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A Novel Constraints Model of Credibility-Fuzzy for Reliability Redundancy Allocation Problem by Simplified Swarm Optimization

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Abstract: A novel constraints model of credibility-fuzzy for the reliability redundancy allocation problem (RRAP) is studied in this work. The RRAP that must simultaneously decide reliability and redundancy of components is an effective approach in improving the system reliability. In practice various systems, the uncertainty condition of components used in the systems, which few studies have noticed this state over the years, is a concrete fact due to several reasons such as production conditions, different batches of raw materials, time reasons, and climatic factors. Therefore, this study adopts the fuzzy theory and credibility theory to solve the components uncertainty in the constraints of RRAP including cost, weight, and volume. Moreover, the simplified swarm optimization (SSO) algorithm has been adopted to solve the fuzzy constraints of RRAP. The effectiveness and performance of SSO algorithm have been experimented by four famous benchmarks of RRAP.

Keywords: reliability redundancy allocation problem (RRAP); credibility-fuzzy theory; simplified swarm optimization (SSO) algorithm

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Citation: Lin, H.C.-S.; Huang, C.-L.; Yeh, W.-C. A Novel Constraints Model of Credibility-Fuzzy for Reliability Redundancy Allocation Problem by Simplified Swarm Optimization. *Appl. Sci.* **2021**, *11*, 10765. https://doi.org/10.3390/app112210765

Academic Editor: Bernabe Dorronsoro

Received: 14 October 2021
Accepted: 12 November 2021
Published: 15 November 2021

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1. Introduction

The reliability enhancement has increasingly become important in most systems because the system reliability is an important measure of system performance in many practical systems. For example, medical systems [1], video systems [2], traffic systems [3], industrial intelligence systems [4], network systems [5–9], wireless sensor networks [10,11], power transmission networks [12], social networks [13], and Internet of Things [14,15]. The evaluation and enhancement of system reliability must be optimized in the early design stage because the enhancement of reliability is the most efficient at this stage [16,17].

With the objective of optimizing system reliability in various systems, the redundancy allocation problems (RAP), which has to only decide the redundancy of components, and the reliability redundancy allocation problems (RRAP), which has to simultaneously decide reliability and redundancy of components, are two well-known methods to improve the system reliability. RRAP that has been studied in this work is more complicated and difficult than RAP because it must determine the reliability and redundancy of components in the systems at the same time [18–20].

The researches of RRAP over the years have mainly used four network systems as benchmarks including series system, the network with series and parallel elements, a complex (bridge) system, and the overspeed protection of a gas turbine system, which are thus adopted in this study, as shown in following Figures 1–4. Yeh et al. researched the bi-objective RRAP including optimizing the system reliability and cost [18]. The cold-standby strategy in RRAP was adopted by several researchers such Yeh, Ardakan and Hamadani, and Mellal and Zio [19,21,22]. Dobani et al. considered the heterogeneous components in RRAP [23]. The heterogeneous components and mixed strategy including active and cold-standby strategy are studied by Ouyang et al. [24]. Huang shown the hybrid swarm algorithms in RRAP [25]. And Garg presented a penalty guide in RRAP [26].

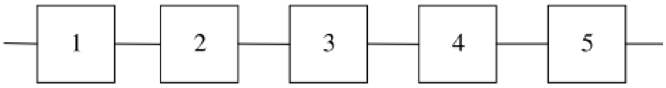

Figure 1. Series system.

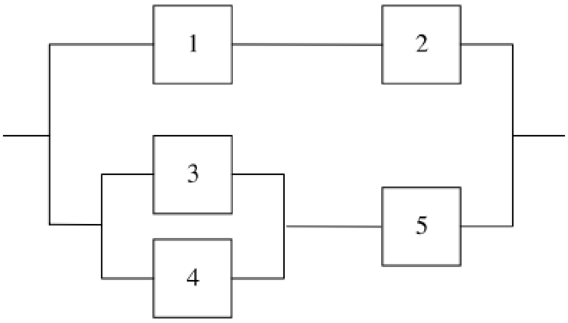

Figure 2. The network with series and parallel elements.

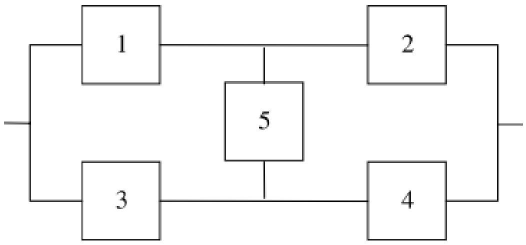

Figure 3. A complex (bridge) system.

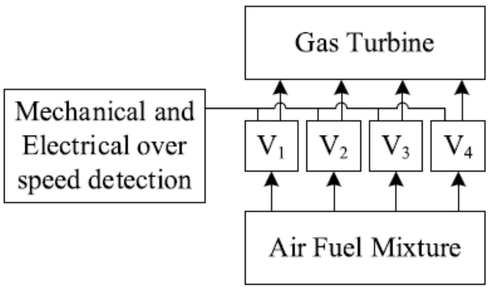

Figure 4. The overspeed protection of a gas turbine system.

