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Integral-Type Edge-Event- and Edge-Self-Triggered Synchronization to Multi-Agent Systems with Lur'e Nonlinear Dynamics

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Abstract: This paper proposes event- and self-triggered control strategies to achieve distributed synchronization for multiple Lur'e systems with unknown static nonlinearities. Firstly, the integral-type edge-event-triggered function is designed here without Zeno behaviors. Compared to the traditional event-triggered schemes, the considered algorithm has the advantages of reducing controller update frequency and sensor energy consumption. Then, the integral-type self-triggered is further investigated, which implements discontinuous monitoring and discontinuous agent listening. Finally, numerical simulations verified the effectiveness and superiority of our policies.

Keywords: synchronization; integral-type event-triggered control; Lur'e nonlinear system; self-triggered control



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1. Introduction

In recent years, distributed control of multi-agent systems has emerged as an essential topic in the field of control theory [1–12], and it has been applied in more and more fields such as the coordinated attitude of multiple spacecraft, formation cooperative control, multi-drone coordination, and multiple robotic arms coordination. The main concern on distributed control is how to coordinate all individuals to achieve synchronization and consensus control under information interaction [1,2,5,6]. Generally, sampled-data control is applied to the studies on synchronization and consensus analysis in actual digital computers, among which the most common solution is periodic sampling based on a time-triggered mechanism. However, consensus analysis and performance requirements typically result in conservative choices of the sampling period (i.e., minimum), which consume excessive system resources.

In contrast to specific time intervals, the other is event-triggered sampling control, whose sampling instants are determined by preset triggering conditions [13–25]. Here, these events are triggered at time points when the norm of a certain measurement error becomes large to the norm of a designed function as time evolves. Earlier studies on event-triggered control include a conference paper on event-triggered control of PID controllers [13]. However, the application and development of event-triggered control theories have not made a good progress for a long time. This is because Zeno behavior often appears in event-triggered control systems, and its related theoretical problems have not been well resolved. Until 2007, Tabuada [14] strictly proved that there is a minimum time-bound between the interval of any two consecutive events to avoid Zeno behaviors, in a class of nonlinear feedback control systems based on state-dependent event-triggered control. Furthermore, self-triggered control strategies were developed in [26–30], in which

each agent predicts its next triggering time at any event instant; that is, continuous or periodic event detections can be avoided. Such schemes aim to replace the resource consumption of all sensors with an increase in computational complexity.

The above event-triggered and self-triggered conditions are all designed to be state-dependent or decaying exponential function-dependent, which demands that the measurement error needs to meet these conditions at any time. To relax such conservative settings, in [31], Girard designed an event-triggered control mechanism determined by a dynamic differential equation. Subsequently, in [32], Mousavi et al. proposed an integral event-triggered condition, which requires that the measurement error only needs to be less than the given threshold function over a certain integration interval. Then, in [33,34], Zhang et al. extend the integral-type method to first-order linear multi-agent systems. The integral-type event-triggered control relaxed the requirement for measurement error and also relaxed the requirement for decreasing Lyapunov functions, further reducing the controller updates.

In this paper, we study the design problem of integral-type edge-event- and edge-self-triggered control for Lur'e uncertain nonlinear systems with unknown static nonlinearities. First, the research on synchronization of Lur'e systems has greatly promoted the development of control theory [35–39]. Recently, distributed synchronization of multi-agent systems with Lur'e node dynamics was investigated in Wen et al. (2013). Robust synchronization of multi-agent systems with uncertain Lur'e-type nonlinear dynamics was studied in Zhao et al. (2013). Then, most distributed event-triggered control policies require each agent to gather its own information and then communicate with its neighbors to achieve global control goals. In actual scenarios, the accurate measurement of self-state information is often difficult to achieve. For instance, the individual's own position, velocity, and other information often need to be obtained with the help of GPS or inertial sensors using of sophisticated algorithmic measurements. Here, it may be easier for agents to implement coordinated control by using relative state information. Therefore, another state measurement scheme is applied. The relative state information between itself and its neighbors, termed edge state information, is measured by means of relative state sensors such as Lidar or laser speedometer mounted on each individual to achieve the control goal. Quite a number of studies on synchronization and consensus analysis for distributed edge-event-triggered control have been represented [40–42].

Hinted by these observations, the integral-type edge-event- and edge-self-triggered control strategies are developed for Lur'e uncertain nonlinear systems in this paper. In the proposed event-triggered strategy, an integral-type triggering function is designed to reduce the controller updates. Moreover, we accomplish the edge-self-triggered control on the above system to avoiding continuous monitoring, in which each controller only updates at certain sampling instants and can effectively reduce controller burden and sensor energy consumption. The main contributions of this work are stated as follows:

1. The dominant motive of this work is to design integral-type edge-event- and edge-self-triggered control strategies for Lur'e uncertain nonlinear systems to seek novel scheduling policies of active sensors. Here, only relative states are employed, while absolute state information is uninvolved. These combined edge-based control policies can provide another approach, where absolute state information is not easily available [16,17,19,20,23].
2. A distributed integral-type edge-event-triggered control algorithm is designed in the proposed control strategies, which relaxes the setting of the measurement error in the traditional event-triggered strategy [13–17,19–23,25] that needs to meet the trigger conditions at all times. The application of Barbalat's Lemma in proof guarantees the convergence. Compared to the studies on distributed integral-type event-triggered control in [33,34], our results on edge states related to each agent are evaluated asynchronously, based on nonlinear dynamic models.
3. The self-triggering condition does not require continuous or periodic detection of measurement error information, which avoids the periodic monitoring of the

sensor [13–17,19–23,25,32,33], further reducing the number of sensor measurement samples and then saving system resources.

The remainder of this paper is organized as follows. Some preliminaries and the problem formulation are given in Section 2. The main results are presented in Sections 3 and 4. Section 5 validates the obtained theoretical results using numerical simulations. Finally, concluding remarks close the paper in Section 6.

2. Preliminaries

2.1. Preliminaries on Graph Theory

Consider an undirected graph \mathcal{G} with \mathcal{N} edges and N nodes, which is denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with node set $\mathcal{V} = \{v_1, v_2, \dots, v_n\}$ and edge set $\mathcal{E} = \{(v_i, v_j) : v_i, v_j \in \mathcal{V}\}$. The set of neighbours of node i is denoted by $N_i = \{v_j \in \mathcal{V} : (v_j, v_i) \in \mathcal{E}\}$, and $|N_i|$ denotes the cardinality of N_i , i.e. $\mathcal{N} = |N_1| + |N_2| + \dots + |N_N|$. Then the weighted adjacency matrix $A \in \mathbb{R}^{N \times N}$ is used to describe the topology of system (2), in which self-loops and multiple information links are not allowed, where $a_{ij} > 0$ if $(v_i, v_j) \in \mathcal{E}$, while $a_{ij} = 0$, otherwise. The graph Laplacian of \mathcal{G} is defined as $L = \{l_{ij}\} \in \mathbb{R}^{N \times N}$, in which $l_{ij} = -a_{ij}$ if $i \neq j$, and $l_{ij} = \sum_{j \in N_i} a_{ij}$ otherwise. We now introduce several important matrices in the graph theory. The matrix which relates the nodes to the edges is called incidence matrix, denoted by $H = \{h_{ki}\} \in \mathbb{R}^{N \times N}$ [23]. By choosing arbitrary orientation for all edges for an undirected graph, the entries of its incidence matrix are defined as $h_{ki} = 1$ if the k th edge sinks at node i , or $h_{ki} = -1$ if the k th edge leaves node i , or $h_{ki} = 0$ otherwise. Therefore, the corresponding Laplacian matrix is represented as $L = H^T H$ [4,43].

