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Cybersecurity against the Loopholes in Industrial Control Systems Using Interval-Valued Complex Intuitionistic Fuzzy Relations

Abdul Nasir [1], Naeem Jan [1], Abdu Gumaei [2,*], Sami Ullah Khan [1] and Fahad R. Albogamy [3]

[1] Department of Mathematics, Institute of Numerical Sciences, Gomal University, Dera Ismail Khan 29050, Pakistan; theabdulnasir@gmail.com (A.N.); naeem.phdma73@iiu.edu.pk (N.J.); gomal85@gmail.com (S.U.K.)
[2] STC's Artificial Intelligence Chair, Department of Information Systems, College of Computer and Information Sciences, King Saud University, Riyadh 11543, Saudi Arabia
[3] Computer Sciences Program, Turabah University College, Taif University, P.O. Box 11099, Taif 21944, Saudi Arabia; f.alhammdani@tu.edu.sa
* Correspondence: agumaei.c@ksu.edu.sa

Abstract: Technology is rapidly advancing and every aspect of life is being digitalized. Since technology has made life much better and easier, so organizations, such as businesses, industries, companies and educational institutes, etc., are using it. Despite the many benefits of technology, several risks and serious threats, called cyberattacks, are associated with it. The method of neutralizing these cyberattacks is known as cybersecurity. Sometimes, there are uncertainties in recognizing a cyberattack and nullifying its effects using righteous cybersecurity. For that reason, this article introduces interval-valued complex intuitionistic fuzzy relations (IVCIFRs). For the first time in the theory of fuzzy sets, we investigated the relationships among different types of cybersecurity and the sources of cyberattacks. Moreover, the Hasse diagram for the interval-valued complex intuitionistic partial order set and relation is defined. The concepts of the Hasse diagram are used to inspect different cybersecurity techniques and practices. Then, using the properties of Hasse diagrams, the most beneficial technique is identified. Furthermore, the best possible selection of types of cybersecurity is made after putting some restrictions on the selection. Lastly, the advantages of the proposed methods are illuminated through comparison tests.

Keywords: cybercrime; cybersecurity; Hasse diagram; interval-valued complex intuitionistic fuzzy relations; interval-valued complex intuitionistic fuzzy sets

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1. Introduction

Mathematical modeling is a process of transforming practical problems and real life events into a mathematical form. Human opinions and views are generally imprecise and unclear. Similarly, experimental errors, inaccuracies and miscalculations lead to inexact results. Hence, uncertainty is an inevitable part of our lives. Information is often complex, unknowable and puzzling. Modeling uncertainty was almost impossible before 1965 when Zadeh [1] introduced the world to a life changing theory of fuzzy sets (FS) and fuzzy logic. This theory opened up new gates for mathematicians because it could model uncertainty such as complexity in human opinions and errors in scientific experimentations. An FS is a set that associates a mapping $\acute{m}$ to each of its elements, where $\acute{m}$ is called the degree of membership that attains values from the $[0, 1]$ interval.

The concept of relations between crisp sets was defined by Klir [2]. Crisp sets are limited to only two possibilities; the elements are either a member of a set or not a member of a set. In other words, crisp sets and their relations only work with yes-and-no-type problems. This theory of sets deals with exact information and cannot be used to model uncertainty. Mendel [3] invented the concepts of relations for FSs, which are called fuzzy relations (FRs). FRs are not just limited to respond in yes-or-no, but they can also specify the strength, grade and level of good relations between any pair of FSs. This depends on its degree of membership: if the value of the degree of membership is near one then the relation is said to be a strong relation, while values near zero are the sign of weak relationship. FRs generalize crisp relations because, if the values of the degrees of membership of each element of an FR are restricted to zero (0) and one (1), then the FR becomes a crisp relation. In 1975, Zadeh [4] generalized FSs and introduced the idea of interval-valued fuzzy sets (IVFSs). An IVFS replaces the single value of the degree of membership in an FS by an interval, whose extremes belong to a $[0, 1]$ interval, i.e., the degree of membership is a subinterval of a $[0, 1]$ interval. The concept of relation was introduced to IVFSs by Bustince and Burillo [5], called the interval-valued fuzzy relation (IVFR). Goguen [6] gave a system of axioms for a relatively simple form of FS theory; Zywica [7] modeled the medical uncertainties using FSs; Roman-Flores et al. [8] proposed a note on Zadeh's extensions; Dubious and Prade [9] discussed uncertainty, bipolarity and gradualness through FSs; Gehrke et al. [10] commented on IVFSs; Bustince [11] applied IVFSs to approximate reasoning; Turksen [12] entitled his work *IVFSs and compensatory AND*.

In 2002, Ramot et al. [13] came up with an innovative idea of switching to complex valued mappings as the degrees of membership of a set. They introduced the complex fuzzy set (CFS) in which the degree of membership takes on values from a unit disk of a complex plane. Since the degrees of membership in a CFS are complex numbers, thus, they involve two parts: the real and imaginary parts. Each of these parts corresponds to a different entity. The real part is known as the amplitude term of the degree of membership, while the imaginary part is said to be the phase term of the degree of membership. Most often, CFSs are used to model phase altering problems. When the phase term or the imaginary part of the degree of membership is set to zero in the CFS, it becomes an FS. Further, Ramot et al. [14] defined complex fuzzy relations (CFRs), which are used to find the relationships between CFSs. Greenfield et al. [15] changed the degree of membership of a CFS from a single number to an interval, thus bringing up the notion of interval-valued complex fuzzy set (IVCFS). Recently, Nasir et al. [16] defined interval-valued complex fuzzy relations as being used for studying the relationships between two or more IVCFSs. Chen et al. [17] studied a neuro-fuzzy architecture employing CFSs; Yazdanbakhsh and Dick [18] reviewed the CFSs; Tamir et al. [19] proposed some applications of CFSs; Dai et al. [20] formulated distance measures between IVCFSs; Greenfield et al. [21] defined the join and meet operations for IVCFSs.

According to the definition of an FS, it only discusses the degree of membership $\acute{m}$, but it also covers the degree of non-membership $\underline{n}$, which is the complement of the degree of membership with respect to unit interval, i.e., $\underline{n} = 1 - \acute{m}$. This hidden feature of FSs led to the definition of the intuitionistic fuzzy set (IFS), proposed by Atanassov [22]. An IFS associates two mappings to each of its elements, which are called the degree of membership and the degree of non-membership. Both of these mappings take on values from $[0, 1]$ interval given that their sum also lies within the range of this interval. The idea of the intuitionistic fuzzy relation (IFR) was concocted by Burillo et al. [23]. In 1999, Atanassov [24] expressed the degree of membership and the degree of non-membership of an IFS in the form of intervals and, hence, developed a new concept called the interval-valued intuitionistic fuzzy set (IVIFS). Complex valued mappings were brought in to IFS theory by Alkouri et al. [25] in 2012, who introduced the notion of the complex intuitionistic fuzzy set (CIFS). Garg and Rani [26] put forward the interval-valued complex intuitionistic fuzzy relation (IVCIFS). Li [27] used IFSs for multiattribute decision-making (MD) models and methods; De et al. [28] applied IFSs in medical diagnosis; Vlachos and Sergiadis [29] applied IFSs to pattern recognition; Lee et al. [30] compared IVFSs, IFSs and bipolar-valued FSs; Grzegorzewski [31] used Hausdorff metric for finding the distances between IVFSs and IFSs; Nasir et al. [32–35] applied complex relations to analysis of economic relationships. Ali et al. [36] studied complex intuitionistic fuzzy classes; Liu and Jiang [37] proposed a new distance measure of IVIFSs and applied it in decision making; Bustince and Burillo [38] devised the correlation of IVIFSs; Nayagam and Sivaraman [39] proposed a method of ranking IVIFSs. Otero et al. [40] proposed the application of fuzzy logic in assessing the security control for organizations; Tariq et al. [41,42] also used fuzzy theory to rank and prioritize the security controls for cloud wireless sensor networks and computing networks; Mokhtari et al. [43] used fuzzy multiple-attribute decision-making techniques for selecting the best control system.

