Article

Early Robust Design—Its Effect on Parameter and Tolerance Optimization

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Featured Application: This paper emphasizes the important role of early robust design for optimal parameter and tolerance design. The theoretically discussed effects on parameter and tolerance optimization are supplemented by an illustrative case study showing the benefits of a robust product concept.

Abstract: The development of complex products with high quality in dynamic markets requires appropriate robust design and tolerancing workflows supporting the entire product development process. Despite the large number of methods and tools available for designers and tolerance engineers, there are hardly any consistent approaches that are applicable throughout all development stages. This is mainly due to the break between the primarily qualitative approaches for the concept stage and the quantitative parameter and tolerance design activities in subsequent stages. Motivated by this, this paper bridges the gap between these two different views by contrasting the used terminology and methods. Moreover, it studies the effects of early robust design decisions with a focus on Suh's Axiomatic Design axioms on later parameter and tolerance optimization. Since most robust design activities in concept design can be ascribed to these axioms, this allows reliable statements about the specific benefits of early robust design decisions on the entire process considering variation in product development for the first time. The presented effects on the optimization of nominal design parameters and their tolerance values are shown by means of a case study based on ski bindings.

Keywords: robust design; parameter optimization; tolerance optimization; early design stages; axiomatic design

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1. Introduction

Modern market environments are characterized by competing requirements and trends, such as high product complexity, shortened development times and high-quality standards [1,2]. In order to ensure quality, the variation in product behavior needs to be reduced, for example, by a suitable tolerance design limiting unavoidable manufacturing induced variation [3]. However, tolerance design is limited by manufacturing constraints and the fact that tight tolerances lead to high manufacturing costs [4]. Therefore, a robust product design that is insensitive to variation is fundamental, especially for complex products. Accordingly, Taguchi suggests a three-step robust design process comprising system, parameter and tolerance design [5].

Given the exponential rise of effort and the decreasing effect of design changes along the product development process [6], the robust system or concept design is an essential basis for an efficient product and parameter design [7,8]. However, robustness activities at the beginning of the product development process are primarily qualitative and characterized by basic guidelines [9]. Most of them can be traced back to the *Independence and Information Axiom* [10] from Suh's Axiomatic Design. This is in contrast with the mainly quantitative approaches in later development stages, such as parameter (often also referred

to as robust design optimization) and tolerance optimization. Thus, there are different views on the robust design problem in early and late development stages.

Despite the obvious interrelation of the single activities [8,11], these divergent views imply that the specific effect of early robust design on subsequent steps is neither transparent nor quantifiable so far. In order to overcome this shortcoming, this contribution analyzes and compares the different views and studies the effect of early robust design principles, represented by Suh's axioms, on parameter and tolerance optimization. Thus, the interrelations between early robust design activities and subsequent robust design steps are explicitly analyzed for the first time, revealing the specific benefits of an early robustness consideration.

The paper is structured as follows. First, Section 2 summarizes related work considering robust design in product development. Then, Section 3 analyzes the similarities and differences between early and late robust design activities and compares the terminology used. Furthermore, the effects of early robust design on the subsequent optimization process are studied. These theoretical relationships are demonstrated with the aid of the case study in Section 4, leading to a discussion of the observed effects in Section 5. Finally, Section 6 gives a conclusion and an outlook.

2. Related Work

2.1. Robust Design in Product Development

In order to control the complexity in product development, the structuring in process models, e.g., according to Pahl & Beitz [12], as well as the formal organization of the solution finding process within the framework of design methods, e.g., in [13], is of great importance. Robust design, as a supportive part of the solution process, plays a decisive role in product success [14] and should therefore be considered at every stage of the product development process [9]. The classification into system, parameter and tolerance design [5] fosters an assignment of robust design approaches to the product development stages requirements definition, concept, embodiment and final design according to the established model of Pahl & Beitz [12]. Moreover, approaches, such as the Variation Risk Management [15], the domain model according to Suh [10] or the Variation Management Framework [16], support the consistent linking of customer requirements with design parameters and manufacturing, thus providing a framework for robust design activities. In addition to these generalized methodologies with a focus on structuring, there are various methods and tools supporting engineers in robust design activities, which are reviewed, e.g., in [9,17–19]. These approaches are presented in detail below. In accordance with the different views mentioned at the beginning of this paper, a distinction is made between early and late robust design approaches.

2.1.1. Early Robust Design

Early robust design activities are assigned to the system respective concept design stage and are particularly crucial for the robustness of the final product [8,20]. Since geometric and quantitative information is lacking at this stage [5], the activities are mainly shaped by robust design guidelines and principles [9,21] or expert assessments. Apart from a few analysis approaches that enable a quantitative robustness evaluation as a basis for concept decisions [22,23], the robust concept design is thus primarily characterized by qualitative aspects. These robustness aspects can be integrated by appropriate principles, which, according to [24], can be divided into the categories: safety (e.g., redundancy), kinematic design and complexity control. A collection of established principles aiming at a robust concept design is given in Table 1. These principles are implemented both by choosing alternative principle solutions [15] or by adapting the concept, e.g., by integrating or differentiating sub-assemblies [9].

Most of these robust design principles essentially follow the basic idea of the two axioms, including the derived corollaries and theorems according to Suh [10], and describe their implementation in the context of robust design: While the *Independence Axiom*

(Axiom 1) intends to achieve a separation or independence of the numerous functional product requirements, the *Information Axiom* (Axiom 2) intends to minimize the information content of the design. In contrast to the empirical or intuitive procedures in conventional development processes, this axiomatic approach thus supports the targeted selection of a good design. [10] As a result, Axiomatic Design has also gained importance in the context of robust design [25]. Even though the compliance with the axioms increases the probability of achieving a robust design, further aspects, such as the inherent contradiction of requirements, should be considered [26]. However, such analyzes already require deeper insights into the relationship between functional requirements. This in turn calls for expert knowledge or preliminary experiments respectively simulative screenings leading to models that are usually associated with late robust design activities. Consequently, early robust design in the concept stage is largely based on principles that can be largely attributed to the Axiomatic Design axioms.

Table 1. Assignment of robust design principles to Suh's axioms.