For the edge $\varepsilon_p \in \mathcal{E}$ with the orientation from agents v_j to v_i , let $r_p(t) = r_{(i,j)}(t) = x_i(t) - x_j(t)$ be the corresponding edge state. We say that system (2) reaches the state synchronization if and only if $\lim_{t \rightarrow \infty} \|r_p(t)\| = 0$, $p = 1, 2, \dots, \mathcal{N}$, for any initial values $r_p(t_0)$. The following assumption and lemma are presented:

Definition 1. [44] Let $S_1, S_2 \in \mathbb{R}^{s \times s}$ be real symmetric matrices such that S_1 is positive semi-definite and $S_1 - S_2$ is positive definite, i.e., $0 \leq S_1 < S_2$. Then $\phi(\cdot)$ is called sector bounded within the sector $[S_1, S_2]$ if it satisfies

$$(\phi(y) - S_1 y)^T (\phi(y) - S_2 y) \leq 0, \tag{1}$$

for all $y \in \mathbb{R}^m$.

Assumption 1. The interaction topology G of system (2) is connected.

Lemma 1. There is $\phi(\cdot)$ called sector bounded within the sector $[S_1, S_2]$, such that

$$\|\phi(y_i) - \phi(y_j)\|^2 \leq \eta \|y_i - y_j\|^2 \leq \eta \|C\|^2 \|x_i - x_j\|^2.$$

ie. $[(H \otimes I)\Phi(y)]^T [(H \otimes I)\Phi(y)] \leq \eta \|C\|^2 R(t)^T R(t)$.

Proof. By (1), we have

$$\phi^T(y)\phi(y) < \phi^T(y)(S_1 + S_2)y - \frac{1}{2} \left(y^T (S_1 S_2 + S_2 S_1) y \right).$$

Using the fact $-2\phi^T(y_i)\phi(y_j) \leq \phi^T(y_i)\phi(y_i) + \phi^T(y_j)\phi(y_j)$, one gets

$$\begin{aligned} \|\phi(y_i) - \phi(y_j)\|^2 &= (\phi(y_i) - \phi(y_j))^T (\phi(y_i) - \phi(y_j)) \\ &= \phi^T(y_i)\phi(y_i) - 2\phi^T(y_i)\phi(y_j) + \phi^T(y_j)\phi(y_j) \\ &\leq 2(\phi^T(y_i)\phi(y_i) + \phi^T(y_j)\phi(y_j)) \\ &\leq 2\phi^T(y_i)(S_1 + S_2)y_i - y_i^T(S_1S_2 + S_2S_1)y_i + 2\phi^T(y_j)(S_1 + S_2)y_j - y_j^T(S_1S_2 + S_2S_1)y_j. \end{aligned}$$

Then we derived that

$$\|\phi(y_i) - \phi(y_j)\|^2 = Y^T \begin{bmatrix} D_3 & 0 & -D_2 & 0 \\ 0 & D_3 & 0 & -D_2 \\ -D_2 & 0 & 0 & 0 \\ 0 & -D_2 & 0 & 0 \end{bmatrix} Y,$$

where $D_2 = -(S_1 + S_2)$, $D_3 = -(S_1S_2 + S_2S_1)$. $Y = (y_i^T, y_j^T, \phi^T(y_i), \phi^T(y_j))^T$. Then in order to prove that $\|\phi(y_i) - \phi(y_j)\|^2 \leq \eta\|y_i - y_j\|^2$, we just need to find the positive constant η that satisfies that

$$\begin{bmatrix} D_3 & 0 & -D_2 & 0 \\ 0 & D_3 & 0 & -D_2 \\ -D_2 & 0 & 0 & 0 \\ 0 & -D_2 & 0 & 0 \end{bmatrix} \leq \eta \begin{bmatrix} I & -I & 0 & 0 \\ -I & I & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

i.e.,

$$S = \begin{bmatrix} D_1 & -\eta I & D_2 & 0 \\ -\eta I & D_1 & 0 & D_2 \\ D_2 & 0 & 0 & 0 \\ 0 & D_2 & 0 & 0 \end{bmatrix} \geq 0,$$

where $D_1 = \eta I + (S_1S_2 + S_2S_1)$. The above matrix S is semi-positive definite if and only if its contract matrix \hat{S} is semi-positive definite.

$$\hat{S} = \begin{bmatrix} D_1 & -\eta I & 0 & 0 \\ -\eta I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

based on the fact that $D_2, -\eta I$ are invertible matrices. The following, we take $\eta > \lambda_m(S_1S_2 + S_2S_1)$ with λ_m the maximum eigenvalue of the matrix $S_1S_2 + S_2S_1$, such that $D_2 > 0$.

Similarly, the $\hat{S} \geq 0$ is equivalent to

$$\begin{bmatrix} D_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \geq 0.$$

Therefore, there's an η here that makes $\|\phi(y_i) - \phi(y_j)\|^2 \leq \eta\|y_i - y_j\|^2$ true. This completes the proof. \square

2.2. Problem Formulation

We consider a event-triggered control of Lur’e systems.

$$\begin{aligned} x'_i(t) &= Ax_i(t) + Bu_i(t) + B_1\phi(y_i(t)), \\ y_i(t) &= Cx_i(t), i \in \nu, \end{aligned} \tag{2}$$

where $x_i(t) \in \mathbb{R}^n$ is the state vector of the i th agent, $u_i(t) \in \mathbb{R}^p$ is the control input, and $y_i(t) \in \mathbb{R}^m$ is the output. Let $A, B, B_1,$ and C be given constant matrices with appropriate dimensions. The function $\phi(y_i(t)) \in \mathbb{R}^p$ is defined as follows.

To this end, the focal issue is how to design a distributed protocol u_i according to an edge-event-triggered rule, which will be detailed in the next section.

3. Integral-Type Edge-Event-Triggered Policy

Here, the control strategy will be introduced. The designed policy on combined relative state measurements is given as follows:

$$u_i(t) = -cKz_i(t_k^i), t \in [t_k^i, t_{k+1}^i), \quad i = 1, 2, \dots, N, \tag{3}$$

where $z_i(t) = \sum_{j \in N_i} a_{ij}(x_i(t) - x_j(t))$. Here, $x_i(t) - x_j(t)$ with $j \in N_i$ represent the relative states between v_i and all neighboring agents v_j with $j \in N_i$.

