



# Article SPICE-Aided Compact Electrothermal Model of Impulse Transformers <sup>†</sup>

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# Featured Application: The presented results can be applied in designing impulse transformers and switch-mode power supplies.

**Abstract:** This article proposes a new form of compact electrothermal model of impulse transformers. The proposed model is dedicated for use with SPICE and it is formulated in the network form. It simultaneously takes into account electrical, thermal, and magnetic phenomena occurring in the considered device. Nonlinearity of the core magnetization characteristics and nonlinearity of the heat transfer efficiency are taken into account in this model. The form of the proposed model is shown. Equations of the presented model are given. Experimental verification of the proposed model is performed for selected impulse transformers. Selected results of the performed investigations are presented.

**Keywords:** compact electrothermal models; impulse transformer; modeling; multi-domain models; SPICE

# 1. Introduction

Impulse transformers are commonly used in many applications, for example in switchmode power converters [1–5]. Such converters require considered components operating at frequency ranging from several to several hundred kilohertz [4,6]. The considered transformers contain two components: a core made of any ferromagnetic material and windings (at least two) made of conductive material [1,3,4].

During the operation of a transformer, different phenomena, magnetic, electrical, and thermal, occur in it. Magnetic phenomena in the core are a result of the excitation of magnetic force connected with the current flow through the transformer windings. On the other hand, alternating magnetic force induces electromotive force (induced voltage) in all the transformer windings. Additionally, temperature influences electrical properties of the windings and magnetic properties of the ferromagnetic core [4,6–8]. Due to thermal phenomena occurring in the transformer, temperatures of the core and the windings can be much higher than the ambient temperature. These phenomena contain self-heating in each transformer components and mutual thermal couplings between these components [9].

Currently, engineers use computer programs, e.g., SPICE or PLECS, to design and analyze power electronic circuits [10–12]. Practical usefulness of the obtained results of computations depends on, e.g., accuracy of the models of applied elements contained in these circuits [13]. Therefore, models of the considered components characterized by a different accuracy are given in the literature. The review of selected transformers models is



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**Copyright:** © 2021 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). given in the papers [14,15]. Additionally, a manner of modeling the thermal properties of selected impulse transformers is described in the papers [6,9,16].

From the literature review, it is visible that the accessible transformers models describe properties of this component in a too-simplified way. This means that such models can correctly describe properties of impulse transformers only at selected operating conditions. The very popular linear model of the ideal transformer containing linearly coupled inductors [1,10,17] is accurate only in the case when the magnetic permeability of the transformer core is constant. This condition is fulfilled when the dependence of magnetic flux density *B* on magnetic force *H* is linear. Such dependence *B*(*H*) is observed when *B* value is considerably lower than saturation flux density. The mentioned simple model neither takes into account thermal phenomena occurring in the transformer nor power losses in the core and in the windings. On the other hand, non-linear transformer models given in [17] make it possible to take into account non-linearity of the magnetization characteristics *B*(*H*) of the ferromagnetic core. Unfortunately, in these models the influence of temperature on transformer characteristics are omitted.

In order to take into account mutual interactions between magnetic and thermal phenomena in computer analyses, the electrothermal models of the components contained in the investigated circuit are needed. For transformers, inductors, or solid-state light sources, such models belong to multi-domain models [18]. Electrothermal models can be formulated in the form of detailed or compact models [19,20]. The detailed models allow computing time-spatial temperature distribution in the modeled device, but computations with the use of such models are time-consuming. In turn, compact models make it possible to compute a waveform of one internal temperature characterizing the whole device and computations using such models in a short time.

Electrothermal transformer models are given in the literature [7,14]. Models used in these papers are based on the ferromagnetic core model proposed in [21] and extended by the equations describing the temperature influence on the dependence B(H). A disadvantage of the models described in the cited papers is that they are a very simplified manner of describing thermal properties of transformers. In the paper [7], only one temperature is used to characterize all the transformer components. It is assumed that the core and the windings have the same temperature. In contrast, in the paper [14], different values of the core temperature  $T_C$  and temperature of all the windings  $T_W$  were used. It is also worth noticing that in the paper [9], some measurements results are shown, which prove that temperatures of transformer windings can visibly differ between each other. Such differences can exceed even 20 °C for a toroidal transformer. On the other hand, a difference between temperature of primary winding and temperature of the core for the same transformer can exceed even 40 °C [9]. Temperature distribution on the surface of each of the mentioned parts of a transformer is quasi uniform and differences between measured values of temperature on these parts are smaller than 4 °C.

This paper is an extended version of the conference paper [22]. It presents a compact electrothermal model of the impulse transformer. In comparison to the paper [22], the description of the elaborated model is extended and additional measurements and computation results of different transformers are shown and discussed. Particularly, results of investigations of toroidal and planar transformers are presented. An influence of thermal phenomena, frequency, and load resistance on characteristics of the mentioned transformers are investigated.

The presented model is formulated as a subcircuit for SPICE. It takes into account the mutual interaction of electrical, magnetic, and thermal phenomena which occur in the transformer. Additionally, differences of the core temperature and temperature of each winding are taken into account in this model. While characterizing thermal phenomena, both the self-heating phenomena in the core and in the windings and mutual thermal couplings between each pair of components of the transformer are taken into account. In the proposed model, the non-linearity of both magnetizing core characteristic and nonlinearity of the heat transfer using the thermal model proposed in [9] are taken into account, too. The correctness of the elaborated model is verified experimentally for transformers with different cores and different windings.

