



Jie Huang, Ke-Yu Pan, Xue-Lei Feng * D and Yong Shen *

Key Laboratory of Modern Acoustics, Institute of Acoustics, Nanjing University, Nanjing 210093, China; hjacoustics@smail.nju.edu.cn (J.H.); kypan@smail.nju.edu.cn (K.-Y.P.)

* Correspondence: xlfeng@nju.edu.cn (X.-L.F.); yshen@nju.edu.cn (Y.S.); Tel.: +86-025-8359-3416 (Y.S.)

Abstract: Nonlinear acoustic damping is a key nonlinearity in miniature loudspeakers when the air velocity is at a high amplitude. Measurement of nonlinear acoustic damping is beneficial for predicting and analyzing the performance of miniature loudspeakers. However, the general measuring methods for acoustic impedance, such as the standing-wave tube method or the impedance tube method, are not applicable in this scenario because the nonlinear acoustic damping in miniature loudspeakers is coupled with other system nonlinearities. In this study, a measurement method based on nonlinear system identification was constructed to address this issue. The nonlinear acoustic damping was first theoretically analyzed and then coupled in an equivalent circuit model (ECM) to describe the full dynamics of miniature loudspeakers. Based on the ECM model, the nonlinear acoustic damping was identified using measured electrical data and compared with theoretical calculations. The satisfactory agreement between the identification and theoretical calculations confirms the validity of the proposed identification method.

Keywords: nonlinear acoustic damping; nonlinear system identification; miniature loudspeaker; equivalent circuit model

check for **updates**

Citation: Huang, J.; Pan, K.-Y.; Feng, X.-L.; Shen, Y. Analysis and Identification of Nonlinear Acoustic Damping in Miniature Loudspeakers. *Appl. Sci.* **2021**, *11*, 7713. https:// doi.org/10.3390/app11167713

Academic Editor: Lamberto Tronchin

Received: 23 July 2021 Accepted: 18 August 2021 Published: 21 August 2021

Publisher's Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.



Copyright: © 2021 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). 1. Introduction

It has been proven that nonlinear acoustic damping is a dominant nonlinearity in miniature loudspeakers and has a significant influence on their performance [1–3]. Klippel showed that the acoustic damping in ports is nonlinear and can generate distortion at the port resonance when describing the nonlinearities in micro-speakers [1]. Sun et al. predicted the nonlinear resistance and distortion of miniature loudspeakers by considering the nonlinear acoustic damping caused by the gaps and orifices in a magnet system [2]. Huang et al. analyzed the total harmonic distortion of miniature loudspeakers that are used in mobile phones and found that nonlinear acoustic damping can lead to significant harmonic distortions at both lower and higher frequency bands [3].

While previous works have been devoted to the theoretical analysis of nonlinear acoustic damping and its influence on performance, less attention has been paid to the real measurement of the nonlinear acoustic damping in miniature loudspeakers. The nonlinearities of acoustic damping on orifices or resonators have been investigated theoretically and experimentally for many years in sound absorption theory [4–12]. Sivian measured the acoustic impedance of orifices under various particle velocities and found that the nonlinear resistance of the orifice is proportional to the velocity [4]. Dickey et al. investigated the nonlinear damping of perforated tubes using an extended impedance tube method and proposed a time-domain computational model to predict the acoustic behavior of perforated tubes at a high sound pressure level using a two-microphone method [8,10]. It was found by the authors that the radius of the curvature of the edges of the open end of the tube has a crucial influence on the amplitude of nonlinear acoustic damping. Nonlinear acoustic damping can be obtained with accuracy using the measurement method used in

these works. However, the general measuring methods of the acoustic impedance used in these works, such as the standing-wave tube method and the impedance tube method, are not applicable in this scenario because the nonlinear acoustic damping in miniature loudspeakers is coupled with other nonlinearities in the entire loudspeaker system [3,13]. In this study, a measurement method based on nonlinear system identification was constructed to address this issue. The system identification method has been widely used in the electroacoustics industry to measure the nonlinear parameters of loudspeakers and miniature loudspeakers [14–17]. Current research is limited to identifying the nonlinearities in the electrical and mechanical domains. To the best of our knowledge, no work to date has focused on identifying the nonlinearities in the acoustical domain. Thus, in this study, nonlinear acoustic damping was first theoretically analyzed and then coupled in an equivalent circuit method (ECM) model to describe the full dynamics of miniature loudspeakers. Based on the ECM model, nonlinear acoustic damping was identified using measured electrical data.