We introduce the following integral-type triggering condition. Define the state measurement error between the sampled state and the current state by

$$e_i(t) = z_i(t_k^i) - z_i(t), t \in [t_k^i, t_{k+1}^i). \tag{4}$$

Each triggering instant t_k^i is determined by

$$t_{k+1}^i = \inf\{t > t_k^i : f_i(t) \geq 0\}, \quad i \in \mathbb{N}, \tag{5}$$

where

$$f_i(t) = \int_{t_i}^t \|e_i(s)\|^2 ds - \int_{t_i}^t k_i \|z_i(t_k^i)\|^2 + \beta e^{-\alpha s} ds. \tag{6}$$

Here, k_i, β and α are positive constants to be designed.

Denote $R(t) = \{r_p(t)\}_{\mathcal{N} \times 1} = (H \otimes I)x(t)$. Under protocol (3), the closed-loop system can be represented by

$$\begin{aligned} R'(t) &= (I_N \otimes A)R(t) - c(HH^T \otimes BK)R(t) - c(H \otimes BK)e(t) + (H \otimes B_1)\Phi(y) \\ y(t) &= (I_N \otimes C)x(t), \end{aligned} \tag{7}$$

where $x(t) = (x_1^T(t), x_2^T(t), \dots, x_N^T(t))^T, e(t) = (e_1^T(t), e_2^T(t), \dots, e_N^T(t))^T, y(t) = (y_1^T(t), y_2^T(t), \dots, y_N^T(t))^T$. We note that $z(t) = (E \otimes I_n)R(t), E = \{e_{ij}\} \in \mathbb{R}^{N \times N}$. The entries of matrix E are defined as $e_{ij} = 1$ if the j th entry of $R(t)$ is $r_i(t) - r_j(t)$, or $e_{ij} = -1$ if the j th entry of $R(t)$ is $r_j(t) - r_i(t)$, or $e_{ij} = 0$ otherwise. ie. $E = H^T, z(t) = (H^T \otimes I_n)R(t)$. According to (1), $\Phi(y) = (\phi^T(y_1), \dots, \phi^T(y_N))^T$ satisfies $[\Phi(y) - (I_N \otimes S_1)y]^T[\Phi(y) - (I_N \otimes S_2)y] \leq 0$. Then $\Phi(\cdot)$ is called sector bounded within the sector $[I_N \otimes S_1, I_N \otimes S_2]$.

The following theorem presents some sufficient conditions for achieving synchronism tracking of system (2) with integral event-triggered information.

Theorem 1. For a given positive real constant $a, \eta > \lambda_m(S_1S_2 + S_2S_1), \gamma = \frac{2k_{max}}{1-2k_{max}}, k_{max} < \frac{1}{2}$, if there exists a symmetric positive definite $X \in \mathbb{R}^{n \times n}$, a matrix $Y \in \mathbb{R}^{p \times n}$ and positive real constants c , such that the linear matrix inequality

$$\begin{bmatrix} \bar{A} + \bar{B} & I_N \otimes X & H \otimes X & I_N \otimes B_1 & H \otimes BY \\ I_N \otimes X & -\frac{1}{\eta \|C\|^2} I_{N \times n} & 0 & 0 & 0 \\ H^T \otimes X & 0 & -\frac{1}{a\gamma} I_{N \times n} & 0 & 0 \\ I_N \otimes B_1^T & 0 & 0 & -I_{N \times m} & 0 \\ H^T \otimes Y^T B^T & 0 & 0 & 0 & -\frac{a}{c^2} I_N \otimes \bar{X} \end{bmatrix}, \tag{8}$$

is negative definite matrix, where $\bar{A} = I_N \otimes (XA^T + AX), \bar{B} = -c[HH^T \otimes (BY + Y^T B^T)], \bar{X} = I_n - 2X$, then the system (7) with $K = YX^{-1}$ consensus tracking for sector bounded within the sector $[I_N \otimes S_1, I_N \otimes S_2]$ under the integral-type edge-event-triggering condition (6). Furthermore, no Zeno behavior occurs for all $t > t_0$.

Proof. Choose the following Lyapunov function:

$$V(t) = R^T(t)(I_N \otimes P)R(t), \tag{9}$$

where P is a positive-definite matrix with $P = X^{-1}$. Then, taking the time derivative of $V(t)$ along the trajectory of (7) yields

$$\begin{aligned} V'(t) &= R^T(t) \left((I_N \otimes A^T P) + (I_N \otimes PA) - c(HH^T \otimes PBK) - c(HH^T \otimes K^T B^T P) \right) R(t) \\ &\quad - R^T(t) (2c(H \otimes PBK))e(t) + R^T(t) (2(H \otimes PB_1))\Phi(y) \\ &\leq R^T(t) \left((I_N \otimes A^T P) + (I_N \otimes PA) - c(HH^T \otimes PBK) - c(HH^T \otimes K^T B^T P) \right) \\ &\quad + \frac{c^2}{a} (HH^T \otimes PBK K^T B^T P) R(t) + ae^T(t)e(t) + R^T(t) [2(H \otimes PB_1)]\Phi(y), \end{aligned} \tag{10}$$

where we use the fact that $-R^T[2(H \otimes PBK)]e \leq \frac{1}{a}R^T(HH^T \otimes PBK K^T B^T P)R + ae^T e$ for any positive real constant a to derive the above inequality. However, a has to be fixed due to the product of a and X . \square

In addition, it follows from the event condition (6) that

$$\int_{t_0}^t \|e(s)\|^2 ds \leq \frac{1}{1-2k_{max}} \int_{t_0}^t 2k_{max} \|z(s)\|^2 + \beta N e^{-\alpha s} ds, \tag{11}$$

Noting that $k_{max} = \max_{i \in \{1,2,\dots,N\}} k_i < \frac{1}{2}$ and $z(t) = (H^T \otimes I_n)R(t)$. Then, integrating (10) from t_0 to t , one gets

$$\begin{aligned} V(t) - V(t_0) &\leq \int_{t_0}^t R^T(s) \left((I_N \otimes A^T P) + (I_N \otimes PA) - c(HH^T \otimes PBK) - c(HH^T \otimes K^T B^T P) \right) \\ &\quad + \frac{c^2}{a} (HH^T \otimes PBK K^T B^T P) R(s) + ae^T(s)e(s) + R^T(s) (2(H \otimes PB_1))\Phi(y) ds \\ &\leq \int_{t_0}^t R^T(s) \left((I_N \otimes A^T P) + (I_N \otimes PA) - c(HH^T \otimes PBK) - c(HH^T \otimes K^T B^T P) \right) \\ &\quad + \frac{c^2}{a} (HH^T \otimes PBK K^T B^T P) + \frac{2ak_{max}}{1-2k_{max}} (E^T E \otimes I_n) R(s) + R^T(s) (2(H \otimes PB_1))\Phi(y) \\ &\quad + \frac{a\beta N}{1-2k_{max}} e^{-\alpha s} ds. \end{aligned} \tag{12}$$

