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Nonlinear Vibration Isolation via a NiTiNOL Wire Rope

Mu-Qing Niu  and Li-Qun Chen *

School of Science, Harbin Institute of Technology, Shenzhen 518055, China; niumuqing@hit.edu.cn

* Correspondence: chenliqun@hit.edu.cn

Featured Application: The vibration isolations of precision processing equipment, measuring instruments and spacecraft payloads are potential applications of the work.

Abstract: Vibration isolators with both stiffness and damping nonlinearities show promise for exhibiting compound advantages for broadband vibration isolation. A nonlinear isolator with a NiTiNOL wire rope is proposed with cubic stiffness, hysteretic damping, and pinching effects induced by geometric constraints, inner frictions, and phase transitions, respectively. A combined method of a beam constraint model and a Bouc-Wen model is presented to characterize the restoring force of the NiTiNOL wire rope. The frequency responses of the nonlinear isolator were analyzed through a harmonic balance method with an alternating frequency/time domain technique. The generalized equivalent stiffness and the generalized equivalent damping ratio were defined for a comprehensive understanding of the nonlinear characteristics. The isolator exhibited a stiffness-softening-hardening characteristic. The pinching effect, the Bouc-Wen hysteresis, and the cubic stiffness mainly influenced the equivalent stiffness at the initial value, the small displacements, and the large displacements, respectively. The rate-independent damping ratio increased and then decreased with increasing displacement, and the parameters influenced the damping ratio change in different ways. Compared to an isolator with a steel wire rope, the isolator with a NiTiNOL wire rope exhibited less initial stiffness and a stronger damping effect, and thus, better vibration isolation performance. The relationships of the peak displacement transmissibility and the resonant frequency with the excitation amplitude were both non-monotonic due to the non-monotonic changes of the stiffness and the damping ratio. The minimum peak transmissibility, the lowest resonant frequency, and their corresponding excitation amplitudes depended on the isolator parameters. The isolator demonstrated stiffness-softening and stiffness-hardening types of jump phenomena with different parameters.



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1. Introduction

Vibration isolators are widely used in mechanical engineering, civil engineering, and aerospace technology. Nonlinear vibration isolators [1–3] are superior to linear vibration isolators, with better broadband isolation performance. The stiffness nonlinearity endows the isolator with a low-frequency vibration isolation ability without weakening the load capacity. Damping nonlinearity is beneficial for suppressing the resonance without degrading the high-frequency isolation efficiency. Isolators with both stiffness and damping nonlinearities show promise for presenting compound advantages.

Nonlinear stiffness can be accomplished with linear springs with different orientations [4–6], buckling beams [7–9], spring-roller-cam mechanisms [10–12], and magnetisms [13–15]. Nonlinear damping can be induced by geometric constraints [16–20], frictions [21,22], materials with intrinsic nonlinearities [23,24], and eddy current effects [25,26]. Vibration isolators with both stiffness and damping nonlinearities have been proposed in recent years. Lu et al. [16] proposed an isolator with nonlinear stiffness and damping derived from horizontal linear springs and dampers, respectively. Cheng et al. [17] proposed

a quasi-zero-stiffness isolator with nonlinear damping, and the stiffness and damping nonlinearities resulted from geometric constraints. Liu et al. [18] investigated a quasi-zero-stiffness isolator with a spring-roller-cam mechanism integrated with nonlinear damping accomplished with two horizontal dampers. Bian et al. [19] studied the nonlinear damping characteristics of the limb-like structured or X-structured isolators. Yan et al. [20] proposed a quasi-zero-stiffness isolator with nonlinear damping using a three-link mechanism. In these isolators, the damping nonlinearity is derived from deliberately designed geometric constraints, and thus, the isolator structures are complicated. In addition, the damping is viscous in most cases, and there is a lack of research regarding the nonlinear stiffness isolator enhanced with non-viscous damping.

Generally, the nonlinear damping force is approximated as an explicit function of the velocity and displacement for dynamic analysis. Huang et al. [27] investigated a vibration isolation system with real-power nonlinearities in restoring and damping forces through a multiple scales method. Ho et al. [28] studied a vibration isolator with high-order stiffness and cubic damping via an output frequency response function approach. Liu et al. [29] investigated the effects of the hardening and softening of the damping on a quasi-zero-stiffness vibration isolator with an averaging method. Donmez et al. [22] performed a harmonic balance analysis on a quasi-zero-stiffness isolation system with dry friction damping, expressed as a piecewise function. However, some hysteretic damping induced by frictions and phase-change materials is history-dependent and characterized by phenomenological models, such as the Bouc-Wen model [30,31]. The Bouc-Wen model expression is both piecewise and implicit. There are two major approaches to dealing with the dynamic system with the Bouc-Wen model. The first approach is to take a piecewise fitting on the Bouc-Wen-type hysteretic force, and thus, the implicit expression can be transformed into a piecewise explicit expression. Okuizumi et al. [30] and Casalotti et al. [31] performed multiple scales analyses on hysteretic systems with piecewise fittings on the Bouc-Wen model, respectively. The second approach is to adopt an alternating frequency/time domain technique [32], and the implicit function is numerically solved in the time domain. This study was focused on a modified Bouc-Wen model. The expression is more complicated than the classic Bouc-Wen model, and it is difficult to perform a piecewise fitting on the hysteretic force. The alternating frequency/time domain technique is more suitable and its effectiveness for the modified Bouc-Wen model remains to be verified.

Wire ropes produce nonlinear damping due to inner frictions, and the damping is rate-independent. The wire ropes have been adopted in nonlinear dampers. Barbieri et al. [33] studied an asymmetric Stockbridge damper and the steel wire ropes are utilized as cantilever beams. Carpineto et al. [34] proposed a tuned mass damper using steel wire ropes with both ends clamped, and the nonlinear damping force was characterized with a Bouc-Wen model. Compared to steel wire ropes, NiTiNOL wire ropes are more efficient in energy dissipation due to the phase transitions of the material. Carboni et al. [35,36] proposed a vibration absorber accomplished with NiTiNOL-steel wire rope assemblies and revealed a pinching effect derived from NiTiNOL. Zhang et al. [37] integrated a NiTiNOL wire rope with a nonlinear energy sink to improve energy dissipation. Zheng et al. [38] adopted a NiTiNOL wire rope for structural vibration suppression. With the wire ropes styled as a ring, the load capacity was enhanced and showed promise for vibration isolation. Leblouba et al. [39] studied the shear cyclic behavior of polycal wire rope isolators. Lu et al. [40] attached a NiTiNOL wire rope to a ring-type isolator and enhanced the isolation performances. Salvatore et al. [41] investigated a compact nonlinear isolator with initially curved wire ropes. The ring-type or semi-ring-type isolator presented an asymmetric restoring force, and it was not conducive to vibration isolation. We [42] have proposed a nonlinear isolator with compliant mechanisms and straight steel wire ropes with a linear spring enhancing the load capacity and verified the superiority of the symmetric restoring force for vibration isolation. An isolator with straight NiTiNOL wire ropes showed promise for exhibiting a symmetric restoring force and stronger damping effects. The pinching effect of NiTiNOL endowed the isolator with more nonlinear behaviors. It is essential to

perform a comprehensive comparison of the nonlinear characteristics and the isolation performances between the isolators with steel wire ropes and those with NiTiNOL wire ropes. Furthermore, the stiffness and damping of the isolators with wire ropes are not only nonlinear but also non-monotonically varying. This leads to the unique issue of “the most suitable excitation amplitude” for the isolators. The aim of this research was to reveal the parameter influences on the most suitable excitation amplitude.