Suh's Axiom	Robust Design Principle	Classification
Axiom 1	prevention of overconstrained systems by proper definition of constraints and degree of freedom [27]	kinematic design
	unambiguous design of interfaces, e.g., by sufficient clearance to avoid unintended contact [8]	kinematic design
	division of tasks [28]	complexity control
	decoupling and uncoupling [29]	complexity control
Axiom 2	shortening of force-transmission paths [28]	kinematic design
	minimizing the number of design parameters [21]	complexity control
	shielding from the cause of variation (e.g., heat) [21]	complexity control

2.1.2. Late Robust Design

In contrast, late robust design, which is assigned to the embodiment and detail design stage, uses primarily quantitative approaches for parameter and tolerance design as well as robust design optimization. Thus, this traditional robust design mainly deals with Design of Experiments (DoE), statistics and optimization [5,30,31]. Therefore, transfer functions, which represent the relation between design parameters and product behavior, are determined by experiments or simulations and evaluated [32]. The product behavior is mapped by the quantifiable parameters respectively characteristics. In this context, they are often called Key Characteristics (KC) representing the relevant product requirements, e.g., via a distance between two adjacent surfaces [33]. The subsequent evaluation of the resulting KC enables the determination of the expected quality. Therefore, quality metrics, such as statistical moments, e.g., mean or variance, the scrap rate, the yield or process capabilities, are suitable [34]. In addition, Taguchi's quality loss functions enable the quantification of the impact of varying KCs on customers' perceived quality [35], thus linking to associated costs [34]. Further metrics, such as the Signal-to-Noise Ratio (SNR) [36] or sensitivity indices [37,38], allow a quantification of the robustness [39], supporting engineers in the stringent optimization of parameter and tolerance values [40]. During parameter design, the nominal design parameter values, e.g., the length of a lever, are first modified so that the KC becomes insensitive to design parameter variations. Considering the inherent variance of design parameters, this leads to a mean shift in the KC resulting in improved quality without influencing product costs [25]. During the subsequent tolerance design, the admissible variation of design parameters is defined with respect to the predefined KC tolerance. However, the modification of design parameter tolerances is directly linked to manufacturing costs [41], which leads to a tolerance-cost conflict. Since established approaches for solving this conflict are not universally applicable [34], tolerance values are commonly adjusted iteratively in a time-consuming process, often failing to achieve the optimal values [42–45].

In order to overcome these drawbacks, computer-aided, optimization-based approaches are increasingly used in parameter and tolerance design [34,46]. Therefore, besides individual approaches, existing tools from research and industry can be utilized [47,48]. This requires a transfer of the search for optimal nominal design parameter and tolerance values into a mathematical optimization problem [25,34]. Although the basic formulation of the parameter and tolerance design problem is largely the same [46], they differ in the definition of objectives and constraints in combination with the used quality and robustness metrics, see Figure 1. In robust parameter optimization, often called robust (design) optimization, the robustness of the solution is either added to (a) the constraints or to (b) + (c) the objective(s) [25]. The parameter optimization tries to satisfy both the requirement for the mean value of the resulting scattering KC and the requirement for robustness or sensitivity of the KC around this mean value. While some robustness metrics, such as the quality loss or the SNR, combine both aspects (Figure 1b), a separate consideration as shown in Figure 1c leads to a multi-objective optimization problem [25,49], requiring the selection of trade-off solutions from a Pareto front or a predefined weighting of the aspects [50]. Similar to parameter optimization, there are different formulations of the optimization problem in optimal tolerance design or tolerance (-cost) optimization, see Figure 1, whereby the tolerance-related costs are usually minimized under consideration of a minimum quality constraint [44,51], see Figure 1e. The incorporation of the quality loss as an additional objective leads to a multi-objective optimization problem and is known as robust tolerance design, see Figure 1f [52]. Although the main focus of parameter and tolerance design is different and commonly considered in two separated, subsequent design stages, various approaches combine parameter and tolerance design, enabling a concurrent robust optimization of parameters and tolerances, e.g., [53–55].

	robust parameter optimization			tolerance-cost optimization		
objective(s)	(a) $\Delta\mu$	(b) R	(c) $\Delta\mu, R$	(d) Q	(e) C	(f) C, R
constraint(s)	R, D	$\Delta\mu, D$	D	C	Q	(Q)
variable	$X + \Delta X$: nominal design parameter			t : tolerance value		

$\Delta\mu$: mean shift between KC target value and mean of scattering KC

R : robustness metrics, e.g. KC variance, sensitivity, SNR, quality loss,...

Q : quality metrics, e.g. yield, scrap rate, process capability,...

C : tolerance-related costs

D : design constraints, e.g. permissible parameter ranges

Figure 1. Optimization-based robust parameter and tolerance design in comparison.

2.2. Interaction between Early and Late Robust Design

Due to their different characters, early and late robust design are usually considered separately in the literature. However, some research addresses the effect of Suh's axioms on late robust design approaches, stressing the positive effect of the sequential application of the *Independence and Information Axiom* on the achievable robustness [25,56]. For example, the evaluation of the results of a computer-aided robustness analysis under the consideration of the *Independence Axiom* supports the definition of robust locating schemes with uncoupled KCs [29,57]. Other approaches propagate a simplified tolerance design for uncoupled assemblies, without specifically addressing the effects of coupling on the resulting tolerance values [58,59]. While independence only differentiates between coupled, decoupled or uncoupled, the information content is mapped using quantitative metrics that represent the extent of information required to fulfill the respective requirements of KCs [10,60]. Since this information content can also be interpreted as the probability of

fulfillment [10], a link between proper tolerance design and low information content has already been shown in [61]. Moreover, the relationship between couplings and resulting trade-offs in subsequent optimization was discussed [62]. However, a specific focus on the effect of Axiomatic Design axioms is missing.