In this paper, the Cartesian product (CP) of two IVCIFSs is introduced. Furthermore, the innovative conception of an interval-valued complex intuitionistic fuzzy relation (IVCIFR) is defined by using the concept of the CP of IVCIFSs. Moreover, the types of IVCIFRs have been defined, including the interval-valued complex intuitionistic converse fuzzy relation, interval-valued complex intuitionistic equivalence fuzzy relation, interval-valued complex intuitionistic partial order fuzzy relation, interval-valued complex intuitionistic total order fuzzy relation, interval-valued complex intuitionistic composite fuzzy relation and many more. Every definition is held by the examples. In addition, some results have been proved for the type of IVCIFRs. Besides these, the Hasse diagrams for interval-valued complex intuitionistic partial order fuzzy sets and relations have also been presented. Furthermore, the ideas related to a Hasse diagram: maximum element, minimum element, maximal element, minimal element, supremum, infimum, upper and lower bounds are discussed. The novel concepts introduced in this study, i.e., IVCIFSs and IVCIFRs are superior to the pre-existing frameworks of FSs, IFSs, IVFSs, IVIFSs, CFSs, CIFSs and IVCFSs. Since IVCIFRs analyze the relationships among the IVCIFSs, so they are composed of the degrees of membership and non-membership, which are in the form of intervals with complex numbers. These notions can handle uncertainty much better than other mentioned concepts. The advantage of the interval-values is that they cover the little mistakes, ambiguities and errors made by decision makers and experts. In addition, their intuitionistic-type structure talks about the degree of membership and the degree of non-membership. Due to the complex valued mappings, they are also capable of handling information with multiple variables.

Every now and then, enterprises, corporations, industries and business companies shift their structures and organizations to a digital system. These computerized systems are extremely beneficial, but they are also vulnerable to many risky attacks and threats, which are known as cybercrimes. Different techniques, practices, tools and methods called cybersecurities, are used to tackle cybercrimes. At times, it can be challenging to detect the type of cybercrime, thus leaving many uncertainties. Similarly, there are also hesitations in employing the right cybersecurity techniques to rescue the business or companies from attacks as there are so many options out there to overcome these threats and risks. The selection of the most appropriate security technique may be difficult due to uncertainties and an inability to make the right decision. Therefore, in order to overcome all these uncertainties, we used the fuzzy theory. This article mathematically analyzes the relationships among cybersecurities and the sources of cyberattacks in an industrial control system (ICS), such as the effectiveness and ineffectiveness of cybersecurity against a certain source. Moreover, the current article also proposes a method to inspect different types of cybersecurity and choose the best one for an organization or network. This innovative method is based on the concepts of Hasse diagrams and interval-valued complex intuitionistic partial order fuzzy relations. Furthermore, the proposed methods are compared with other similar methods that pre-exist in the literature. The dominance and the reliability of the introduced methods are verified using numerical problems as the complex relations have not yet been discovered in fuzzy set theory. Henceforth, there is a great opportunity for potential research work to be carried out to explore these structures.

The arrangement of the remaining the paper is as follows:

Section 2 reviews some predefined concepts of fuzzy set theory that are used as the basis for the paper. Section 3 introduces the innovative conceptions of IVCIFRs, CPs between two IVCIFSs, the types of IVCIFRs and some theorems are also proved. In Section 4, we deliberate on the Hasse diagram for interval-valued complex intuitionistic partial order fuzzy sets and relations, including some useful properties and definitions related to Hasse diagrams. Section 5 offers two applications of IVCIFSs and IVCIFRs. The first application investigates cybercrimes, cyber-securities and the sources of penetration in an industrial control system (ICS). The second application uses Hasse diagrams and interval-valued complex intuitionistic partial order fuzzy relations to find the best cybersecurity technique. The existing structures in the field of fuzzy set theory are compared with the proposed structures in Section 6. Finally, the paper ends with the conclusion.

2. Preliminaries

In this section, some basic definitions and their examples are presented such as fuzzy set (FS), interval-valued fuzzy set (IVFS), complex fuzzy set (CFS), interval-valued complex fuzzy set (IVCFS), Cartesian product (CP) of two IVCFSs, interval-valued complex fuzzy relation (IVCFR), intuitionistic fuzzy set (IFS), complex intuitionistic fuzzy set (CIFS), and interval-valued complex intuitionistic fuzzy set (IVCIFS).

Definition 1. *Ref.* [1] *A fuzzy set (FS) $\breve{Ȩ}$ in a universe T is of the following form*

$$\breve{Ȩ} = \{(x, \acute{m}(x)) : x \in T\}$$

where $\acute{m} : \breve{Ȩ} \to [0,\ 1]$ *is a mapping called degree of membership.*

Example 1. $\breve{Ȩ} = \{(s, 0), (t, 0.63), (u, 0.97), (v, 0.26), (w, 0.18), (x, 1),\ (y, 0.85), (z, 0.35)\}$ *is an FS.*

Definition 2. *Ref.* [4] *An interval-valued fuzzy set (IVFS) $\breve{Ȩ}$ in a universe T is of the following form*

$$\breve{Ȩ} = \left\{\left(x, \left[\acute{m}^{-}(x), \acute{m}^{+}(x)\right]\right) : x \in T\right\}$$

where $\acute{m}^{-} : \breve{Ȩ} \to [0,\ 1]$ *and* $\acute{m}^{+} : \breve{Ȩ} \to [0,\ 1]$ *are the mappings called lower and upper degrees of membership, respectively.*

Example 2. $\breve{Ȩ} = \left\{ \begin{array}{c} (u, [0.32, 0.68]), (v, [0.16, 0.26]), (w, [0.58, 0.63]), \\ (x, [0.55, 0.66]), (y, [0.23, 0.49]), (z, [0.87, 1]) \end{array} \right\}$ *is an IVFS.*

Definition 3. *Ref.* [13] *A complex fuzzy set (CFS) $\breve{Ȩ}$ in a universe T is of the following form*

$$\breve{Ȩ} = \{(x, \acute{m}_{\mathbb{C}}(x)) : x \in T\}$$

where $\acute{m}_{\mathbb{C}} : \breve{Ȩ} \to \mathcal{Z} \ni 0 \leq |\mathcal{Z}| \leq 1$ *and $\mathcal{Z}$ is a complex number. The mapping $\acute{m}_{\mathbb{C}}$ is called degree of membership. Equivalently, the CFS can also be represented in the following form*

$$\breve{Ȩ} = \left\{\left(x, \alpha(x)e^{i2\rho(x)\pi}\right) : x \in T\right\}$$

where $\alpha : \breve{Ȩ} \to [0,\ 1]$ *and* $\rho : \breve{Ȩ} \to [0,\ 1]$ *are mappings called amplitude term and phase term of degree of membership and* $i = \sqrt{-1}$.

Example 3. $\breve{Ȩ} = \left\{ \begin{array}{c} \left(u, 0.15e^{i2(0.58)\pi}\right), \left(v, 0.75e^{i2(0.23)\pi}\right), \left(w, 0.65e^{i2(0.42)\pi}\right), \\ \left(x, 0e^{i2(1)\pi}\right), \left(y, 1e^{i2(0)\pi}\right), \left(z, 0.93e^{i2(0.37)\pi}\right) \end{array} \right\}$ *is a CFS.*

Definition 4. *Ref.* [15] *An interval-valued complex fuzzy set (IVCFS) $\breve{Ȩ}$ in a universe T is of the following form*

$$\breve{Ȩ} = \left\{\left(x, \left[\acute{m}_{\mathbb{C}}^{-}(x), \acute{m}_{\mathbb{C}}^{+}(x)\right]\right) : x \in T\right\}$$

where $\acute{m}_{\mathbb{C}}^{-} : \breve{Ȩ} \to \mathcal{Z}$ and $\acute{m}_{\mathbb{C}}^{+} : \breve{Ȩ} \to \mathcal{Z} \ni 0 \leq |\mathcal{Z}| \leq 1$ and $\mathcal{Z}$ *is a complex number. The mappings* $\acute{m}_{\mathbb{C}}^{-}$ *and* $\acute{m}_{\mathbb{C}}^{+}$ *are called the lower and upper degrees of membership, respectively. Equivalently, the IVCFS can also be represented in the following form*

$$\breve{Ȩ} = \left\{ \left(x, \left[\alpha^{-}(x), \alpha^{+}(x)\right] e^{i2[\rho^{-}(x), \rho^{+}(x)]\pi} \right) : x \in T \right\}$$

where $\alpha^{-} : \breve{Ȩ} \to [0,\ 1]$, $\alpha^{+} : \breve{Ȩ} \to [0,\ 1]$, $\rho^{-} : \breve{Ȩ} \to [0,\ 1]$ *and* $\rho^{+} : \breve{Ȩ} \to [0,\ 1]$ *are mappings called the lower amplitude term, upper amplitude term, lower phase term and upper phase term of degree of membership and* $i = \sqrt{-1}$.