In this paper, we propose an isolator with a NiTiNOL wire rope with both ends and the middle part clamped. The isolator exhibited nonlinear stiffness and nonlinear damping due to geometric constraints, frictions, and phase transitions of the material. The isolator was modeled with a modified Bouc-Wen model and analyzed with a harmonic balance method. A comparison was made between isolators with steel and those with NiTiNOL wire rope. The most suitable excitation amplitude and the parameter influences were revealed. The paper is organized as follows. The nonlinear isolator and its dynamic model are introduced in Section 2. The harmonic balance analysis and the stiffness and damping ratio analysis are presented in Section 3. The numerical simulation results are presented in Section 4 with further discussion. Conclusions are drawn in Section 5.

2. Models

Figure 1 shows a vibration isolator with a NiTiNOL wire rope. The wire rope is horizontally suspended in tension. Both ends of the wire rope are fixed on the base with two clamping blocks, and the middle of the wire rope is fixed on a guiding block. A guiding rod is assembled on the base with screw threads, and thus, its height can be adjusted. The guiding block and the guiding rod are assembled with a linear bearing, and thus, the guiding block can move only vertically. A linear spring is assembled between the guiding block and the guiding rod. The moving part of the system includes the guiding block, and the middle clamping block and the payload are fixed to it. At the initial state, the gravity of the moving part is balanced by the linear spring, and the wire rope is horizontal. A vertical displacement excitation Y_b is applied to the base, and the vertical responding displacement of the payload is Y_p . The relative displacement is $Y = Y_p - Y_b$.

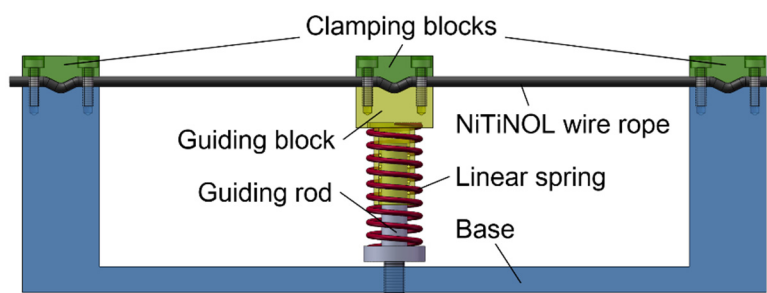


Figure 1. Vibration isolator with a NiTiNOL wire rope.

The restoring force of the wire rope was modeled with a combined method. The wire rope was cut from the middle and divided into two segments. Firstly, each segment was regarded as a cantilever beam fixed on the base with a constant section, and the inner frictions and the phase transitions of NiTiNOL were ignored. Thus, this part of the restoring force is due to the linear elastic deformation with geometric constraints, and it was modeled with a physical method. The length, the area of the section, the moment of inertia, and Young’s modulus of the beam are denoted as L , A , I , and E , respectively. The free end of the beam is attached to the payload and the external loads applied to the free end are denoted as F_{wx} , F_{wy} , and M_{wz} , respectively. The elastic deformations of the free end are denoted as X_w , Y_w , and θ_w , respectively. The free end is fixed on the payload, and thus, the rotation angle $\theta_w \equiv 0$.

At the initial state ($Y_w = 0$), the beam is horizontal with an axial force F_{wx0} . The restoring force due to the linear elastic deformation was modeled based on a beam con-

straint model [43]. More details can be found in our previous research [42], and the vertical restoring force is expressed as

$$F_{wy} = \left(\frac{12EI}{L^3} + \frac{6F_{wx0}}{5L} - \frac{F_{wx0}^2 L}{700EI} \right) Y_w + \left(\frac{18EA}{25L^3} - \frac{3F_{wx0}A}{1750IL} \right) Y_w^3 \quad (1)$$

Secondly, the inner frictions and the phase transitions of NiTiNOL were modeled with a phenomenological method. The hysteretic effect was characterized by a Bouc-Wen model, and an exponential function was added to describe the pinching effect [35]. This part of the restoring force is denoted as Z_w and is expressed as

$$Z'_w = \left\{ K_{wd} \left(1 - \xi e^{-\frac{Y_w^2}{Y_b}} \right) - [G_w + B_w \operatorname{sgn}(Y'_w Z_w)] |Z_w|^{n_b} \right\} Y'_w \quad (2)$$

where K_{wd} , G_w , B_w , and n_b are the parameters of the Bouc-Wen model, and ξ and Y_b are the parameters of the pinching effect function. The symbol (') denotes the derivative with respect to time. The total restoring force of the wire rope is the summation of the two parts and

$$F_r = 2F_{wy} + 2Z_w = K_{w1} Y_w + K_{w3} Y_w^3 + Z \quad (3)$$

$$Z' = \left\{ K_d \left(1 - \xi e^{-\frac{Y_w^2}{Y_b}} \right) - [G + B \operatorname{sgn}(Y'_w Z)] |Z|^{n_b} \right\} Y'_w \quad (4)$$

where

$$\begin{cases} K_{w1} = \frac{24EI}{L^3} + \frac{12F_{wx0}}{5L} - \frac{F_{wx0}^2 L}{350EI}, & K_{w3} = \frac{36EA}{25L^3} - \frac{3F_{wx0}A}{875IL}, \\ K_d = 2K_{wd}, & G = 2^{1-n_b} G_w, \quad B = 2^{1-n_b} B_w \end{cases}$$

With the model of the wire rope, the dynamic equation of the isolator shown in Figure 1 can be established. Assume that the excitation on the base is harmonic and

$$Y_b = A_b \cos(\omega_b T) \quad (5)$$

where A_b , ω_b , and T are the amplitude, the angular frequency, and the time, respectively. The mass of the payload and the stiffness of the linear spring are denoted as M and K_s , respectively. The dynamic equation of the isolator can be expressed as

$$MY'' + (K_s + K_{w1})Y + K_{w3}Y^3 + Z = MA_b\omega_b^2 \cos(\omega_b T) \quad (6)$$

where Z is expressed in Equation (4) and $Y_w = Y$. The introduction of the dimensionless parameters leads to

$$\eta^2 \ddot{y} + k_1 y + k_3 y^3 + \delta z = \eta^2 \cos t \quad (7)$$

$$\dot{z} = \left\{ 1 - \xi e^{-\frac{y^2}{y_b}} - [\gamma + \beta \operatorname{sgn}(\dot{y} z)] |z|^{n_b} \right\} \dot{y} \quad (8)$$

where

$$\begin{cases} K_c = K_s + K_{w1} + K_d & w_c = \sqrt{\frac{K_c}{M}}, \quad \eta = \frac{w_b}{w_c}, \quad t = w_b T, \quad y = \frac{Y}{A_b}, \quad k_1 = \frac{K_s + K_{w1}}{K_c} \\ k_3 = \frac{K_{w3} A_b^2}{K_c}, \quad z = \frac{Z}{K_d A_b}, \quad \delta = \frac{K_d}{K_c}, \quad y_b = \frac{Y_b}{A_b}, \quad \gamma = G A_b^{n_b} K_d^{n_b-1}, \quad \beta = B A_b^{n_b} K_d^{n_b-1} \end{cases} \quad (9)$$

and K_c and ω_c are the characteristic stiffness and the characteristic frequency, respectively. The dot and double-dot symbols on the parameters denote the derivative and the second-order derivative with respect to the dimensionless time, respectively. The amplitudes of the absolute and relative displacements of the payload are denoted as A_p and A , respectively, and their dimensionless forms are $a_p = A_p/A_b$ and $a = A/A_b$, respectively. Therefore, the displacement transmissibility of the isolator is $T_D \equiv a_p$.