The form of the elaborated model is described in Section 2. Section 3 describes the investigated transformers. The measurements and computations results are presented and discussed in Section 4.

# 2. Model Form

The proposed transformer model belongs to compact electrothermal models [7,23,24] and makes it possible to simultaneously compute voltages and currents of this device, the temperature of its core and each winding, and the magnetic force and magnetic flux density.

Figure 1 shows the network representation of this model. The presented model consists of three blocks: the core model describing its magnetizing characteristic, the windings model describing voltages induced in them, and the non-linear thermal model describing temperatures of the transformer components. Terminals marked as 1a and 1b represent connectors of the primary winding, whereas terminals 2a and 2b—connectors of the secondary winding. Terminals  $X_1$  and  $X_2$  are used to model electrical properties of the core. Terminals labeled H, B,  $T_C$ ,  $T_{W1}$ , and  $T_{W2}$  of voltage-controlled sources correspond to magnetic force, magnetic flux density, temperature of the core, temperature of the primary winding, and temperature of secondary winding, respectively.



Figure 1. Network form of the proposed transformer model.

In practical measurements of the electrical properties of the ferromagnetic core, dedicated terminals in the form of metal rings or screwed wires shown in [25] are used.

In the proposed model, thermal and magnetic variables correspond to voltages and currents computed in SPICE. Relations between these quantities are presented in Table 1.

Name of Variable	Symbol	SPICE Voltage	
Magnetic force	Н	v(H)	
Magnetic flux density	В	v(B)	
Magnetization	М	v(M)	
Power losses in the core	P <sub>loss</sub>	v(Ploss)	
Voltage between the core terminals	υ	$v(X_1,X_2)$	
Temperature of the core	T <sub>C</sub>	V(TC)	
Temperature of the primary winding	$T_{W1}$	V(TW1)	
Temperature of the secondary winding	$T_{W2}$	V(TW2)	

Table 1. Relations between thermal and magnetic variables and SPICE voltages.

# 2.1. Core Model

In the core model, the voltage on terminal B corresponds to magnetic flux density, the voltage on terminal H to magnetic force, and the voltage on terminal  $P_{loss}$  to power dissipated in the core, whereas terminals  $X_1$  and  $X_2$  represent the border of the core. The controlled voltage source  $E_4$  is used to compute magnetization on the primary magnetizing curve  $M_a$  given by [26]

$$M_a = \left[\frac{B_{S0} \cdot (1 + \alpha_{BS} \cdot (T_C - T_0))}{\mu_0} - H_S\right] \cdot f\left(\frac{H + \alpha_M \cdot M_a}{A}\right) \tag{1}$$

where  $B_{S0}$  is saturation flux density at the reference temperature  $T_0$  and magnetic force  $H_S$ ,  $\alpha_{BS}$ —the temperature coefficient of saturation flux density,  $\mu_0$ —magnetic permeability of free air,  $T_C$ —core temperature,  $\alpha_M$ —the parameter of coupling of the walls of magnetic domains, and A—the parameter of thermal energy.

All the abovementioned parameters also appear in the Jiles–Atherton model. In this model, f(x) is the Langevin function, which is non-continuous for x = 0. In order to eliminate this disadvantage, in the proposed model, this function is described by the empirical formula [26]

$$f(x) = \frac{x}{\sqrt{x^2 + 1}} \left[ 1 - 0.9 \cdot \exp\left(-\frac{|x|}{2.5}\right) - 0.1 \cdot \exp\left(-\frac{|x|}{25}\right) \right]$$
(2)

Voltage drop on resistor  $R_3$  is proportional to time derivative  $dM_a/dt$ . Controlled voltage source  $E_5$  makes it possible to compute flux density with the use of the following formula

$$B = \mu_0 \cdot (H + M \cdot C_R \cdot y) \tag{3}$$

where  $C_R$  denotes capacitance of capacitor of the same name, *y* describes the Curie phenomenon with the empirical formula

$$y = \begin{cases} 1 \ if \ T_C < T_{Curie} \\ 1 - (T_C - T_{Curie}) \cdot 0.006 K^{-1} \ if \ T_{Curie} < T_C < T_{Curie} + 17K \\ 0 \ if \ T_C > T_{Curie} + 17K \end{cases}$$
(4)

and  $T_{Curie}$  denotes the Curie temperature. In turn, voltage on resistor R<sub>2</sub> is proportional to time derivative dB/dt.

Controlled voltage source  $E_H$  represents magnetic force H and voltage drop on resistor  $R_1$  is proportional to dH/dt.