Example 4. $\breve{Ȩ} = \left\{ \begin{array}{c} \left(w, [0.28, 0.36]e^{i2[0.43,0.61]\pi}\right), \left(x, [0.17, 0.23]e^{i2[0.67,0.81]\pi}\right), \\ \left(y, [0.45, 0.50]e^{i2[0.04,0.21]\pi}\right), \left(z, [0.89, 0.98]e^{i2[0,0.16]\pi}\right) \end{array} \right\}$ *is an IVCFS.*

Definition 5. *Ref.* [16] *Let* $\breve{Ȩ} = \left\{ \left(x, \left[\alpha_{\breve{Ȩ}}^{-}(x), \alpha_{\breve{Ȩ}}^{+}(x)\right] e^{i2[\rho_{\breve{Ȩ}}^{-}(x), \rho_{\breve{Ȩ}}^{+}(x)]\pi} \right) : x \in T \right\}$ *and* $\dot{F} = \left\{ \left(y, \left[\alpha_{\dot{F}}^{-}(y), \alpha_{\dot{F}}^{+}(y)\right] e^{i2[\rho_{\dot{F}}^{-}(y), \rho_{\dot{F}}^{+}(y)]\pi} \right) : y \in T \right\}$ *be two IVCFSs in a universe T, then their Cartesian product is given as*

$$\breve{Ȩ} \times \dot{F} = \left\{ \left((x,y), \left[\alpha_{\breve{Ȩ}\times\dot{F}}^{-}(x,y), \alpha_{\breve{Ȩ}\times\dot{F}}^{+}(x,y)\right] e^{i2[\rho_{\breve{Ȩ}\times\dot{F}}^{-}(x,y), \rho_{\breve{Ȩ}\times\dot{F}}^{+}(x,y)]\pi} \right) : x \in \breve{Ȩ}, y \in \dot{F} \right\}$$

where $\alpha_{\breve{Ȩ}\times\dot{F}}^{-}(x,y) = \min\left\{\alpha_{\breve{Ȩ}}^{-}(x), \alpha_{\dot{F}}^{-}(y)\right\}$, $\alpha_{\breve{Ȩ}\times\dot{F}}^{+}(x,y) = \min\left\{\alpha_{\breve{Ȩ}}^{+}(x), \alpha_{\dot{F}}^{+}(y)\right\}\rho_{\breve{Ȩ}\times\dot{F}}^{-}(x,y) = \min\left\{\rho_{\breve{Ȩ}}^{-}(x), \rho_{\dot{F}}^{-}(y)\right\}$ *and* $\rho_{\breve{Ȩ}\times\dot{F}}^{+}(x,y) = \min\left\{\rho_{\breve{Ȩ}}^{+}(x), \rho_{\dot{F}}^{+}(y)\right\}$.

Definition 6. *Ref.* [16] *The interval-valued complex fuzzy relation (IVCFR) is a subset of the Cartesian product of any two IVCFSs, i.e.,* $\bar{Ṛ} \subseteq \breve{Ȩ} \times \dot{F}$, *where* $\breve{Ȩ}$ *and* $\dot{F}$ *are IVCFSs and* $\bar{Ṛ}$ *denotes the IVCFR.*

Example 5. *The Cartesian product of two IVCFSs*

$$\breve{Ȩ} = \left\{ \begin{array}{c} \left(s, [0.28, 0.36]e^{i2[0.43,0.61]\pi}\right), \left(t, [0.17, 0.23]e^{i2[0.67,0.81]\pi}\right), \\ \left(u, [0.45, 0.50]e^{i2[0.04,0.21]\pi}\right), \left(v, [0.89, 0.98]e^{i2[0,0.16]\pi}\right) \end{array} \right\} \text{ and}$$

$$\dot{F} = \left\{ \begin{array}{c} \left(w, [0.53, 0.64]e^{i2[0.29,0.37]\pi}\right), \left(x, [0.73, 0.79]e^{i2[0.43,0.52]\pi}\right), \\ \left(y, [0.19, 0.35]e^{i2[0.24,0.35]\pi}\right), \left(z, [0.39, 0.55]e^{i2[0.45,0.55]\pi}\right) \end{array} \right\} \text{ is}$$

$$\breve{Ȩ} \times \dot{F} = \left\{ \begin{array}{c} \left((s,w), [0.28, 0.36]e^{i2[0.29,0.37]\pi}\right), \left((s,x), [0.28, 0.36]e^{i2[0.43,0.52]\pi}\right), \\ \left((s,y), [0.19, 0.35]e^{i2[0.24,0.36]\pi}\right), \left((s,z), [0.28, 0.36]e^{i2[0,0.16]\pi}\right), \\ \left((t,w), [0.17, 0.23]e^{i2[0.29,0.37]\pi}\right), \left((t,x), [0.17, 0.23]e^{i2[0.43,0.52]\pi}\right), \\ \left((t,y), [0.17, 0.23]e^{i2[0.24,0.35]\pi}\right), \left((t,z), [0.17, 0.23]e^{i2[0.45,0.55]\pi}\right), \\ \left((u,w), [0.45, 0.50]e^{i2[0.04,0.21]\pi}\right), \left((u,x), [0.45, 0.50]e^{i2[0.04,0.21]\pi}\right), \\ \left((u,y), [0.19, 0.35]e^{i2[0.04,0.21]\pi}\right), \left((u,z), [0.39, 0.50]e^{i2[0.04,0.21]\pi}\right), \\ \left((v,w), [0.53, 0.64]e^{i2[0.29,0.37]\pi}\right), \left((v,x), [0.73, 0.79]e^{i2[0,0.16]\pi}\right), \\ \left((v,y), [0.19, 0.35]e^{i2[0,0.16]\pi}\right), \left((v,z), [0.39, 0.55]e^{i2[0,0.16]\pi}\right) \end{array} \right\}$$

The IVCFR $\bar{\text{Ṛ}}$. between the IVCFSs $\breve{\text{Ȩ}}$ and $\dot{\text{F}}$ is given

$$\bar{\text{Ṛ}} = \left\{ \begin{array}{c} \left((s,w),[0.28,0.36]e^{i2[0.29,0.37]\pi}\right), \left((s,x),[0.28,0.36]e^{i2[0.43,0.52]\pi}\right), \\ \left((t,x),[0.17,0.23]e^{i2[0.43,0.52]\pi}\right), \left((u,w),[0.45,0.50]e^{i2[0.04,0.21]\pi}\right), \\ \left((v,y),[0.19,0.35]e^{i2[0,0.16]\pi}\right), \left((v,z),[0.39,0.55]e^{i2[0,0.16]\pi}\right) \end{array} \right\}.$$

Definition 7. *Ref.* [22] *An intuitionistic fuzzy set (IFS) $\breve{\text{Ȩ}}$ in a universe T is of the following form*

$$\breve{\text{Ȩ}} = \{(x, \acute{m}(x), \underline{n}(x)) : x \in T\}$$

where $\acute{m} : \breve{\text{Ȩ}} \to [0, 1]$ and $\underline{n} : \breve{\text{Ȩ}} \to [0, 1]$ are mappings called the degree of membership and degree of non-membership, respectively, given that $0 \leq \acute{m}(x) + \underline{n}(x) \leq 1$.

Example 6. $\breve{\text{Ȩ}} = \{(u, 0.86, 012), (w, 0.47, 0.36), (x, 0.34, 0.59), (y, 0.61, 0.28), (z, 0.16, 0.73)\}$ *is an IFS.*

Definition 8. *Ref.* [25] *A complex intuitionistic fuzzy set (CIFS) $\breve{\text{Ȩ}}$ in a universe T is of the following form*

$$\breve{\text{Ȩ}} = \{(x, \acute{m}_{\mathbb{C}}(x), \underline{n}_{\mathbb{C}}(x)) : x \in T\}$$

where $\acute{m}_{\mathbb{C}} : \breve{\text{Ȩ}} \to \mathcal{Z}$ and $\underline{n}_{\mathbb{C}} : \breve{\text{Ȩ}} \to \mathcal{Z} \ni 0 \leq |\mathcal{Z}| \leq 1$ and $\mathcal{Z}$ is a complex number. The mappings $\acute{m}_{\mathbb{C}}$ and $\underline{n}_{\mathbb{C}}$ are called degree of membership and degree of non-membership, respectively, given that $0 \leq |\acute{m}_{\mathbb{C}}(x)| + |\underline{n}_{\mathbb{C}}(x)| \leq 1$. Equivalently, the CIFS can also be represented in the following form

$$\breve{\text{Ȩ}} = \left\{ \left(x, \alpha_{\acute{m}}(x)e^{i2\rho_{\acute{m}}(x)\pi}, \alpha_{\underline{n}}(x)e^{i2\rho_{\underline{n}}(x)\pi}\right) : x \in T \right\}$$

where $\alpha_{\acute{m}} : \breve{\text{Ȩ}} \to [0, 1]$, $\alpha_{\underline{n}} : \breve{\text{Ȩ}} \to [0, 1]$, $\rho_{\acute{m}} : \breve{\text{Ȩ}} \to [0, 1]$ and $\rho_{\underline{n}} : \breve{\text{Ȩ}} \to [0, 1]$ are mappings called amplitude term of degree of membership, amplitude term of degree of non-membership, phase term of degree of membership and phase term of degree of non-membership and $i = \sqrt{-1}$, given that $0 \leq \alpha_{\acute{m}}(x) + \alpha_{\underline{n}}(x) \leq 1$ and $0 \leq \rho_{\acute{m}}(x) + \rho_{\underline{n}}(x) \leq 1$.