The remainder of this paper is organized as follows: Section 2 summarizes the theoretical analysis of the nonlinear acoustic damping and establishes an ECM model of miniature loudspeakers with consideration of the nonlinear acoustic damping. In Section 3, a measurement method based on nonlinear system identification is proposed, and its implementation is introduced in detail to identify the nonlinear acoustic damping in miniature loudspeakers. Section 4 describes the experiments conducted to obtain the nonlinear acoustic damping and compares the results with theoretical calculations. Some discussion is presented in this section. Finally, conclusions are drawn in the last section.

2. Miniature Loudspeaker Model Considering Nonlinear Acoustic Damping

The miniature loudspeakers discussed in this paper are limited to moving-coil type transducers. Figure 1 shows a cross-sectional view of a typical miniature loudspeaker. The polar piece, magnet, and under yoke together form a magnet system and generate a magnetic field around the voice coil. When current or voltage is applied to the voice coil, Lorentz force generates and drives the mechanical motion of the diaphragm and the dome to radiate sound. A front cover with an orifice is mounted on the top of a miniature loudspeaker to protect the diaphragm from direct contact with other objects.



Figure 1. Cross-sectional view of a moving-coil miniature loudspeaker.

2.1. Theoretical Analysis of Nonlinear Acoustic Damping in Miniatue Loudspeakers

When a miniature loudspeaker works under a large signal condition, the air velocity through the orifice on the front cover is sufficiently large. It is well-known that the acoustic damping of an orifice becomes nonlinear with high-amplitude air velocity. Flow separation occurs near the orifice, resulting in nonlinear acoustic damping in a miniature loudspeaker.

As depicted in Figure 2, the deviation in the nonlinear acoustic damping can be obtained by applying the conservation of momentum to a control volume around the orifice of the miniature loudspeaker [7,9]:

$$\rho_0 S_0 l_{eq} \frac{dv_0}{dt} + \rho_0 (v_l^2 S_l - v_0^2 S_0) + \tau_\omega S_\omega = (p_0 - p_l) S_0 \tag{1}$$

where ρ_0 denotes the air density, l_{eq} represents the equivalent length of the orifice, and τ_{ω} is the frictional wall shear stress distributed over the orifice-wetted area S_{ω} ; v_0 and P_0 denote the air velocity and the sound pressure at the starting surface of the control volume, respectively; and v_l and P_l denote the air velocity and the sound pressure at the ending surface of the control volume, respectively. The terms in Equation (1) describe the inertia force, momentum convection, momentum loss caused by the sheer force, and pressure forced on the control volume, respectively. The air velocity through the starting and ending surfaces can be related using the discharge coefficient C_D with $v_l = v_0/C_D$. The discharge coefficient represents the average volume flow rate entering/exiting the control volume [9]. Another relationship between v_0 and v_l can be obtained from the conservation of mass within the control volume, $v_0S_0 = v_1S_1$. By dividing both sides of Equation (1) by S_0 , the equation can be rewritten as:

$$\Delta p = \rho_0 l_{eq} \left. \frac{dv_0}{dt} + R_I v_0 S_0 + \rho_0 \frac{1 - C_D}{C_D} v_0 \right| v_0 \right|$$
(2)

where $R_l = \frac{8\pi\mu l_{eq}}{S_0^2}$ denotes the equivalent linear acoustic damping coefficient caused by $\tau_{\omega}S_{\omega}$ with μ being the viscosity coefficient of air [18]. The absolute value of v_0 is used to ensure that Equation (2) is applicable to both the in-flow and out-flow half cycles.