Voltage in the node *M* represents magnetization of the core. The output current of the controlled current source  $G_1$  is given by [26]

$$G_{1} = -\frac{(M_{a} - M \cdot C_{1}) \cdot \text{sgn}(dH/dt)}{C_{1} \cdot H_{C0} \cdot [1 + \alpha_{HC} \cdot (T_{C} - T_{0})]} \cdot \frac{dH}{dt} - \frac{C}{C_{1} \cdot (1 + C)} \cdot \frac{dM_{a}}{dt} + M \cdot \frac{C_{R}}{R_{C}}$$
(5)

where function sgn(z) computes the sign of the argument z,  $H_{C0}$ —coercion magnetic force at the reference temperature  $T_0$ , C characterizes elastic deformations of domains walls,  $\alpha_{HC}$ —the temperature coefficient of coercion magnetic force, and  $C_1$  and  $R_C$  are capacitance of the capacitor and resistance of the resistor occurring in the core model.

Controlled voltage sources  $E_{alf}$ ,  $E_C$ ,  $E_{\mu}$ , and  $E_{A1}$  are used to compute the values of the parameters  $\alpha$ , *C*,  $\mu$ , and *A* occurring in equations describing the *B*(*H*) curve. The dependences of each of the mentioned parameters on temperature *T<sub>C</sub>* are given by linear functions.

The circuit containing controlled voltage source  $E_{11}$ , diode  $D_1$ , resistors  $R_4$  and  $R_5$ , and capacitors  $C_4$  and  $C_5$  is used to compute the average (voltage on capacitor  $C_5$ ) and the maximum values of *B* (voltage on capacitor  $C_4$ ), respectively. Voltage on the source  $E_{11}$  is proportional to *B*. The controlled voltage source  $E_{DB1}$  computes the amplitude of magnetic flux density  $B_m$ .

The voltage source  $E_P$  models power losses in the core given by the equation of the form [26]

$$E_p = V_e \cdot \left(\frac{B_{pp}}{2}\right)^{\beta - \alpha} \cdot \left(1 + \alpha_P \cdot (T_C - T_m)^2\right) \cdot \frac{P_{V0}}{T} \int_0^T \left|\frac{dB}{dt}\right|^{\alpha} dt$$
(6)

where  $P_{V0}$  denotes core power losses per unit of volume, T—a period of the waveform B(t),  $B_{pp}$ —peak-to-peak value of this waveform,  $V_e$ —equivalent core volume,  $\alpha_P$ —the square temperature coefficient of power losses,  $T_m$ —temperature corresponding to the minimum of power losses, while  $\alpha$  and  $\beta$  are parameters describing the influence of frequency f and  $B_m$  on core power losses.

In the described model, the elements making it possible to compute electrical core characteristics at its external electric stimulation are also included: voltage source  $V_1$ , the controlled voltage source  $E_R$ , and resistor  $R_6$ .

Resistor  $R_6$  represents the minimum value of the core resistance R. The core current i is monitored by voltage source  $V_1$ , whereas voltage source  $E_R$  models the dependence  $R(i, T_C)$  with the use of the following formula [25]

$$E_{R} = \left[ \left( m_{1} \cdot \exp\left(-\frac{i}{n_{1}}\right) + m_{2} \right) \cdot \exp\left(B_{1} \cdot \left(2 - \exp\left(-\frac{i}{n_{2}}\right)\right) \cdot \left(\frac{1}{T_{C}} - \frac{1}{T_{0}}\right) \right) + \left(m_{3} \cdot \exp\left(-\frac{i}{n_{3}}\right) + m_{4}\right) \cdot \exp\left(B_{2} \cdot \left(\frac{1}{T_{C}} - \frac{1}{T_{0}}\right) \right) \right]$$

$$\cdot i \cdot R_{0} \cdot \left(m_{5} + m_{6} \cdot \exp\left(-\frac{i}{n_{4}}\right) \right)$$

$$(7)$$

where  $R_0$  represents the core resistance R at temperature  $T_C = T_0$  and current i tending to infinity,  $B_1$  and  $B_2$  model a slope of the dependence of core resistance on temperature, parameters  $m_1$ ,  $m_2$ ,  $m_3$ ,  $m_4$ ,  $m_5$ , and  $m_6$  describe an influence of core current on its resistance, whereas parameters  $n_1$ ,  $n_2$ ,  $n_3$ , and  $n_4$  model an influence of core current on a slope of the dependence R(i).

#### 2.2. Windings Model

The model of the windings contains some elements representing properties of the primary winding and the secondary winding. Resistor  $R_{S1}$  models the primary winding series resistance at the temperature  $T_0$ , the controlled voltage source  $E_{RS1}$  describes an influence of temperature of this winding  $T_{W1}$  on this resistance. The controlled voltage source  $E_{V1}$  computes voltage induced in the primary winding; the controlled current sources  $G_{L1}$  and  $G_R$  represent magnetizing current core power losses. The controlled voltage source  $E_{RMS1}$  computes RMS value ( $V_{RMS1}$ ) of the primary winding current. Voltage sources  $V_{L1}$  and  $V_{111}$  have a zero value and they monitor the value of currents

flowing through them. In order to model properties of the secondary winding, three elements are used. Voltage induced in this winding is described by  $E_{V2}$ , and winding series resistance by  $R_{S2}$  and  $E_{RS2}$ .