Example 7. $\breve{\text{Ȩ}} = \left\{ \begin{array}{c} \left(w, 0.35e^{i2(0.42)\pi}, 0.26e^{i2(0.31)\pi}\right), \left(x, 0.51e^{i2(0.16)\pi}, 0.38e^{i2(0.84)\pi}\right), \\ \left(y, 0.71e^{i2(0.22)\pi}, 0.19e^{i2(0.49)\pi}\right), \left(z, 0.09e^{i2(0.17)\pi}, 0.03e^{i2(0.75)\pi}\right) \end{array} \right\}$ *is a CIFS.*

Definition 9. *Ref.* [26] *An interval-valued complex intuitionistic fuzzy set (IVCIFS) $\breve{\text{Ȩ}}$ in a universe T is of the following form*

$$\breve{\text{Ȩ}} = \left\{ \left(x, \left[\acute{m}^-_{\mathbb{C}}(x), \acute{m}^+_{\mathbb{C}}(x)\right], \left[\underline{n}^-_{\mathbb{C}}(x), \underline{n}^+_{\mathbb{C}}(x)\right]\right) : x \in T \right\}$$

where $\acute{m}^-_{\mathbb{C}} : \breve{\text{Ȩ}} \to \mathcal{Z}$, $\acute{m}^+_{\mathbb{C}} : \breve{\text{Ȩ}} \to \mathcal{Z}$, $\underline{n}^-_{\mathbb{C}} : \breve{\text{Ȩ}} \to \mathcal{Z}$ and $\underline{n}^+_{\mathbb{C}} : \breve{\text{Ȩ}} \to \mathcal{Z} \ni 0 \leq |\mathcal{Z}| \leq 1$ and $\mathcal{Z}$ is a complex number. The mappings $\acute{m}^-_{\mathbb{C}}$ and $\acute{m}^+_{\mathbb{C}}$ are called the lower and upper degrees of membership, respectively, while the mappings $\underline{n}^-_{\mathbb{C}}$ and are called the lower and upper degrees of non-membership, respectively, given that $0 \leq \left|\acute{m}^+_{\mathbb{C}}(x)\right| + \left|\underline{n}^+_{\mathbb{C}}(x)\right| \leq 1$. Equivalently, the IVCIFS can also be represented in the following form

$$\breve{\text{Ȩ}} = \left\{ \left(x, \left[\alpha^-_{\acute{m}}(x), \alpha^+_{\acute{m}}(x)\right]e^{i2[\rho^-_{\acute{m}}(x), \rho^+_{\acute{m}}(x)]\pi}, \left[\alpha^-_{\underline{n}}(x), \alpha^+_{\underline{n}}(x)\right]e^{i2[\rho^-_{\underline{n}}(x), \rho^+_{\underline{n}}(x)]\pi}\right) : x \in T \right\}$$

where $\alpha^-_{\acute{m}} : \breve{\text{Ȩ}} \to [0, 1]$, $\alpha^+_{\acute{m}} : \breve{\text{Ȩ}} \to [0, 1]$, $\rho^-_{\acute{m}} : \breve{\text{Ȩ}} \to [0, 1]$, $\rho^+_{\acute{m}} : \breve{\text{Ȩ}} \to [0, 1]$ and $\alpha^-_{\underline{n}} : \breve{\text{Ȩ}} \to [0, 1]$, $\alpha^+_{\underline{n}} : \breve{\text{Ȩ}} \to [0, 1]$, $\rho^-_{\underline{n}} : \breve{\text{Ȩ}} \to [0, 1]$, $\rho^+_{\underline{n}} : \breve{\text{Ȩ}} \to [0, 1]$ are mappings called the lower amplitude term, upper amplitude term, lower phase term, upper phase term of degree of membership and lower ampli-

tude term, upper amplitude term, lower phase term, upper phase term of degree of non-membership, respectively, $i = \sqrt{-1}$. *Given that* $0 \leq \alpha^{+}_{\acute{m}} + \alpha^{+}_{\underline{n}} \leq 1$ *and* $0 \leq \rho^{+}_{\acute{m}} + \rho^{+}_{\underline{n}} \leq 1$.

Example 8. $\breve{Ȩ} = \left\{ \begin{array}{c} \left(w, [0.28, 0.36]e^{i2[0.43,0.53]\pi}, [0.48, 0.61]e^{i2[0.32,0.42]\pi}\right), \\ \left(x, [0.07, 0.12]e^{i2[0.24,0.29]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right), \\ \left(y, [0.63, 0.71]e^{i2[0.03,0.15]\pi}, [0.11, 0.23]e^{i2[0.73,0.82]\pi}\right) \\ \left(z, [0.37, 0.46]e^{i2[0.15,0.25]\pi}, [0.32, 0.43]e^{i2[0.53,0.68]\pi}\right) \end{array} \right\}$ *is an IVCIFS.*

3. Interval-Valued Complex Fuzzy Relations

This section introduces the novel concepts of the Cartesian product of two interval-valued complex intuitionistic fuzzy sets (IVCIFSs), an interval-valued complex intuitionistic fuzzy relation (CIVIFR) and its types. For each definition a suitable example is given. Moreover, some interesting results for IVCIFRs have also been proved.

Definition 10. *Let* $\breve{Ȩ} = \left\{ \left(\begin{array}{c} x, \left[\alpha^{-}_{(\breve{Ȩ})\acute{m}}(x), \alpha^{+}_{(\breve{Ȩ})\acute{m}}(x)\right] e^{i2[\rho^{-}_{(\breve{Ȩ})\acute{m}}(x), \rho^{+}_{(\breve{Ȩ})\acute{m}}(x)]\pi}, \\ \left[\alpha^{-}_{(\breve{Ȩ})\underline{n}}(x), \alpha^{+}_{(\breve{Ȩ})\underline{n}}(x)\right] e^{i2[\rho^{-}_{(\breve{Ȩ})\underline{n}}(x), \rho^{+}_{(\breve{Ȩ})\underline{n}}(x)]\pi} \end{array} \right) : x \in T \right\}$ *and* $\dot{\mathrm{F}} = \left\{ \left(\begin{array}{c} y, \left[\alpha^{-}_{(\dot{\mathrm{F}})\acute{m}}(y), \alpha^{+}_{(\dot{\mathrm{F}})\acute{m}}(y)\right] e^{i2[\rho^{-}_{(\dot{\mathrm{F}})\acute{m}}(y), \rho^{+}_{(\dot{\mathrm{F}})\acute{m}}(y)]\pi}, \\ \left[\alpha^{-}_{(\dot{\mathrm{F}})\underline{n}}(y), \alpha^{+}_{(\dot{\mathrm{F}})\underline{n}}(y)\right] e^{i2[\rho^{-}_{(\dot{\mathrm{F}})\underline{n}}(y), \rho^{+}_{(\dot{\mathrm{F}})\underline{n}}(y)]\pi} \end{array} \right) : y \in T \right\}$ *be two IVCFSs in a universe* T, *then their Cartesian product is given as*

$$\breve{Ȩ} \times \dot{\mathrm{F}} = \left\{ \left(\begin{array}{c} (x,y), \left[\alpha^{-}_{(\breve{Ȩ}\times\dot{\mathrm{F}})\acute{m}}(x,y), \alpha^{+}_{(\breve{Ȩ}\times\dot{\mathrm{F}})\acute{m}}(x,y)\right] e^{i2[\rho^{-}_{(\breve{Ȩ}\times\dot{\mathrm{F}})\acute{m}}(x,y), \rho^{+}_{(\breve{Ȩ}\times\dot{\mathrm{F}})\acute{m}}(x,y)]\pi}, \\ \left[\alpha^{-}_{(\breve{Ȩ}\times\dot{\mathrm{F}})\underline{n}}(x,y), \alpha^{+}_{(\breve{Ȩ}\times\dot{\mathrm{F}})\underline{n}}(x,y)\right] e^{i2[\rho^{-}_{(\breve{Ȩ}\times\dot{\mathrm{F}})\underline{n}}(x,y), \rho^{+}_{(\breve{Ȩ}\times\dot{\mathrm{F}})\underline{n}}(x,y)]\pi} \end{array} \right) : x \in \breve{Ȩ}, y \in \dot{\mathrm{F}} \right\}$$