From Equation (2), the nonlinear acoustic damping in miniature loudspeakers can be obtained as:

$$R_a = R_l + \rho_0 \frac{1 - C_D}{C_D} \frac{|v_0|}{S_0}$$
(3)

which clearly shows that the acoustic damping of an orifice contains a linear part engendered by viscous losses and a nonlinear part resulting from flow separation. Notably, the discharge coefficient C_D is known to be sensitive to the shape of the orifice. In this study, $C_D \approx 0.68$ was used in the theoretical calculations with consideration of the squared-edge orifice. Some details about the discharge coefficients are discussed in Section 4.



Figure 2. Control volume used in nonlinear acoustic damping deviation.

2.2. ECM Model of Miniature Loudspeakers Considering Nonlinear Acoustic Damping

The ECM model describing the full dynamics of a miniature loudspeaker is shown in Figure 3. In the electrical domain, the voice coil is modeled as an inductor L_e and a resistor R_e . Mechanical resistance R_m , moving mass M_m , and mechanical stiffness K_m were used to model the moving part of the miniature loudspeaker. Force factor *Bl* represents the coupling

effect between the electrical and mechanical systems. In the acoustic domain, inductor M_a is employed to represent the acoustical mass of the air pushed by the diaphragm and dome. In particular, a nonlinear parameter R_a is used to model the nonlinear acoustic damping near the orifice of the miniature loudspeaker. The effective radiation area A of the diaphragm behaves as a transformer connecting the mechanical and acoustical domains.



Figure 3. ECM model of miniature loudspeakers considering nonlinear acoustic damping.

In the large-signal domain, mechanical stiffness K_m , mechanical resistance R_m , and force factor Bl are nonlinear and dependent on the displacement/velocity of the voice coil [11]. Based on the ECM model shown in Figure 3, the following loop equations can be obtained:

$$u = R_e i + L_e \frac{di}{dt} + Bl(x)v \tag{4}$$

$$Bl(x)i = M_m \frac{dv}{dt} + R_m(v)v + K_m(x)x + pA$$
(5)

$$p = M_a \frac{d(vA)}{dt} + R_a(v)vA \tag{6}$$

where u is the applied voltage, i denotes the current, x is the displacement of the voice coil, v represents the velocity of the voice coil, and p is the sound pressure on the diaphragm and dome. By substituting Equation (6) into Equation (5), we obtain

$$Bl(x)i = M'_{m}\frac{dv}{dt} + (R_{m}(v) + R_{ma}(v))v + K_{m}(x)x$$
(7)

where $M'_m = (M_m + M_a A^2)$ is the equivalent moving mass and $R_{ma}(v) = R_a(v)A^2$ is the equivalent mechanical resistance. The loop equations can be discretized into the following forms according to Equations (4) and (7) using the forward Euler method:

$$u[n] = R_e i[n] + L_e \frac{i[n+1] - i[n]}{T} + Bl(x[n])v[n]$$
(8)

$$Bl(x[n])i[n] = M'_{m} \frac{v[n+1] - v[n]}{T} + (R_{m}(v[n]) + R_{ma}(v[n]))v[n] + K_{m}(x[n])x[n]$$
(9)

with *T* being the discrete time interval.

3. Identification Strategy for Nonlinear Acoustic Damping

3.1. Nonlinear Parameters

To identify nonlinear acoustic damping, other parameters in the loop equations must be simultaneously identified. Nonlinearities in the electrical and mechanical domains can be expressed as polynomial expansion functions [13], as follows:

$$K_m(x) = k_0 + k_1 x + k_2 x^2 + k_3 x^3 + k_4 x^4$$
(10)

$$Bl(x) = b_0 + b_1 x + b_2 x^2 + b_3 x^3 + b_4 x^4$$
(11)

$$R_m(v) = r_0 + r_1 v + r_2 v^2 + r_3 v^3 + r_4 v^4$$
(12)

From Equations (7) and (9), we can readily determine that the nonlinear acoustic damping, $R_a(v)$, is coupled with the nonlinear mechanical resistance, $R_m(v)$. Therefore, the nonlinear mechanical resistance described by Equation (12) should be measured in advance, which can be realized by measuring the miniature loudspeaker using an electroacoustic test instrument, such as the Klippel R&D System [14], when the front cover is removed.

