

Article

A Discrete Model of Rock-Soil Medium Response in the Determination of Horizontal Strain Values

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Abstract: This paper presents a method for determining the standard deviation and variation coefficient for both the predicted and measured values of horizontal strain that were caused by underground mining operations. The solution was based on a discrete model of the rock–soil medium response to the effects of mining operations. The “elementary horizontal strain increment” variable was random and could be described using the normal distribution. It was also assumed that the average horizontal strain values could be described using the solution given by Budryk and Knothe. The obtained solution allows for a much more comprehensive analysis of the results of deformation forecasts that are obtained with a view toward the protection of buildings that are located on the surface.

Keywords: horizontal strain; variation coefficient; discrete model; surface deformations



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1. Introduction

Mining operations that take place in rock masses generally lead to a disturbance of the initial stress condition in the rock mass. The tendency of the rock mass to reach a new state manifests itself as movements of the rock mass surrounding the operations, extending up to the ground level.

These movements can result in damage to buildings on the surface, which can even pose a threat to human life. Scientific research concerning the mathematical description of rock mass and surface movements that are caused by mining has been carried out intensively since the early 20th century.

The geometric–integral methods that were formulated during this period by Schmitz in 1923 [1], Keinhorst in 1925 [2], Bals in 1931/1932 [3], Beyer in 1945 [4], Knothe in 1953 [5], Kochmański [6], the Ruhrkohle method in 1961 [7] or the stochastic-medium-based solutions by Liwiniszyn [8–11] and Smolarski [12] enable a fairly correct description of the expected average values of vertical and horizontal displacements. Each of the mentioned methods is a specific calculation methodology that is based on the so-called influence function. These methods take into account, in a relatively simple and transparent way, the most important elements that affect the rock mass and the ground surface movements. In the case of longwall coal mining, these include the geometry of the mining area (longwall), thickness of the seam, mining depth, excavation advance rate, excavation system and properties of rock mass located above the seam.

These models are a more or less successful description of reality, but as in situ measurements show, they also differ from the actual values. One of the reasons is the rock mass itself, which is very complicated with undefined heterogeneity (anisotropy), where it is

variable in its geometric structure, strength, resistance, and deformation properties, which change during mining operations. Regarding the reaction of the rock–soil medium comprising the rock mass located atop the seam, Smolarski [12] formulated the concept of the “capricious medium”, thus signalling the necessity of a different approach to the description of rock mass deformation caused by mining operations as a purely deterministic approach.

Horizontal strain is the basic indicator of surface deformation that describes the danger to buildings. The differences that are observed in practice between the measured maximum values of horizontal strain, both tensile and compressive, and the previously predicted values, are often relatively large. This is the cause of many conflicts between surface users, mining authorities and mining companies (mines) concerning mining damage, including opinions that these deformations cannot be predicted based on calculations.

The given geometric–integral methods describe the subsidence distribution using two parameters, namely, the vertical scale parameter and the horizontal scale parameter of the influence range of mining operations.

The relative parameters of these two scales are:

- Vertical scale: operation coefficient (subsidence) a ;
- Horizontal scale: cotangent function of the influence range angle β .

The absolute parameters include:

- Vertical scale: the maximum full basin subsidence $s_{max} = a \cdot g$;
- Horizontal scale: the influence range of the mining operations $R = H \cdot \cot\beta$;

where

g —thickness of the excavated seam (m);

H —depth of the mining operation (m).

The relative parameters of the vertical scale of the respective theory or the geometric–integral methods are both conceptually (by definition) and quantitatively equal. On the other hand, the relative parameters of the scale of the horizontal influence range are different. They depend on the influence function of the calculation method and the definition of the edge of the subsidence basin (or the horizontal range of influence of the mining operation).

Among the mentioned calculation methods, Knothe theory [5] is the method that has been widely used for over 60 years. It is used in forecast calculations of surface movements that are caused by mining operations, not only in Poland [13–18] but also in many other countries; these include Germany [19–23], the Czech Republic [24–26], China [27–31], the USA [32–34] and other regions [35–39].

The Ruhrkohle method that is used in Germany is based on the Knothe solution, including the influence function as a Gaussian function. It differs from Knothe theory only by a different definition of the angle describing the horizontal influence range of mining operations. Since the influence functions of both methods are appropriately parametrised Gaussian functions, it is not even theoretically possible to determine the absolute limit of the horizontal influence range, i.e., the so-called closest point at which the subsidence value reaches zero, unlike in the case of the methods proposed by Schmitz, Keinhorst, Bals and Bayer. A relationship (Equation (1)) exists between the relative scale parameter of the horizontal range of operation influence between both methods:

$$\cot\beta = \sqrt{\frac{\pi}{k}} \cot\gamma \quad (1)$$

where

$k = -\ln 0.01$;

β —the angle of the range of the main influence in the Knothe method;

γ —the angle of the range of the boundary influence in the Ruhrkohle method.