Similarly, by the fact that $R^T(2(H \otimes PB_1))\Phi(y) \leq R^T(I_N \otimes PB_1B_1^T P)R + ((H \otimes I_m)\Phi(y))^T((H \otimes I_m)\Phi(y))$, we have

$$\begin{aligned}
 V(t) - V(t_0) &\leq \int_{t_0}^t R^T(s) \left((I_N \otimes A^T P) + (I_N \otimes PA) - c(HH^T \otimes PBK) - c(HH^T \otimes K^T B^T P) \right. \\
 &\quad \left. + \frac{c^2}{a}(HH^T \otimes PBK K^T B^T P) + \frac{2ak_{max}}{1 - 2k_{max}}(HH^T \otimes I_n) + (I_N \otimes PB_1B_1^T P) \right) R(s) \quad (13) \\
 &\quad + ((H \otimes I_m)\Phi(y))^T((H \otimes I_m)\Phi(y)) + \frac{a\beta N}{1 - 2k_{max}} e^{-as} ds,
 \end{aligned}$$

and then, based on Lemma 1, we get

$$\begin{aligned}
 V(t) - V(t_0) &\leq \int_{t_0}^t R^T(s) \left(((I_N \otimes A^T P) + (I_N \otimes PA) - c(HH^T \otimes PBK) - c(HH^T \otimes K^T B^T P) \right. \\
 &\quad \left. + \frac{c^2}{a}(HH^T \otimes PBK K^T B^T P) + \frac{2ak_{max}}{1 - 2k_{max}}(HH^T \otimes I_n) + (I_N \otimes PB_1B_1^T P) \right. \quad (14) \\
 &\quad \left. + \eta \|C\|^2 I_{N \times n} \right) R(s) + \frac{a\beta N}{1 - 2k_{max}} e^{-as} ds.
 \end{aligned}$$

The following, we will prove that $(I_N \otimes A^T P) + (I_N \otimes PA) - c(HH^T \otimes PBK) - c(HH^T \otimes K^T B^T P) + \frac{c^2}{a}(HH^T \otimes PBK K^T B^T P) + \frac{2ak_{max}}{1 - 2k_{max}}(HH^T \otimes I_n) + (I_N \otimes PB_1B_1^T P) + \eta \|C\|^2 I_{N \times n} < 0$ holds.

On the other hand, from (8), we have that

$$\begin{bmatrix} \bar{A} + \bar{B} & I_N \otimes X & H \otimes X & I_N \otimes B_1 & H \otimes BY \\ I_N \otimes X & -\frac{1}{\eta \|C\|^2} I_{N \times n} & 0 & 0 & 0 \\ H^T \otimes X & 0 & -\frac{1}{a\gamma} I_{N \times n} & 0 & 0 \\ I_N \otimes B_1^T & 0 & 0 & -I_{N \times m} & 0 \\ H^T \otimes Y^T B^T & 0 & 0 & 0 & -\frac{a}{c^2} I_N \otimes X^2 \end{bmatrix}, \quad (15)$$

is a negative definite matrix, due to the fact that $-X^2 \leq I_n - 2X$ and $(I_n - X)^2 = I_n - 2X + X^2 \geq 0$. By premultiplying and postmultiplying the matrix inequality with $\text{diag}(I_N \otimes P, I_{N \times n}, I_{N \times n}, I_{N \times m}, I_N \otimes P)$, along with $K = YX^{-1}$.

$$\begin{bmatrix} \hat{A} + \hat{B} & I_{N \times n} & H \otimes I_n & I_N \otimes PB_1 & H \otimes PBK \\ I_{N \times n} & -\frac{1}{\eta \|C\|^2} I_{N \times n} & 0 & 0 & 0 \\ H^T \otimes I_n & 0 & -\frac{1}{a\gamma} I_{N \times n} & 0 & 0 \\ I_N \otimes B_1^T P & 0 & 0 & -I_{N \times m} & 0 \\ H^T \otimes K^T B^T P & 0 & 0 & 0 & -\frac{a}{c^2} I_{N \times n} \end{bmatrix}, \quad (16)$$

is negative definite, where $\hat{A} = I_N \otimes A^T P + PA$, $\hat{B} = -c[HH^T \otimes (PBK + K^T B^T P)]$.

Then, using the Schur complement lemma, we obtain that

$$\begin{bmatrix} \hat{A} + \hat{B} + \hat{C} & I_{N \times n} & H \otimes I_n & I_N \otimes PB_1 \\ I_{N \times n} & -\frac{1}{\eta \|C\|^2} I_{N \times n} & 0 & 0 \\ H^T \otimes I_n & 0 & -\frac{1}{a\gamma} I_{N \times n} & 0 \\ I_N \otimes B_1^T P & 0 & 0 & -I_{N \times m} \end{bmatrix} < 0, \quad (17)$$

where $\hat{C} = \frac{c^2}{a}(HH^T \otimes PBK K^T B^T P)$. Similarly, we have that

$$\begin{bmatrix} \hat{A} + \hat{B} + \hat{C} + I_N \otimes PB_1B_1^T P + a\gamma(HH^T \otimes I_n) & 0 \\ 0 & -\frac{1}{\eta \|C\|^2} I_{N \times n} \end{bmatrix} < 0, \quad (18)$$

being equivalent to $\hat{A} + \hat{B} + \hat{C} + I_N \otimes PB_1B_1^T P + a\gamma(HH^T \otimes I_n) + \eta \|C\|^2 I_{N \times n} < 0$. i.e., $(I_N \otimes A^T P) + (I_N \otimes PA) - c(HH^T \otimes PBK) - c(HH^T \otimes K^T B^T P)$

+ $\frac{c^2}{a}(HH^T \otimes PBKK^T B^T P) + a\gamma(HH^T \otimes I_n) + (I_N \otimes PB_1 B_1^T P) + \eta \|C\|^2 I_{N \times n} < 0$. Therefore, the above matrix is negative definite, with defined as $-\hat{P}$. Then

$$\begin{aligned} V(t) - V(t_0) &\leq \int_{t_0}^t -R^T(s)\hat{P}R(s) + \frac{a\beta N}{1 - 2k_{max}} e^{-\alpha s} ds \\ &\leq \int_{t_0}^t -\lambda_{min}(\hat{P})\|R(s)\|^2 + \frac{a\beta N}{1 - 2k_{max}} e^{-\alpha s} ds \end{aligned} \tag{19}$$

where $\lambda_{min}(\hat{P})$ is the minimum eigenvalue of the matrix \hat{P} . Inequality (19) yields

$$\int_{t_0}^t \|R(s)\|^2 ds \leq \frac{V(t_0)\alpha(1 - 2k_{max}) + a\beta N e^{-\alpha t_0}}{\alpha(1 - 2k_{max})\lambda_{min}(\hat{P})} \tag{20}$$

based on the fact that $V(t) \geq 0, e^{-\alpha t} > 0$, As a result, $\int_{t_0}^t \|R(s)\|^2 ds$ has an upper bound. Furthermore, we implies that $V(t)$ is bounded, and hence, $V'(t)$ is also bounded. Thus, we can further have that $\frac{d^2}{dt^2} \int_{t_0}^t \|R(s)\|^2 ds$ is bounded. According to Barbalat’s Lemma, one has that

$$\lim_{t \rightarrow \infty} \frac{d}{dt} \int_{t_0}^t \|R(s)\|^2 ds = \lim_{t \rightarrow \infty} \|R(s)\|^2 = 0. \tag{21}$$

Therefore, the synchronization is achieved.