where $\alpha^{-}_{(\breve{Ȩ}\times\dot{\mathrm{F}})\acute{m}}(x,y) = \min\left\{\alpha^{-}_{(\breve{Ȩ})\acute{m}}(x), \alpha^{-}_{(\dot{\mathrm{F}})\acute{m}}(y)\right\}$, $\alpha^{+}_{(\breve{Ȩ}\times\dot{\mathrm{F}})\acute{m}}(x,y) = \min\left\{\alpha^{+}_{(\breve{Ȩ})\acute{m}}(x), \alpha^{+}_{(\dot{\mathrm{F}})\acute{m}}(y)\right\}$ $\rho^{-}_{(\breve{Ȩ}\times\dot{\mathrm{F}})\acute{m}}(x,y) = \min\left\{\rho^{-}_{(\breve{Ȩ})\acute{m}}(x), \rho^{-}_{(\dot{\mathrm{F}})\acute{m}}(y)\right\}$ *and* $\rho^{+}_{(\breve{Ȩ}\times\dot{\mathrm{F}})\acute{m}}(x,y) = \min\left\{\rho^{+}_{(\breve{Ȩ})\acute{m}}(x), \rho^{+}_{(\dot{\mathrm{F}})\acute{m}}(y)\right\}$ *and* $\alpha^{-}_{(\breve{Ȩ}\times\dot{\mathrm{F}})\underline{n}}(x,y) = \max\left\{\alpha^{-}_{(\breve{Ȩ})\underline{n}}(x), \alpha^{-}_{(\dot{\mathrm{F}})\underline{n}}(y)\right\}$, $\alpha^{+}_{(\breve{Ȩ}\times\dot{\mathrm{F}})\underline{n}}(x,y) = \max\left\{\alpha^{+}_{(\breve{Ȩ})\underline{n}}(x), \alpha^{+}_{(\dot{\mathrm{F}})\underline{n}}(y)\right\}$ $\rho^{-}_{(\breve{Ȩ}\times\dot{\mathrm{F}})\underline{n}}(x,y) = \max\left\{\rho^{-}_{(\breve{Ȩ})\underline{n}}(x), \rho^{-}_{(\dot{\mathrm{F}})\underline{n}}(y)\right\}$ *and* $\rho^{+}_{(\breve{Ȩ}\times\dot{\mathrm{F}})\underline{n}}(x,y) = \max\left\{\rho^{+}_{(\breve{Ȩ})\underline{n}}(x), \rho^{+}_{(\dot{\mathrm{F}})\underline{n}}(y)\right\}$.

Definition 11. *The interval-valued complex intuitionistic fuzzy relation (IVCIFR) is a subset of the Cartesian product of any two IVCIFSs, i.e.,* $\bar{Ṛ} \subseteq \breve{Ȩ} \times \dot{\mathrm{F}}$, *where* $Ȩ$ *and* $\dot{\mathrm{F}}$ *are IVCIFSs and* $\bar{Ṛ}$ *denotes the IVCIFR.*

Example 9. *The Cartesian product of two IVCFSs*

$$\breve{Ȩ} = \left\{ \begin{array}{c} \left(u, [0.07, 0.12]e^{i2[0.24,0.29]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right), \\ \left(v, [0.63, 0.71]e^{i2[0.03,0.15]\pi}, [0.11, 0.23]e^{i2[0.73,0.82]\pi}\right), \\ \left(w, [0.37, 0.46]e^{i2[0.15,0.25]\pi}, [0.32, 0.43]e^{i2[0.53,0.68]\pi}\right) \end{array} \right\} \text{ and}$$

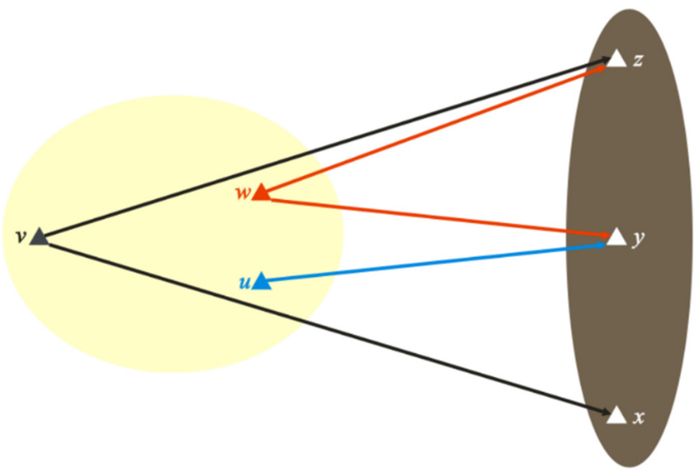
$$\dot{\mathrm{F}} = \left\{ \begin{array}{c} \left(x, [0.26, 0.34]e^{i2[0.15,0.25]\pi}, [0.38, 0.49]e^{i2[0.63,0.74]\pi}\right), \\ \left(y, [0.52, 0.58]e^{i2[0.30,0.42]\pi}, [0.18, 0.37]e^{i2[0.06,0.13]\pi}\right), \\ \left(z, [0.28, 0.36]e^{i2[0.43,0.53]\pi}, [0.48, 0.61]e^{i2[0.32,0.42]\pi}\right), \end{array} \right\} \text{ is}$$

$$\breve{Ȩ}\times\dot{F}=\left\{\begin{array}{c}\left((u,x),[0.07,0.12]e^{i2[0.15,0.25]\pi},[0.56,0.66]e^{i2[0.63,0.74]\pi}\right),\\ \left((u,y),[0.07,0.12]e^{i2[0.24,0.29]\pi},[0.56,0.66]e^{i2[0.57,0.71]\pi}\right),\\ \left((u,z),[0.07,0.12]e^{i2[0.24,0.29]\pi},[0.56,0.66]e^{i2[0.57,0.71]\pi}\right),\\ \left((v,x),[0.26,0.34]e^{i2[0.03,0.15]\pi},[0.38,0.49]e^{i2[0.73,0.82]\pi}\right),\\ \left((v,y),[0.52,0.58]e^{i2[0.03,0.15]\pi},[0.18,0.37]e^{i2[0.73,0.82]\pi}\right),\\ \left((v,z),[0.28,0.36]e^{i2[0.03,0.15]\pi},[0.48,0.61]e^{i2[0.73,0.82]\pi}\right),\\ \left((w,x),[0.26,0.34]e^{i2[0.15,0.25]\pi},[0.38,0.49]e^{i2[0.63,0.74]\pi}\right),\\ \left((w,y),[0.37,0.46]e^{i2[0.15,0.25]\pi},[0.32,0.43]e^{i2[0.53,0.68]\pi}\right),\\ \left((w,z),[0.28,0.36]e^{i2[0.15,0.25]\pi},[0.48,0.61]e^{i2[0.53,0.68]\pi}\right)\end{array}\right\}$$

The IVCIFR $\bar{Ṛ}$ between the IVCFSs $\breve{Ȩ}$ and $\dot{F}$ is

$$\bar{Ṛ}=\left\{\begin{array}{c}\left((u,y),[0.07,0.12]e^{i2[0.24,0.29]\pi},[0.56,0.66]e^{i2[0.57,0.71]\pi}\right),\\ \left((v,x),[0.26,0.34]e^{i2[0.03,0.15]\pi},[0.38,0.49]e^{i2[0.73,0.82]\pi}\right),\\ \left((v,z),[0.28,0.36]e^{i2[0.03,0.15]\pi},[0.48,0.61]e^{i2[0.73,0.82]\pi}\right),\\ \left((w,y),[0.37,0.46]e^{i2[0.15,0.25]\pi},[0.32,0.43]e^{i2[0.53,0.68]\pi}\right),\\ \left((w,z),[0.28,0.36]e^{i2[0.15,0.25]\pi},[0.48,0.61]e^{i2[0.53,0.68]\pi}\right)\end{array}\right\}$$

The IVCIFR $\bar{Ṛ}$ between the IVCFSs $\breve{Ȩ}$ and $\dot{F}$ is given as below (Figure 1),



(u, y)	$[0.07,0.12]e^{i2[0.24,0.29]\pi},[0.56,0.66]e^{i2[0.57,0.71]\pi}$
(v, x)	$[0.26,0.34]e^{i2[0.03,0.15]\pi},[0.38,0.49]e^{i2[0.73,0.82]\pi}$
(v, z)	$[0.28,0.36]e^{i2[0.03,0.15]\pi},[0.48,0.61]e^{i2[0.73,0.82]\pi}$
(w, y)	$[0.37,0.46]e^{i2[0.15,0.25]\pi},[0.32,0.43]e^{i2[0.53,0.68]\pi}$
(w, z)	$[0.28,0.36]e^{i2[0.15,0.25]\pi},[0.48,0.61]e^{i2[0.53,0.68]\pi}$

Figure 1. Interval-valued complex intuitionistic fuzzy relation.