If the above relationship is satisfied, both methods, i.e., the Knothe theory and the Ruhrkohle method, lead to identical calculation results, i.e., these methods are formally identical. Both methods still constitute the basis for surface deformation forecasts that are

prepared for mining authorities as a part of obtaining permits for mining operations. They are also an effective tool in the process of planning mining in terms of minimising its impact on the buildings and structures that are located on the surface and the mine facilities, such as mine shafts and main adits. For this and other reasons, appropriate solutions of the mathematical model of horizontal deformation contained in Knothe theory provided the basis for the thoughts and considerations presented in this work.

As stated earlier, horizontal strain is the key indicator of surface deformation that describes the threat to buildings and structures. Analysis of the results of section length measurements along the in situ measurement lines clearly shows that the horizontal strain index displays high fluctuations (randomness). On the one hand, this is a result of the accuracy of measurements performed and, on the other hand, of the fluctuations that are related to the reaction of the rock–soil medium. The analyses performed indicate that the value of the fluctuation resulting from the reaction of the rock–soil medium is at least an order of magnitude greater than the value of the fluctuation related to the accuracy of the performed measurements.

The uncertainty of the forecasts that are made can be evaluated using two different approaches. The first of these is related to the preferred computational model. The basis of this approach accepts the relative parameters of the vertical scale and the horizontal scale of the influence range as real random numbers with defined values of standard deviation. This limits the uniqueness of the predicted deformation values, but it also has a much deeper meaning, as it is a much better approximation of the expected reality compared with a deterministic forecast. Here, we should mention the Monte Carlo simulation method [40] or the simple variance increment method (the uncertainty propagation law). In this approach, we replace the point estimate with a range estimate with a defined probability.

A different approach to solving the uncertainty problem was proposed by Batkiewicz [41], who assumed that the horizontal deformation that is observed in a single measurement section is a result of the forming fissures, no matter how small they are. Assuming that the random variable of the fissure width is described by the normal distribution and the random variable of the number of fissures is described by the binomial distribution, he concluded that the variation coefficient of the horizontal strain can be described using Equation (2):

$$M_\varepsilon = \frac{K}{\sqrt{l}} \quad (2)$$

where

l —the length of the measurement section (m);

K —a certain constant that is dependent on the geological and mining conditions of the undertaken mining operation.

The work started by Batkiewicz was continued by Popiołek [42,43], Pielok [44], Klein [45,46], Bartosik-Sroka and Sroka [47], Stoch [48] and Kowalski [49], among others.

The work by Popiołek is particularly noteworthy among the scientific works cited above. The findings contained therein concerning the variation coefficient of the predicted maximum values of horizontal strain (tension and compression) and the influence of the length of the measurement section and geological and mining conditions are still valid and commonly used. The results of the above works are presented in detail later in this paper.

2. Theoretical Model of Deformation Accumulation

A rock that is subjected to a load accumulates the generated stress (energy), which is released discretely and recorded as an elementary, discrete deformation increase once it exceeds a certain value that is characteristic for the given rock type. This cyclical “accumulation–release of energy” process characterises a significant part of the deformation process. It is caused by the structure of the rock material and its internal “resistance”. This process is related not only to the behaviour of rock samples during strength tests that are performed in a laboratory but also to the deformation of the rock–soil medium that is caused by underground mining operations. Online extensometric measurements

of horizontal deformations that were carried out in mining areas and many years of experience of the authors in the field of measurements and analysis of both continuous and discontinuous deformations clearly confirmed such a course of the horizontal strain accumulation (Figure 1).

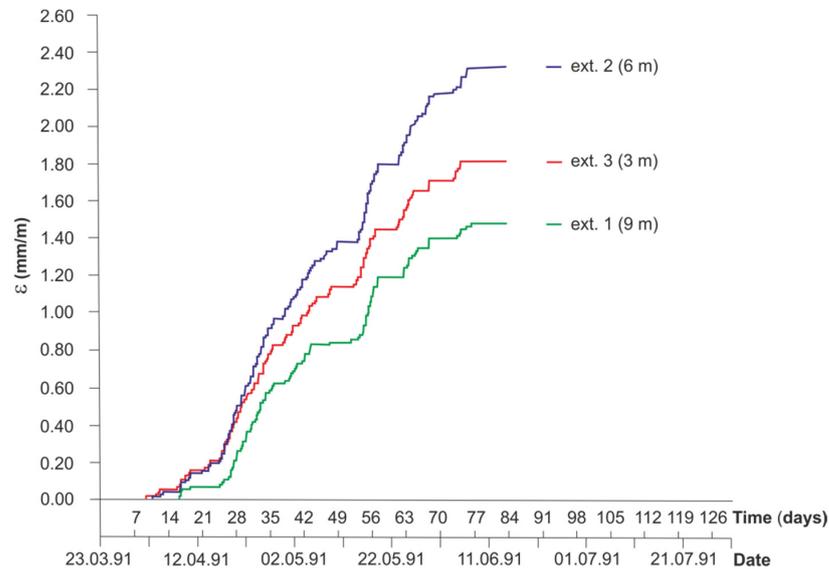


Figure 1. The accumulation of horizontal strain that was recorded from 29 March 1991 to 14 June 1991 using three extensometers installed at Station 2 Fahrradweg at Kapellen (own study).