The controlled voltages sources  $E_{PW1}$  and  $E_{PW2}$  are also included in the windings model. These sources represent power losses in the primary and in the secondary windings, respectively. The output voltages are given as follows

$$E_{PW1} = \left[1 + \alpha_{\rho} \cdot (T_{W1} - T_0)\right] \cdot V_{RS1} \cdot i_1 + R_{acW1} \cdot \left[1 + \alpha_{\rho} \cdot (T_{W1} - T_0)\right] \cdot i_1^2 \tag{8}$$

$$E_{PW2} = \left[1 + \alpha_{\rho} \cdot (T_{W2} - T_0)\right] \cdot V_{RS2} \cdot i_2 + R_{acW2} \cdot \left[1 + \alpha_{\rho} \cdot (T_{W2} - T_0)\right] \cdot i_2^2 \tag{9}$$

where  $\alpha_{\rho}$  denotes the temperature coefficient of copper resistivity  $\rho$ ,  $i_1$  and  $i_2$ —currents of the windings,  $V_{RS1}$  and  $V_{RS2}$ —voltage drops on resistors  $R_{S1}$  and  $R_{S2}$ , and  $R_{acW1}$  and  $R_{acW2}$  are resistances of both the windings for alternating current described with the dependence [27], in which *k* is equal to 1 or 2.

$$R_{acWi} = R_{Si} \cdot \frac{y_1}{2} \cdot \left[ \frac{\sinh(y_1) + \sin(y_1)}{\cosh(y_1) - \cos(y_1)} + (2 \cdot m - 1)^2 \cdot \frac{\sinh(y_1) - \sin(y_1)}{\cosh(y_1) + \cos(y_1)} \right]$$
(10)

In Equation (10),  $R_{Sk}$  is resistance of *k*-th winding for DC and  $y_1$  is the relative layer thickness of the winding given by [27]

$$y_1 = h \cdot \sqrt{\frac{k_w \cdot \pi \cdot \mu_0 \cdot \mu_w \cdot f}{\rho}} \tag{11}$$

where  $k_W$  is the coefficient of copper layer filling factor, *h*—effective thickness of the winding layer, and  $\mu_w$  is relative magnetic permeability of copper.

## 2.3. Non-Linear Thermal Model

The compact non-linear thermal model, whose network representation is presented in Figure 2, is used to compute temperature of all the transformer components:  $T_{W1}$ ,  $T_{W2}$ , and  $T_C$ . It takes into account both self-heating in the core, in the primary winding, and in the secondary winding and mutual thermal couplings between each pair of the mentioned components.



Figure 2. Network form of a compact non-linear thermal model.

In Figure 2, nine subcircuits are visible. Three of them (shown on the left-hand side) are used to compute three temperatures:  $T_{W1}$ ,  $T_{W2}$ , and  $T_C$ . They correspond to voltages in

nodes denoted using the same symbols. In turn, current sources  $I_C$ ,  $I_{W1}$ , and  $I_{W2}$  model power dissipated in the core and in each winding.

The form of this model corresponds to a non-linear thermal model of semiconductor devices proposed in [28]. In the described model, differences in the value of temperature of the core and each winding are taken into account. Additionally, the influence of the dissipated power of the effectiveness of the removal of the heat generated in the transformer is taken into account. While formulating the presented model, it was assumed that the dissipated power influences only thermal resistances  $R_{th}$  in the thermal model, whereas it does not influence thermal capacitances.

Circuits containing capacitors and the controlled current sources model self-transient thermal impedances:  $Z_{thW1}(t)$  of the primary winding ( $C_{W11}, \ldots, C_{W1n}, G_{W11}, \ldots, G_{W1n}$ ),  $Z_{thW2}(t)$  of the secondary winding ( $C_{W21}, \ldots, C_{W2n}, G_{W21}, \ldots, G_{W2n}$ ), and  $Z_{thC}(t)$  of the core ( $C_{C1}, \ldots, C_{Cn}, G_{C1}, \ldots, G_{Cn}$ ). Voltage on each of these circuits corresponds to an excess of temperature of the transformer components above the ambient temperature  $T_a$  due to self-heating. Controlled voltage sources  $E_1$ ,  $E_2$ , and  $E_3$  represent an increase in the temperature of the transformer components resulting from mutual thermal couplings between these components. Voltage on the source  $E_1$  is equal to the sum of voltages in nodes  $T_{W11}$  and  $T_{WC1}$ , voltage on the source  $E_2$  to the sum of voltages in nodes  $T_{W21}$  and  $T_{WC2}$ , and voltage on the source  $E_3$  to the sum of voltages in nodes  $T_{CW1}$  and  $T_{CW2}$ . Voltage sources  $V_1$ ,  $V_2$ , and  $V_3$  correspond to the temperature  $T_a$ .

Another six subcircuits describe mutual thermal couplings between the components of the transformer. Each of these subcircuits represents appropriate transfer transient thermal impedance. Current sources represent power dissipated in the primary winding ( $I_{W12}$  and  $I_{W1C}$ ), the secondary winding ( $I_{W21}$  and  $I_{W2C}$ ), and the core ( $I_{C1}$  and  $I_{C2}$ ).