According to Equation (3), nonlinear acoustic damping R_a is a function of the absolute value of v_0 , that is, the air velocity through the orifice. By applying the law of conservation of mass, the air velocity through the orifice can be related to the velocity of the voice coil, $v_0S_0 = vA$. Thus, the nonlinear acoustic damping R_a and the equivalent mechanical resistance R_{ma} can be expressed as follows:

$$R_a(v) = R_l + \rho_0 \frac{1 - C_D}{C_D} \frac{|v|A}{S_0^2}$$
(13)

$$R_{ma}(v) = R_a(v)A^2 = R_l A^2 + \rho_0 \frac{1 - C_D}{C_D} \frac{|v|A^3}{S_0^2}$$
(14)

Based on Equation (14), we can use two parameters to describe $R_{ma}(v)$ in a concise form, as follows:

$$R_{ma}(v) = c_0 + c_1 |v| \tag{15}$$

As discussed in the next subsection, all nonlinear parameters in the loop equations should be differentiable. However, an issue arises when Equation (15) is used directly in the identification process because Equation (15) has no derivative with respect to velocity at zero owing to the absolute value function. A differentiable approximation of Equation (15) can be used in the identification process, as follows:

$$R_{ma}(v) = c_0 + c_1 \left(\frac{v}{1 + e^{-\delta v}} - \frac{v}{1 + e^{\delta v}} \right)$$
(16)

where δ is the coefficient with a positive value that controls the degree of approximation. An example of the comparison between Equations (15) and (16) is shown in Figure 4 with $\delta = 100$, $c_0 = 0$, and $c_1 = 1$. Negligible differences are found only near the zero point.



Figure 4. Comparison between Equations (15) and (16).

3.2. Identification Strategy

Parameter identification can be regarded as a multiple parameter optimization problem in which the model is closely adjusted to the physical plant. Figure 5 shows a block diagram of the identification process. Based on the above analysis, the parameters that need to be identified are $[w_p] = [R_e L_e b_{0-4} k_{0-4} c_0 c_1]$, with p = 1, 2, ... 14. The equivalent moving mass is measured in advance and assumed to be constant during the entire optimization process. The nonlinear mechanical resistance $R_m(v)$ is also measured in advance and directly fed into the identification process.



Figure 5. Block diagram of the identification process.

In the proposed identification process, the measured electrical data were used as the training set to identify the parameters. The difference between the measured voltage u_m and the predicted voltage u_p using the measured current i_m via Equation (8) was chosen as the error signal in the identification process. The cost function *J* was defined as the mean square of the error signal (MSE). Thus,

$$e[n] = u_m[n] - u_p[n] \tag{17}$$

$$J = E\left[e[n]^2\right] \tag{18}$$

where *E* is the expectation operator. The discretized velocity v[n] can be updated recursively from Equation (9) as

$$v[n+1] = v[n] + \frac{T}{M'_m} \{Bl(x[n])i[n] - (R_m(v[n]) + R_{ma}(v[n]))v[n] - K_m(x[n])x[n]\}$$
(19)

The displacement *x*[*n*] can be updated as

$$x[n+1] = x[n] + Tv[n]$$
(20)

The parameters $[w_p]$ are updated along the negative error gradient direction,

$$w_p[n+1] = w_p[n] - \frac{1}{2}\mu_p \frac{\partial J}{\partial w_p}[n] = w_p[n] - \mu_p e[n] \frac{\partial e[n]}{\partial w_p}$$
(21)

with μ_p being the step size for parameter w_p . In Equation (21), the gradient of the cost function is approximated by the gradient of the instantaneous value of the error. The error gradient $\frac{\partial e[n]}{\partial w_p}$ can be calculated from Equations (8) and (17) as follows:

$$\frac{\partial e[n]}{\partial w_p} = -i_m[n]\frac{\partial R_e[n]}{\partial w_p} - \frac{i_m[n+1] - i_m[n]}{T}\frac{\partial L_e[n]}{\partial w_p} - v[n]\frac{\partial Bl(x[n])}{\partial w_p} - Bl(x[n])\frac{\partial v[n]}{\partial w_p}$$
(22)