2.3. Discussion of the State-of-the-Art and Research Question

As it has been highlighted, robust design activities in product development change dramatically when part and product geometry is defined. Thus, the objective of a holistic system improvement in early robust design turns into a minimization of the sensitivity of KCs in later design stages. Despite the enormous importance of a robust concept, robust design is primarily applied in late product development stages in practice [20] because of missing quantitative information in the concept stage, leading to difficulties in the specific implementation of design principles and the verification of their effectiveness [63]. Motivated by the present discrepancy, this paper aims at bridging the gap between early and late robust design to attain a common understanding as well as in-depth knowledge of the respective effects. This intended enhanced awareness of the high importance of both early and late robust design is regarded as a prerequisite for future research and application of early robust design in the context of a comprehensive robust design methodology. In this respect, the following research question is to be answered: How does early robust design, represented by the *Independence and Information Axiom*, affect the optimization of parameter and tolerance values in subsequent robust design stages?

3. Linking Early Robust Design and Optimal Parameter and Tolerance Design

A central challenge in linking early and late robust design is that the early stages are primarily characterized by a large number of different robust design principles. However, their effects are usually superimposed by a simultaneous application of several principles and also overlaid by the concretizing design and geometry definition in the ongoing product development process. Therefore, instead of going into detail about the effects of individual principles, they are considered jointly via the two axioms of Axiomatic Design, as suggested in Table 1. Furthermore, in order to minimize the negative influence of design concretization uncertainties, the focus is primarily on the effect on the optimization process rather than on the resulting values. The answering of the research question first requires a common understanding of early and late robust design with corresponding boundary conditions, aims and terminology. Their compilation, categorization and comparison reveal the relations between the different views thus forming the basis for their deeper analysis. In a second step, according to the claim in [56], the **effect** of *Independence and Information Axiom* in concept design on the subsequent robust parameter optimization according to Figure 1c and tolerance-cost optimization according to Figure 1e is analyzed.

3.1. Comparison of Characteristic Aspects

In the following, the characteristics of approaches for early robust design, parameter optimization, and tolerance optimization are compared, taking into account the associated terminology. For the comprehensive collection of influencing factors, the 7 M of the Ishikawa diagram known from quality management [64,65] were considered first. This collection is completed by a cross-check with the aspects developed within the PSI framework, which enables a comprehensive description of design activities under various conditions [66]. Inspired by Taguchi's P-diagram [5], the collected factors are categorized in input, output, control as well as noise factors and are shown as a black box in Figure 2.

Figure 2. Aspects considered in the comparison of the approaches.

Table 2 compares the domains early robust design, robust parameter optimization and tolerance-cost optimization by means of these aspects. The focus here is not on a precise classification of all terms used in the respective domains or a sharp differentiation, which is not feasible anyway due to the large variety of partially combining approaches. Instead, the aim is to give a common understanding of the essential aspects of the three domains so that engineers from different domains can gain an insight into the other one and thus understand the interrelations. Accordingly, Table 2 is limited to the two widely used methods of robust parameter optimization (with mean shift and robustness objective) and tolerance-cost optimization (with cost objective) highlighted in Figure 1.

First of all, the comparison of the characteristics shows a significant difference between early robust design and the subsequent domains primarily due to the different levels of part and product geometry definition, which is also represented by the changing *dimension*. Along with the increasing detailed *functional system description*, the *extent* of the optimization in late design stages is narrowed down and more specific. However, the apparently lower *number of perspectives* respectively complexity within the optimization domains is compensated by an increasing *level of detail*, see Table 2 section *domain characteristics*.

Moreover, the *engineers* and the *environments or departments* dealing with approaches from the respective domains change, starting with concept developers up to multidisciplinary tolerance experts incorporating manufacturing and inspection knowledge. The individual engineers are increasingly supported by computer-aided *facilities* and *specific methods* as product development progresses, see Table 2 section *control factors*.

This goes along with a change of the inherent *uncertainties*, transforming from primarily epistemic uncertainties regarding conceptual design decisions to aleatory or irreducible uncertainties, e.g., resulting from the virtual mapping accuracy of reality [67]. Accordingly, a high degree of primarily *informal knowledge* and design knowledge or experience is necessary for a successful concept and early robust design, see Table 2 section *noise factors*.

This results from the poor *availability of formal knowledge* in early development stages. It is also reflected in the primary available *information* that forms the starting point for the respective approaches in the domains. They are increasingly specific and comprehensive. However, this concretization also leads to an increasing restriction of the scope for action or improvement. While in early robust design, only the requirements are considered as rather qualitative *boundary conditions*, the optimizations are subject to clear quantitative constraints, see Table 2 section *input*.

Table 2. Comparison of the domains early robust design, robust parameter and tolerance-cost optimization.

Aspect	Early Robust Design	Robust Parameter Optimization	Tolerance-Cost Optimization
domain characteristics			
extent	entire system	specific aspects	specific aspects
number of perspectives	high (design, parameter, variation)	medium (parameter, variation)	medium (variation, manufacturing)
dimension	primarily 2D	2D/3D	2D/3D
functional system description	qualitative	quant. transfer function (implicit/explicit)	quant. transfer function (implicit/explicit), quant. tolerance-cost function, (quality loss function)
level of detail	low	medium	high
control factors			
engineer	design engineer	product simulation engineer, design engineer	tolerance engineer, design engineer
environment / department	design, concept development	design	tolerance management
facilities	paper, visualization tools	computer-aided tools	computer-aided tools
specific method	n.a. (workflow)	optimization	optimization
noise factors			
uncertainty	epistemic uncertainty in designer decision	reduced ambiguity, aleatory uncertainty	primarily aleatory uncertainty
informal knowledge	epistemic design knowledge	n.a.	n.a.
input			
formal knowledge availability	low	high	high
information—data	requirements, initial concept and product structure	embodiment design	nominal design, manufacturing information
information—documents	sketch, graph	product model, (CAD/drawing)	product model, (CAD/drawing)
boundary conditions	requirements	design constraints, parameter ranges, noise	quality requirements, tolerance ranges, KC
output			
quantitative objective	n.a.	KC (mean shift and robustness)	tolerance-related costs
objective metrics	n.a.	robustness metrics	tangible and intangible costs
general aim	robust concept design	robust nominal design parameters	cost-optimal tolerance values

Accordingly, *quantitative objectives* as well as *objective metrics* are clearly defined in optimization, see Table 2 section *output*. Despite the fact that the basic idea and the characteristics of the optimization domains are very similar, they differ in the definition of the objectives and constraints (also see Figure 1) and thus also in the mathematical formulation of the optimization problem. While KC variation is considered as an objective in robust parameter optimization, predefined limits of KCs usually form the constraint of tolerance-cost optimization, see Table 2.