3. Methods

3.1. Harmonic Balance Analysis

A harmonic balance method was adopted to solve Equations (7) and (8). The restoring force of the wire rope, as shown in Equation (3), is an odd function, and thus, there are only odd harmonics in the responses. Only the first- and third-order harmonics were considered for approximate analysis. The amplitudes of higher-order harmonics are less than 1/10 of the first- and third-order harmonics, provided that the responding amplitude is not so high ($a < 3$). Assume that

$$y = a_1 \cos t + b_1 \sin t + a_3 \cos 3t + b_3 \sin 3t \quad (10)$$

$$z = c_1 \cos t + d_1 \sin t + c_3 \cos 3t + d_3 \sin 3t \quad (11)$$

where $a_1, b_1, a_3, b_3, c_1, d_1, c_3$, and d_3 are the coefficients. The Fourier expansion of y^3 can be obtained from a recursive method introduced in our previous research [42], and

$$y^3 = p_1 \cos t + q_1 \sin t + p_3 \cos 3t + q_3 \sin 3t \quad (12)$$

where

$$\begin{cases} p_1 = \frac{3}{4}(a_1^3 + a_1^2 a_3 + 2a_1 a_3^2 + a_1 b_1^2 + 2a_1 b_3^2 - a_3 b_1^2 + 2a_1 b_1 b_3) \\ q_1 = \frac{3}{4}(b_1^3 - b_1^2 b_3 + 2b_1 b_3^2 + b_1 a_1^2 + 2b_1 a_3^2 + b_3 a_1^2 - 2b_1 a_1 a_3) \\ p_3 = \frac{1}{4}(a_1^3 + 3a_3^3 + 6a_1^2 a_3 - 3a_1 b_1^2 + 6a_3 b_1^2 + 3a_3 b_3^2) \\ q_3 = \frac{1}{4}(-b_1^3 + 3b_3^3 + 6b_1^2 b_3 + 3b_1 a_1^2 + 6b_3 a_1^2 + 3b_3 a_3^2) \end{cases} \quad (13)$$

According to Equation (7), there are

$$\begin{cases} c_1 = \frac{1}{\delta}[(1 + a_1)\eta^2 - k_1 a_1 - k_3 p_1] \\ d_1 = \frac{1}{\delta}(b_1 \eta^2 - k_1 b_1 - k_3 q_1) \\ c_3 = \frac{1}{\delta}(9a_3 \eta^2 - k_1 a_3 - k_3 p_3) \\ d_3 = \frac{1}{\delta}(9b_3 \eta^2 - k_1 b_3 - k_3 q_3) \end{cases} \quad (14)$$

Therefore, when the parameters (a_1, b_1, a_3, b_3, η) are determined, z can be calculated. Define a residual r , according to Equation (8), as

$$r = \dot{z} - \left\{ 1 - \zeta e^{-\frac{y^2}{y_b}} - [\gamma + \beta \operatorname{sgn}(\dot{y}z)] |z|^{n_b} \right\} \dot{y} \quad (15)$$

and its Fourier expansion is expressed as

$$r = u_1 \cos t + v_1 \sin t + u_3 \cos 3t + v_3 \sin 3t \quad (16)$$

An alternating frequency/time domain method [41] was adopted to calculate the parameters u_1, v_1, u_3 , and v_3 . A discrete time sequence from $t = 0$ to $t = 2\pi$ was selected, and $y(t)$, $z(t)$, and $r(t)$ can be calculated as discrete sequences based on Equations (10), (11), and (15), respectively. The parameters (u_1, v_1, u_3, v_3) were obtained by applying the Fourier transform to $r(t)$. If the determined parameters (a_1, b_1, a_3, b_3, η) are the exact solutions of Equations (7) and (8), the parameters (u_1, v_1, u_3, v_3) are all 0. Thus, solving the equations is equivalent to the least-mean-square problem of searching (a_1, b_1, a_3, b_3, η) to achieve the minimum value of $\sqrt{u_1^2 + v_1^2 + u_3^2 + v_3^2}$. An arc-length continuation method [44] was adopted to solve the problem. The absolute displacement of the payload was calculated as

$$y_p = (a_1 + 1) \cos t + b_1 \sin t + a_3 \cos 3t + b_3 \sin 3t \quad (17)$$

Therefore, the frequency responses of $a_p(T_D)$ could be acquired.

3.2. Stiffness and Damping Ratio Analysis

For a comprehensive understanding of the stiffness and damping nonlinearities of the isolator, the generalized equivalent stiffness and a generalized equivalent damping ratio were defined. According to the characteristics of the Bouc-Wen hysteresis, the area of the hysteretic loop is related to β/γ , and there is no hysteresis with $\beta = 0$. Therefore, an anhysteretic form of Equation (8) is defined as

$$\dot{z}_{an} = \left[1 - \xi e^{-\frac{y^2}{y_b^2}} - (\gamma + \beta)|z_{an}|^{n_b} \right] \dot{y} \quad (18)$$

The anhysteretic force z_{an} can be calculated through numerical integration of Equation (18), and it is a single-valued function characterizing the stiffness nonlinearity induced by both the Bouc-Wen hysteresis and the pinching effect. The total anhysteretic restoring force of the isolator is

$$f_{an} = k_1 y + k_3 y^3 + \delta z_{an} \quad (19)$$

The generalized equivalent stiffness is

$$k_e = \frac{df_{an}}{dy} = k_1 + 3k_3 y^2 + \delta \frac{dz_{an}}{dy} \quad (20)$$

The first two terms on the right-hand side indicate the traditional equivalent stiffness due to elastic deformations of the wire rope and the linear spring. The third term indicates the influences of the inner frictions and the phase transitions of the wire rope on the stiffness. Thus, the stiffness is defined as the generalized equivalent stiffness. The dimensional generalized equivalent stiffness is $K_e = k_e K_c$.

The generalized equivalent damping ratio is defined by the ratio of the dissipated energy and stored energy. The dimensionless restoring force is expressed as

$$f_r = k_1 y + k_3 y^3 + \delta z \quad (21)$$

and the dimensional restoring force is $F_r = f_r K_c / A_b$. The dissipated energy in one period is

$$E_d = \int_{-a}^a f_r dy - \int_a^{-a} f_r dy \quad (22)$$

The stored energy is derived from the anhysteretic restoring force, and the maximum stored energy in a period is

$$E_s = \int_0^a f_{an} dy \quad (23)$$

The generalized equivalent damping ratio is given as

$$\zeta_e = \frac{E_d}{4\pi E_s} \quad (24)$$

It should be noted that the generalized equivalent damping ratio is different from the traditional damping ratio in two aspects. Firstly, the stored energy is calculated from the anhysteretic restoring force. Thus, it not only contains the elastic stored energy, but it is also influenced by the inner frictions and the phase transitions. Secondly, the damping effects induced by the inner frictions and the phase transitions are independent of the frequency. Therefore, there is no η in the denominator of Equation (24).

4. Results and Discussion

4.1. Characteristic Indicators

Numerical simulations were performed to study the characteristic indicators of the isolator, including the restoring force, the generalized equivalent stiffness, and the generalized equivalent damping ratio. The parameters of the wire ropes were derived from the identification results in [45], using a NiTiNOL wire rope and a steel wire rope for comparison. Table 1 shows the dimensional parameters of the isolators.

Table 1. Dimensional parameters of the isolators with a NiTiNOL wire rope and a steel wire rope.