In fact, the status of components in the actual systems is uncertain due to many reasons in the practical environment including production conditions, different batches of raw materials, time reasons, and climatic factors. However, over the years, the RRAP studies such as the above researches have not considered the uncertainty conditions of components [18–26]. Though Muhuri et al. considered interval type-2 fuzzy numbers to solve the uncertainty terms of components including reliability, cost, and weight in RRAP [27]. However, Muhuri et al.'s research does not study the typical four benchmarks of RRAP and the components uncertainty of volume is not considered. Therefore, this work comprehensively studies the uncertainties of systems components in the constraints of RRAP including cost, weight, and volume applied to the famous four benchmarks shown in Figures 1–4.

The fuzzy number that was originally invented by Zadeh in 1965 [28] has been shown to be effective for solving the uncertainty condition, thus, this work adopts the fuzzy number for the uncertainties of systems components in the constraints of RRAP including cost, weight, and volume. However, the fuzzy numbers that applied to the constraints of RRAP including cost, weight, and volume finally must be transferred to a crisp value. Liu and Iwamura in 1998 first extended the chance-constrained programming, which let the constraints meet a certain probability value of confidence level such as p and was first developed by Charnes & Cooper in 1959 [29], to the fuzzy environment called the fuzzy chance-constrained programming [30]. After that, Liu in 2004 developed the credibility theory [31] based on the fuzzy chance-constrained programming that has been successfully approved its effectiveness for transferring the fuzzy constraints to crisp constraints at a confidence level in several fields such as budget planning [32], and RAP [33]. So far, there are no scholars apply the fuzzy number and credibility theory to the comprehensive consideration of uncertainty in the constraints of RRAP. Therefore, this study adopts the fuzzy theory and credibility theory to solve the components uncertainty in the constraints of RRAP including cost, weight, and volume.

The RRAP has been shown to be a NP-hard problem [18], thus, numerous meta-heuristic algorithms are adopted to solve various RRAP researches such as genetic algorithm (GA) [21,23,27], particle swarm optimization (PSO) [22,24], stochastic fractal search (SFS) [21], cuckoo search (CS) [25,26], artificial bee colony algorithm (ABC) [34], and simplified swarm optimization (SSO) [18–20,24]. In various kinds of metaheuristic algorithms, the SSO, which was originally developed by Yeh in 2009 [35], is a famous artificial intelligence and swarm algorithm and has been proved its superior performance in many areas such as RRAP [18–20,24], RAP [16,33], Wireless Sensor Networks [9], Internet of Things [15], and disassembly sequencing problem [36]. Therefore, in this study, the SSO algorithm has been adopted to solve and optimize the system reliaiblity for RRAP with considering the components uncertainty in the constraints of RRAP including cost, weight, and volume.

To summarize the above description, a flowchart as shown in Figure 5 is included to explain the processing of developing the results and how the system is working.

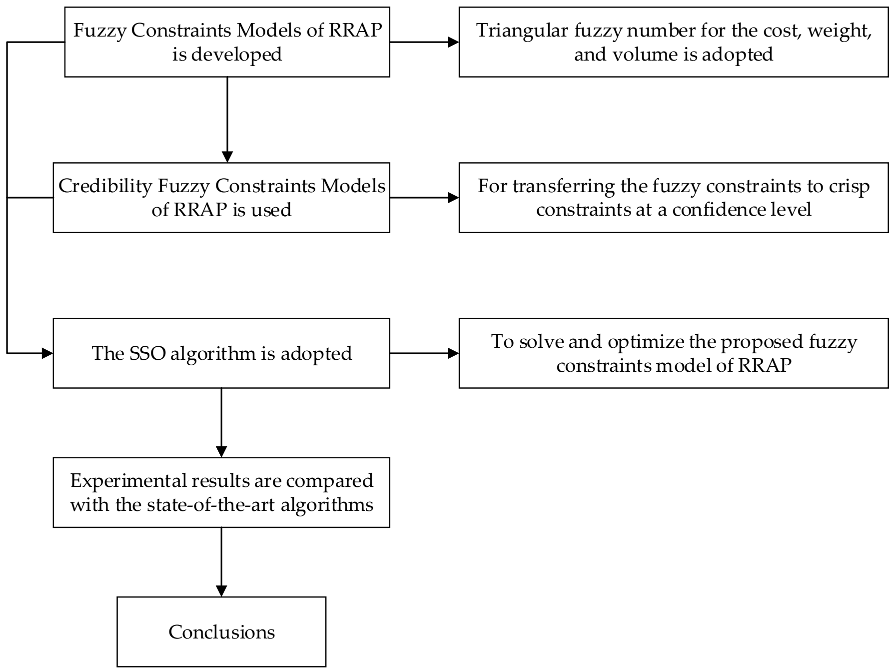

Figure 5. The flowchart of how the system is working.

The other chapters of this study are allocated as described. The RRAP models for the four benchmarks are introduced in Section 2. Section 3 has represented the proposed fuzzy constraints models of RRAP. Section 4 has shown the adopted SSO algorithm. The numerical experiments are displayed in Section 5. Finally, the conclusion is represented in Section 6.

2. The RRAP Models

The RRAP models, while aim to maximize the system reliability under the non-linear constraints of cost, weight, and volume, for the four benchmarks including series system, the network with series and parallel elements, a complex (bridge) system, and the overspeed protection of a gas turbine system as shown in Figures 1–4 are discussed in this section.