In summary, it can be stated that the three domains differ significantly despite the consistency in the optimization approaches. This becomes particularly clear when comparing the *general aims* of the respective approaches, namely robust concept design, robust nominal design parameters and cost-optimal tolerance values. As the comparison in Table 2 shows, the various aspects in the domains are therefore considered differently. However, it can be seen that the domains depend on each other and that the common aim of improved robustness of the product to variation is achieved at best by a sequential combination of the domains. This inevitably requires knowledge about the interactions between the corresponding approaches.

3.2. Effects of Robust Concept Design on Parameter and Tolerance Optimization

Consequently, this section analyzes the effect of early robust design decisions on robust parameter and tolerance-cost optimization. However, since the principles underlying the early robust design are mostly attributable to a desired uncoupling or reduced information content according to the *Independence or Information Axiom*, see Table 1, the following analysis focuses on the effect of uncoupling and information reduction in concept design on the subsequent optimization process. Figure 3 illustrates the relationship between the axioms and the resulting design by means of a simplified stacked blocks case study (bottom) with corresponding matrix and graph representation. Compared to the usual notation in Axiomatic Design, this representation is adapted to the robust concept design context by using KC instead of functional requirement and the assumption that information reduction is achieved by reducing the number of elements contributing to a KC. Although this assumption is not always valid in general [68], it is reasonable in the context of robust concept design [23], since further information is usually lacking at this stage, which leads to a higher probability of shorter tolerance chains or less information content in the case of fewer elements.

Figure 3. Influence of the *Independence Axiom* (Axiom 1) and the *Information Axiom* (Axiom 2) on concept design.

The analysis of the effects for the optimization requires a detailed look on the respective mathematical optimization problems, already briefly discussed in Figure 1. In robust parameter design, the main goal is to improve the global system robustness by a conscious selection of the nominal design parameters X_i within predefined lower and upper boundaries X_i^{lb} and X_i^{ub} . Focusing on case (c) in Figure 1, the mean shift $\Delta\mu_k$ from the target value and the robustness metric R_k , such as variance or the sensitivity, are forming the objectives covering the individual K KCs. Thereby, the exact mathematical formulation and solution of the optimization problem strongly depend on the chosen robustness metric. In the literature, different methods were presented to statistically evaluate the effects of aleatory uncertainties on the system robustness. Thus, the effects of variation of the currently chosen nominal design parameters, e.g., with respect to its general tolerances [69], on the KC can be measured in the optimization using these metrics. Additional inequality constraints g_i and equality constraints h_j are necessary to cover design restrictions and to avoid technically infeasible solutions.

In the subsequent tolerance-cost optimization, it is the aim to identify the least-cost tolerances t_i within predefined tolerance ranges [34], see Figure 1e. Tolerance-cost optimization thus corresponds to the minimization of the total costs C_{sum} as the sum of all tolerance-related costs $C_i(t_i)$ [34]. Tolerance analysis is incorporated into the constraints to

assure product quality with respect to the predefined K KCs. In doing so, a single scrap rate or non-conformance rate for each KC as well as a total product scrap rate $\hat{z}_{\text{tot}}$ are commonly used to ensure product functionality [70,71]. The optimization problems can generally be defined as follows [46,71]:

<p style="text-align: center;">optimal parameter design:</p> <p>Minimize $\Delta\mu_k,$</p> <p style="text-align: center;">$R_k(\mathbf{X}) \forall k = 1, \dots, K,$</p> <p>subject to $g_i(\mathbf{X}) \leq 0, \forall i = 1, \dots, I,$</p> <p style="text-align: center;">$h_j(\mathbf{X}) = 0, \forall j = 1, \dots, J.$</p>	<p style="text-align: center;">least-cost tolerance design:</p> $C_{\text{sum}}(\mathbf{t}) = \sum_{i=1}^I C_i(t_i), \quad (1)$ $\hat{z}_k \leq z_{\max,k} \forall k = 1, \dots, K, \quad (2)$ $\hat{z}_{\text{tot}} \leq z_{\max}, \quad (3)$
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with $\Delta\mu_k$: mean shift between target value and mean of scattering of KC k ; R_k : robustness metric for KC k ; $C_{\text{sum}}(\mathbf{t})$: total costs for a specific set of tolerances; g_i : inequality constraints; h_j : equality constraints; $\hat{z}_k$: single scrap rate for KC k ; $\hat{z}_{\text{tot}}$: total product scrap rate.

As a consequence, the optimization problem and its solution mainly depend on the number (K) and interrelation of the KCs as well as their mathematical description. While in parameter design, the KCs are influencing the objective(s), the aim of optimal tolerance design leads to a shift of their consideration in the constraints, see Equation (1). Thus, the two axioms with the associated uncoupling and information reduction in the concept design affect the robust parameter optimization as well as the tolerance-cost optimization. Figure 4 gives a comprehensive summary of these effects.

The uncoupling of the KCs in the concept design (Axiom 1) results, as already demonstrated in the center in Figure 3, in completely independent KCs, each of which can be described separately. Thus, the transfer functions usually used to describe the mathematical relationship between varying parameters and the respective KC are also uncoupled so that all the parameters of the product always influence only one KC. This separate formulation of the underlying functional relation representing the product behavior simplifies the overall optimization problem. Thus, the multi-objective robust parameter optimization, which is necessary to consider the K interrelated KCs, see Equation (1), can be split into several uncoupled optimization problems for each individual KC. As a consequence, trade-offs, which usually require a weighting and prioritization of the analyzed KCs or a manual selection of the optimal result, e.g., from a Pareto front, are omitted. Thus, unambiguous optimal solutions result for each KC, since no compromises between potentially contradicting KCs are necessary. In addition, this leads to a reduced number of parameters that need to be considered in each single optimization. In the example in Figure 3, the number is thus lowered from five to three parameters each, leading to a splitting of the five-dimensional optimization space into two three-dimensional spaces. In the case of tolerance-cost optimization, the simplified optimization problem affects the constraint definition, see Equations (2) and (3) and Table 2. The uncoupling of the KCs leads to an elimination of the dependencies between the individual constraints, thus enabling a separate optimization on the basis of the single transfer functions. This, consequently, leads to a less restricted search space since the individual tolerances can be chosen with respect to just one single KC. The resulting additional optimization potential allows an independent selection of nominal design parameters and tolerances. Regardless of any sampling-based or optimization algorithm-related anomalies, this finally contributes to better optimization results in terms of robustness and costs.