Next, we show that no Zone behavior occurs. If there is a lower bound on the inter-event intervals of any agent in the integral-type event condition (6), Zeno-free behavior can be assured. Let us consider the derivative of $e_i(t)$ over the interval $[t_k^i, t_{k+1}^i)$.

$$\begin{aligned} e_i'(t) &= -z_i'(t) \\ &= -\sum_{j \in N_i} (x_i'(t) - x_j'(t)) \\ &= -Az_i(t) + cBK \sum_{j \in N_i} (z_i(t_k^i) - z_j(t_{k(t)}^j)) - B_1 \sum_{j \in N_i} (\phi(y_i) - \phi(y_j)) \\ &\leq \|A\| \|e_i(t)\| + \Xi_k^i, \end{aligned} \tag{22}$$

where $t_{k(t)}^j$ denotes the latest triggering time instant of agent j for $t < t_k^i$, with $\Xi_k^i = \max_{t \in [t_k^i, t_{k+1}^i)} \|-Az_i(t_k^i) + cBK \sum_{j \in N_i} [z_i(t_k^i) - z_j(t_{k(t)}^j)]\| + \Delta$. The matrix $B_1 \sum_{j \in N_i} [\phi(y_i) - \phi(y_j)]$ is bounded by Δ , since $\phi(y_i) - \phi(y_j)$ goes to zero as $t \rightarrow \infty$.

Then, the derivative of $\|e_i(t)\|^2$ satisfies

$$\begin{aligned} \frac{d}{dt} \|e_i(t)\|^2 &\leq 2\|e_i(t)\| \|e_i'(t)\| \\ &\leq 2\|e_i(t)\| (\|A\| \|e_i(t)\| + \Xi_k^i) \\ &\leq (2\|A\| + 1)\|e_i(t)\|^2 + (\Xi_k^i)^2, \end{aligned} \tag{23}$$

Using the comparison principle, one can further obtain that

$$\begin{aligned} \|e_i(t)\|^2 &\leq (\Xi_k^i)^2 \int_{t_k^i}^t e^{(2\|A\|+1)(t-s)} ds \\ &\leq (\Xi_k^i)^2 (t - t_k^i) e^{(2\|A\|+1)(t-t_k^i)}. \end{aligned} \tag{24}$$

On the other hand, when triggering condition (6) is satisfied, the next event time t_{k+1}^i is obtained from $f_i(t) = 0$. The sufficient condition is obtained to guarantee $f_i(t) \leq 0, t \in [t_k^i, t_{k+1}^i)$:

$$\begin{aligned} \int_{t_k^i}^t \|e_i(s)\|^2 ds &\leq (\Xi_k^i)^2 \int_{t_k^i}^t (s - t_k^i) e^{(2\|A\|+1)(s-t_k^i)} ds \\ &\leq \frac{(\Xi_k^i)^2}{2\|A\|+1} (t - t_k^i) \left(e^{(2\|A\|+1)(t-t_k^i)} - 1 \right), \end{aligned} \tag{25}$$

Accordingly, with Equation (25), one obtains

$$\begin{aligned} \int_{t_k^i}^t \|e_i(s)\|^2 ds &\leq \frac{(\Xi_k^i)^2}{2\|A\|+1} (t - t_k^i) \left(e^{(2\|A\|+1)(t-t_k^i)} - 1 \right) \\ &\leq k_i \|z_i(t_k^i)\|^2 (t - t_k^i) \\ &\leq \int_{t_k^i}^t k_i \|z_i(t_k^i)\|^2 + \beta e^{-\alpha s} ds, \end{aligned} \tag{26}$$

Letting $\tau_i = t - t_k^i$ and combining with the condition (26), one gets

$$\frac{(\Xi_k^i)^2}{2\|A\|+1} \left(e^{(2\|A\|+1)\tau_i} - 1 \right) = k_i \|z_i(t_k^i)\|^2. \tag{27}$$

In order to show $\tau_i > 0$, we proceed by contradiction. Suppose that $\tau_i < 0$, the left side of (27) is negative, whereas the right side of (27) is positive. Furthermore, if $\tau_i = 0$, the left side of (27) is zero, which is not equal to $k_i \|z_i(t_k^i)\|^2$. Equation (27) implicitly defines τ_i as a positive lower bound between any two consecutive event instants. Hence, no Zeno behavior occurs. The proof is completed.

Remark 1. To select the appropriate parameters $a, \eta > \lambda_m(S_1S_2 + S_2S_1), \gamma = \frac{2k_{max}}{1-2k_{max}}, k_{max} < \frac{1}{2}$ to guarantee the condition (8), we analyze the matrix. Let $a = c^2$ and $\gamma = \frac{1}{c^3}$ i.e., $k = \frac{1}{2c^3+2} < \frac{1}{2}$. It follows that matrix (8) can be represented as

$$\begin{bmatrix} \bar{A} + \bar{B} & I_N \otimes X & H \otimes X & I_N \otimes B_1 & H \otimes BY \\ I_N \otimes X & -\frac{1}{\eta\|C\|^2} I_{N \times n} & 0 & 0 & 0 \\ H^T \otimes X & 0 & -cI_{N \times n} & 0 & 0 \\ I_N \otimes B_1^T & 0 & 0 & -I_{N \times m} & 0 \\ H^T \otimes Y^T B^T & 0 & 0 & 0 & -I_N \otimes \bar{X} \end{bmatrix}. \tag{28}$$

Therefore, only the sufficiently large c and $\eta > \lambda_m(S_1S_2 + S_2S_1)$ are needed to ensure that the inequality holds.