The RRAP aims to maximize the system reliability, which has to simultaneously decide reliability and redundancy of components under the non-linear constraints of cost, weight, and volume, is modelled as following Equations (1) and (2).

$$\text{Maximize } R_s(\boldsymbol{R}, \boldsymbol{N}) \tag{1}$$

$$\text{Subject to } Cst_j(\boldsymbol{R}, \boldsymbol{N}) \leq U_j \tag{2}$$

Based on the RRAP formulations in Equations (1) and (2), model the four benchmarks of RRAP including series system, the network with series and parallel elements, a complex (bridge) system, and the overspeed protection of a gas turbine system as following Equations (3)–(18).

Benchmark 1. Series system in Figure 1 [24,34,37].

$$\text{Maximize } R_s(\boldsymbol{R}, \boldsymbol{N}) = \prod_{i=1}^{N_v} R_i \tag{3}$$

$$\text{Subject to } Cst_1(\boldsymbol{R}, \boldsymbol{N}) = \sum_{i=1}^{N_v} \alpha_i(-1000/\ln r_i)^{\beta_i}(n_i + \exp(n_i/4)) \leq U_c \tag{4}$$

$$Cst_2(\boldsymbol{R}, \boldsymbol{N}) = \sum_{i=1}^{N_v} w_i n_i \exp(n_i/4) \leq U_w \tag{5}$$

$$Cst_3(\boldsymbol{R}, \boldsymbol{N}) = \sum_{i=1}^{N_v} w_i v_i^2 n_i^2 \leq U_v \tag{6}$$

$$0 \leq r_i \leq 1$$

The objective function to maximize the system reliability for the series system in Figure 1 is shown in above Equation (3). The constraints of cost, weight and volume are shown in above Equations (4)–(6), respectively.

Benchmark 2. Network with series and parallel elements in Figure 2 [24,34,37].

$$\text{Maximize } R_s(\boldsymbol{R}, \boldsymbol{N}) = 1 - (1 - R_1R_2)\{1 - [1 - (1 - R_3)(1 - R_4)]R_5\} \tag{7}$$

$$\text{Subject to } Cst_1(\boldsymbol{R}, \boldsymbol{N}) = \sum_{i=1}^{N_v} \alpha_i(-1000/\ln r_i)^{\beta_i}(n_i + \exp(n_i/4)) \leq U_c \tag{8}$$

$$Cst_2(\boldsymbol{R}, \boldsymbol{N}) = \sum_{i=1}^{N_v} w_i n_i \exp(n_i/4) \leq U_w \tag{9}$$

$$Cst_3(\boldsymbol{R}, \boldsymbol{N}) = \sum_{i=1}^{N_v} w_i v_i^2 n_i^2 \leq U_v \tag{10}$$

$$0 \leq r_i \leq 1$$

The objective function to maximize the system reliability for the network with series and parallel elements in Figure 2 is shown in above Equation (7). The constraints of cost, weight and volume are shown in above Equations (8)–(10), respectively.
Benchmark 3. Complex (bridge) system in Figure 3 [24,34,37].

$$\text{Maximize } R_s(\boldsymbol{R},\ \boldsymbol{N}) = \begin{array}{l} R_1R_2 + R_3R_4 + R_1R_4R_5 + R_2R_3R_5 - \\ R_1R_2R_3R_4 - R_1R_2R_3R_5 - R_1R_2R_4R_5 - \\ R_1R_3R_4R_5 - R_2R_3R_4R_5 + 2R_1R_2R_3R_4R_5 \end{array} \tag{11}$$

$$\text{Subject to } Cst_1(\boldsymbol{R},\ \boldsymbol{N}) = \sum_{i=1}^{N_v} \alpha_i(-1000/\ln r_i)^{\beta_i}(n_i + \exp(n_i/4)) \leq U_c \tag{12}$$

$$Cst_2(\boldsymbol{R},\ \boldsymbol{N}) = \sum_{i=1}^{N_v} w_i n_i \exp(n_i/4) \leq U_w \tag{13}$$

$$Cst_3(\boldsymbol{R},\ \boldsymbol{N}) = \sum_{i=1}^{N_v} w_i v_i^2 n_i^2 \leq U_v \tag{14}$$

$$0 \leq r_i \leq 1$$

The objective function to maximize the system reliability for the complex (bridge) system in Figure 3 is shown in above Equation (11). The constraints of cost, weight and volume are shown in above Equations (12)–(14), respectively.
Benchmark 4. The overspeed protection of a gas turbine system in Figure 4 [24,34,37].

$$\text{Maximize } R_s(\boldsymbol{R},\ \boldsymbol{N}) = \prod_{i=1}^{N_v} [1 - (1 - r_i)^{n_i}] \tag{15}$$

$$\text{Subject to} Cst_1(\boldsymbol{R},\ \boldsymbol{N}) = \sum_{i=1}^{N_v} \alpha_i(-1000/\ln r_i)^{\beta_i}(n_i + \exp(n_i/4)) \leq U_c \tag{16}$$

$$Cst_2(\boldsymbol{R},\ \boldsymbol{N}) = \sum_{i=1}^{N_v} w_i n_i \exp(n_i/4) \leq U_w \tag{17}$$

$$Cst_3(\boldsymbol{R},\ \boldsymbol{N}) = \sum_{i=1}^{N_v} v_i n_i^2 \leq U_v \tag{18}$$

$$0.5 \leq r_i \leq 1 - 10^{-6}, 1 \leq n_i \leq 10$$

The objective function to maximize the system reliability for the overspeed protection of a gas turbine system in Figure 4 is shown in above Equation (15). The constraints of cost, weight and volume are shown in above Equations (16)–(18), respectively.