Figure 4. Effects of concept design changes according to *Independence* and *Information Axiom* on robust parameter and tolerance-cost optimization.

According to the hypothesis stated above, reducing the number of elements involved in a KC (Axiom 2) in the conceptual design stage indicates a smaller number of varying parameters in the transfer function describing the KC in the subsequent optimization process. The lower number of parameters contributing to the respective KCs leads to a reduced variation of the KCs and thus to a less noisy optimization problem. This entails a reduced dimensionality of the search space as well as a lower complexity of the transfer functions. However, the level of complexity reduction induced by the *Information Axiom* differs significantly from the simplification described in the previous paragraph. Instead of eliminating the interactions between different KCs and transfer functions, Axiom 2 yields a reduction of the interactions of the parameters within a transfer function. However, the resulting reduced interactions in the parameter and tolerance definition also lead to basically better optimization results.

Thus, there is a significant effect of the axiomatic conceptual design changes on robust parameter optimization as well as the tolerance-cost optimization usually applied in the subsequent stages. While both concept changes associated with the axioms have a positive effect on the optimization, the application of the *Independence Axiom* is to be prioritized higher at first, since the complexity reduction caused by uncoupling may include a reduced information content in the then separated optimization. This is consistent with the Axiomatic Design philosophy according to Suh [10].

4. Application

The following case study enables the verification and exemplary quantification of the effect of uncoupling and information reduction in concept design on the subsequent robust parameter and tolerance-cost optimization. The basis for this analysis is four alternative design concepts, each with significant differences according to the fundamental *Independence and Information Axiom*. The subsequent optimization of these alternatives is mainly based on the initial concept sketches with a low level of detail in order to keep the impact of the further design detailing to a minimum and thus to clearly show the effect. After a short presentation of the case study, the optimization process is described and finally the results are discussed.

4.1. Presentation of the Case Study

The case study is based on the fundamental requirements of a ski binding used in alpine skiing. Its task is to fasten the skier's ski boot to the ski and to safely release this binding in the event of an imminent accident in order to prevent injuries [72]. Further important aspects such as ergonomic handling or individual adjustability of the binding are intentionally neglected in this study to ensure the focus on the effects of the axioms to be analyzed. The safety-relevant determination of corresponding release moments is of great importance in alpine skiing and is comprehensively regulated in standards such as [73] and [74]. The requirements described therein can be simplified to a horizontal force F_y to ensure contact pressure in the toe area of the binding and a vertical force F_z that ensures the release of the binding by a rotation around the x-axis, see Figure 5(top). Based on the normative requirements, the following specifications were defined, allowing a maximal scrap rate $z_{\max} = 2700$ ppm: $F_y = 1250 \text{ N} \pm 75 \text{ N}$, $F_z = 2400 \text{ N} \pm 100 \text{ N}$.

Boundary condition and basic structure

Concept alternatives

Figure 5. Boundary condition and basic structure of simplified ski binding (top) and summary of studied concept alternatives (bottom).

Based on the basic structure of the ski binding, see Figure 5(top), two basic theoretical concepts were first developed to meet the requirements. For reasons of clarity, the representation of these concepts (Binding 1a and Binding 2a) in Figure 5 is limited to the heel area of the binding, since the toe area in both concepts corresponds to the basic structure shown

at the top of Figure 5. In addition, the concept alternatives (b) in the lower part of Figure 5 are studied. These differ from Binding 1 and Binding 2 in the removal of additional soles attached to the ski boot and the integration of the front and heel binding parts, which are attached separately to the ski in the concept variants a (Figure 5(top)), into a common binding plate, see Figure 5(bottom). A comprehensive presentation of the four analyzed concepts can be found in Figures A1 and A2 in Appendix A.

These simplified concepts are intended to differ from established ones in order to ensure a similar level of detail and thus a good comparability between the concepts. Consequently, the case study neither claims to represent the holistic product development in the sense of Axiomatic Design, nor strives for an optimal concept, parameter and tolerance design for real-world use and should therefore be regarded primarily as an academic use case. The four alternative concepts shown in Figure 5 are briefly explained and analyzed with respect to the axioms below. Concept Binding 1a applies the two safety-relevant forces to the ski boot by means of a compression spring and an inclined plane. However, the incline leads to coupling of the KCs F_y and F_z . In contrast, according to the *Independence Axiom* (Axiom 1), the forces in Binding 2a are provided by two separate springs. By reducing or integrating elements, the variants b each represent simplified concepts with lower information content corresponding to the *Information Axiom* (Axiom 2). As it can be seen from the comprehensive mathematical functional description of the concepts in the Appendix A (see Equations (A1)–(A8)), this reduces the number of parameters contributing to the two KCs from 10 to 8 for Binding 1 and from 14 to 11 for Binding 2. This shows that the total information content is not necessarily reduced by uncoupling, as additional elements are often needed for this step. In compliance with Suh [10], the best design is first an uncoupled one and second with minimum information content. Thus, a comparison of the information content of Binding 1 and Binding 2 is not meaningful.

4.2. Parameter and Tolerance–Cost Optimization

To find the optimal robust parameter and tolerance design, the optimization problems from Equations (1)–(3) are solved in MATLAB R2019b using advanced optimization and sampling techniques. The utilized optimization process is thoroughly described in [70,71]. Additional information needed for the optimization of the concepts, such as design parameters and tolerances, is summarized in Table A1 in Appendix A.