By minimizing cost function *J*, all parameters can be identified, including c_0 and c_1 . Once c_0 and c_1 are determined, the nonlinear acoustic damping can be acquired according to Equations (13)–(15). That is,

$$R_a(v_0) = \frac{c_0}{A^2} + c_1 \frac{S_0}{A^3} |v_0|$$
(23)

3.3. Identification Algorithm

The identification algorithm is summarized as follows:

- 1. Obtain M'_m and $R_m(v)$ from the normal electroacoustic testing system and keep them unchanged during the identification process.
- 2. Initialize the parameter vector $[w_p]$ as $[w_p(0)] = [R_e(0) L_e(0) b_{0-4}(0) k_{0-4}(0) c_0(0) c_1(0)]$ and start the iteration from n = 1.
- 3. Select an appropriate step size vector $[\mu_p]$ and specify a convergence indicator ε , which is defined as the relative deviation of the parameters of adjacent frames.
- 4. Solve $u_p[n]$ using the measured current $i_m[n]$ as well as the current parameter vector $[w_p[n+1]]$, and calculate the gradient of the instantaneous value of the error $\frac{\partial e[n]}{\partial w_n}$.
- 5. Update $w_{v}[n+1]$ from Equation (21) and set the search iteration to n = n + 1.
- 6. Check for termination. If ε is smaller than a tiny tolerance, or the maximum number of iterations is reached, end the iteration process; otherwise, restart from step (4).
- 7. Calculate the nonlinear acoustic damping using Equation (23) with the identified values of c_0 and c_1 .

4. Experimental Validation and Discussion

Experimental validations were conducted to verify the effectiveness of the identification algorithm proposed in Section 3 for nonlinear acoustic damping. A miniature loudspeaker with dimensions of $12 \times 16 \times 2.8$ mm was selected as the experimental object. The opening area of the orifice on the front cover was selected as the control variable. Three front covers with different opening areas were fabricated and mounted in turn on the same miniature loudspeaker. Nonlinear acoustic damping and other nonlinearities were identified. The geometric parameters of the orifices on the front covers are listed in Table 1.

Table 1. Parameters of the orifices and the miniature loudspeaker.

Parameters	Orifice 1	Orifice 2	Orifice 3
Diameter (mm)	2.00	3.00	4.00
Thickness (mm)	1.00	1.00	1.00
$M_m'(g)$	$1.31 imes 10^{-4}$	$1.21 imes10^{-4}$	$1.16 imes10^{-4}$
$R_m(v)$	1.007 imes 10	$^{-2}v^2 - 1.724 \times 10^{-3}v + 1.$	$658 imes 10^{-1}$

4.1. Measurement Setup

Before starting the identification process, the equivalent moving mass of the miniature loudspeaker with three different front covers was measured using Klippel R&D System (Klippel GmbH, Germany), as listed in Table 1, along with the nonlinear mechanical resistance. Pink noise signals with rated power were applied to the miniature loudspeaker, ensuring that the miniature loudspeaker worked under large signal condition. The voltage and current data were recorded at the terminals of the miniature loudspeaker and used for parameter identification. The acquisition device was a B&K PULSE (Brüel & Kjær, Denmark), and the sampling rate was set to 65,536 Hz.

4.2. Results and Discussions

4.2.1. Nonlinear Acoustic Damping

Figure 6 compares the nonlinear acoustic damping obtained from the identification algorithm and the theoretical calculations using Equation (3). Satisfactory agreements can be observed between the identified results and theoretical calculations for the three different orifice cases. It can be concluded that the proposed identification algorithm is effective for identifying the nonlinear acoustic damping in miniature loudspeakers. Furthermore, by comparing the identified values of the nonlinear acoustic damping in the three different orifice cases, it is evident that the front cover with the smallest orifice has the largest nonlinearity in the nonlinear acoustic damping. This phenomenon can be explained

by the theoretical analysis of Equation (3), where $R_a(v_0)$ is inversely proportional to the area of the orifices.



Figure 6. Comparison between the identified and calculated values of the nonlinear acoustic damping $R_a(v_0)$.