Material	K_s (N/mm)	K_{w1} (N/mm)	K_{w3} (N/mm ³)	K_d (N/mm)	ξ	Y_b (mm ²)	G (mm ⁻¹)	B (mm ⁻¹)	n_b
NiTiNOL	2.26	2.17	0.53	217	0.9	215.57	0.591	0.501	1
Steel	2.26	5.66	0.53	21	0	/	0.500	0.544	1

The restoring forces of the isolators with different materials of the wire rope are shown in Figure 2. Compared to the steel wire rope, the NiTiNOL wire rope demonstrated a larger hysteric loop, indicating a stronger energy dissipation ability. This is because NiTiNOL dissipates more energy due to the phase transition. Furthermore, the NiTiNOL wire rope presented a flatter curve around the balance location ($Y = 0$). This phenomenon was a result of the pinching effect.

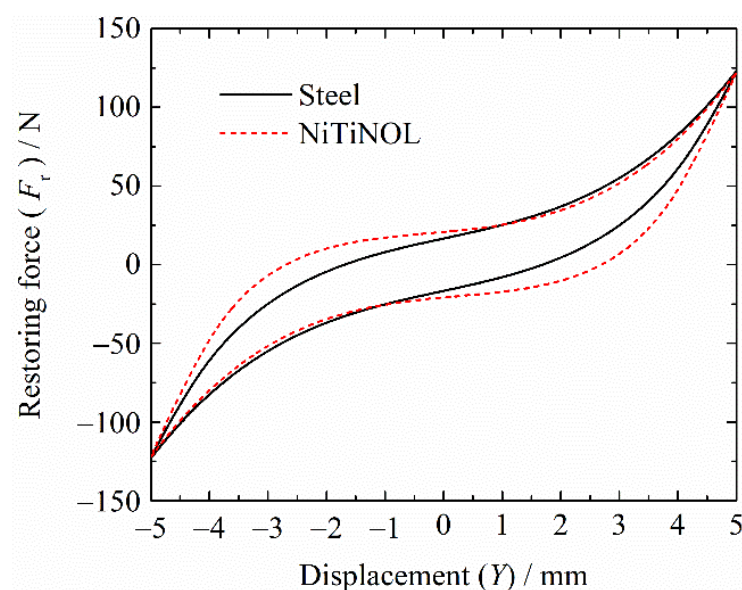


Figure 2. Restoring forces of the isolators with a steel wire rope and a NiTiNOL wire rope.

For further comparison, the generalized equivalent stiffness and damping ratios of the isolators with steel and NiTiNOL wire ropes were calculated through Equations (20) and (24). As shown in Figure 3a, the initial equivalent stiffness at $Y = 0$ of the NiTiNOL wire rope was much lower than that of the steel wire rope due to the pinching effect. As shown in Figure 3b, the equivalent damping ratio of the NiTiNOL wire rope was higher than that of the steel wire rope due to the energy dissipation of the phase transition.

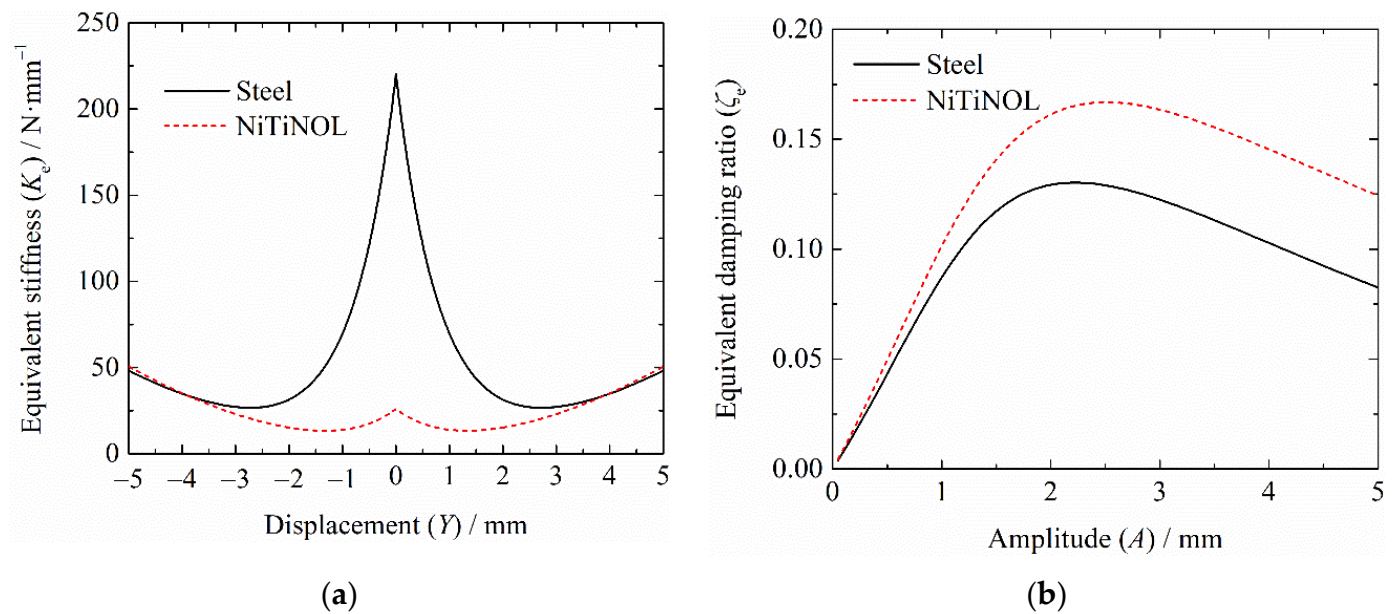


Figure 3. The characteristic indicators of the isolators with a steel wire rope and a NiTiNOL wire rope. (a) The generalized equivalent stiffness and (b) the generalized equivalent damping ratio.

The influences of the parameters on the characteristic indicators of the isolator with a NiTiNOL wire rope were studied. The dimensional generalized equivalent stiffness of the isolator is presented in Figure 4. The black solid lines show the results calculated from the parameters shown in Table 1. The equivalent stiffness was symmetric with the balance location. There was a positive initial stiffness induced by the Bouc-Wen hysteresis. The stiffness decreased dramatically and then increased with increasing displacement. Thus, the isolator exhibited a stiffness-softening-hardening characteristic. As shown in Figure 4a, with higher cubic stiffness, the equivalent stiffness increased more dramatically with large displacements. It should be noted that, with $k_3 = 0$, there was still a weak stiffness-hardening characteristic due to the pinching effect, as shown with the red dashed line. As shown in Figure 4b, the Bouc-Wen model parameter $\beta + \gamma$ mainly influenced the stiffness-softening characteristic with small displacements. The stiffness softening was more dramatic with a higher $\beta + \gamma$ value. With $\beta + \gamma = 0$, the isolator exhibited a stiffness-hardening characteristic at all displacements. The generalized equivalent stiffness was independent of the individual values of β and γ , provided that $\beta + \gamma$ was the same. As shown in Figure 4c, the pinching parameter ξ mainly influenced the initial stiffness. With a lower ξ value, the initial stiffness was higher and the stiffness softening was more dramatic. With $\xi = 1$, the initial stiffness was equal to $k_1 K_c$, and it was independent with the Bouc-Wen model. As shown in Figure 4d, the pinching parameter y_b influenced the stiffness hardening induced by the pinching effect. With a lower y_b value, this part of the stiffness-hardening effect was stronger.