The assumption that this discrete course of the measured deformation is caused by the internal resistance of the measuring device was shown to be false. The performed tests of the measuring device showed that this deformation picture was dominated by the reaction of the rock–soil medium.

To estimate the uncertainty of such a deformation process, let us suppose that a single elementary deformation increment $\Delta\varepsilon$ is a random variable with a normal distribution with a defined expected (average) value h and a standard deviation σ_h . Furthermore, let us assume that this applies to the entire deformation process, i.e., to both tensile and compressive deformations (Equation (3)):

$$|\Delta\varepsilon| \sim N(h, \sigma_h) \tag{3}$$

For the schematic course of deformation accumulation over time presented in Figure 2, the standard deviation of any single value $\varepsilon(t)$ can be determined according to Equation (4):

$$\sigma_{\varepsilon(t)} = \sigma_h \cdot \sqrt{n(t)} \tag{4}$$

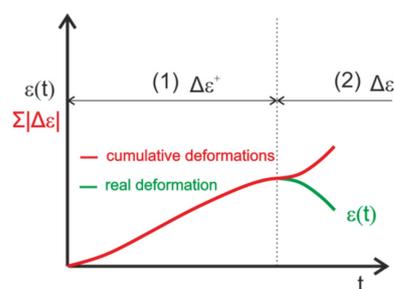


Figure 2. A schematic representation of deformation accumulation over time (own study).

where

$n(t)$ —the number of elementary discrete reactions up to time t .

The average number of elementary discrete responses can be estimated using Equation (5):

$$n(t) = \frac{\sum_0^t |\Delta\varepsilon|}{h} \tag{5}$$

where

$\sum_0^t |\Delta\varepsilon|$ —cumulative value of the deformation increases at time t .

For the first section of the curve $\sum_0^t |\Delta\varepsilon|$ (Figure 2), we therefore obtain Equation (6):

$$\sigma_{\varepsilon(t)} = \frac{\sigma_h}{\sqrt{h}} \cdot \sqrt{\varepsilon(t)} = c \cdot \sqrt{\varepsilon(t)} \tag{6}$$

where

c —a certain constant that is dependent on the type and properties of the deformed rock–soil medium.

It can be seen from the above considerations that for the first phase of the influence of progressing mining operations, i.e., the phase with increasing tensile strain, the standard deviation is proportional to the square root of the predicted or measured strain.

The variation coefficient of the strain therefore assumes the form of Equation (7):

$$M_{\varepsilon(t)} = \frac{\sigma_{\varepsilon(t)}}{\varepsilon(t)} = \frac{c}{\sqrt{\varepsilon(t)}} \tag{7}$$

The solutions presented above are illustrated by the following example in which the profile of horizontal deformation that is caused by mining operations of a single longwall (Figure 3) corresponds to the full characteristics of the deformation profile in the direction of the mining operation, which includes the phases of tensile and compressive deformation. Let us assume that the analysed object is located at the point P (Figure 3).

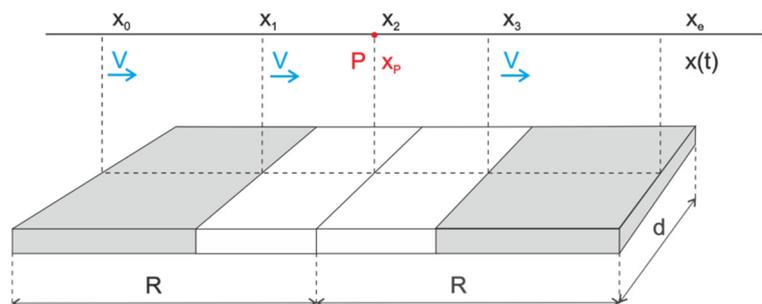


Figure 3. The schematic profile and geometry of longwall mining for the case of a full deformation profile at point P (own study), (V - velocity of exploitation).

In the case of the operation that is shown schematically in Figure 3, the distribution of horizontal deformation over time, assuming that the influences occur immediately according to the solution by Budryk and Knothe, [5,50], is described by Equation (8):

$$\varepsilon_p(t) = \varepsilon_{max}(d) \cdot \sqrt{2\pi}e \cdot \frac{x_p - x(t)}{R} \cdot \exp\left(-\pi \frac{(x_p - x(t))^2}{R^2}\right) \tag{8}$$

where

$$\varepsilon_{max}(d) = \varepsilon_{max} \cdot \int_{-\frac{d}{2R}}^{+\frac{d}{2R}} \exp(-\pi\lambda^2) d\lambda$$

ϵ_{max} —the maximum value of horizontal deformation for the so-called full basin (mm/m);
 $x(t)$ —the location coordinate of the mining front (longwall face) at time t ($x_0 \leq x(t) \leq x_e$);
 x_p —the location coordinate of the analysed object;
 d —length of the longwall face (m);
 R —the so-called radius of the main influences range (m).