Remark 2. For the triggering condition (6), only the integral of $\|e_i(t)\|^2$ from each triggering instant t_k^i to the time t , needs to be less than a designed variable in time intervals, and $\|e_i(t)\|^2$ does not need to be less than a given variable at any time. That is, the proposed mechanism relaxes the requirement in traditional event-triggered control [13–17,19–23,25]. Then, we design the following nonintegral-type event condition

$$\begin{aligned} t_{k'+1}^i &= \inf \{ t > t_{k'}^i : f_i'(t) \geq 0 \}, i \in \mathbb{N}, \\ f_i'(t) &= \|e_i(t)\|^2 - \|z_i(t_k^i)\|^2 - \beta e^{-\alpha t} \geq 0. \end{aligned} \tag{29}$$

where $\{t_{k'}^i\}_{k' \in \mathbb{Z}_{\geq 0}}$ represents the triggering instant sequence of agent v_i . Under the system (2) and control protocol (3), it can achieve synchronization asymptotically, and its minimum inter-event interval is less than Theorem 1.

Remark 3. The triggering function (6) has a hybrid form, which contains time-dependent item $\int_{t_k^i}^t \beta e^{-\alpha s} ds$ and state-dependent item $\int_{t_k^i}^t \|z_i(t_k^i)\|^2 ds$. By (19) and (20), $\int_{t_k^i}^t \beta e^{-\alpha s} ds$ ensures a positive lower bound of the inter-event intervals. There are two parameters β and α influencing the convergence rate of agents' states. Sufficiently small β and sufficiently large α can reduce the lower bound of the inter-event intervals, but make it converge quickly. Then, using technical studies on these influences, we can qualitatively adjust these parameters in practical applications.

Remark 4. The proposed Lyapunov function is positive, while its derivative is not negative all the times. Moreover, the integral $\int_{t_0}^t \frac{dV(s)}{ds} ds$ is nonnegative at some time due to the existence of the time-dependent item $\int_{t_k^i}^t \beta e^{-\alpha s} ds$ in the proposed triggering function, which is different from the results on single nonlinear systems. Therefore, the application of Barbalat's Lemma can guarantee the convergence results.

4. Integral-Type Edge-Self-Triggered Policy

In the event-triggered formulation, it becomes apparent that continuous monitoring of the measurement error norm $\|e(t)\|$ is required to check condition (6). In the following self-triggered control, this requirement is relaxed. Specifically, the next time t_{k+1}^i at which control law is updated is predetermined at the previous event time t_k^i and no state or error measurement is required in between the control updates. Define an increasing positive sequence $k_0^i, k_1^i, \dots, k_{\bar{q}}^i$, taking into account the effects of neighboring agent updates, with $t_k^i = t_{k_0^i}$ and $t_{k_q^i} \leq t_{k_{q+1}^i}$. By the inequality (22), the measurement error $e_i(t)$ satisfies

$$\|e_i(t)\| \leq \mathcal{A}\|e_i(t)\| + \Xi_{k_q^i}^i, \tag{30}$$

where

$$\Xi_{k_q^i}^i = \left\| -Az_i(t_k^i) + cBK \sum_{j \in N_i} (z_i(t_k^i) - z_j(t_{k_q^i})) \right\| + \Delta, \tag{31}$$

and $\mathcal{A} = \|A\|$. Solving such inequality in $t \in [t_{k_q^i}, t_{k_{q+1}^i}]$, $q = 0, 1, \dots, \bar{q} - 1$, we have

$$\|e_i'(t)\|^2 \leq e^{2\mathcal{A}t} \left(\mathcal{M}_{k_q^i}^i \right)^2 + 2\mathcal{M}_{k_q^i}^i \Xi_{k_q^i}^i \left(e^{\mathcal{A}(2t-t_{k_q^i})} - e^{\mathcal{A}t} \right) + \frac{(\Xi_{k_q^i}^i)^2 \left(e^{\mathcal{A}(t-t_{k_q^i})} - 1 \right)^2}{\mathcal{A}^2}, \tag{32}$$

where

$$\mathcal{M}_{k_q^i}^i = \sum_{r=0}^{q-1} \frac{\Xi_{k_r^i}^i \left(e^{-\mathcal{A}k_r^i} - e^{-\mathcal{A}k_{r+1}^i} \right)}{\mathcal{A}}. \tag{33}$$

Here, $\Xi_{k_q}^i$ and $\mathcal{M}_{k_q}^i$ are fix up for any $t \in [t_{k_q}^i, t_{k_{q+1}}^i]$, $q = 0, 1, \dots, \bar{q} - 1$. Defined a variable $\tau_k^i = \tau_k^i(t) = t - t_k^i$, we can now compute

$$\int_{t_k^i}^{t_k^i + \tau_k^i} \|e_i(s)\|^2 ds \leq \frac{(\Xi_{k_q}^i)^2}{\mathcal{A}^2} (\tau_k^i)^2 + \left(e^{2\mathcal{A}(t_k^i + \tau_k^i)} - e^{2\mathcal{A}t_k^i} \right) \left(\frac{(\mathcal{M}_{k_q}^i)^2}{2\mathcal{A}} + \frac{\mathcal{M}_{k_q}^i \Xi_{k_q}^i e^{-\mathcal{A}t_{k_q}^i}}{\mathcal{A}} + \frac{(\Xi_{k_q}^i)^2 e^{-2\mathcal{A}t_{k_q}^i}}{2\mathcal{A}^3} \right) - \left(e^{\mathcal{A}(t_k^i + \tau_k^i)} - e^{\mathcal{A}t_k^i} \right) \left(\frac{2\mathcal{M}_{k_q}^i \Xi_{k_q}^i}{\mathcal{A}} + \frac{2(\Xi_{k_q}^i)^2 e^{-\mathcal{A}t_{k_q}^i}}{\mathcal{A}^3} \right), \tag{34}$$

By (6) and (34), the self-triggered function is as follows:

$$\tilde{f}(\tau_k^i) = \frac{(\Xi_{k_q}^i)^2}{\mathcal{A}^2} (\tau_k^i)^2 + \left(e^{2\mathcal{A}(t_k^i + \tau_k^i)} - e^{2\mathcal{A}t_k^i} \right) \left(\frac{(\mathcal{M}_{k_q}^i)^2}{2\mathcal{A}} + \frac{\mathcal{M}_{k_q}^i \Xi_{k_q}^i e^{-\mathcal{A}t_{k_q}^i}}{\mathcal{A}} + \frac{(\Xi_{k_q}^i)^2 e^{-2\mathcal{A}t_{k_q}^i}}{2\mathcal{A}^3} \right) - \left(e^{\mathcal{A}(t_k^i + \tau_k^i)} - e^{\mathcal{A}t_k^i} \right) \left(\frac{2\mathcal{M}_{k_q}^i \Xi_{k_q}^i}{\mathcal{A}} + \frac{2(\Xi_{k_q}^i)^2 e^{-\mathcal{A}t_{k_q}^i}}{\mathcal{A}^3} \right) - k_i \|z_i(t_k^i)\|^2 \tau_k^i - \frac{\beta}{\alpha} e^{-at_k^i} (1 - e^{-a\tau_k^i}), \tag{35}$$

For each $i = 1, 2, \dots, N$ the self-triggered ruling defines the next update time as following: if there is a $\tau_k^i > 0$ such that $\tilde{f}(\tau_k^i) = 0$, the next update time t_{k+1}^i take place at most τ_k^i time units after t_k^i . Agent v_i also checks the condition (35) when its neighbors updated. Otherwise, if the inequality $\tilde{f}(\tau_k^i) < 0$ holds for all $\tau_k^i > 0$, then agent v_i waits until the next update of the control law of one of its neighbors to recheck its condition.