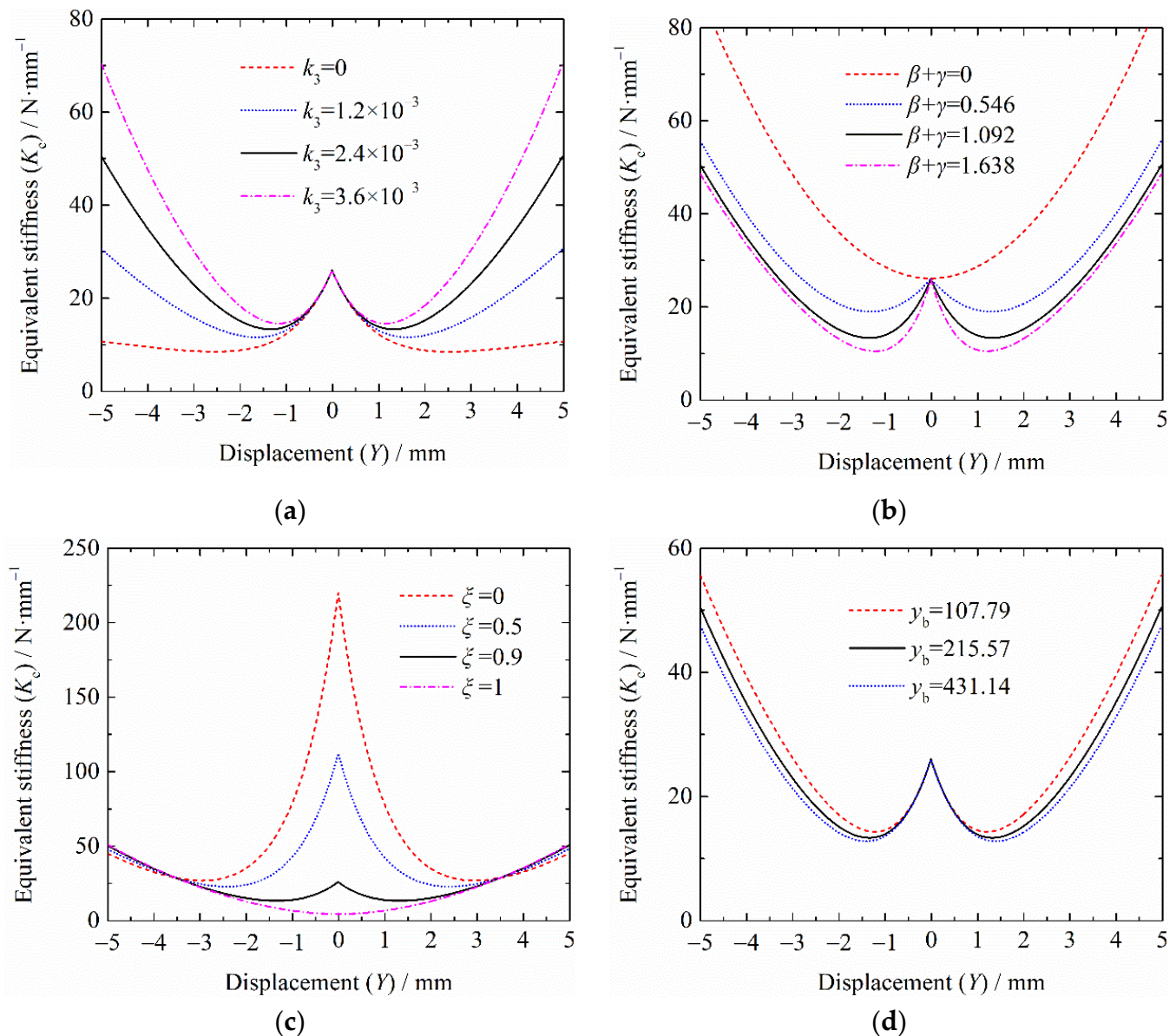


Figure 4. The generalized equivalent stiffness of the isolator with different (a) cubic stiffness k_3 , (b) Bouc-Wen model parameters $(\beta + \gamma)$, (c) pinching parameters ζ , and (d) pinching parameters y_b .

The generalized equivalent damping ratios of the isolator were calculated with Equation (24) and are shown in Figure 5. The black solid lines show the results calculated with the parameters shown in Table 1. The generalized equivalent damping ratio was independent of the frequency, and it increased and then decreased with an increasing vibration amplitude. As shown in Figure 5a, with a constant $\beta + \gamma$ value, a higher β value led to a higher damping ratio, and a lower amplitude occurred at the peak damping ratio. As shown in Figure 5b, with a constant β/γ , a higher $\beta + \gamma$ value led to a higher peak damping ratio and a lower corresponding amplitude. As shown in Figure 5c, with a higher ζ value, the damping ratio was smaller. As shown in Figure 5d, with a higher y_b value, the damping ratio was smaller.

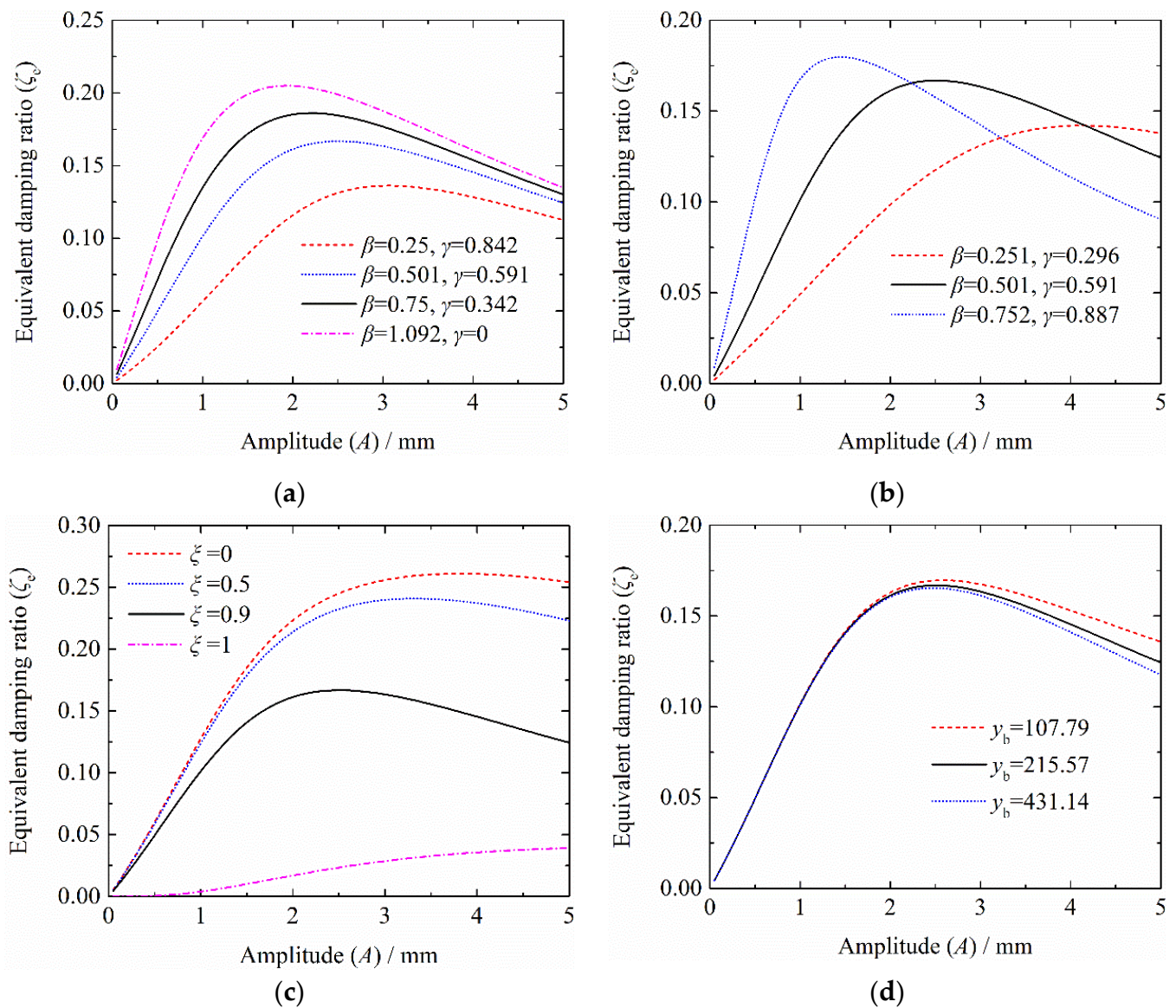


Figure 5. The generalized equivalent damping ratio of the isolator with different (a) Bouc-Wen model parameters (β/γ), (b) Bouc-Wen model parameters ($\beta + \gamma$), (c) pinching parameters ζ , and (d) pinching parameters y_b .