NOTE: For convenience, throughout this article, x and (x,y) will be used to denote $\left(x,\left[\alpha^{-}_{(\breve{Ȩ})\acute{m}}(x),\alpha^{+}_{(\breve{Ȩ})\acute{m}}(x)\right]e^{i2[\rho^{-}_{(\breve{Ȩ})\acute{m}}(x),\rho^{+}_{(\breve{Ȩ})\acute{m}}(x)]\pi},\left[\alpha^{-}_{(\breve{Ȩ})\underline{n}}(x),\alpha^{+}_{(\breve{Ȩ})\underline{n}}(x)\right]e^{i2[\rho^{-}_{(\breve{Ȩ})\underline{n}}(x),\rho^{+}_{(\breve{Ȩ})\underline{n}}(x)]\pi}\right)$ and $\left(\begin{array}{c}(x,y),\left[\alpha^{-}_{(\breve{Ȩ}\times\breve{Ȩ})\acute{m}}(x,y),\alpha^{+}_{(\breve{Ȩ}\times\breve{Ȩ})\acute{m}}(x,y)\right]e^{i2[\rho^{-}_{(\breve{Ȩ}\times\breve{Ȩ})\acute{m}}(x,y),\rho^{+}_{(\breve{Ȩ}\times\breve{Ȩ})\acute{m}}(x,y)]\pi},\\ \left[\alpha^{-}_{(\breve{Ȩ}\times\breve{Ȩ})\underline{n}}(x,y),\alpha^{+}_{(\breve{Ȩ}\times\breve{Ȩ})\underline{n}}(x,y)\right]e^{i2[\rho^{-}_{(\breve{Ȩ}\times\breve{Ȩ})\underline{n}}(x,y),\rho^{+}_{(\breve{Ȩ}\times\breve{Ȩ})\underline{n}}(x,y)]\pi}\end{array}\right)$, respectively, otherwise it will be mentioned.

Definition 12. *Let Ḝ be an IVCIFS in a universe T and Ṝ be an IVCIFR on Ḝ. Then,*

1. *Ṝ is known as an IVCI reflexive fuzzy relation (IVCI-reflexive-FR) on Ḝ if* $(x, x) \in$ Ṝ, $\forall x \in$ Ḝ.
2. *Ṝ is known as an IVCI irreflexive fuzzy relation (IVCI-irreflexive-FR) on Ḝ if* $(x, x) \notin$ Ṝ, $\forall x \in$ Ḝ.
3. *Ṝ is known as an IVCI symmetric fuzzy relation (IVCI-symmetric-FR) on Ḝ if* $\forall x, y \in$ Ḝ, $(x, y) \in$ Ṝ $\Rightarrow (y, x) \in$ Ṝ.
4. *Ṝ is known as an IVCI antisymmetric fuzzy relation (IVCI-antisymmetric-FR) on Ḝ if* $\forall x, y \in$ Ḝ, $(x, y) \in$ Ṝ *and* $(y, x) \in$ Ṝ $\Rightarrow x = y$.
5. *Ṝ is known as an IVCI asymmetric fuzzy relation (IVCI-asymmetric-FR) on Ḝ* if $\forall x, y \in$ Ḝ, $(y, x) \in$ Ṝ $\Rightarrow (x = y \notin)$Ṝ.
6. *Ṝ is known as an IVCI complete fuzzy relation (IVCI-complete-FR) on Ḝ if* $\forall x, y \in$ Ṝ, $(x, y) \in$ Ṝ *or* $(y, x) \in$ Ṝ.
7. *Ṝ is known as an IVCI transitive fuzzy relation on (IVCI-transitive-FR) Ḝ if* $\forall x, y, z \in$ Ḝ, $(x,y) \in$ Ṝ *and* $(y, z) \in$ Ṝ $\Rightarrow (x, z) \in$ Ṝ.
8. *Ṝ is known as an IVCI equivalence fuzzy relation (IVCI-equivalence-FR) on Ḝ if Ṝ is IVCI-reflexive-FR, IVCI-symmetric-FR and IVCI-transitive-FR on Ḝ.*
9. *Ṝ is known as an IVCI preorder fuzzy relation (IVCI-preorder-FR) on Ḝ if Ṝ is IVCI-reflexive-FR and IVCI-transitive-FR on Ḝ.*
10. *Ṝ is known as an IVCI strict order fuzzy relation (IVCI-strict order-FR) on Ḝ if Ṝ is IVCI-irreflexive-FR and IVCI-transitive-FR on Ḝ.*
11. *Ṝ is known as an IVCI partial order fuzzy relation (IVCI-partial order-FR) on Ḝ if Ṝ is IVCI-preorder-FR and IVCI-antisymmetric-FR on Ḝ.*
12. *Ṝ is known as an IVCI linear order fuzzy relation (IVCI-linear order-FR) on Ḝ if Ṝ is IVCI-partial order-FR and IVCI-complete-FR on Ḝ.*

Example 10. *For an IVCIFS* Ḝ $= \left\{ \begin{array}{l} \left(u, [0.07, 0.12]e^{i2[0.24,0.29]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right), \\ \left(v, [0.63, 0.71]e^{i2[0.03,0.15]\pi}, [0.11, 0.23]e^{i2[0.73,0.82]\pi}\right), \\ \left(w, [0.37, 0.46]e^{i2[0.15,0.25]\pi}, [0.32, 0.43]e^{i2[0.53,0.68]\pi}\right) \end{array} \right\}$

the Cartesian product Ḝ × Ḝ is

$$\breve{E} \times \breve{E} = \left\{ \begin{array}{l} \left((u, u), [0.07, 0.12]e^{i2[0.15,0.25]\pi}, [0.56, 0.66]e^{i2[0.63,0.74]\pi}\right), \\ \left((u, v), [0.07, 0.12]e^{i2[0.03,0.15]\pi}, [0.56, 0.66]e^{i2[0.73,0.82]\pi}\right), \\ \left((u, w), [0.07, 0.12]e^{i2[0.15,0.25]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right), \\ \left((v, u), [0.07, 0.12]e^{i2[0.03,0.15]\pi}, [0.56, 0.66]e^{i2[0.73,0.82]\pi}\right), \\ \left((v, v), [0.63, 0.71]e^{i2[0.03,0.15]\pi}, [0.11, 0.23]e^{i2[0.73,0.82]\pi}\right), \\ \left((v, w), [0.37, 0.46]e^{i2[0.03,0.15]\pi}, [0.32, 0.43]e^{i2[0.73,0.82]\pi}\right), \\ \left((w, u), [0.07, 0.12]e^{i2[0.15,0.25]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right), \\ \left((w, v), [0.37, 0.46]e^{i2[0.03,0.15]\pi}, [0.32, 0.43]e^{i2[0.73,0.82]\pi}\right), \\ \left((w, w), [0.37, 0.46]e^{i2[0.15,0.25]\pi}, [0.32, 0.43]e^{i2[0.53,0.68]\pi}\right) \end{array} \right\}$$

Then,

1. The IVCI-equivalence-FR $\bar{\underset{\cdot}{\mathrm{R}}}_1$ on $\breve{\text{Ȩ}}$ is given as

$$\bar{\underset{\cdot}{\mathrm{R}}}_1 = \left\{ \begin{array}{l} \left((u,u), [0.07, 0.12]e^{i2[0.15,0.25]\pi}, [0.56, 0.66]e^{i2[0.63,0.74]\pi}\right), \\ \left((u,w), [0.07, 0.12]e^{i2[0.15,0.25]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right), \\ \left((v,v), [0.63, 0.71]e^{i2[0.03,0.15]\pi}, [0.11, 0.23]e^{i2[0.73,0.82]\pi}\right), \\ \left((w,u), [0.07, 0.12]e^{i2[0.15,0.25]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right), \\ \left((w,w), [0.37, 0.46]e^{i2[0.15,0.25]\pi}, [0.32, 0.43]e^{i2[0.53,0.68]\pi}\right) \end{array} \right\}$$

2. The IVCI-preorder-FR $\bar{\underset{\cdot}{\mathrm{R}}}_2$ on $\breve{\text{Ȩ}}$ is given as

$$\bar{\underset{\cdot}{\mathrm{R}}}_2 = \left\{ \begin{array}{l} \left((u,u), [0.07, 0.12]e^{i2[0.15,0.25]\pi}, [0.56, 0.66]e^{i2[0.63,0.74]\pi}\right), \\ \left((v,u), [0.07, 0.12]e^{i2[0.03,0.15]\pi}, [0.56, 0.66]e^{i2[0.73,0.82]\pi}\right), \\ \left((v,v), [0.63, 0.71]e^{i2[0.03,0.15]\pi}, [0.11, 0.23]e^{i2[0.73,0.82]\pi}\right), \\ \left((w,w), [0.37, 0.46]e^{i2[0.15,0.25]\pi}, [0.32, 0.43]e^{i2[0.53,0.68]\pi}\right) \end{array} \right\}$$