3. Fuzzy Constraints Models of RRAP

This study uses fuzzy theory for modelling the fuzzy constraints of RRAP with considering uncertainty and adopts the credibility theory to modelling credibility fuzzy constraints of RRAP.

3.1. Fuzzy Constraints Models of RRAP

Zadeh in 1965 first developed the fuzzy number [28], which has been shown to be effective for solving the uncertainty condition. The triangular fuzzy number, which has been defined as the following Definition 1, is one of the famous fuzzy number that has been adopted in this study.

Definition 1. *The triangular fuzzy number* $\underline{A}$ = (a_1, a_2, a_3), *where* $a_1 < a_2 < a_3$ *and* a_i *belongs to the real numbers for* i = 1, 2, 3. *The relationship of the membership function* $u_{\underline{A}}(a)$ *and the triangular fuzzy number* $\underline{A}$ = (a_1, a_2, a_3) *has been shown in Equation (19).*

$$u_{\underline{\underline{A}}}(a) = \begin{cases} 0, & if\ a < a_1 \\ (a - a_1)/(a_2 - a_1), & if\ a_1 \leq a < a_2 \\ 1, & if\ a = a_2 \\ (a_3 - a)/(a_3 - a_2), & if\ a_2 < a < a_3 \\ 0, & if\ a_3 < a \end{cases} \tag{19}$$

Accordingly, this work adopts the triangular fuzzy number for the uncertainties of systems components in the constraints of RRAP including cost, weight, and volume, which are defined as following the triangular fuzzy number for the cost, weight, and volume:

1. The triangular fuzzy number for the cost $\underline{c}$ = (c_1, c_2, c_3), where $c_1 < c_2 < c_3$ and c_i belongs to the real numbers for i = 1, 2, 3.
2. The triangular fuzzy number for the weight $\underline{w}$ = (w_1, w_2, w_3), where $w_1 < w_2 < w_3$ and w_i belongs to the real numbers for i = 1, 2, 3.
3. The triangular fuzzy number for the volume $\underline{v}$ = (v_1, v_2, v_3), where $v_1 < v_2 < v_3$ and v_i belongs to the real numbers for i = 1, 2, 3.

Therefore, the constraints of the four benchmarks of RRAP including series system, the network with series and parallel elements, a complex (bridge) system, and the overspeed protection of a gas turbine system as shown in Equations (5) and (6), Equations (9) and (10), Equations (13) and (14), and Equations (17) and (18) are accordingly modelled to the triangular fuzzy number for the constraints of RRAP as following Equations (20)–(27). However, the components cost is not used within the cost constraints so that there is no need to convert Equations (4), (8), (12), and (16) to the fuzzy constraints.
Benchmark 1. Series system in Figure 1.

$$\text{Subject to } Cst_2(\boldsymbol{R},\ \boldsymbol{N}) = \sum_{i=1}^{N_v} \underline{\underline{w}}_i n_i \exp(n_i/4) \leq U_w \tag{20}$$

$$Cst_3(\boldsymbol{R},\ \boldsymbol{N}) = \sum_{i=1}^{N_v} \underline{\underline{w}}_i \underline{\underline{v}}_i^2 n_i^2 \leq U_v \tag{21}$$

The triangular model of RRAP fuzzy weight and volume constraints for the series system in Figure 1 are shown in above Equations (20) and (21), where $\underline{w}$ = (w_1, w_2, w_3) and $\underline{v}$ = (v_1, v_2, v_3) are the triangular fuzzy number for weight and volume, respectively.
Benchmark 2. Network with series and parallel elements in Figure 2.

$$\text{Subject to } Cst_2(\boldsymbol{R},\ \boldsymbol{N}) = \sum_{i=1}^{N_v} \underline{\underline{w}}_i n_i \exp(n_i/4) \leq U_w \tag{22}$$

$$Cst_3(\boldsymbol{R},\ \boldsymbol{N}) = \sum_{i=1}^{N_v} \underline{\underline{w}}_i \underline{\underline{v}}_i^2 n_i^2 \leq U_v \tag{23}$$

The triangular model of RRAP fuzzy weight and volume constraints for the network with series and parallel elements in Figure 2 are shown in above Equations (22) and (23), where $\underline{w}$ = (w_1, w_2, w_3) and $\underline{v}$ = (v_1, v_2, v_3) are the triangular fuzzy number for weight and volume, respectively.
Benchmark 3. Complex (bridge) system in Figure 3.

$$\text{Subject to } Cst_2(\boldsymbol{R},\ \boldsymbol{N}) = \sum_{i=1}^{N_v} \underline{\underline{w}}_i n_i \exp(n_i/4) \leq U_w \tag{24}$$

$$Cst_3(\boldsymbol{R},\ \boldsymbol{N}) = \sum_{i=1}^{N_v} \underline{\underline{w}}_i \underline{\underline{v}}_i^2 n_i^2 \leq U_v \tag{25}$$

The triangular model of RRAP fuzzy weight and volume constraints for the complex (bridge) system in Figure 3 are shown in above Equations (24) and (25), where $\underline{w} = (w_1, w_2, w_3)$ and $\underline{v} = (v_1, v_2, v_3)$ are the triangular fuzzy number for weight and volume, respectively.