According to the above analysis, we give Algorithm 1 as follows:

Algorithm 1 Integral-Type Edge-Self-Triggered Control Algorithm

- Step 1. Set the algorithm execution time \mathcal{T} . At initial time $t = t_0$, let $k = 0$, $t_k^i = t_0$, $q = 0$ and $k_q^i = 0$. Let $z_i(t) = z_i(t_0)$; update $u_i(t_0) = -cBKz_i(t_0)$; compute $\Xi_{k_q}^i$ and $\mathcal{M}_{k_q}^i$ by (31) and (33); compute maximum allowable value τ_k^i by (35) such that $\tilde{f}(\tau_k^i) = 0$.
- Step 2. When $0 < t \leq \mathcal{T}$, check the events related to agents v_i, v_j ; perform the following steps:
- i. If any agent updated before $t_k^i + \tau_k^i$, let $q = q + 1$; recompute $\Xi_{k_q}^i$ and $\mathcal{M}_{k_q}^i$ by (31) and (33), such that $\tilde{f}(\tau_k^i) = 0$;
 - ii. If there is no agent updated before $t_k^i + \tau_k^i$, the event related to agent v_j , occurs at $t_k^i + \tau_k^i$; let $k = k + 1$; update the event instant t_k^i ; let $z_i(t) = z_i(t_k^i)$; update $u_i(t_k^i) = -cBKz_i(t_k^i)$; let $q = 0$; compute $\Xi_{k_q}^i$ and $\mathcal{M}_{k_q}^i$ by (31) and (33); compute maximum allowable value τ_k^i by (35) such that $\tilde{f}(\tau_k^i) = 0$.
- Step 3. When $t > \mathcal{T}$, jump out of Algorithm 1.
-

Theorem 2. Under the assumptions and conditions in Theorem 1, the system (2) with the control law (3) and Algorithm 1 can achieve synchronization for any initial condition in \mathbb{R}^N . Furthermore, no Zeno behavior occurs for all $t > t_0$.

Remark 5. The designed distributed integral-type edge-event-triggered policy still requires continuous monitoring of measurement errors. The self-triggered control can solve this problem. That is, the equipped sensors only perform intermittent sampling, which can further reduce the number of sensor measurement samples and then save system resources. Based on the self-triggered policies [26–28], a distributed integral-type edge-self-triggered algorithm is proposed in this paper. This algorithm, in addition to intermittent updates, schedules sensors measurements intermittently.

5. Simulations

In this section, we use an example to illustrate the effectiveness of the results developed in above sections. We consider Chua’s circuit, which is described by the following system:

$$\begin{cases} x'_1(t) = -h(m_1 + 1)x_1 + hx_2 + f(x_1), \\ x'_2(t) = x_1 - x_2 + x_3, \\ x'_3(t) = -gx_2, \end{cases} \tag{36}$$

where $x_1, x_2, x_3 \in \mathbb{R}$. $f(x_1) = -h(m_2 - m_1)(|x_1 + 1| - |x_1 - 1|)$, belonging to the sector $[0, 2]$, represents the change in resistance vs. Choosing the parameters as $h = 0.25$, $g = 1$, $m_1 = 3$, and $m_2 = -1$. Then, we can rewrite (36) in the form of a Lur’e system with control input $u \in \mathbb{R}^3$

$$\begin{cases} x'(t) = Ax + Bu + B_1z, \\ y = Cx, \\ z = \phi(y), \end{cases} \tag{37}$$

with $x = [x_1, x_2, x_3]^T$, $A = \begin{bmatrix} -1 & 0.25 & 0 \\ 1 & -1 & 1 \\ 0 & -1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $B_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $C = [1 \ 0 \ 0]$, and

$\phi(y) = |y + 1| - |y - 1|$. Its Lur’e-type nonlinearity satisfies the sector bounded condition $\phi(y)(\phi(y) - 2y) \leq 0$ with $S_1 = 0$ and $S_2 = 2$. Taking (37) as each agent dynamics, a network of five such agents is shown in Figure 1. The interconnection topology is undirected and connected.

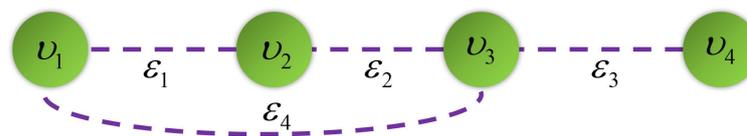


Figure 1. Communication topology.

Let $Y = [1, 0, 0]$ and $\eta = 1$. By using the Matlab LMI Control Toolbox to make the matrix (8) negative, we can compute the symmetric positive definite matrix

$$X = \begin{bmatrix} 0.4995 & -0.0058 & 0.0074 \\ -0.0058 & 0.4346 & 0.0833 \\ 0.0074 & 0.0833 & 0.3940 \end{bmatrix},$$

and $c = 1.0004$, respectively. Take $k_i = 0.1 * 0.01i < \frac{1}{2}$, $\beta = 0.11$, and $\alpha = 1.85$. Here, all the above parameters satisfy the conditions in Theorem 1. The results of the nonintegral-type edge-event-triggered policy (under condition (29)), the integral-type edge-event-triggered policy (under condition (5)) and the integral-type edge-self-triggered policy (under Algorithm 1) are shown in Figures 2–4, respectively. Tables 1 and 2 are the event numbers and average sampling intervals under the triggering mechanisms (5), (29), and Algorithm 1. Obviously, compared with the nonintegral-type event-triggered control in (29), the integral-type edge-event-triggered control can reduce sampling frequency, thereby reducing system resource consumption. Moreover, the integral-type edge-self-

triggered control avoids continuous monitoring of all measurement errors, which further reduces sensor resource consumption at the expense of increased sampling frequency.