The profile of the full horizontal deformation shown in Figure 4 contains three characteristic increase phases, namely:

- Phase 1—the phase of positive elementary discrete deformation increments $\Delta\epsilon^+$ (between points x_0 and x_1);
- Phase 2—the phase of negative elementary discrete deformation increments $\Delta\epsilon^-$ (between points x_1 and x_3);
- Phase 3—the phase of positive elementary discrete increments of deformation $\Delta\epsilon^+$

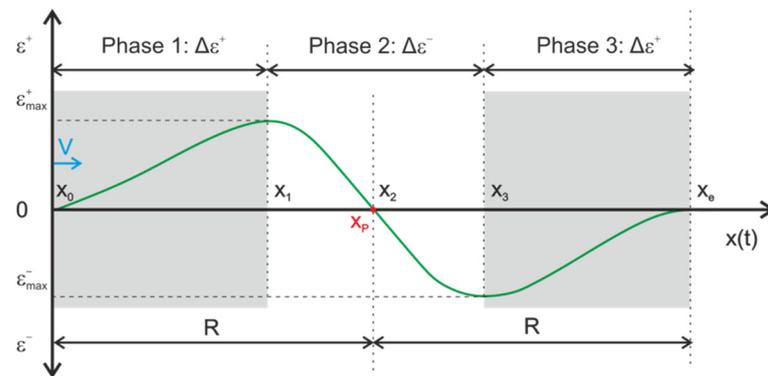


Figure 4. Full characteristics of the deformation profile for the point P for the longwall operations that are shown in Figure 3 (own study).

As discussed earlier, it is not the actual strain but the cumulative strain profile, i.e., the sum of absolute increment values $\Delta\epsilon$ until t , that comprises the basis for the uncertainty assessment of the predicted horizontal strain.

The difference between the normal deformation profile and the cumulative profile is shown in Figure 5.

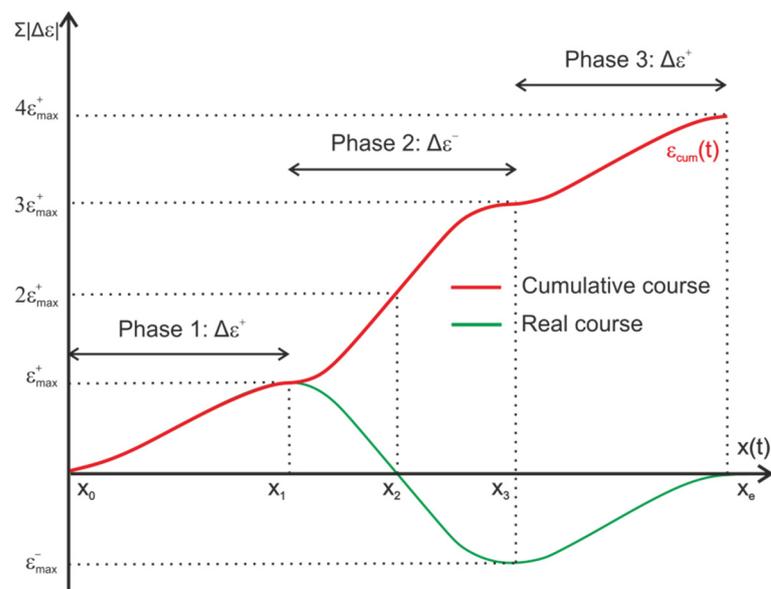


Figure 5. Difference between the actual deformation profile and the cumulative profile (own study).

Each value of the actual deformation at time t corresponds to a precisely defined value of the cumulative strain $\epsilon_{cum}(t)$. In phase 1 of the deformation profile, the actual profile of

the tensile strain values is consistent with the cumulative profile. The maximum value of the tensile strain is the end point of phase 1. Thus, for the maximum value of the tensile strain, we obtain Equations (9) and (10):

$$\sigma_{\varepsilon_{max}^+} = c\sqrt{\varepsilon_{max}^+} \quad (9)$$

$$M_{\varepsilon_{max}^+} = \frac{c}{\sqrt{\varepsilon_{max}^+}} \quad (10)$$

At the beginning of phase 2, the phase of the negative elementary strain increments and the profiles of the actual strain and cumulative strain functions begin to diverge. The maximum value of compressive strain is the end point of the actual strain profile in phase 2. The corresponding value of the cumulative strain, according to the solution from Budryk and Knothe [5,50] for the so-called full basin and the full deformation profile in which the maximum values of tensile and compressive strain are equal to each other, is three times greater. The standard deviation and the variation coefficient of the maximum compressive strain value are therefore described by Equations (11)–(13). In Equation (12), the absolute value of ε_{max}^+ equals ε_{max}^- due to the assumption in Figure 4 of the infinite half-plane.