Benchmark 4. The overspeed protection of a gas turbine system in Figure 4.

$$\text{Subject to} Cst_2(\boldsymbol{R},\ \boldsymbol{N}) = \sum_{i=1}^{N_v} \underline{\underline{w}}_i n_i \exp(n_i/4) \leq U_w \tag{26}$$

$$Cst_3(\boldsymbol{R},\ \boldsymbol{N}) = \sum_{i=1}^{N_v} \underline{\underline{v}}_i n_i^2 \leq U_v \tag{27}$$

The triangular model of RRAP fuzzy weight and volume constraints for the overspeed protection of a gas turbine system in Figure 4 are shown in above Equations (26) and (27), where $\underline{w} = (w_1, w_2, w_3)$ and $\underline{v} = (v_1, v_2, v_3)$ are the triangular fuzzy number for weight and volume, respectively.

3.2. Credibility Fuzzy Constraints Models of RRAP

The credibility theory first developed by Liu in 2004 [31], which was extended from the fuzzy chance-constrained programming originally developed by Liu and Iwamura in 1998 [30] and has been successfully approved its effectiveness for transferring the fuzzy constraints to crisp constraints at a confidence level, is defined as the following Equation (28):

$$\text{Subject to } Cr\{\underline{A} \mid Cst(n, \underline{A}) \leq U\} \geq p \tag{28}$$

where $Cr\{\bullet\}$ means the credibility of the event $\{\bullet\}$ and p means the confidence level.

By the credibility theory, the Equation (28) is transferred to Equation (29) called the credibility constraints.

$$(2p-1)\sum_{i=1}^{N_v} a_{i,3} n_i + 2(1-p)\sum_{i=1}^{N_v} a_{i,2} n_i \leq U \tag{29}$$

Finally, the crisp value is obtained by the credibility constraints in Equation (29), which is transferred from the fuzzy constraints from Equation (28).

According to the credibility constraints in Equation (29), the triangular fuzzy number for the constraints of RRAP as shown in Equations (20)–(27) can be modelled as the following Equations (30)–(37), where the ε and γ are the predefined confidence level.

Benchmark 1. Series system in Figure 1.

$$\text{Subject to } (2\varepsilon-1)\sum_{i=1}^{N_v} w_{i,3} n_i \exp(n_i/4) + 2(1-\varepsilon)\sum_{i=1}^{N_v} w_{i,2} n_i \exp(n_i/4) \leq U_w \tag{30}$$

$$(2\lambda-1)\sum_{i=1}^{N_v} w_{i,3} v_{i,3}^2 n_i^2 + 2(1-\lambda)\sum_{i=1}^{N_v} w_{i,2} v_{i,2}^2 n_i^2 \leq U_v \tag{31}$$

Benchmark 2. Network with series and parallel elements in Figure 2.

$$\text{Subject to } (2\varepsilon-1)\sum_{i=1}^{N_v} w_{i,3} n_i \exp(n_i/4) + 2(1-\varepsilon)\sum_{i=1}^{N_v} w_{i,2} n_i \exp(n_i/4) \leq U_w \tag{32}$$

$$(2\lambda-1)\sum_{i=1}^{N_v} w_{i,3} v_{i,3}^2 n_i^2 + 2(1-\lambda)\sum_{i=1}^{N_v} w_{i,2} v_{i,2}^2 n_i^2 \leq U_v \tag{33}$$

Benchmark 3. Complex (bridge) system in Figure 3.

$$\text{Subject to } (2\varepsilon-1)\sum_{i=1}^{N_v} w_{i,3} n_i \exp(n_i/4) + 2(1-\varepsilon)\sum_{i=1}^{N_v} w_{i,2} n_i \exp(n_i/4) \le U_w \tag{34}$$

$$(2\lambda-1)\sum_{i=1}^{N_v} w_{i,3} v_{i,3}^2 n_i^2 + 2(1-\lambda)\sum_{i=1}^{N_v} w_{i,2} v_{i,2}^2 n_i^2 \le U_v \tag{35}$$

Benchmark 4. The overspeed protection of a gas turbine system in Figure 4.

$$\text{Subject to } (2\varepsilon-1)\sum_{i=1}^{N_v} w_{i,3} n_i \exp(n_i/4) + 2(1-\varepsilon)\sum_{i=1}^{N_v} w_{i,2} n_i \exp(n_i/4) \le U_w \tag{36}$$

$$(2\lambda-1)\sum_{i=1}^{N_v} v_{i,3} n_i^2 + 2(1-\lambda)\sum_{i=1}^{N_v} v_{i,2} n_i^2 \le U_v \tag{37}$$

The above constraints in Equations (30)–(37) all are the crisp value that has been successfully transferred from the triangular fuzzy constraints models of RRAP. Therefore, this study applies the fuzzy number and credibility theory to the comprehensive consideration of uncertainty in the constraints of RRAP including cost, weight, and volume.

4. The SSO

The SSO algorithm, which was originally developed by Yeh in 2009 [35] and is a famous artificial intelligence, swarm algorithm and evolutionary algorithm, has been adopted to solve and optimize the proposed fuzzy constraints model of RRAP while aims to optimize the system reliability that the importance of enhancing system reliability as well as artificial intelligence, swarm algorithm and evolutionary algorithm can be proved by the related research by numerous literatures [38–44].