4.2. Frequency Responses

The frequency responses of the displacement transmissibility (T_D) were studied to demonstrate the overall isolation performance of the isolators. The peak transmissibility (dimensionless resonant amplitude) and the resonant frequency were calculated. Generally, lower peak transmissibility indicates a stronger damping effect, and a lower resonant frequency indicates a stronger low-frequency vibration isolation ability.

The frequency responses of the isolators with the NiTiNOL and the steel wire ropes were obtained from the harmonic balance analysis introduced in Section 3.1. Figure 6 shows the frequency responses of the isolators with the parameters shown in Table 1. The responding displacement transmissibility with different excitation amplitudes is shown in Figure 6a. With $A_b = 0.7$ mm, both isolators demonstrated frequency responses with resonant peaks. The peak transmissibility and the resonant frequency of the isolator with the NiTiNOL wire rope were lower, indicating a better isolation performance. With $A_b = 1$ mm, the isolator with the NiTiNOL wire rope presented a resonant peak, while the isolator with the steel wire rope exhibited an unbounded response. This is a unique

phenomenon for nonlinear isolators, and this is because the damping produced by the steel wire rope was not enough to suppress the resonance.

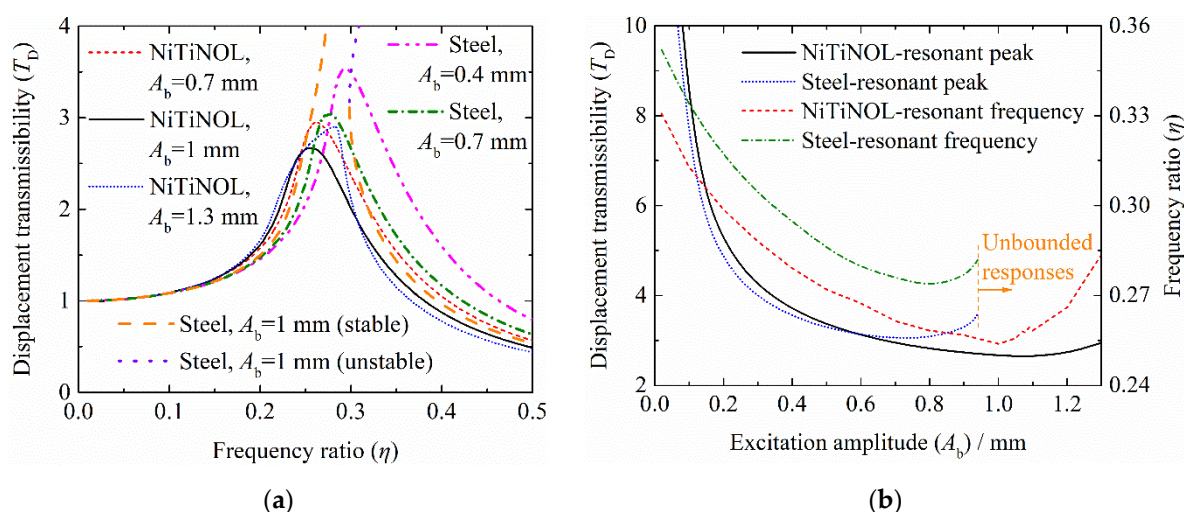


Figure 6. The frequency responses of the isolators. (a) The displacement transmissibility with different excitation amplitudes, and (b) the relationships among the peak transmissibility, the resonant frequency, and the excitation amplitude.

The responses of the isolator with the NiTiNOL wire rope with different excitation amplitudes were compared. With $A_b = 1$ mm, the resonant frequency and the peak transmissibility are the lowest. This indicates that the relationships among the peak transmissibility, the resonant frequency, and the excitation amplitude are not monotonic, as shown in Figure 6b. With an increasing excitation amplitude, both the peak transmissibility and the resonant frequency decreased and then increased. With $A_{b-pmin} = 1.08$ mm, the peak transmissibility reached the lowest value ($T_{pmin} = 2.65$), and thus, A_{b-pmin} was identified as the most suitable excitation amplitude of the isolator for suppressing the resonance. With $A_{b-rmin} = 1$ mm, the resonant frequency reached the lowest value ($\eta_{rmin} = 0.25$), and thus, A_{b-rmin} was identified as the most suitable excitation amplitude of the isolator for low-frequency vibration. The isolator with the steel wire rope also demonstrated non-monotonic relationships among the peak transmissibility, the resonant frequency, and the excitation amplitude, as shown in Figure 6b. The non-monotonic relationships were due to the non-monotonic equivalent stiffness and damping ratio of the isolators, as shown in Figure 3.

The frequency responses of the isolator with the NiTiNOL wire rope with $A_b = 1$ mm and different isolator parameters were studied. The frequency responses were also calculated from direct integration of Equations (7) and (8) based on the Runge–Kutta method. The results calculated from the harmonic balance analysis (denoted as “HBM results” in the following figures) coincide well with the Runge–Kutta method, based on the numerical results (denoted as “RK results” in the following figures). This verifies the effectiveness of the alternating frequency/time domain technique in dealing with the modified Bouc–Wen model, and that the third-order assumption of Equation (10) is reasonable.

The frequency responses with different values of β/γ are shown in Figure 7. As discussed in Section 4.1, β/γ influenced the equivalent damping ratio, while it did not affect the equivalent stiffness. Thus, with a higher β/γ value, the peak transmissibility was lower, while the resonant frequency change was not obvious. Table 2 shows the lowest peak transmissibility T_{pmin} , the lowest resonant frequency η_{rmin} , and the corresponding excitation amplitudes, A_{b-pmin} and A_{b-rmin} . Higher β/γ values led to lower T_{pmin} , higher A_{b-pmin} , and higher A_{b-rmin} values, while the η_{rmin} value stayed approximately the same.

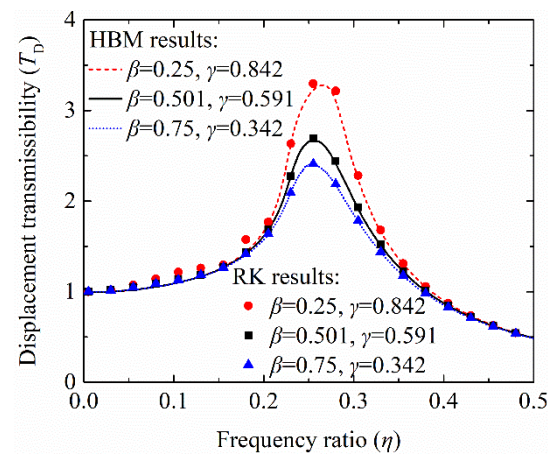


Figure 7. The frequency responses with $A_b = 1$ mm and different values of β/γ .