$$\sigma_{\varepsilon(t)} = c\sqrt{\sum_0^t |\Delta\varepsilon|} \quad (11)$$

$$\sigma_{\varepsilon_{max}^-} = c\sqrt{3 \cdot |\varepsilon_{max}^-|} = c\sqrt{3 \cdot \varepsilon_{max}^+} \quad (12)$$

$$M_{\varepsilon_{max}^-} = c\sqrt{\frac{3}{\varepsilon_{max}^+}} = M_{\varepsilon_{max}^+} \cdot \sqrt{3} \quad (13)$$

By comparing the variation coefficient of the maximum compressive strain and tensile strain values, we can see that these values are different and their ratio for a single mining operation is given in Equation (14):

$$\frac{M_{\varepsilon_{max}^-}}{M_{\varepsilon_{max}^+}} = \sqrt{3} = 1.73 \quad (14)$$

Therefore the determination uncertainty of the maximum compressive strain value is much greater than for the tensile strain. The analyses of deformation measurement results show that in the case of a full basin, the maximum compressive strain values are greater than the maximum tensile strain values.

According to Niedojadło [51], the ratio of the average values of these strains is about 1.2.

By accepting this value, we obtain Equation (15):

$$\frac{M_{\varepsilon_{max}^-}}{M_{\varepsilon_{max}^+}} = \frac{\sqrt{3.2}}{1.2} = 1.49 \quad (15)$$

By comparing the obtained results with the earlier results of empirical studies by Popiołek [42,43], Kwiatek [52], Stoch [48] and Kowalski [49] that are shown in Table 1, it should be noted that at least the results obtained on the basis of relatively simple theoretical assumptions that describe the ratio of the variation coefficient of the maximum compressive strain value to the variation coefficient of the maximum tensile strain value are correct.

Table 1. Summary of results from past research that is related to the variation coefficient values for the maximum values of tensile and compressive strain (own study).

Author	$M_{\varepsilon_{max}^+}$ (%)	$M_{\varepsilon_{max}^-}$ (%)	$\frac{M_{\varepsilon_{max}^-}}{M_{\varepsilon_{max}^+}}$
Popiołek [42,43]	20.0	30.0	1.50
Kwiatek [52]	20.0	30.0	1.50
Stoch [48]	20.5	25.9	1.26
Kowalski [49]	19.0	32.0	1.68

According to the theoretical solution from Budryk and Knothe [5,50], as well as in situ observations, the deformation profile in phase 3 approaches the average final value of zero. The solution presented in this paper shows that this zero value carries the largest standard deviation and explains the post-mining remnants of the flat bottom of the subsidence basin (Figure 6), introduced by Batkiewicz [41,53] on the basis of in situ observations.

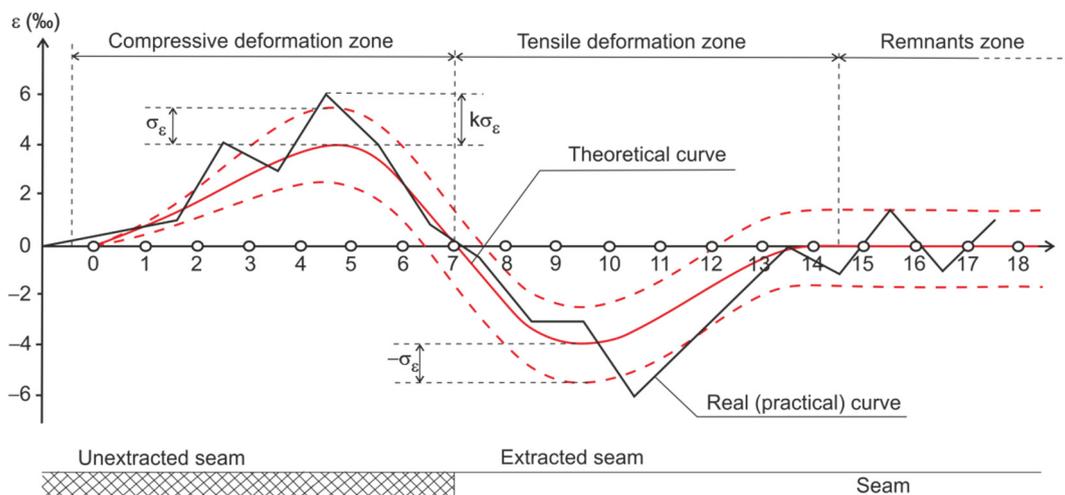


Figure 6. The graph of the theoretical and practical curves of specific horizontal deformations with the area of their standard deviations marked (own study).

The considerations presented above applied to deformation forecasts for a single longwall working.