The update mechanism (UM) of SSO is shown in following Equation (38), in which C_g, C_p, and C_w are the predefined parameters. When randomly generate a random number ρ that belongs to uniform distribution [0, 1], the solution of the new generation ($x_{i,j}^{g+1}$, where g, i, and j are the gth generation, the ith solution, and the jth variable, respectively) may be equal to one of the following four solutions according to the UM in Equation (38):

1. $x_{i,j}^{g+1}$ equals to the global best (G_j) if ρ falls to [0, C_g];
2. $x_{i,j}^{g+1}$ equals to the local best ($P_{i,j}$) if ρ falls to [C_g, C_p];
3. $x_{i,j}^{g+1}$ equals to the solution of last generation ($x_{i,j}^{g}$) if ρ falls to [C_p, C_w];
4. $x_{i,j}^{g+1}$ equals to a new solution x that is randomly selected from [l_j, u_j], where l_j and u_j are the lower bound and upper bound of the jth variable, respectively.

$$x_{i,j}^{g+1} = \begin{cases} G_j & if\ \rho \in [0, C_g] \\ P_{i,j} & if\ \rho \in [C_g, C_p] \\ x_{i,j}^{g} & if\ \rho \in [C_p, C_w] \\ x & if\ \rho \in [C_w, 1] \end{cases} \tag{38}$$

Conclude the UM of SSO above, the pseudo code is shown as following steps:

Step 1.	The solution X_i is randomly generated for i = 1, 2, . . . , Nsol, the fitness function F(X_i) is found, let X_i = P_i, and find the global best G.
Step 2.	Let i = 1.
Step 3.	X_i is updated according to Equation (38) and the fitness function F(X_i) is found.
Step 4.	P_i = X_i if X_i is better than P_i. Otherwise, go to Step 6.
Step 5.	G = P_i if P_i is better than G.
Step 6.	i = i + 1 if i < Nsol and go to Step 3.
Step 7.	g = g + 1 if g < Ngen for g = 1, 2, . . . , Ngen and go to Step 2. Otherwise, halt.

5. Numerical Experiments

Modelling the fuzzy constraints including cost, weight and volume under consideration of uncertainty for the four benchmarks of RRAP aiming to maximize the system reliability, the experimental results are solved by the adopted SSO algorithm. The similar literature does not exist resulting our work cannot be compared to the other in literature. However, the performance is compared with the state-of-the-art algorithms including PSO, hybrid of PSO and SSO (named PSSO), and GA. Thus, the parameters of these algorithms are defined as following:

SSO: Cg = 0.4, Cp = 0.7, Cw = 0.9
PSO: C1 = C2 = 2, W = 0.9
GA: Cr = 0.8 (crossover rate), Cm = 0.2, elite selection

For fairness, the following conditions and data settings of the four algorithms including the adopted SSO, PSO, PSSO, and GA are consistent. While performing the experiments, the algorithms are coded using DEV C++ with 64-bit Windows 10, implemented on an Intel Core i7-6650U CPU @ 2.20 GHz 2.21-GHz notebook with 64 GB of memory. Total 1000 generations are ran and 50 solutions are obtained for each algorithm. The confidence level for the credibility constraints of weight and volume in Equations (30)–(37) has been defined as $\varepsilon = 0.95$, and also $\lambda = 0.95$.

The data, which the weight and volume of components have been converted to the triangular fuzzy number, used in the four benchmarks of RRAP is shown in following Tables 1–3.

Table 1. Data for benchmark 1 and benchmark 3 [24,34,37].

Subsystem i	$10^5\alpha_i$	β_i	$w_iv_i^2$	w_i	V	C	W
1	2.330	1.5	(0.8, 1, 1.2)	(6.5, 7, 7.5)			
2	1.450	1.5	(1.8, 2, 2.2)	(7.5, 8, 8.5)			
3	0.541	1.5	(2.8, 3, 3.2)	(7.5, 8, 8.5)	110	175	200
4	8.050	1.5	(3.8, 4, 4.2)	(5.5, 6, 6.5)			
5	1.950	1.5	(1.8, 2, 2.2)	(8.5, 9, 9.5)			

Table 2. Data for benchmark 2 [24,34,37].

Subsystem i	$10^5\alpha_i$	β_i	$w_iv_i^2$	w_i	V	C	W
1	2.500	1.5	(1.8, 2, 2.2)	(3, 3.5, 4)			
2	1.450	1.5	(3.8, 4, 4.2)	(3.5, 4.0, 4.5)			
3	0.541	1.5	(4.8, 5, 5.2)	(3.5, 4.0, 4.5)	180	175	100
4	0.541	1.5	(7.8, 8, 8.2)	(3, 3.5, 4)			
5	2.100	1.5	(3.8, 4, 4.2)	(4, 4.5, 5)			

Table 3. Data for benchmark 4 [24,34,37].

Subsystem i	$10^5\alpha_i$	β_i	v_i	w_i	V	C	W
1	1	1.5	(0.8, 1, 1.2)	(5.5, 6, 6.5)			
2	2.3	1.5	(1.8, 2, 2.2)	(5.5, 6, 6.5)	250.0	400.0	500.0
3	0.3	1.5	(2.8, 3, 3.2)	(7.5, 8, 8.5)			
4	2.3	1.5	(1.8, 2, 2.2)	(6.5, 7, 7.5)			

The system reliability and running time are obtained to compare the performance of the adopted SSO algorithm with PSO, PSSO and GA for the four benchmarks of RRAP. The statistics of experimental results including minimum (Min), maximum (Max), average, and standard deviation (STD) of the system reliability and running time and also the convergent generation number for the best solution (denoted as $\text{Gen}_{\text{convergent}}$) of the system reliability and running time are analyzed and shown in following Tables 4–11.