3. The IVCI-strict order-FR $\bar{\underset{\cdot}{\mathrm{R}}}_3$ on $\breve{\text{Ȩ}}$ is given as

$$\bar{\underset{\cdot}{\mathrm{R}}}_3 = \left\{ \begin{array}{l} \left((v,u), [0.07, 0.12]e^{i2[0.03,0.15]\pi}, [0.56, 0.66]e^{i2[0.73,0.82]\pi}\right), \\ \left((w,u), [0.07, 0.12]e^{i2[0.15,0.25]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right), \\ \left((w,v), [0.37, 0.46]e^{i2[0.03,0.15]\pi}, [0.32, 0.43]e^{i2[0.73,0.82]\pi}\right) \end{array} \right\}$$

4. The IVCI-partial order-FR $\bar{\underset{\cdot}{\mathrm{R}}}_4$ on $\breve{\text{Ȩ}}$ is given as

$$\bar{\underset{\cdot}{\mathrm{R}}}_4 = \left\{ \begin{array}{l} \left((u,u), [0.07, 0.12]e^{i2[0.15,0.25]\pi}, [0.56, 0.66]e^{i2[0.63,0.74]\pi}\right), \\ \left((v,u), [0.07, 0.12]e^{i2[0.03,0.15]\pi}, [0.56, 0.66]e^{i2[0.73,0.82]\pi}\right), \\ \left((v,v), [0.63, 0.71]e^{i2[0.03,0.15]\pi}, [0.11, 0.23]e^{i2[0.73,0.82]\pi}\right), \\ \left((w,u), [0.07, 0.12]e^{i2[0.15,0.25]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right), \\ \left((w,w), [0.37, 0.46]e^{i2[0.15,0.25]\pi}, [0.32, 0.43]e^{i2[0.53,0.68]\pi}\right) \end{array} \right\}$$

5. The IVCI-linear order-FR $\bar{\underset{\cdot}{\mathrm{R}}}_5$ on $\breve{\text{Ȩ}}$ is given as

$$\bar{\underset{\cdot}{\mathrm{R}}}_5 = \left\{ \begin{array}{l} \left((u,u), [0.07, 0.12]e^{i2[0.15,0.25]\pi}, [0.56, 0.66]e^{i2[0.63,0.74]\pi}\right), \\ \left((v,u), [0.07, 0.12]e^{i2[0.03,0.15]\pi}, [0.56, 0.66]e^{i2[0.73,0.82]\pi}\right), \\ \left((v,v), [0.63, 0.71]e^{i2[0.03,0.15]\pi}, [0.11, 0.23]e^{i2[0.73,0.82]\pi}\right), \\ \left((w,u), [0.07, 0.12]e^{i2[0.15,0.25]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right), \\ \left((w,v), [0.37, 0.46]e^{i2[0.03,0.15]\pi}, [0.32, 0.43]e^{i2[0.73,0.82]\pi}\right), \\ \left((w,w), [0.37, 0.46]e^{i2[0.15,0.25]\pi}, [0.32, 0.43]e^{i2[0.53,0.68]\pi}\right) \end{array} \right\}$$

Definition 13. *For an IVCIFR $\bar{\underset{\cdot}{\mathrm{R}}}$, the converse relation $\bar{\underset{\cdot}{\mathrm{R}}}^c$ of $\bar{\underset{\cdot}{\mathrm{R}}}$ is defined as,* $\bar{\underset{\cdot}{\mathrm{R}}}^c = \{(y,x) : (x,y) \in \bar{\underset{\cdot}{\mathrm{R}}}\}$.

Example 11. *If* $\bar{\text{Ṛ}} = \left\{ \begin{array}{l} \left((u,y), [0.07, 0.12]e^{i2[0.24,0.29]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right), \\ \left((v,x), [0.26, 0.34]e^{i2[0.03,0.15]\pi}, [0.38, 0.49]e^{i2[0.73,0.82]\pi}\right), \\ \left((v,z), [0.28, 0.36]e^{i2[0.03,0.15]\pi}, [0.48, 0.61]e^{i2[0.73,0.82]\pi}\right), \\ \left((w,y), [0.37, 0.46]e^{i2[0.15,0.25]\pi}, [0.32, 0.43]e^{i2[0.53,0.68]\pi}\right), \\ \left((w,z), [0.28, 0.36]e^{i2[0.15,0.25]\pi}, [0.48, 0.61]e^{i2[0.53,0.68]\pi}\right) \end{array} \right\}$ *is an IVCIFR between IVCIFSs* $\check{\text{Ȩ}} = \left\{ \begin{array}{l} \left(u, [0.07, 0.12]e^{i2[0.24,0.29]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right), \\ \left(v, [0.63, 0.71]e^{i2[0.03,0.15]\pi}, [0.11, 0.23]e^{i2[0.73,0.82]\pi}\right), \\ \left(w, [0.37, 0.46]e^{i2[0.15,0.25]\pi}, [0.32, 0.43]e^{i2[0.53,0.68]\pi}\right) \end{array} \right\}$ *and* $\dot{\text{F}} = \left\{ \begin{array}{l} \left(x, [0.26, 0.34]e^{i2[0.15,0.25]\pi}, [0.38, 0.49]e^{i2[0.63,0.74]\pi}\right), \\ \left(y, [0.52, 0.58]e^{i2[0.30,0.42]\pi}, [0.18, 0.37]e^{i2[0.06,0.13]\pi}\right), \\ \left(z, [0.28, 0.36]e^{i2[0.43,0.53]\pi}, [0.48, 0.61]e^{i2[0.32,0.42]\pi}\right), \end{array} \right\}$*, then the converse relation* $\bar{\text{Ṛ}}^c$ *of* $\bar{\text{Ṛ}}$ is

$$\bar{\text{Ṛ}}^c = \left\{ \begin{array}{l} \left((x,v), [0.26, 0.34]e^{i2[0.03,0.15]\pi}, [0.38, 0.49]e^{i2[0.73,0.82]\pi}\right), \\ \left((y,u), [0.07, 0.12]e^{i2[0.24,0.29]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right), \\ \left((y,w), [0.37, 0.46]e^{i2[0.15,0.25]\pi}, [0.32, 0.43]e^{i2[0.53,0.68]\pi}\right), \\ \left((z,v), [0.28, 0.36]e^{i2[0.03,0.15]\pi}, [0.48, 0.61]e^{i2[0.73,0.82]\pi}\right), \\ \left((z,w), [0.28, 0.36]e^{i2[0.15,0.25]\pi}, [0.48, 0.61]e^{i2[0.53,0.68]\pi}\right) \end{array} \right\}$$

The IVCI-equivalence-FRs sets off the concept of IVCIF-equivalences classes, which are defined as follows.

Definition 14. *For an IVCI-equivalence-FR* $\bar{\text{Ṛ}}$*, the IVCIF equivalence class of* x *modulo* $\bar{\text{Ṛ}}$ *is defined as,* $\bar{\text{Ṛ}}[x] = \{y|(y,x) \in \bar{\text{Ṛ}}\}$.

Example 12. *If* $\bar{\text{Ṛ}} = \left\{ \begin{array}{l} \left((u,u), [0.07, 0.12]e^{i2[0.15,0.25]\pi}, [0.56, 0.66]e^{i2[0.63,0.74]\pi}\right), \\ \left((u,w), [0.07, 0.12]e^{i2[0.15,0.25]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right), \\ \left((v,v), [0.63, 0.71]e^{i2[0.03,0.15]\pi}, [0.11, 0.23]e^{i2[0.73,0.82]\pi}\right), \\ \left((w,u), [0.07, 0.12]e^{i2[0.15,0.25]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right), \\ \left((w,w), [0.37, 0.46]e^{i2[0.15,0.25]\pi}, [0.32, 0.43]e^{i2[0.53,0.68]\pi}\right), \end{array} \right\}$ *is an IVCIFR on an IVCIFS* $\check{\text{Ȩ}} = \left\{ \begin{array}{l} \left(u, [0.07, 0.12]e^{i2[0.24,0.29]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right), \\ \left(v, [0.63, 0.71]e^{i2[0.03,0.15]\pi}, [0.11, 0.23]e^{i2[0.73,0.82]\pi}\right), \\ \left(w, [0.37, 0.46]e^{i2[0.15,0.25]\pi}, [0.32, 0.43]e^{i2[0.53,0.68]\pi}\right) \end{array} \right\}$*, then the IVCIF-equivalences class of*

1. *x modulo* $\bar{\text{Ṛ}}$ is given as

$$\bar{\text{Ṛ}}[u] = \left\{ \begin{array}{l} \left(u, [0.07, 0.12]e^{i2[0.24,0.29]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right), \\ \left(w, [0.37, 0.46]e^{i2[0.15,0.25]\pi}, [0.32, 0.43]e^{i2[0.53,0.68]\pi}\right) \end{array} \right\}$$