The discrepancy between the actual deformation and the profile of accumulated deformation increases in the case of long-term forecasts that include the influence of multiple longwall workings or for multi-seam mining operations. It can thus be concluded that deformation forecasts for the final state of the long-term forecasting period bear a much greater uncertainty and, as the long-term experience of the authors shows, these forecasts sometimes do not make much sense.

The presented solutions related to standard deviations and variation coefficients of the predicted values of horizontal deformation include a constant c , which is a property of the deformed soil–rock medium according to the adopted model of deformation accumulation. With the future practical applications of the solution presented here in mind, the following part of the paper includes an attempt made at estimating the value of the constant c for the conditions of the Upper Silesian Basin.

3. An Attempt to Determine the Value of the Constant c Based on Past Experience

The value estimation for the constant c was based on the existing empirical findings and experiments. According to the works by Popiołek [42,43], Greń and Popiołek [54],

Stoch [48] and Kowalski [49], the average value of the variation coefficient of the maximum value of tensile strain is approximately constant and amounts to Equation (16):

$$M_{\epsilon^+} = 0.20 \tag{16}$$

It can be seen from the above that the average value of the variation coefficient of the maximum tensile strain is independent of the maximum tensile strength value.

Equation (16) is therefore qualitatively contradictory to the theoretical solution obtained in this work (Equation (7)).

In our opinion, this is caused by the fact that the analyses of the extensive experimental material covering many cases of mining operations that were performed by the aforementioned authors included only the determination of the variation coefficient separately for each individual case, followed by the determination of the average value using the arithmetic mean method.

Therefore, the works by Popiołek [42] and Greń and Popiołek [54] show that the value range of the variation coefficient M_{ϵ^+} for 49 individually analysed cases can be described by Inequality (17):

$$0.04 \leq M_{\epsilon^+} \leq 0.54 \tag{17}$$

The limits of this inequality differ significantly from the average value of 0.20 that is preferred in the literature.

In order to make an approximate estimation of the constant c , let us assume that Inequality (17) is correct for the average mining conditions in the Upper Silesian Basin. The range of possible values of the constant c for this assumption could be determined using Equation (18):

$$c = 0.2\sqrt{\bar{\epsilon}_{max}^+} \tag{18}$$

where

$\bar{\epsilon}_{max}^+$ —the average maximum value of the tensile deformation for the average mining conditions in the Upper Silesian Basin.

The value of $\bar{\epsilon}_{max}^+$ was calculated according to the research work by Popiołek [42,43] using Equation (19):

$$\bar{\epsilon}_{max}^+ = 0.48 \frac{s_{max}}{R} = 0.48 \frac{a \cdot g}{H \cdot \cot \beta} \left(\frac{m}{m} \right) \tag{19}$$

The value of the constant c was determined by assuming the following data:

- Relative vertical scale parameter: $a = 0.8$;
- Relative horizontal range scale parameter: $\cot \beta = 0.5$;
- Average seam thickness: $g = 2.0$ m;
- Mining depth range: $400 \leq H \leq 800$ m.

The calculation results for the value of the constant c as a function of the mining depth, assuming full subsidence basin, are presented in Table 2.

Table 2. Calculation results for the value of the constant c (own study).

Exploitation Depth (m)	$c(\sqrt{\frac{m}{m}})$
400	0.0124
500	0.0111
600	0.0101
700	0.0094
800	0.0088
	$\bar{c} = 0.0104$

The calculations show that for a representative depth range from 400 to 800 m, the value of the constant c varied between 0.0088 and 0.0124 $(m/m)^{0.5}$.

Acceptance of the mean value led to Equations (20) and (21):

$$\sigma_{\epsilon_{max}^+} = 0.010\sqrt{\epsilon_{max}^+} \tag{20}$$

$$M_{\varepsilon^+} = \frac{0.010}{\sqrt{\varepsilon_{max}^+}} \quad (21)$$

For deformation values expressed in millimetres per metre or per mille (‰), the value of the constant c was 0.320.

4. Influence of Measurement Base Length on the Uncertainty of Horizontal Deformation

Batkiewicz [41,53] assumed that relative horizontal elongations (hereinafter: horizontal deformations) that are caused by mining operations are a result of the discontinuous reaction of the rock mass to the influence of mining. According to Batkiewicz, fissures form in the rock mass, even if they are extremely small. According to this assumption, the horizontal strain that is measured at a section with a length l is a function of the fissure width and the number of fissures. Assuming that the random variable fissure width is described by the normal distribution and the random variable of the number of fissures is described by the binomial distribution and that these variables are mutually independent, Batkiewicz obtained Equation (22) for the variation coefficient of the average value of the horizontal strain:

$$M_{\varepsilon} = \frac{2}{\sqrt{3}} \sqrt{\frac{1}{n}} \quad (22)$$

where

n —number of fissures.

Assuming that the number of fissures is proportional to the length of the measured section l , Batkiewicz formulated the basic relationship given in Equation (2) that links the variation coefficient of the horizontal strain value with the length of the measurement section.