All self and transfer transient thermal impedances are given by [19]

$$Z_{th}(t) = R_{th} \cdot \left[ 1 - \sum_{i=1}^{N} a_i \cdot \exp\left(-\frac{t}{\tau_{thi}}\right) \right]$$
(12)

The dependence of thermal resistance  $R_{th}$  on power p dissipated in the heating transformer component is expressed by the empirical formula

$$R_{th} = R_{th0} \cdot \left[1 + a \cdot \exp\left(-\frac{p}{b}\right)\right]$$
(13)

where  $R_{th0}$  is the minimum thermal resistance value, while *a* and *b* are model parameters. Parameter *a* describes the range of change in the thermal resistance value, whereas parameter b characterizes the slope of the dependence  $R_{th}(p)$ .

Thermal resistance is modelled by the controlled current sources  $G_i$ . The output current of this source is described by

$$G_i = \frac{V_{Gi}}{a_i \cdot R_{th}} \tag{14}$$

where  $V_{Gi}$  is voltage drop on this source.

#### 3. Investigated Devices

In order to verify the correctness of the proposed compact non-linear electrothermal model of an impulse transformer, many measurements and computations were performed. In these investigations, different constructions of impulse transformers were used. The planar transformer and classical transformers with ferromagnetic cores made of different ferromagnetic materials and characterized by different shapes and dimensions were considered. The investigated devices are briefly presented below.

Figure 3 presents the investigated planar transformer with the ferrite core E22/6/16R made of 3F3 [29] material. The spiral windings were performed on the FR-4 PCB of the thin equal to 1 mm with copper layer 35  $\mu$ m thin. The secondary and primary windings

Contraction of the second seco

and the secondary of 4 turns, 1 mm wide.

**Figure 3.** Investigated planar transformer with an E22/6/16R core made of 3F3 material.

Many of the investigated transformers contain ring cores made of different materials. Cores made of following materials were used: powdered iron—material –26 [30] (called RTP), nanocrystals—material M-070 [31] (called RTN), and ferrites—material F-867 [32] (called RTF). The cores have an external diameter of about 26 mm, an internal diameter of about 15 mm, and a height of about 11 mm. The transformers with small cores contained two windings with 20 turns of copper wire in enamel of a diameter 0.8 mm. Images of some investigated transformers with ring cores are shown in Figure 4.

had the shape of an oval spiral. The primary winding consisted of 3 turns, 2.5 mm wide,



Figure 4. Investigated transformer with ring cores: (a) RTN, (b) RTP, and (c) RTF.

## 4. Investigations Results

The correctness of the proposed transformer model was verified experimentally. Some characteristics of transformers described in Section 3 were computed and measured. In Section 4.1, transformers with ring cores are considered, whereas in Section 4.2, the results of investigations of the planar transformer are presented.

In the figures presented in this section, the results of the computations obtained by means of the proposed model are marked with solid lines, whereas the results of the measurements are shown with points. Additionally, some computation results obtained using the model given in the paper [15] are denoted with a dashed line. In the investigations, the results of which are presented in this section, the transformers operated exceeding their primary winding by sinusoidal signal of frequency *f* and amplitude  $V_m$ . The load of the secondary winding was resistor  $R_0$ .

# 4.1. Ring Transformers

Figure 5 illustrates the influence of load resistance  $R_0$  on energy efficiency  $\eta$  of the considered ring transformers. This efficiency was equal to the quotient of the average value of the product of voltage on the secondary winding  $V_{W2}$  and the current of this winding  $I_{W2}$  by the average value of the product of the voltage on the primary winding  $V_{W1}$  and the current of this winding  $I_{W1}$ .

The obtained characteristics  $\eta(R_0)$  were decreasing functions for all the transformers. The highest energy efficiency was obtained for the transformer containing the nanocrystalline core (RTN), whereas the lowest was for the transformer containing the core made of powdered iron (RTP). At the mentioned operating conditions, for the transformer with the RTP core, the values of  $\eta$  decreased over 14 times (from 85% to only just 6%). A decrease in



energy efficiency for high values of  $R_0$  resulted from a high value of idling current, which is the main component of current flowing through the primary winding.

Figure 5. Dependences of energy efficiency of the ring transformers on load resistance.

As one can notice, the computations results obtained using the proposed model were convergent with the measurements results for all the considered transformers in the whole range of  $R_0$  changes. In contrast, the results obtained using the model from [15] fit well the measurements results only for the RTP core, whereas they differed from the results of measurements even by 15% for the other transformers.

Figure 6 illustrates an influence of load resistance on the core temperature at  $V_m = 67$  V and f = 100 kHz. As seen, the dependence  $T_C(R_0)$  was an increasing function. The core temperature achieved even 110 °C for the core RTP at  $R_0 = 1$  k $\Omega$ .



Figure 6. Dependences of the core temperature on load resistance.

The obtained shape of the characteristics  $T_C(R_0)$  shows that the most important component of transformer power losses was dissipated in the core [27]. The values of the temperature T<sub>C</sub> computed using the new model fit the measurements results well. Differences between them were lower than 5 °C. In contrast, the results of computations obtained using the model from the paper [15] differed visibly from the results of measurements. Such differences reached as far as 50 °C.

Figure 7 illustrates the dependence of the primary winding temperature  $T_{W1}$  on load resistance at frequency f = 100 kHz.



Figure 7. Dependences of the primary winding temperature on load resistance.