Figure 7 depicts the identified values of the equivalent mechanical resistance $R_{ma}(v)$. The measured data of the mechanical resistance $R_m(v)$ are also plotted in Figure 7 for comparison. It can be observed that $R_{ma}(v)$ increases in accordance with the velocity of the voice coil. Furthermore, the equivalent mechanical resistance $R_{ma}(v)$ caused by the nonlinear acoustic damping is comparable to the mechanical resistance $R_m(v)$, especially in the small orifice case. This finding proves that the nonlinear acoustic damping is the dominant nonlinearity in miniature loudspeakers.



Figure 7. Comparison between the identified values of the equivalent mechanical resistance $R_{ma}(v)$ and the measured mechanical resistance $R_m(v)$.

4.2.2. Discharge Coefficient

As mentioned in Section 3, the discharge coefficient is used to describe the flow separation phenomena near the orifice. The discharge coefficient is an experimentally determined quantity and is known to be sensitive to the orifice shape and surface conditions. Several different values of the discharge coefficient have been used in the literature for squared-edge orifice cases. Zinn [6] employed a value of 0.61 in his theoretical calculations,

and Hersh et al. obtained a value of 0.64 for high sound pressure condition [9]. Moreover, Förner et al. obtained a value of 0.7 based on numerical simulation results [11].

The discharge coefficient can be obtained from the identification process proposed in this study. Based on Equations (14) and (15), the discharge coefficient can be derived as $C_D = \frac{\rho_0 A^3}{\rho_0 A^3 + c_1 S_0^2}$. The identified values of the discharge coefficient and values used in literatures are listed in Table 2. The average value obtained from the identification process is 0.698, which is very close to the values used in Förner's work.

Table 2. Discharge coefficients obtained from system identification.

Orifice 1	Orifice 2	Orifice 3	Average	Zinn	Hersh	Förner
0.682	0.685	0.726	0.698	0.61	0.64	0.7

4.2.3. Other Parameters

Figure 8 depicts the identified nonlinear stiffness $K_m(x)$ and nonlinear force factor Bl(x) of the miniature loudspeaker when different front covers were considered. The good consistency of the identified nonlinearities between different front covers demonstrates the robustness of the proposed identification algorithm. Although one would expect these two parameters to be fully consistent across all experiments, the results in Figure 8 are reasonable because a moving coil miniature loudspeaker is an analog device whose parameters may be influenced by its working conditions. The nonlinear stiffness and force factor measured using traditional electroacoustic test instruments when the front cover is removed are also plotted in Figure 8 for comparison. The satisfactory agreement between the measured data and the identified data confirms the validity of the proposed ECM model.



Figure 8. Comparison of the (a) nonlinear stiffness $K_m(x)$ and (b) nonlinear force factor Bl(x).

5. Conclusions

The nonlinear acoustic damping originating from the flow separation near the orifice of the front cover of a miniature loudspeaker was theoretically analyzed. An ECM model was established by considering the nonlinear acoustic damping to describe the full dynamics of the miniature loudspeaker. An identification algorithm was proposed based on the ECM model to identify the nonlinear acoustic damping in miniature loudspeakers. Experiments and calculations confirmed the effectiveness of the proposed identification algorithm. The identified nonlinear acoustic damping results agreed well with the theoretical calculations. In addition, the discharge coefficients obtained from the identification process are consistent with the results described in the literature. The identification algorithm proposed in this study is an effective tool for measuring the nonlinear acoustic damping in miniature loudspeakers. Based on the theoretical analysis and experimental identifications of nonlinear acoustic damping, the compensation of the resulting distortion is worthy of further study.

Author Contributions: Conceptualization, J.H.; data curation, J.H. and K.-Y.P.; formal analysis, J.H.; funding acquisition, X.-L.F. and Y.S.; investigation, J.H.; methodology, J.H.; project administration, X.-L.F. and Y.S.; resources, J.H. and Y.S.; software, J.H. and K.-Y.P.; supervision, X.-L.F. and Y.S.; validation, J.H.; visualization, J.H.; writing—original draft, J.H.; writing—review and editing, K.-Y.P., X.-L.F. and Y.S. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by National Natural Science Foundation of China (Grant No. 11904330).