Equation (2) shows that the greater the length of the measurement section, the lower the value of the variation coefficient of the observed/measured value of the horizontal strain.

Because of this fact, Batkiewicz proposed that the average maximum strain value should be determined from the average strain value for a length of the measurement section that is equal to the main influence range radius R .

According to Batkiewicz, the maximum values of horizontal strain that are determined using this method are characterized by a relatively low variation coefficient that ranges from a few to several percent. The relationship between the average maximum horizontal strain $\bar{\varepsilon}_{max}$ and the average value determined for a section with length l is given by Equation (23):

$$\bar{\varepsilon}_{max} = \sqrt{\frac{2\pi}{e}} \cdot \bar{\varepsilon}_{max} \quad (l = R) \quad (23)$$

As a conclusion of these considerations, Batkiewicz stated that the predicted values of horizontal strain should be reported with their standard deviations or variation coefficients, as this leads to a more realistic assessment of the impact of mining operations on the surface buildings and structures.

Among the research in this field, the works by Popiołek [42,43], which still form the basis for the uncertainty assessment of forecasts and the horizontal strain values that are measured in situ, are particularly noteworthy.

According to Popiołek, the variation coefficient of the horizontal tensile strain, which includes the influence of the measurement section length, can be described by Equation (24):

$$M_{\varepsilon}(l) = \frac{5}{\sqrt{l}} (0.14 + 0.0006 \cdot H - 0.0008 \cdot N - 0.06 \cdot s_{max}) \quad (24)$$

where

H —depth of the mining operations (m);

N —thickness of the loose rock overburden (m);

s_{max} —the maximum subsidence value (m);

l —measurement section length.

Popiołek also reported the empirically obtained findings on the average values of the variation coefficient of maximum values of tensile and compressive strain, as shown in Equations (25) and (26):

$$M_{\varepsilon_{max}^+} = 0.2 \quad (25)$$

$$M_{\varepsilon_{max}^-} = 0.3 \quad (26)$$

The values indicated above were fully confirmed in later works, among others, by Stoch [48] and Kowalski [49].

Popiołek [42] analysed the measurement results that were obtained for changes in the length of the measurement sections along the so-called measurement lines, particularly in parts involving tensile deformations. This analysis included a total of 49 cases involving coal (35), iron ore (10) and copper ore (4) mining. This extensive empirical material related to various geological and mining conditions, various mining systems (including longwall mining with a roof collapse and hydraulic backfilling, pillar and chamber mining with a roof collapse, hydraulic backfilling and dry backfilling and strip mining with hydraulic backfilling), a wide range of thickness of the excavated deposit (from 1.0 to 7.0 m) and a wide range of mining depths (from 35 to 600 m).

On the basis of in situ measurements and the theoretical solution provided by Budryk and Knothe [5,50], Bartosik-Sroka and Sroka [47] presented an analysis of the dependence of the variation coefficient of the horizontal strain M_ε on the values of the absolute parameters of the vertical scale ($a \cdot g$), the scale of the horizontal range R of the subsidence basin and the length of the measurement section l .

The considerations assumed that the variation coefficient value can be described using a regression function in the form of the product model given in Equation (27):

$$M_\varepsilon(a \cdot g, R, l) = \mu \cdot (a \cdot g)^{n_1} \cdot R^{n_2} \cdot l^{n_3} \cdot \xi \quad (27)$$

where

μ —a certain, constant value;

n_1, n_2, n_3 —power coefficients;

ξ —a multiplicative, random coefficient with the expected value of 1 and a certain, finite variance.

As a result of the analysis and iterative calculations, the following values of regression function coefficients were obtained:

$$\mu = 0.6,$$

$$n_1 = -0.5,$$

$$n_2 = 1.0,$$

$$n_3 = -0.5$$

Which led to Equation (28):

$$M_\varepsilon(l) = 0.6 \frac{R}{\sqrt{a \cdot g \cdot l}} (\%) \quad (28)$$

The results of this unpublished work were presented by Pielok [44] in the German scientific journal *Das Markscheidewesen*.

According to the formulated model of the discrete accumulation of horizontal strain of a rock–soil medium for the influence of mining operations, the variation coefficient that is related to the measurement section length can be determined according to Equation (29):

$$M_\varepsilon(l) = \frac{c}{\sqrt{\Delta\varepsilon_{max}(l)}} \quad (29)$$

where

$\Delta\varepsilon_{max}(l)$ —the maximum strain increase along the length of the measurement section.