Table 2. The lowest peak transmissibility/resonant frequency and the corresponding excitation amplitudes with different values of β/γ .

	T_{pmin}	A_{b-pmin}	η_{rmin}	A_{b-rmin}
$\beta = 0.25, \gamma = 0.842$	3.29	1.00 mm	0.26	0.99 mm
$\beta = 0.501, \gamma = 0.591$	2.65	1.08 mm	0.25	1.00 mm
$\beta = 0.75, \gamma = 0.342$	2.37	1.16 mm	0.25	1.16 mm

The frequency responses of the isolator with $A_b = 1$ mm and different values of $\beta + \gamma$ are presented in Figure 8. With higher $\beta + \gamma$ values, the peak transmissibility was lower due to a higher equivalent damping ratio, as shown in Figure 5b. The resonant frequency was lower due to the lower equivalent stiffness, as shown in Figure 4b. As shown in Table 3, higher $\beta + \gamma$ values led to lower T_{pmin} , lower A_{b-pmin} , and lower η_{rmin} values, while the relationship between A_{b-rmin} and $\beta + \gamma$ was not monotonic.

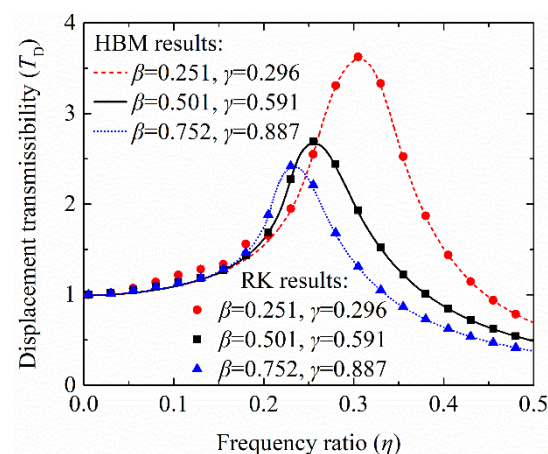


Figure 8. The frequency responses of the isolator with different values of $\beta + \gamma$.

Table 3. The lowest peak transmissibility/resonant frequency and the corresponding excitation amplitudes with different values of $\beta + \gamma$.

	T_{pmin}	A_{b-pmin}	η_{rmin}	A_{b-rmin}
$\beta = 0.251, \gamma = 0.296$	3.48	1.22 mm	0.30	0.82 mm
$\beta = 0.501, \gamma = 0.591$	2.65	1.08 mm	0.25	1.00 mm
$\beta = 0.752, \gamma = 0.887$	2.39	0.92 mm	0.23	0.92 mm

The frequency responses of the isolator with $A_b = 1$ mm and different values of ζ are shown in Figure 9. With higher ζ values, the peak transmissibility was higher due to a smaller equivalent damping ratio, as shown in Figure 5c. The resonant frequency was lower due to the lower equivalent stiffness, as shown in Figure 4c. As shown in Table 4, higher ζ values led to higher T_{pmin} , lower A_{b-pmin} , lower η_{rmin} , and lower A_{b-rmin} values.

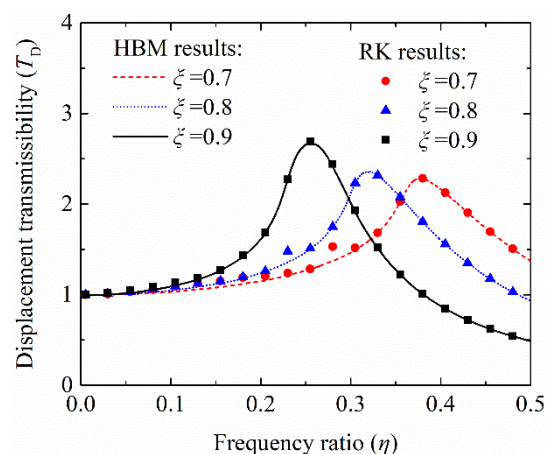


Figure 9. The frequency responses of the isolator with different values of ζ .

Table 4. The lowest peak transmissibility/resonant frequency and the corresponding excitation amplitudes with different values of ζ .

	T_{pmin}	A_{b-pmin}	η_{rmin}	A_{b-rmin}
$\zeta = 0.7$	1.81	2.20 mm	0.33	2.10 mm
$\zeta = 0.8$	2.04	1.74 mm	0.30	1.72 mm
$\zeta = 0.9$	2.65	1.08 mm	0.25	1.00 mm

The influences of the linear spring stiffness (K_s) on the responses were equivalent to the influences of the dimensionless parameter k_1 . The frequency responses of the isolator with $A_b = 1$ mm and different values of k_1 are presented in Figure 10, including $k_1 = 0.02$ ($K_s = 2.26$ N/mm), $k_1 = 0.025$ ($K_s = 13.33$ N/mm), and $k_1 = 0.03$ ($K_s = 24.40$ N/mm). With higher k_1 (K_s) values, both the peak transmissibility and the resonant frequency became higher, while the load capacity of the isolator was enhanced. As shown in Table 5, higher k_1 (K_s) values led to higher T_{pmin} , lower A_{b-pmin} , higher η_{rmin} , and lower A_{b-rmin} values.

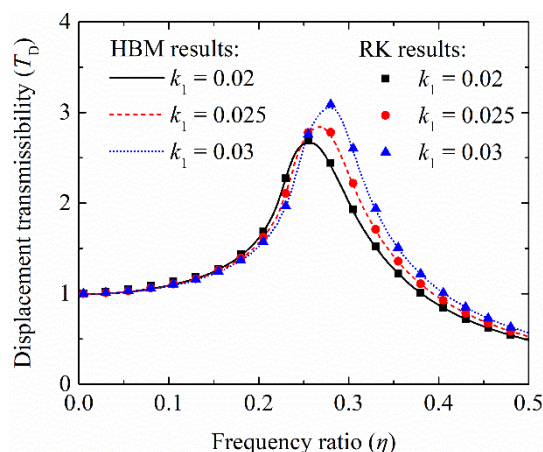
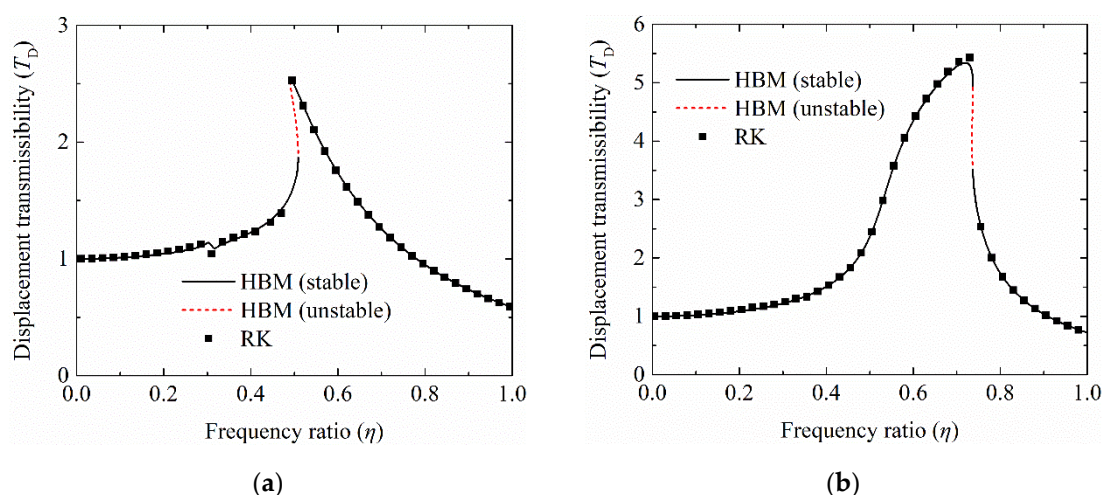


Figure 10. The frequency responses of the isolator with different values of k_1 .