Although there exist different approaches and measures for robust parameter optimization, the quality loss function is exemplarily used in this case study to define the objectives illustrating the influence of early robust design: $QL_k = \kappa_k \cdot (\sigma_k^2 + (\mu_k - m_k)^2)$ for each k KC, where $\kappa_k = \kappa = 1$ corresponds to the loss constant, σ_k^2 to the variance, μ_k to the mean and m_k to the target value [36]. In tolerance–cost optimization, the quality assurance is measured by the non-conformance rate representing the relative fraction of assemblies that do not meet the requirements. A non-parametric, empirical estimation technique is used to realistically consider the dependencies between the KCs in tolerance–cost optimization according to [71] forming the inequality conditions of Equations (2) and (3). With respect to computational efficiency, instead of ordinary Monte Carlo Sampling, the Latin Hypercube Sampling is used for tolerance analysis to evaluate the influence of uncertainty on the system behavior in terms of the quality loss and non-conformance rate. A sample size of 10,000 and 100,000 is set for the robust parameters respectively tolerance–cost optimization, as a compromise between validity and computational effort. The individual costs are defined by $C_i = 1/t_i$ forming the total costs C_{sum} as the objective according to Equation (1), where t_i are the individual tolerances, and the costs are given here in dollars for clarity.

For optimization, metaheuristic algorithms are preferred for the sampling-based robust parameter and tolerance-cost optimization since they can handle the stochastic effects of the sampling in the optimization. As a consequence, the Non-dominated Sorting Genetic Algorithm II is used for multi-objective robust parameter optimization and a single-objective Genetic Algorithm is applied for single-objective robust parameter and tolerance-

cost optimization. All optimization runs were repeated five times to increase the probability of finding the global optimum. In order to ensure comparability between the results of tolerance-cost optimization, the initially defined nominal values from Appendix A were used in tolerance-cost optimization, thus neglecting the results from previous robust parameter optimization. Corresponding to the theoretical description in Figure 4, this enables a separate consideration of the effects on robust parameter optimization and tolerance-cost optimization.

4.3. Discussion of the Results

Figure 6 shows the results from the **robust parameter optimization** for the four concepts (Binding 1a, 1b, 2a, 2b) presented in Figure 5. The quality loss values QL_{F_y} and QL_{F_z} represent the concept robustness with respect to the KCs F_y and F_z . As already described in Section 4.2, the quality loss value unites the influence of the variance and the mean shift of the respective KC. The multi-objective optimization of the concept Binding 1 with coupled KCs leads to ambiguous results and optimized nominal parameter values that are represented in a Pareto front each for concept 1a and 1b. The inverse relation between QL_{F_y} and QL_{F_z} requires the subsequent selection of a suitable trade-off solution with the lowest possible quality loss values. The comparison between Binding 1a and Binding 1b with a reduced number of parameters according to the *Information Axiom* shows reduced scattering of the KCs and a shift of the Pareto front into an area with lower quality loss values for the simplified concept. This unlocks additional potential. Therefore, the quality loss value for the exemplarily chosen trade-off solutions is reduced from 1470 to 1119 for QL_{F_y} and from 8043 to 6773 for QL_{F_z} . In a similar way, the robustness of the simplified concept Binding 2b is significantly higher than that of Binding 2a. The comparison of the results of Binding 1 with those of the uncoupled concepts of Binding 2 shows that considering the *Independence Axiom* results in optimal solutions respectively respective parameter values instead of Pareto fronts with trade-off solutions, see Figure 6. However, a quantitative comparison of the resulting quality loss values between the coupled and uncoupled concepts is not useful for general statements or a proof due to the changed boundary conditions associated with the concept change.

Figure 6. Parameter design: Optimal results for the four ski binding concepts.

Figure 7 shows the initial status for the **tolerance-cost optimization** carried out in addition to the robust parameter optimization discussed above. Considering only the initial tolerance settings from Table A1, the scattering of F_y is significantly lower for the simplified concept alternatives b with fewer parameters contributing to the KC, see Equations (A1), (A3), (A5) and (A7). This effect of the *Information Axiom* becomes particularly evident in the reduced non-conformance rates $\hat{z}$ in ppm (parts per million).

Figure 7. Tolerance design: Distribution of F_y for the four ski binding concepts for the initial tolerance design with $t_i = t_i^{ub}$ for all tolerances according to Table A1 with upper (USL) and lower specification limit (LSL) according to Equation (A9).

This positively affects the subsequent tolerance-cost optimization since the tolerances in the simplified concepts b have to be narrowed to a lesser extent in order to maintain the required quality target. Consequently, the consideration of the *Information Axiom* in the concept design reduced the optimized total tolerance costs C_{sum} from 37.42\$ (Binding 1a) to 27.00\$ (Binding 1b) and from 68.13\$ (Binding 2a) to 48.90\$ (Binding 2b). This can be explained by the reduced number of components involved resulting in fewer contributors in the determination of the KCs of the concept alternatives b. In tolerance-cost optimization, this means that the tolerances of the individual parameters require fewer limitations in systems with reduced information content, thus leading to lower tolerance costs. Figure 8 shows the optimized tolerance costs for the individual parameters for the four concept alternatives according to Figures A1 and A2. Since a comparison of tolerance costs for individual parameters is not appropriate due to the reallocation that takes place during optimization, the average costs of the parameters are compared. Therefore, the average tolerance cost per parameter for Binding 1 is reduced from 4.16\$ to 3.86\$ and for Binding 2 from 5.68\$ to 5.43\$. Thus, the application of the *Information Axiom* also leads to significantly better results in tolerance-cost optimization. However, similar to the robust parameter optimization, a quantitative comparison of the costs between the concepts 1 and 2 is not meaningful. This is due to the fact that uncoupling is often associated with major design changes. For example, the uncoupled concept of the case study contains significantly more parameters thus contributing to higher costs.

Figure 8. Tolerance design: Optimal tolerance-related costs for the four ski binding concepts.

5. Discussion

The previous sections have shown that there are considerable differences in the approaches and terminology between the early robust design and the subsequent optimizations but also interactions. In particular, it was shown in Figure 4 that an uncoupling and information reduction in the concept design associated with the *Independence and Information Axiom* has a positive impact on the subsequent optimization processes and thus also on the results. In addition to mathematical optimization, these findings are also valid for the manual optimization process by trial and error, whereby the specific results are additionally affected by human interaction here.