For the transformers with RTP and RTF cores, the dependence  $T_{w1}(R_0)$  was an increasing function, whereas for the transformer with RTN core, the local minimum at  $R_0 = 100 \Omega$  could be observed. Big differences (equal even to 70 °C) in the values of temperature  $T_{W1}$  for different transformers were observed at high values of load resistance. This means that this value was a result of mutual thermal couplings between the primary winding and the core, because in this range of  $R_0$ , the power was dissipated mostly in the core. The results obtained with the proposed model fit the results of measurements well only for the transformer with the RTP core in the range of high  $R_0$  values. The differences between the measurements and computations results did not exceed 3 °C. In contrast, the results obtained using the model given in [15] differed from the measurements results by 15 °C.

Comparing results presented in Figures 6 and 7, one can observe that using the model described in the paper [15], one can obtain overestimated values of the transformer core and underestimated values of the transformer primary winding. This problem results from omission in the cited paper of an influence of dissipated power on the values of thermal resistances in the transformer model and from omitting differences in temperature of each winding. Such a problem was not observed for the proposed model.

Figure 8 shows the measured and computed dependences of the transformer output voltage  $V_{out}$  on resistance  $R_0$  at f = 100 kHz.



Figure 8. Dependences of the transformers' output voltage on load resistance.

As can be seen, the computations results obtained using the new model and the model given in [15] differed from the measurements results by several percentage points. The dependence  $V_{out}(R_0)$  is an increasing function. One can notice that a change of the material of the transformer core could cause even a double change in the transformer output voltage.

A four times change in the transformer output voltage was also visible as a result of a change in the value of the resistance  $R_0$ .

Figure 9 illustrates the influence of frequency on the temperature  $T_C$ . The presented dependences were obtained at  $R_0 = 100 \Omega$  and  $V_m = 67$  V.



**Figure 9.** Dependences of the temperature  $T_C$  on frequency.

It can be observed that the highest values of temperature  $T_C$  were obtained at  $f \approx 100$  kHz. The maximum of about 100 °C could be observed for the core RTP. For high f values, the core temperature of each transformer was nearly room temperature. This was a result of an increase in f causing a decrease in the amplitude of magnetic flux density and in power losses in the core.

Figure 10 shows the measured and computed dependences of the transformer output voltage  $V_{out}$  on frequency at  $R_0 = 100 \Omega$ . As can be seen, the values of voltage  $V_{out}$  were the highest for the RTN core, whereas the lowest was for the RTP core. The differences between these values were the biggest for low f and at f = 10 kHz, they exceeded 40 V. An acceptable agreement between the obtained measurements and computations results was obtained.



Figure 10. Dependences of the transformer output voltage on frequency.

Comparing the computation results obtained using the new model and presented in Figure 10, it is apparent that the best accuracy of modeling was obtained for the RTP core. A good agreement between the results of measurements and computations performed using the new model was also obtained for the other transformers. In contrast, for the model from [15], an acceptable agreement between the measurements and computations results could not be obtained in many cases.

#### 4.2. Planar Transformers

Figure 11 illustrates the influence of frequency on the measured and computed values of the core temperature  $T_C$  of a planar transformer at two sets of operating conditions characterized by: (a) load resistance  $R_0 = 100 \Omega$  and supply voltage amplitude  $V_m = 30 V$ , (b)  $R_0 = 470 \Omega$  and  $V_m = 45 V$ .



**Figure 11.** Dependences of the temperature  $T_C$  on frequency.

As can be seen, the temperature  $T_C$  decreased with frequency and assumed higher values for the higher of the considered load resistance values. With an increase in f from 40 to 150 kHz, temperature  $T_C$  decreased by 20 °C. For  $R_0 = 100 \Omega$ , the values of temperature  $T_C$  computed with the new model and the model from the paper [15] were practically the same, whereas for  $R_0 = 470 \Omega$ , these values differed from each other by 15 °C.

Figure 12 presents the influence of f on the temperature of the secondary winding  $T_{W2}$  at selected values of  $R_0$  and  $V_m$ .



Figure 12. Dependences of temperature of the secondary winding on frequency.

The temperature  $T_{W2}$  decreased in function of the frequency. Comparing the values of  $T_{W2}$  and  $T_C$  shown in Figures 11 and 12, it can be seen that in the considered operating conditions, the core was warmer than the secondary winding. This means that power of a higher value was dissipated in the core.

Figures 11 and 12 show that the results of computations obtained using the proposed model were convergent to measurements results for f > 50 kHz. At frequency f = 40 kHz and lower considered values of load resistance, the computations results were overestimated by 20 °C. This means that in this range, the proposed model does not correctly describe power dissipated in the transformer components, especially in the second winding.



Figure 13 presents the results of measurements and computations of the transformer energy efficiency  $\eta$  as the function of frequency f with selected values of load resistance  $R_0$ .

Figure 13. Dependences of the transformer energy efficiency on frequency.

As can be seen, the energy efficiency of the transformer increased with an increase in frequency and it decreased if load resistance increased. The efficiency values obtained with resistance  $R_0$  equal to 100  $\Omega$  and 1 k $\Omega$  differed by three times. A decrease in the efficiency in the range of high load resistance resulted from the high value of no-load current, which became the dominant component of the primary winding current. Increasing the frequency from 40 to 150 kHz increased the energy efficiency by up to 15%.