Table 4. The statistics of system reliability for benchmark 1.

Statistics	SSO	PSO	PSSO	GA
Max (Best)	**0.931235373**	0.862435401	0.892945886	0.747861922
$Gen_{convergent}$	498	659	**190**	999
Min (Worse)	**0.867673755**	0.661536336	0.663374484	0.036621228
Average	**0.920391358**	0.759988588	0.769599348	0.246966909
STD	**0.011218917**	0.047793320	0.049490632	0.162324076

The best solution marked in bold.

Table 5. The statistics of running time (units used: seconds) for benchmark 1.

Statistics	SSO	PSO	PSSO	GA
Min (Best)	**0.17100**	0.18700	0.17200	0.29600
$Gen_{convergent}$	797	649	**23**	999
Max (Worse)	**0.26200**	0.28400	0.26500	0.47200
Average	**0.19778**	0.21274	0.20944	0.33396
STD	0.02417	0.02284	**0.02151**	0.03690

The best solution marked in bold.

Table 6. The statistics of system reliability for benchmark 2.

Statistics	SSO	PSO	PSSO	GA
Max (Best)	**0.9999759790**	0.9998579620	0.9998130200	0.9956341980
$Gen_{convergent}$	628	576	**468**	999
Min (Worse)	**0.9998471140**	0.9960312840	0.9967661500	0.0915097220
Average	**0.9999484850**	0.9989969110	0.9990356900	0.4793065570
STD	**0.0000255121**	0.0007723344	0.0006460497	0.3281670328

The best solution marked in bold.

Table 7. The statistics of running time (units used: seconds) for benchmark 2.

Statistics	SSO	PSO	PSSO	GA
Min (Best)	**0.169000**	0.173000	0.176000	0.290000
$Gen_{convergent}$	884	545	**222**	999
Max (Worse)	**0.224000**	0.263000	0.235000	0.360000
Average	**0.193840**	0.209380	0.209820	0.327020
STD	0.014315	0.015369	**0.013011**	0.018947

The best solution marked in bold.

Table 8. The statistics of system reliability for benchmark 3.

Statistics	SSO	PSO	PSSO	GA
Max (Best)	**0.999886930**	0.999567032	0.999680340	0.999073625
$Gen_{convergent}$	**100**	376	489	999
Min (Worse)	**0.999592066**	0.994620740	0.993449271	0.055013273
Average	**0.999814268**	0.998070072	0.997827517	0.427838158
STD	**0.000064727**	0.001102467	0.001329463	0.304351767

The best solution marked in bold.

Table 9. The statistics of running time (units used: seconds) for benchmark 3.

Statistics	SSO	PSO	PSSO	GA
Min (Best)	**0.172000**	0.187000	0.182000	0.298000
$Gen_{convergent}$	954	**319**	450	999
Max (Worse)	**0.219000**	0.225000	0.225000	0.345000
Average	**0.193260**	0.203340	0.202720	0.320140
STD	0.012240	**0.010277**	0.010500	0.012249

The best solution marked in bold.

Table 10. The statistics of system reliability for benchmark 4.

Statistics	SSO	PSO	PSSO	GA
Max (Best)	**0.999953926**	0.999802172	0.999891579	0.998805583
$Gen_{convergent}$	141	821	**6**	999
Min (Worse)	**0.999552667**	0.998303175	0.998482347	0.857600927
Average	**0.999903330**	0.999337146	0.999418706	0.965427958
STD	**0.000084258**	0.000349942	0.000315562	0.029557328

The best solution marked in bold.

Table 11. The statistics of running time (units used: seconds) for benchmark 4.

Statistics	SSO	PSO	PSSO	GA
Min (Best)	**0.13900**	0.14900	0.15600	0.25800
$Gen_{convergent}$	**629**	768	644	999
Max (Worse)	**0.17400**	0.20400	0.20000	0.33200
Average	**0.15506**	0.17540	0.17604	0.27862
STD	**0.00809**	0.01009	0.00999	0.01268

The best solution marked in bold.

For the benchmark 1, the experimental results are analyzed as below and shown in Tables 4 and 5.

1. The adopted SSO has the best performance including Max, Min, Average, and STD of system reliability, i.e., **0.931235373**, **0.867673755**, **0.920391358**, and **0.011218917**.
2. The adopted SSO has the best performance including Min, Max, and Average of running time, i.e., **0.17100**, **0.26200**, and **0.19778**.
3. The PSSO has the best performance in STD of running time, i.e., **0.02151**.
4. The PSSO has the best convergent generation number of both system reliability and running time, i.e., **190**, and **23**.

For the benchmark 2, the experimental results are analyzed as below and shown in Tables 6 and 7.

1. The adopted SSO has the best performance including Max, Min, Average, and STD of system reliability, i.e., **0.9999759790**, **0.9998471140**, **0.9999484850**, and **0.0000255121**.
2. The adopted SSO has the best performance including Min, Max, and Average of running time, i.e., **0.169000**, **0.224000**, and **0.193840**.
3. The PSSO has the best performance in STD of running time, i.e., **0.013011**.
4. The PSSO has the best convergent generation number of both system reliability and running time, i.e., **468**, and **222**.

For the benchmark 3, the experimental results are analyzed as below and shown in Tables 8 and 9.