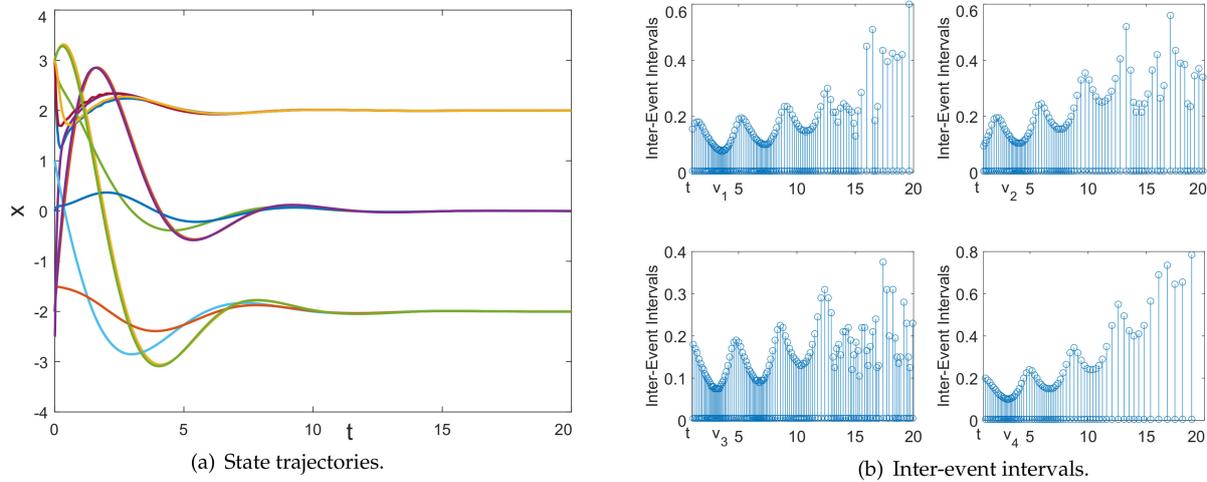


Figure 2. Nonintegral-type edge-event-triggered policy in (29).

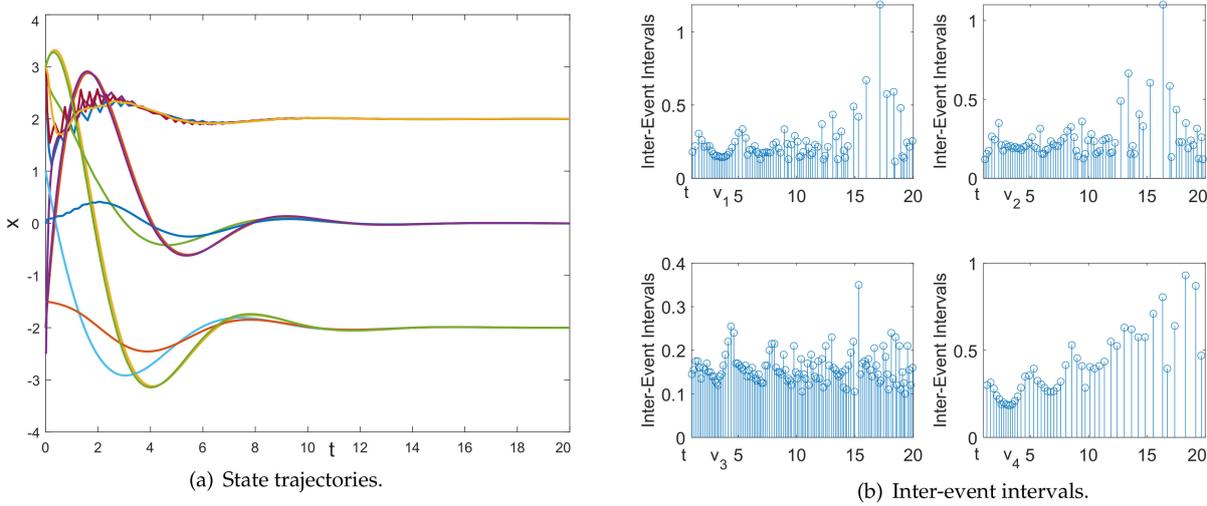


Figure 3. Integral-type edge-event-triggered policy in (5).

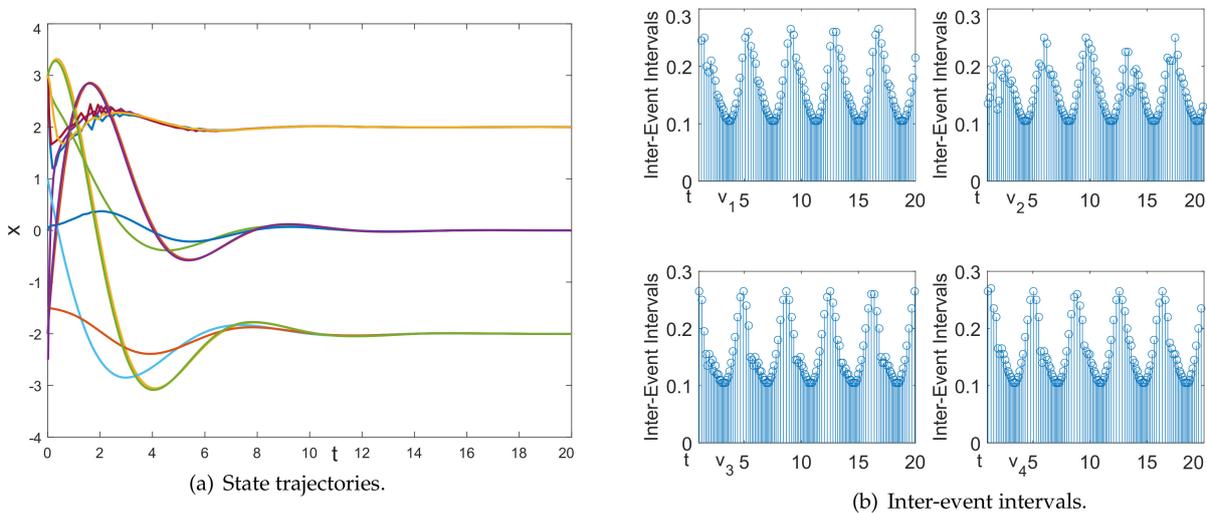


Figure 4. Integral-type edge-self-triggered policy in Algorithm 1.

Table 1. The event numbers under the triggering mechanisms (29), (5), and Algorithm 1.

Agents	v_1	v_2	v_3	v_4	Total Numbers
The mechanism in (29)	230	200	258	156	844
The mechanism in (5)	86	86	127	53	352
Algorithm 1	131	138	132	128	529

Table 2. The average sampling intervals under the triggering mechanisms (29), (5), and Algorithm 1.

Agents	v_1	v_2	v_3	v_4	Total Numbers
The mechanism in (29)	0.087	0.100	0.078	0.128	0.095
The mechanism in (5)	0.233	0.233	0.157	0.377	0.227
Algorithm 1	0.153	0.145	0.152	0.156	0.151

6. Conclusions

In this paper, the synchronization problem of multi-agent systems with Lur'e nonlinear dynamics has been studied via integral-type edge-event- and edge-self-triggered policies. In event-triggered control, the provided LMI condition ensures the convergence of the considered system. Additionally, Zeno-free triggering has also been strictly proven. This study relaxes the conditions of the traditional event-triggered mechanism, thereby resulting in lower sampling frequencies. Besides, the integral-type self-triggering is further designed to avoid continuous monitoring required in integral-type edge-event-triggered control, thereby saving sensor resources. Further studies involve directed graphs, prescribed performance control, and reinforcement learning.

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