Figure 14 presents the non-isothermal electrical characteristic of the planar core power supplied by direct current. In this case, in the computer analysis, the core was connected to the supplied networks using connectors  $X_1$  and  $X_2$  of the proposed model.



Figure 14. Non-isothermal electrical characteristics of the planar core.

As can be observed, the obtained dependence v(i) possessed the maximum at current i = 50 mA. The obtained shape of the considered characteristic was a result of a self-heating phenomenon and an increasing function of the core resistance on temperature  $T_C$  [25].

#### 5. Conclusions

This article presents the new electrothermal compact model of the impulse transformer. This model belongs to multi-domain models and it is dedicated for use with SPICE. In this model, magnetic phenomena occurring in the core, electrical phenomena in the winding, and thermal phenomena in the transformer components and between these components are simultaneously taken into account. In particular, the non-linearity of the removal of the heat generated in the transformer components is included in the model by the use of analytic formulae describing dependences of self and transfer thermal resistances on dissipated power.

Using the proposed model, it is possible to compute waveforms of voltages and the current of each winding, waveforms of magnetic force and magnetic flux density in the core, and waveforms of power dissipated in each winding and in the core. Taking into account thermal phenomena, waveforms of temperature of the core and of each winding can be also computed.

Accuracy and practical usefulness of the new model were verified experimentally for the transformers with ring cores made of different ferromagnetic materials and a planar transformer with the ferrite core. The demonstrated results of computations and measurements proved that the new model correctly describes the influence of frequency and load resistance on the transformer output voltage, its energy efficiency, and temperatures of the core and each winding. The observed differences between the computations and measurements results were smaller than in other models. Of course, many of the simple transformer models described in the literature do not take into account thermal phenomena and such models were not compared with the proposed model.

The accuracy of computations obtained using the proposed model was also satisfied from the point of view of scientific investigations and industry applications. Typically, the difference between computations and measurements results did not exceed a few Celsius degrees (for temperature) or a few percent for voltage and energy efficiency.

The disadvantage of the proposed model is long computation time indispensable to compute the transformer characteristics shown in this paper. In Table 2, the computation times of the mentioned characteristics are compared for the considered transformers. It is apparent that the computation time depends on the values of the model parameters, which correspond to the type and shape of the ferromagnetic core. The shortest computation times were obtained for the RTF ring core, whereas the longest were for the RTP core. Of course, the computation time will be shorter when fast computers are be used.

Results Presented in Figures	Transformer with the Core			
	Ring RTN	<b>Ring RTF</b>	Ring RTP	Planar 3F3
Figures 5–8	3435.5 s	2335.5 s	10,083.5 s	-
Figures 9 and 10	13,736 s	3826.8 s	107,224.8 s	-
Figures 11–13	-	-	-	16,477.7 s

Table 2. Computation times of the presented characteristics.

The presented investigations result also show the strong influence of the selection of the ferromagnetic material used to build the transformer core on the electrical and thermal properties of the transformers. For example, the transformer containing the RTP core is characterized by lower energy efficiency than transformers with other cores. Additionally, the core temperature of such transformers is monotonically increasing function of load resistance. For RTP cores, the transformer output voltage is much smaller (even twice) than for RTN or RTF cores.

The described model and presented investigations results can be usable for designers of electronic circuits. The presented findings can be also used in didactics to illustrate the influence of selected factors on the characteristics of impulse transformers.