1. The adopted SSO has the best performance including Max, Min, Average, and STD of system reliability, i.e., **0.999886930**, **0.999592066**, **0.999814268**, and **0.000064727**.
2. The adopted SSO has the best performance including Min, Max, and Average of running time, i.e., **0.172000**, **0.219000**, and **0.193260**.
3. The PSO has the best performance in STD of running time, i.e., **0.010277**.
4. The SSO has the best convergent generation number of system reliability, i.e., **100**.
5. The PSO has the best convergent generation number of running time, i.e., **319**.

For the benchmark 4, the experimental results are analyzed as below and shown in Tables 10 and 11.

1. The adopted SSO has the best performance including Max, Min, Average, and STD of system reliability, i.e., **0.999953926**, **0.999552667**, **0.999903330**, and **0.000084258**.
2. The adopted SSO has the best performance including Min, Max, Average, and STD of running time, i.e., **0.13900**, **0.17400**, **0.15506**, and **0.00809**.
3. The PSSO has the best convergent generation number of system reliability, i.e., **6**.

4. The SSO has the best convergent generation number of running time, i.e., **629**.

According to the above experimental results and analysis, the adopted SSO has the best performance including Min, Max, Average, and STD of both system reliability and running time for all four benchmarks of RRAP under fuzzy constraints. The reason why PSSO fails to find the good results because it is without selecting better parameters. However, The SSO, PSO, and PSSO have equal shares on the best convergent generation number of system reliability and running time for the four benchmarks of RRAP under fuzzy constraints.

6. Conclusions

In general, few of the RRAP have noticed the uncertainty condition of components used in the systems. However, the uncertainty condition of components does exist in practice various systems so that the fuzzy theory and credibility theory used to solve the components uncertainty in the constraints of RRAP including cost, weight, and volume are investigated in this work. Therefore, this study models the fuzzy constraints of RRAP while aims to maximize the system reliability.

Moreover, the RRAP has been shown to be a NP-hard problem, thus, this study adopted SSO algorithm to solve the fuzzy constraints model of RRAP. The four famous benchmarks of RRAP including series system, the network with series and parallel elements, a complex (bridge) system, and the overspeed protection of a gas turbine system are experimented by the proposed fuzzy constraints models and solved by the adopted SSO algorithm. The experimental results show that:

For the benchmarks 1, 2 and 3, the proposed SSO has the best performance including Max, Min, Average, and STD of system reliability as well as has the best performance including Min, Max, and Average of running time.

For the benchmark 4, the proposed SSO has the best performance including Max, Min, Average, and STD of system reliability as well as has the best performance including Min, Max, Average, and STD of running time.

Thus, the experimental results show that the adopted SSO algorithm successfully solves the proposed fuzzy constraints models of RRAP and obtains feasible, effective, and the best solutions in terms of the system reliability and running time.

The proposed method is planned to implement with real world case study in the future work.

Author Contributions: Conceptualization, H.C.-S.L., C.-L.H., W.-C.Y.; methodology, H.C.-S.L.; software, H.C.-S.L.; validation, H.C.-S.L.; formal analysis, H.C.-S.L., C.-L.H., W.-C.Y.; investigation, H.C.-S.L., C.-L.H.; resources, H.C.-S.L.; data curation, H.C.-S.L.; writing—original draft preparation, H.C.-S.L., C.-L.H., W.-C.Y.; supervision, H.C.-S.L.; project administration, H.C.-S.L.; funding acquisition, H.C.-S.L. All authors have read and agreed to the published version of the manuscript.

Funding: This research was supported in part by the Ministry of Science and Technology, R.O.C (MOST 107-2221-E-007-072-MY3, MOST 110-2221-E-007-107-MY3, MOST 109-2221-E-424-002 and MOST 110-2511-H-130-002).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Acknowledgments: I wish to thank the anonymous editor and the reviewers for their constructive comments and recommendations, which have significantly improved the presentation of this paper.

Conflicts of Interest: The authors declare no conflict of interest.

Nomenclature

$R_s(\boldsymbol{R}, \boldsymbol{N})$	The system reliability
$\boldsymbol{R}$	$\boldsymbol{R} = (r_1, r_2, \ldots, r_{\mathrm{Nv}})$ is the solution vector of components reliability for each subsystem $i = 1, 2, \ldots, \mathrm{Nv}$ in the system
$\boldsymbol{N}$	$\boldsymbol{N} = (n_1, n_2, \ldots, n_{\mathrm{Nv}})$ is the solution vector of components redundancy level for each subsystem $i = 1, 2, \ldots, \mathrm{Nv}$ in the system
r_i	The components reliability used in the ith subsystem, where $i = 1, 2, \ldots, \mathrm{Nv}$
n_i	The components redundancy level used in the ith subsystem, where $i = 1, 2, \ldots, \mathrm{Nv}$
$Cst_j(\boldsymbol{R}, \boldsymbol{N})$	The jth constraint
U_j	The upper bound for the jth constraint
R_i	The reliability of the ith subsystem, where $i = 1, 2, \ldots, \mathrm{Nv}$
c_i, w_i, v_i	The cost, weight, and volume of components used in the ith subsystem, where $i = 1, 2, \ldots, \mathrm{Nv}$
U_c, U_w, U_v	The upper bound for the constraints of cost, weight, and volume, respectively
α_i, β_i	The parameters of the ith subsystem, where $i = 1, 2, \ldots, \mathrm{Nv}$

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