Table 5. The lowest peak transmissibility/resonant frequency and the corresponding excitation amplitudes with different values of k_1 .

	T_{pmin}	A_{b-pmin}	η_{rmin}	A_{b-rmin}
$k_1 = 0.2$	2.65	1.08 mm	0.25	1.00 mm
$k_1 = 0.25$	2.84	1.00 mm	0.26	0.95 mm
$k_1 = 0.3$	3.03	0.92 mm	0.27	0.92 mm

The isolator with strong nonlinearity exhibited diverse nonlinear phenomena, such as jump phenomena. Figure 11 presents two types of jump phenomena, with the stable branches shown as black solid lines, and the unstable branches shown as red dashed lines. Generally, the jump phenomenon tends to exist with weak damping, namely a low β/γ value. With low k_3 , high $\beta + \gamma$, and low ξ values, the equivalent stiffness decreased dramatically at small displacements, and the stiffness-softening type of jump phenomenon could occur. As shown in Figure 11a, with $k_1 = 0.02$, $k_3 = 0$, $\beta = 0.1$, $\gamma = 1.9$, $\xi = 0$, and $A_b = 1$ mm, the resonance peak bends to the left and the jump-down frequency is lower than the resonant frequency. With high k_3 and low $\beta + \gamma$ values, the equivalent stiffness increased dramatically at large displacements, and the stiffness-hardening type of jump phenomenon could occur. As shown in Figure 11b, with $k_1 = 0.02$, $k_3 = 0.014$, $\beta = 0.1$, $\gamma = 0.4$, $\xi = 0.5$, $y_b = 215.57$, and $A_b = 1$ mm, the resonance peak bends to the right and the jump-down frequency is higher than the resonant frequency.

**Figure 11.** The frequency responses with jump phenomena. (a) Stiffness softening and (b) stiffness hardening.

To verify the analysis method of the frequency responses, the calculation results were compared with the experimental and numerical results of a damper with NiTiNOL wire ropes used in [35]. The wire ropes are horizontal and symmetrically arranged, and the structure is similar to the proposed isolator, as shown in Figure 1, without the linear spring. Therefore, the proposed modeling and analysis methods were applied to the damper. The damper was tested under swept-frequency experiments with a constant acceleration amplitude, and the responding displacement amplitude was measured. Similar to Equation (6), the dynamic model of the damper in the experiment can be expressed as

$$MY'' + K_{w1}Y + Z = MU_b \cos(2\pi f_b T) \quad (25)$$

where U_b and f_b are the amplitude and the frequency of the excitation acceleration, respectively, and Z is expressed in Equation (4). The damper was tested in two conditions, and the identified parameters are shown in Table 6 [35].

Table 6. The identified parameters of the damper experiments [35].

M	K_{w1}	U_b	K_d	ζ	γ_b	G	B	n_b
6.46 kg	4.18 kN/m	6.35 m/s ²	58.5 kN/m	0.587	112 mm ²	0.544 kN ^{-n_b+1} /mm	0.823 kN ^{-n_b+1} /mm	1.2
6.46 kg	4.47 kN/m	7.68 m/s ²	100 kN/m	0.294	950 mm ²	2 kN ^{-n_b+1} /mm	1.831 kN ^{-n_b+1} /mm	1.2

The frequency responses of the damper with NiTiNOL wire ropes under two excitation acceleration amplitudes are shown in Figure 12. The results calculated from the proposed harmonic balance analysis essentially coincide with the Runge–Kutta-based numerical results and the experimental results demonstrated in [35]. The main differences appeared in two ways. The experimental results with $U_b = 7.68$ m/s² exhibited the stiffness-softening phenomenon, while both the proposed method and the numerical integration failed to characterize the phenomenon. The harmonic balance analysis failed to present the super-harmonic resonances at about $f_b = 2.4$ Hz, as demonstrated in the numerical results due to the truncation error of Equations (10) and (11). The numerical results and the experimental results verify the effectiveness of the proposed harmonic balance analysis.

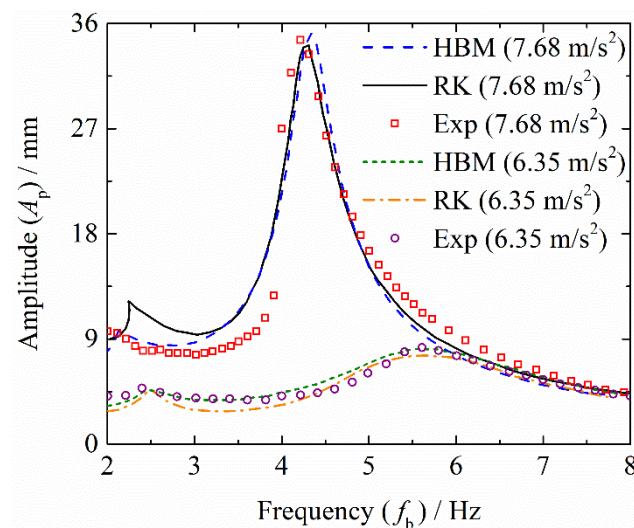


Figure 12. The frequency responses of the damper under the excitation acceleration amplitudes of 6.35 m/s² and 7.68 m/s² [35]. “HBM”, “RK”, and “Exp” denote the results of the harmonic balance analysis, the Runge–Kutta-based numerical integration, and the experiments, respectively.

5. Conclusions

In this study, we proposed a nonlinear isolator with a horizontal NiTiNOL wire rope, providing nonlinear stiffness and nonlinear damping, and a vertical linear spring, enhancing the load capacity. The nonlinearities were derived from geometric constraints, inner frictions, and phase transitions of the material. The restoring force model of the wire rope was established by a combined method of a beam constraint model and a Bouc–Wen model. The dynamic model of the nonlinear isolator was constructed and the frequency responses were analyzed through a harmonic balance method with an alternating frequency/time domain technique. The generalized equivalent stiffness and generalized equivalent damping ratio were defined considering the influences of the hysteretic damping and the pinching effect. Numerical simulations allowed us to draw the following conclusions.

The isolator exhibited a stiffness-softening-hardening characteristic. In a dimensionless form, the initial stiffness, the stiffness softening at small displacements, and the stiffness hardening at large displacements mainly depended on the pinching parameter ζ , the Bouc–Wen parameter $\beta + \gamma$, and the cubic stiffness k_3 , respectively. The generalized equivalent damping ratio increased and then decreased with increasing displacement. The Bouc–Wen

parameters and the pinching parameter ξ showed obvious influences on the damping ratio. Both the stiffness and the damping changes were non-monotonic. Compared to the isolator with a steel wire rope, the NiTiNOL wire rope decreased the initial equivalent stiffness and increased the damping. The isolator with the NiTiNOL wire rope enhanced the vibration isolation performance.

The peak displacement transmissibility and the resonant frequency of the isolator were related to the excitation amplitude, and the relationships were non-monotonic. There were excitation amplitudes corresponding to the minimum peak transmissibility and the lowest resonant frequency, respectively, and they are referred to as the most suitable excitation amplitudes of the isolator. The isolator parameters affected these indexes in different ways depending on their influences on the stiffness and damping characteristics. With low cubic stiffness, high Bouc-Wen hysteresis, and a weak pinching effect, the isolator tended to demonstrate a stiffness-softening type of jump phenomenon, and the stiffness-hardening type of jump phenomenon tended to occur with contrary parameters.

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