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#### References

- 1. Ericson, R.; Maksimovic, D. Fundamentals of Power Electronics; Kluwer Academic Publishers: Norwell, MA, USA, 2001.
- 2. Kazimierczuk, M.K. Pulse-Width Modulated DC-DC Power Converters; John Wiley & Sons: Hoboken, NJ, USA, 2008.
- 3. Rashid, M.H. Power Electronic Handbook; Academic Press: Cambridge, MA, USA; Elsevier: Amsterdam, The Netherlands, 2007.
- 4. van den Bossche, A.; Valchev, V.C. *Inductors and Transformers for Power Electronics*; Taylor & Francis Group/CRC Press: Boca Raton, FL, USA, 2005.
- 5. Kazimierczuk, M.K. High–Frequency Magnetic Components; John Wiley & Sons: Hoboken, NJ, USA, 2014.
- 6. Górecki, K.; Rogalska, M. The compact thermal model of the pulse transformer. *Microelectron. J.* 2014, 45, 1795–1799. [CrossRef]
- Wilson, P.R.; Ross, J.N.; Brown, A.D. Simulation of magnetic component models in electric circuits including dynamic thermal effects. *IEEE Trans. Power Electron.* 2002, 17, 55–65. [CrossRef]
- 8. Tsli, M.A.; Amoiralis, E.I.; Kladas, A.G.; Souflaris, A.T. Power transformer thermal analysis by using an advanced coupled 3D heat transfer and fluid flow FEM model. *Int. J. Therm. Sci.* **2012**, *53*, 188–201. [CrossRef]
- Górecki, K.; Detka, K.; Górski, K. Compact thermal model of the pulse transformer taking into account nonlinearity of heat transfer. *Energies* 2020, 13, 2766. [CrossRef]
- 10. Basso, C.P. Switch-Mode Power Supply SPICE Cookbook; McGraw-Hill: New York, NY, USA, 2001.
- 11. Maksimovic, D.; Stankovic, A.M.; Thottuvelil, V.J.; Verghese, G.C. Modeling and simulation of power electronic converters. *Proc. IEEE* **2001**, *89*, 898–912. [CrossRef]
- 12. Górecki, P.; Wojciechowski, D. Accurate computation of IGBT junction temperature in PLECS. *IEEE Trans. Electron Devices* **2020**, 67, 2865–2871. [CrossRef]
- 13. Górecki, K.; Zarębski, J. The method of a fast electrothermal transient analysis of single-inductance dc-dc converters. *IEEE Trans. Power Electron.* **2012**, 27, 4005–4012. [CrossRef]
- 14. Godlewska, M.; Górecki, K.; Górski, K. Modelling the pulse transformer in SPICE. *IOP Conf. Series Mater. Sci. Eng.* 2016, 104, 012011. [CrossRef]
- 15. Górecki, K.; Godlewska, M. Modelling characteristics of the impulse transformer in a wide frequency range. *Int. J. Circuit Theory Appl.* **2020**, *48*, 750–761. [CrossRef]
- 16. Penabad-Duran, P.; Lopez-Fernandez, X.M.; Turowski, J. 3D non-linear magneto-thermal behavior on transformer covers. *Electr. Power Syst. Res.* **2015**, *121*, 333–340. [CrossRef]
- 17. Wilamowski, B.; Jager, R.C. Computerized Circuit Analysis Using SPICE Programs; McGraw-Hill: New York, NY, USA, 1997.
- 18. Poppe, A.; Farkas, G.; Gaál, L.; Hantos, G.; Hegedüs, J.; Rencz, M. Multi-Domain Modelling of LEDs for Supporting Virtual Prototyping of Luminaires. *Energies* 2019, *12*, 1909. [CrossRef]
- 19. Górecki, K.; Zarębski, J.; Górecki, P.; Ptak, P. Compact thermal models of semiconductor devices—A review. *Int. J. Electron. Telecommun.* **2019**, *65*, 151–158.
- D'Alessandro, V.; Rinaldi, N. A critical review of thermal models for electro-thermal simulation. *Solid State Electron.* 2002, 46, 487–496. [CrossRef]
- 21. Jiles, D.C.; Atherton, D.L. Theory of ferromagnetic hysteresis. J. Magn. Magn. Mater. 1986, 61, 48–60. [CrossRef]
- 22. Górecki, K.; Górski, K. Compact electrothermal model of an impulse transformer for SPICE. In Proceedings of the 14th International Conference on Compatibility, Power Electronics and Power Engineering CPE-POWERENG, Setubal, Portugal, 8–10 July 2020; pp. 82–87.
- Mawby, P.A.; Igic, P.M.; Towers, M.S. Physically based compact device models for circuit modelling applications. *Microelectron. J.* 2001, 32, 433–447. [CrossRef]
- Maxim, A.; Andreu, D.; Boucher, J. Novel behavioral method of SPICE macromodeling of magnetic components including the temperature and frequency dependencies. In Proceedings of the IEEE Application Power Electronics Conference and Exposition APEC'98, Anaheim, CA, USA, 15–19 February 1998; Volume 1, pp. 393–399.
- Górecki, K.; Górski, K. Modelling nonisothermal electrical characteristics of ferrite cores. J. Phys. Conf. Ser. 2018, 1033, 012005. [CrossRef]
- 26. Górecki, K.; Rogalska, M.; Zarębski, J.; Detka, K. Modelling characteristics of ferromagnetic cores with the influence of temperature. *J. Phys. Conf. Ser.* 2014, 494, 012016. [CrossRef]
- 27. Barlik, R.J.; Nowak, K.M. Energoelektronika. Elementy Podzespoły, Układy; Oficyna Wydawnicza Politech: Warsaw, Poland, 2014; ISBN 978-83-7814-062-7.
- 28. Górecki, K.; Górecki, P. Nonlinear compact thermal model of the IGBT dedicated to SPICE. *IEEE Trans. Power Electron.* **2020**, *35*, 13420–13428. [CrossRef]
- 3F3 Material Specification. *Ferroxcube*. 2008. Available online: http://ferroxcube.home.pl/prod/assets/3f3.pdf (accessed on 20 August 2021).

- 30. Iron Powder Toroidal Core for Power Applications. Power Conversion & Line Filter Applications, Micrometels Iron Powder Cores. Available online: https://www.acalbfi.com/nl/Magnetic-components/Cores/Iron-Powder/p/Iron-powder-toroidal-core-for-power-applications/000000BNV (accessed on 20 August 2021).
- 31. Product Specification for Inductive Components. Available online: https://www.magnetec.de/wp-content/uploads/2018/11/ m-070.pdf (accessed on 20 August 2021).
- 32. F-867 Material Characteristics. Available online: http://sklep.remagas.pl/images/W%C5%82a%C5%9Bciwo%C5%9Bci\_materia%C5%82u\_F-867.pdf (accessed on 20 August 2021).