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

References

- 1. Klippel, W. Micro-speakers—Hybrids between headphones and loudspeakers. In Proceedings of the Conference on Headphone Technology of the Audio Engineering Society, Aalborg, Denmark, 24–26 August 2016.
- 2. Sun, C.; An, F.; Liu, B.; Wu, S. The prediction of nonlinear resistance and distortion for a miniature loudspeaker with vented cavities. *J. Acoust. Soc. Am.* **2019**, *146*, 4315–4321. [CrossRef] [PubMed]
- 3. Huang, J.; Feng, X.; Chen, S.; Shen, Y. Analysis of total harmonic distortion of miniature loudspeakers used in mobile phones considering nonlinear acoustic damping. *J. Acoust. Soc. Am.* **2021**, *149*, 1579–1588. [CrossRef] [PubMed]
- 4. Sivian, L.J. Acoustic impedance of small orifices. J. Acoust. Soc. Am. **1935**, 7, 94–101. [CrossRef]
- 5. Thurston, G.B.; Hargrove, L.E.; Cook, B.D. Nonlinear properties of circular orifices. J. Acoust. Soc. Am. 1957, 29, 992–1001. [CrossRef]
- 6. Zinn, B. A theoretical study of non-linear damping by Helmholtz resonators. J. Sound Vib. 1970, 13, 347–356. [CrossRef]
- Dickey, N.S.; Selamet, A.; Novak, J.M. The effect of high-amplitude sound on the attenuation of perforated tube silencers. J. Acoust. Soc. Am. 2000, 108, 1068–1081. [CrossRef] [PubMed]
- 8. Atig, M.; Dalmont, J.-P.; Gilbert, J. Termination impedance of open-ended cylindrical tubes at high sound pressure level. *Comptes Rendus Mécanique* **2004**, 332, 299–304. [CrossRef]
- 9. Hersh, A.S.; Walker, B.E.; Celano, J.W. Helmholtz resonator impedance model, part 1: Nonlinear behavior. *AIAA J.* **2003**, *41*, 795–808. [CrossRef]
- 10. Buick, J.M.; Atig, M.; Skulina, D.J.; Campbell, D.M.; Dalmont, J.P.; Gilbert, J. Investigation of non-linear acoustic losses at the open end of a tube. *J. Acoust. Soc. Am.* 2011, 129, 1261–1272. [CrossRef] [PubMed]
- 11. Förner, K.; Tournadre, J.; Martínez-Lera, P.; Polifke, W. Scattering to higher harmonics for quarter-wave and Helmholtz res-onators. *AIAA J.* 2017, 55, 1194–1204. [CrossRef]
- 12. Komkin, A.; Bykov, A.; Mironov, M. Experimental study of nonlinear acoustic impedance of circular orifices. *J. Acoust. Soc. Am.* **2020**, *148*, 1391–1403. [CrossRef] [PubMed]
- 13. Klippel, W. Tutorial: Loudspeaker nonlinearities—Causes, parameters, symptoms. J. Audio Eng. Soc. 2006, 54, 907–939.
- 14. Klippel, W. Distortion analyzer—A new tool for assessing and improving electrodynamic transducer. In Proceedings of the 108th Convention of the Audio Engineering Society, Paris, France, 19–22 February 2000.
- 15. Klippel, W. Measurement of large-signal parameters of electrodynamic transducer. In Proceedings of the 108th Convention of the Audio Engineering Society, New York, NY, USA, 24–27 September 1999.
- 16. Bright, A. Active Control of Loudspeakers: An Investigation of Practical Applications. Ph.D. Thesis, Technical University of Denmark, Lyngby, Denmark, 2002.
- 17. Tian, X.; Shen, Y.; Chen, L.; Zhang, Z. Identification of nonlinear fractional derivative loudspeaker model. *J. Audio Eng. Soc.* 2020, 68, 355–363. [CrossRef]
- 18. Huang, J.H.; Her, H.-C.; Shiah, Y.C.; Shin, S.-J. Electroacoustic simulation and experiment on a miniature loudspeaker for cellular phones. *J. Appl. Phys.* 2008, 103, 33502. [CrossRef]