calculating the vehicle velocity using tire traces and scattered objects, and improves the efficiency and accuracy of the calculation results.

The method proposed in this study is only applicable to the special accident form of corner rear-end collisions of vehicles. In the simulation experiments, the collision angle was  $15^\circ$ , which has some limitations. Therefore, follow-up research should expand the range of angles to explore the relationship between different velocities, different angles, and collision deformation. The basic principles and general rules of this study can still be used as reference for similar studies.

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