According to the solution by Budryk and Knothe, the maximum value of the increase $\Delta\varepsilon_{\max}(l)$ can be determined according to Equation (30):

$$\Delta\varepsilon_{\max}(l) = \sqrt{2\pi e} \cdot \varepsilon_{\max} \cdot \frac{l}{R} \quad \text{for } \frac{l}{R} \leq 0.2 \quad (30)$$

By substituting Equation (30) into Equation (29), we obtained Equation (31):

$$M_{\varepsilon}(l) = \frac{1}{\sqrt[4]{2\pi e}} \cdot \frac{c}{\sqrt{\varepsilon_{\max} \cdot \frac{l}{R}}} \quad (31)$$

Assuming that the average value of the maximum tensile strain for the solid basin can be described using the Popiołek equation (Equation (19)) and that the average value of the constant c is 0.010, we finally obtained Equation (32):

$$M_{\varepsilon}(l) = 0.7 \frac{R}{\sqrt{a \cdot g \cdot l}} = 0.7 \frac{H \cdot \cot \beta}{\sqrt{a \cdot g \cdot l}} \quad (\%) \quad (32)$$

This formula is consistent with the empirically derived formula that was provided by Bartosik-Sroka and Sroka [44].

Using the previously presented theoretical solution (Equation (10)), Equation (30) could be represented as Equation (33):

$$M_{\varepsilon}(l) = \frac{1}{\sqrt[4]{2\pi e}} \cdot M_{\varepsilon_{\max}^+} \cdot \sqrt{\frac{R}{l}} = 0.50 \cdot M_{\varepsilon_{\max}^+} \cdot \sqrt{\frac{R}{l}} \quad (33)$$

Assuming after Popiołek [43] that the variation coefficient of the maximum tensile strain is constant and equal to $M_{\varepsilon_{\max}^+} = 0.2$, we obtained Equation (34):

$$M_{\varepsilon}(l) = 0.10 \cdot \sqrt{\frac{R}{l}} = \frac{0.10}{\sqrt{L}} \quad (34)$$

where:

$L = \frac{l}{R}$ —the standardised length of the measurement section (m).

It can be concluded from the above that the value of the variation coefficient for the measured maximum value of the tensile strain depends on the measurement section length and the value of the absolute scale factor of the horizontal impact range of the mining operations.

According to the approximation in Equation (34), assuming that $l = R$, we obtained the value of the variation coefficient as 0.1 or 10%. This result is fully consistent with the findings of Batkiewicz [41], who, after analysing the results of in situ measurements, stated that the values of the variation coefficient of the horizontal strain for the measurement section length equal to the influence range radius vary from a few to a maximum of several percent.

Detailed calculations that were performed for the obtained theoretical solution for the depth range of 400 to 800 m (Table 3) led to the assumption of $l = R$ for the variation coefficient in the range between 0.08 and 0.12.

Table 3. Detailed calculations for the obtained theoretical solution carried out for the depth range from 400 to 800 m (own study).

Exploitation Depth (m)	ε (m/m)	M_{ε^+}	$M_{\varepsilon} (l = R)$
400	0.0038	0.1614	0.08
600	0.0026	0.1976	0.10
800	0.0019	0.2282	0.12

The theoretical dependence of the variation coefficient of the measured values of horizontal deformation on the measurement section length that is presented in this paper is qualitatively consistent with the theoretical solution of Batkiewicz [41] and the empirical results of the research by Popiołek [42] and Bartosik-Sroka and Sroka [47].

The presented theoretical solution (Equation (32)) indicates that the value of the variation coefficient $M_\varepsilon(l)$ increases with the excavation depth H and decreases with the increase of the maximum subsidence s_{max} . This is qualitatively consistent with the empirical formula provided by Popiołek (Equation (24)).

5. Conclusions

Due to the high randomness of surface deformations that are caused by underground mining operations, this fact must be taken into account during the assessment of the possible impact of such operations on buildings and structures. The solution presented in this paper allows for the relatively simple estimation of the standard deviation and variation coefficient of the predicted values of horizontal deformation and explains the fluctuation of the deformation values that are measured as a part of deformation monitoring, specifically including the fluctuation dependence on the length of the measurement section.

For the assumed model of the response of the rock–soil medium, which is discrete over time, due to the influence of mining operations, it was obtained that the standard deviation value of the predicted value of horizontal strain $\varepsilon(t)$ is proportional to the square root of the absolute values of the elementary increments of deformation that accumulated over time, i.e., proportional to the square root of the accumulated strain.

The introduction of the cumulative deformation assumption at moment t , which describes the entire course of deformation, allowed for the theoretical determination of the ratio of the coefficients of variation of the maximum values of the compressive strain $M_{\varepsilon-}$ and the tensile strain $M_{\varepsilon+}$. Comparing the theoretical result with the results of empirical research by analysing the results of in situ measurements clearly showed that the presented solution is compatible.

The theoretical results of the deformation variability coefficients depended on the length of the measurement base (Equations (29)–(34)), which, in this case, unexpectedly turned out to be fully consistent with Equation (28), which was obtained via statistical analysis of measurement results.

In light of the presented solutions, long-term forecasts, which often cover periods that range from several years to sometimes even several decades are largely pointless, especially for multiple seam mining operations with highly concentrated extractions.

Therefore, in the case of such forecasts, it is recommended to update them on an ongoing basis, preferably every 1–2 years, as dictated by the experience of the authors.

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