The positive effects attributed to the *Information Axiom* were found in a more robust solution in parameter optimization (Figure 6) as well as in lower optimal tolerance costs and less scattering of KCs (Figure 7). However, a low number of components does not necessarily mean high robustness [68]. This is only the case if each component is accompanied by the same information content, e.g., the same scattering contributing to the KC. Since there is usually hardly any information available in the concept stage and initial assumptions must therefore be made, e.g., on the basis of general tolerances, the assumption of an increase in information associated with additional components is permissible. Although the effect described can change along the product development process, e.g., through the definition of manufacturing processes, the statements made regarding the effect are useful for the concept stage considered in this contribution.

Quantifying the effect associated with the *Independence Axiom* is far more challenging because the uncoupling of KCs is usually accompanied by extensive conceptual design changes. These changes often lead to an increased number of components [68]. Thus, general statements from the comparison of optimal parameter and tolerance values between coupled and uncoupled systems are hardly possible and require individual evaluation. For example, changing the principle solution of the ski binding case study from wedge to lever leads to an increased number of parameters, so that the uncoupled concept is more robust (Figure 6) but has a higher cost optimum (Figure 8). Motivated by the fact that couplings can hardly be avoided in complex products, the development of the Contradiction Index shows that robustness is not directly tied to the coupling [75]. Instead, in the case of a coupling, the decisive factor is whether it is contradicting, leading to trade-offs and thus negatively affecting the robustness. The studies are primarily based on the fact that positive couplings lead to a larger potential solution space compared to contradicting ones, which reduces the necessary number of design iterations and tight tolerances [26]. Although this does not directly affect the definition of the optimization problem, the additional consideration of contradiction would improve the quantitative evaluation of the effect of uncoupling. However, in principle, it remains to be stated that a contradiction is already excluded in the case of an uncoupled design. Moreover, uncoupling in particular has a positive influence on the optimization process, so that, for example, independent and reliable solutions result in the case of the robust parameter optimization, see Figure 6.

6. Conclusions and Outlook

The comprehensive comparison of early robust design and robust parameter as well as tolerance-cost optimization with respect to numerous aspects shows that despite the common goal of a robust product design, there are significant differences in wording, starting point, aim and procedure. Despite the awareness of the importance of early robust design, it remains widely unconsidered as the direct effect or advantage for the subsequent robust design steps are often unclear. Thus, the contribution explicitly analyzes the effect of early robust design on robust parameter and tolerance-cost optimization. Since most activities in the robust concept design can be unified under the two axioms of Axiomatic Design, their effect on subsequent optimization is the focus of this article. Together with the findings from the case study, it can be stated that reducing the complexity of a product concept with the help of the *Information Axiom* has a significant positive effect on the process as well as the results of robust parameter and tolerance-cost optimization. Although

the *Independence Axiom* positively affects the optimization process, the quantitative effect cannot be determined in a generally valid manner, since its implementation is often accompanied by a comprehensive concept change so that comparability is no longer guaranteed. Nevertheless, an uncoupled concept design in the sense of the *Independence Axiom* with low information content according to the *Information Axiom* contributes to an overall optimal robust design. In summary, the contribution addresses the specific effect of early robust design on the subsequent steps for the first time, thus bridging the gap between early and late robust design, which strengthens the motivation for an early robustness consideration.

Further research based on the proposed results could emphasize the significance of the effects. In addition to the pure coupling of functional relations, the effect of contradiction should be analyzed for different cases and optimization problem formulations, especially with regard to the effect on the achieved results. Moreover, further studies with industrial examples would enable a more precise quantification of the effects of the applied axioms and could thus increase the motivation for the practical application of early robust design.

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Abbreviations

The following abbreviations are used in this manuscript:

CAD	Computer-aided design
DoE	Design of Experiments
KC	Key Characteristic
LSL	Lower Specification Limit
QL	Quality Loss
SNR	Signal-to-Noise Ratio
USL	Upper Specification Limit

Appendix A. Detailed Description of the Case Study

In this section, the case study of the ski binding from Figure 5 is described in detail. Two different concepts, namely **Binding 1** and **Binding 2**, are in focus to illustrate the influence of uncoupling in the sense of the *Independence Axiom* on optimal parameter and tolerance design. In addition, for each concept (**a**), a second alternative (**b**) is derived with the aid of reduced information content according to the *Information Axiom*, see Figures A1 and A2.

Appendix A.1. Binding 1

Figure A1. Schematic illustration of the concept of Binding 1 and its two alternatives (a,b).

(a)

$$F_y^{1a} = c \cdot \Delta_1 = X_{15} \cdot (20 - (X_{11} + X_{10} + X_9 - (X_1 + X_2 + \tan(X_3) \cdot X_5 + X_6 + X_7))), \quad (A1)$$

$$F_z^{1a} = F_y^{1a} \cdot \tan(X_3) = X_{15} \cdot (20 - (X_{11} + X_{10} + X_9 - (X_1 + X_2 + \tan(X_3) \cdot X_5 + X_6 + X_7))) \cdot \tan(X_3), \quad (A2)$$

(b)

$$F_y^{1b} = c \cdot \Delta_1 = X_{15} \cdot (20 - (X_8 - (X_1 + X_2 + \tan(X_3) \cdot X_5 + X_6 + X_7))), \quad (A3)$$

$$F_z^{1b} = F_y^{1b} \cdot \tan(X_3) = X_{15} \cdot (20 - (X_8 - (X_1 + X_2 + \tan(X_3) \cdot X_5 + X_6 + X_7))) \cdot \tan(X_3), \quad (A4)$$

with: c spring rate of compression spring; $\Delta_1 = l_{1,0} - s_1$: compression of spring; initial compression length $l_{1,0} = 20$ mm; s_1 : resulting gap.

Appendix A.2. Binding 2

Figure A2. Schematic illustration of the concept of Binding 2 and its two alternatives (a,b).

(a)

$$F_y^{2a} = c_1 \cdot \Delta_1 = X_{15} \cdot (20 - (X_9 + X_{10} + X_{11} - X_1 - X_2 - X_6 - X_7)), \quad (A5)$$

$$F_z^{2a} = \frac{X_{13} \cdot c_2 \cdot \Delta_2}{X_{14} \cdot \cos(\beta_a)} = \frac{X_{13} \cdot X_{16} \cdot (40 - (X_{13} \cdot \tan(\beta_a) + X_{12}))}{X_{14} \cdot \cos(\beta_a)} \quad (A6)$$

$$= \frac{X_{13} \cdot X_{16} \cdot (40 - (X_{13} \cdot \tan(\arcsin(\frac{X_{12} - X_{4a} - X_5}{X_{14}})) + X_{12}))}{X_{14} \cdot \cos(\arcsin(\frac{X_{12} - X_{4a} - X_5}{X_{14}}))},$$

(b)

$$F_y^{2b} = c_1 \cdot \Delta_1 = X_{15} \cdot (20 - (X_8 - X_1 - X_2 - X_6 - X_7)) \quad (A7)$$

$$F_z^{2b} = \frac{X_{13} \cdot c_2 \cdot \Delta_2}{X_{14} \cdot \cos(\beta_b)} = \frac{X_{13} \cdot X_{16} \cdot (40 - (X_{13} \cdot \tan(\beta_b) + X_{12}))}{X_{14} \cdot \cos(\beta_b)} \quad (A8)$$

$$= \frac{X_{13} \cdot X_{16} \cdot (40 - (X_{13} \cdot \tan(\arcsin(\frac{X_{12} - X_{4b}}{X_{14}})) + X_{12}))}{X_{14} \cdot \cos(\arcsin(\frac{X_{12} - X_{4b}}{X_{14}}))},$$

with: $c_{1/2}$ spring rate of compression spring 1/2; $\Delta_1 = l_{1/2,0} - s_{1/2}$: compression of spring; initial compression length $l_{1,0} = 20$ mm, $l_{2,0} = 40$ mm; $s_{1/2}$: resulting gap. The width of the clamping lever is neglected.

Appendix A.3. Specification

The target values for the clamping force $F_{y,0}$ and the release force $F_{z,0}$ with its lower and upper specification limits LSL , USL are inspired by [73,74] with a maximum scrap rate of $z_{\max} = 2700$ ppm:

$$F_{y,0} = 1250 \text{ N} \pm 75 \text{ N}, \quad LSL_y = 1175 \text{ N}, \quad USL_y = 1325 \text{ N}, \quad (A9)$$

$$F_{z,0} = 2400 \text{ N} \pm 100 \text{ N}, \quad LSL_z = 2300 \text{ N}, \quad USL_z = 2500 \text{ N}. \quad (A10)$$

Appendix A.4. Design Parameter

Table A1 gives an overview on the design parameters, their nominal values and tolerances.

Table A1. Summary of the design parameters, its nominal values and tolerances for the ski binding.

Parameter	Description	Nominal Value $X_{i,0}$	Tolerance t_i
X_1	binding heel length 1	5 mm	[0.01;0.20] mm
X_2	binding heel length 2	20 mm	[0.01;0.40] mm
X_3	wedge angle	[10;80]°	[0.01;2.00]°
X_{4a}	ski boot heel size	6 mm	[0.01;0.40] mm
X_{4b}	ski boot heel size	10 mm	[0.01;0.40] mm
X_5	ski boot sole size	4 mm	[0.01;0.40] mm
X_6	ski boot length	302 mm	[0.01;1.00] mm
X_7	binding length 3	30 mm	[0.01;0.60] mm
$X_{8,1/2}$	total binding length	372/362 mm	[0.01;1.00] mm
X_9	binding heel length	140 mm	[0.01;1.00] mm
$X_{10,1/2}$	distance between binding heel and toe	92/82 mm	[0.01;0.60] mm
X_{11}	binding toe length	140 mm	[0.01;1.00] mm
X_{12}	vertical position of pivot lever	[9;15] mm	[0.01;0.40] mm
X_{13}	horizontal position of spring 2	[6;20] mm	[0.01;0.40] mm
X_{14}	clamping lever length	[5;15] mm	[0.01;0.40] mm
X_{15}	spring rate c_1	[50;350] N/mm	5.5 N/mm
X_{16}	spring rate c_2	[50;350] N/mm	5.5 N/mm

If the nominal values or tolerances are considered as optimization variables in the robust parameter or tolerance-cost optimization, the feasible ranges are given, respectively:

- Nominal values: $X_{i,0} \in \mathbb{R}_0^+$, $X_i \in [X_i^{lb}, X_i^{ub}]$,
- Dimensional and angular tolerances $t_i \in \mathbb{R}_0^+$, $t_i \in [t_i^{lb}, t_i^{ub}]$,

For robust parameter optimization, the tolerances are set equal to the upper boundaries $t_i = t_i^{ub}$ of the tolerances in tolerance design with respect to the individual ranges. The upper limits for the dimensional tolerances are chosen in accordance to ISO 2768-m. The spring rate tolerances are chosen as constants with respect to available standard springs. Thus, the fixed tolerance-related costs are neglected in tolerance-cost optimization. All tolerances are considered with a standard normal distribution (mean $\mu_i = X_{i,0}$, standard deviation $\sigma_i = t_i/6$). For tolerance-cost optimization, the subsequent nominal values are used for all variants to ensure the comparability of the optimization results: $X_1 = 5$ mm, $X_2 = 20$ mm, $X_3 = 62.5^\circ$, $X_{4a} = 6$ mm, $X_{4b} = 10$ mm, $X_5 = 4$ mm, $X_6 = 302$ mm, $X_7 = 30$ mm, $X_{8,1/2} = 372/362$ mm, $X_9 = 140$ mm, $X_{10,1/2} = 92/82$ mm, $X_{11} = 140$ mm, $X_{12} = 13$ mm, $X_{13} = 12$ mm, $X_{14} = 8$ mm, $X_{15,1/2} = 98.55/83.33$ N/mm and $X_{16} = 67.00$ N/mm.

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