2. *y modulo* $\bar{\text{Ṛ}}$ *is given as*

$$\bar{\text{Ṛ}}[v] = \left\{ \left(v, [0.63, 0.71]e^{i2[0.03,0.15]\pi}, [0.11, 0.23]e^{i2[0.73,0.82]\pi}\right) \right\}$$

3. *z modulo* $\underset{\cdot}{\bar{R}}$ is given as

$$\underset{\cdot}{\bar{R}}[w] = \left\{ \begin{array}{l} \left(u, [0.07, 0.12]e^{i2[0.24,0.29]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right), \\ \left(w, [0.37, 0.46]e^{i2[0.15,0.25]\pi}, [0.32, 0.43]e^{i2[0.53,0.68]\pi}\right) \end{array} \right\}$$

Definition 15. *For an IVCIFR* $\underset{\cdot}{\bar{R}}$ *on an IVCIFS* $\breve{E}$, *then the IVCI-composite-FR* $\underset{\cdot}{\bar{R}} \circ \underset{\cdot}{\bar{R}}$ *is defined as: for each* $(x, y) \in \underset{\cdot}{\bar{R}}$ *and* $(y, z) \in \underset{\cdot}{\bar{R}} \Rightarrow (x, z) \in \underset{\cdot}{\bar{R}} \circ \underset{\cdot}{\bar{R}}$, $\forall x, y, z \in T$.

Example 13. *For some IVCIFRs*

$$\underset{\cdot}{\bar{R}}_1 = \left\{ \begin{array}{l} \left((u, v), [0.07, 0.12]e^{i2[0.03,0.15]\pi}, [0.56, 0.66]e^{i2[0.73,0.82]\pi}\right), \\ \left((v, u), [0.07, 0.12]e^{i2[0.03,0.15]\pi}, [0.56, 0.66]e^{i2[0.73,0.82]\pi}\right), \\ \left((w, v), [0.37, 0.46]e^{i2[0.03,0.15]\pi}, [0.32, 0.43]e^{i2[0.73,0.82]\pi}\right) \end{array} \right\} \textit{ and}$$

$$\underset{\cdot}{\bar{R}}_2 = \left\{ \begin{array}{l} \left((u, w), [0.07, 0.12]e^{i2[0.15,0.25]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right), \\ \left((v, u), [0.07, 0.12]e^{i2[0.03,0.15]\pi}, [0.56, 0.66]e^{i2[0.73,0.82]\pi}\right), \\ \left((w, v), [0.37, 0.46]e^{i2[0.03,0.15]\pi}, [0.32, 0.43]e^{i2[0.73,0.82]\pi}\right) \\ \left((w, w), [0.37, 0.46]e^{i2[0.15,0.25]\pi}, [0.32, 0.43]e^{i2[0.53,0.68]\pi}\right) \end{array} \right\} \textit{ on an IVCIFS.}$$

$$\breve{E} = \left\{ \begin{array}{l} \left(u, [0.07, 0.12]e^{i2[0.24,0.29]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right), \\ \left(v, [0.63, 0.71]e^{i2[0.03,0.15]\pi}, [0.11, 0.23]e^{i2[0.73,0.82]\pi}\right), \\ \left(w, [0.37, 0.46]e^{i2[0.15,0.25]\pi}, [0.32, 0.43]e^{i2[0.53,0.68]\pi}\right) \end{array} \right\}$$

, then the IVCI-composite-FR $\underset{\cdot}{\bar{R}}_1 \circ \underset{\cdot}{\bar{R}}_2$ *is given as,*

$$\underset{\cdot}{\bar{R}}_1 \circ \underset{\cdot}{\bar{R}}_2 = \left\{ \begin{array}{l} \left((u, u), [0.07, 0.12]e^{i2[0.15,0.25]\pi}, [0.56, 0.66]e^{i2[0.63,0.74]\pi}\right), \\ \left((v, w), [0.37, 0.46]e^{i2[0.03,0.15]\pi}, [0.32, 0.43]e^{i2[0.73,0.82]\pi}\right), \\ \left((w, u), [0.07, 0.12]e^{i2[0.15,0.25]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right), \end{array} \right\}$$

Theorem 1. *An IVCIFR* $\underset{\cdot}{\bar{R}}$ *is an IVCI-symmetric-FR on an IVCIFS* $\breve{E}$ *iff* $\underset{\cdot}{\bar{R}} = \underset{\cdot}{\bar{R}}^c$.

Proof. Assume that $\underset{\cdot}{\bar{R}} = \underset{\cdot}{\bar{R}}^c$, then
$(u, v) \in \underset{\cdot}{\bar{R}} \Rightarrow (v, u) \in \underset{\cdot}{\bar{R}}^c \Rightarrow (v, u) \in \underset{\cdot}{\bar{R}}$.
Thus, $\underset{\cdot}{\bar{R}}$ is an IVCI-symmetric-FR on an IVCIFS $\breve{E}$.
Conversely, suppose that $\underset{\cdot}{\bar{R}}$ is an IVCI-symmetric-FR on an IVCIFS $\breve{E}$, then
$(u, v) \in \underset{\cdot}{\bar{R}} \Rightarrow (v, u) \in \underset{\cdot}{\bar{R}}$.
However, $(v, u) \in \underset{\cdot}{\bar{R}}^c \Rightarrow \underset{\cdot}{\bar{R}} = \underset{\cdot}{\bar{R}}^c$. □

Theorem 2. *An IVCIFR* $\underset{\cdot}{\bar{R}}$ *is an IVCI-transitive-FR on an IVCIFS* $\breve{E}$ iff $\underset{\cdot}{\bar{R}} \circ \underset{\cdot}{\bar{R}} \subseteq \underset{\cdot}{\bar{R}}$.

Proof. Assume that $\underset{\cdot}{\bar{R}}$ is an IVCI-transitive-FR on an IVCIFS $\breve{E}$.
Let $(u, w) \in \underset{\cdot}{\bar{R}} \circ \underset{\cdot}{\bar{R}}$,
Then, by the definition of IVCI-transitivity-FR,
$(u, v) \in \underset{\cdot}{\bar{R}}$ and $(v, w) \in \underset{\cdot}{\bar{R}} \Rightarrow (u, w) \in \underset{\cdot}{\bar{R}} \Rightarrow \underset{\cdot}{\bar{R}} \circ \underset{\cdot}{\bar{R}} \subseteq \underset{\cdot}{\bar{R}}$.
Conversely assume that $\underset{\cdot}{\bar{R}} \circ \underset{\cdot}{\bar{R}} \subseteq \underset{\cdot}{\bar{R}}$, then
for $(u, v) \in \underset{\cdot}{\bar{R}}$ and $(v, w) \in \underset{\cdot}{\bar{R}} \Rightarrow (u, w) \in \underset{\cdot}{\bar{R}} \circ \underset{\cdot}{\bar{R}} \subseteq \underset{\cdot}{\bar{R}} \Rightarrow (u, w) \in \underset{\cdot}{\bar{R}}$.
Thus, $\underset{\cdot}{\bar{R}}$ is an IVCI-transitive-FR on $\breve{E}$. □

Theorem 3. *Suppose* $\underset{\cdot}{\bar{R}}$ *is an IVCI-equivalence-FR on an IVCIFS* $\breve{E}$, *then* $\underset{\cdot}{\bar{R}} \circ \underset{\cdot}{\bar{R}} = \underset{\cdot}{\bar{R}}$.

Proof. Assume that $(u, v) \in \underset{\cdot}{\bar{R}}$,

then by the definition of IVCI-symmetric-FR,

$(v, u) \in \underset{\cdot}{\bar{R}}$.

Now, by using the definition of IVCI-transitive-FR,

$(u, u) \in \underset{\cdot}{\bar{R}}$.

However, by the definition of IVCI-composite-FR,

$(u, u) \in \underset{\cdot}{\bar{R}} \circ \underset{\cdot}{\bar{R}}$.

Thus,

$$\underset{\cdot}{\bar{R}} \subseteq \underset{\cdot}{\bar{R}} \circ \underset{\cdot}{\bar{R}} \tag{1}$$

Conversely, assume that $(u, v) \in \underset{\cdot}{\bar{R}} \circ \underset{\cdot}{\bar{R}}$, then $\exists\, w \in \mathrm{S} \ni (u, w) \in \underset{\cdot}{\bar{R}}$ and $(w, v) \in \underset{\cdot}{\bar{R}}$.

However, it is given that $\underset{\cdot}{\bar{R}}$ is an IVCI-equivalence-FR on $\underset{\cdot}{\check{E}}$, so $\underset{\cdot}{\bar{R}}$ is also an IVCI-transitive-FR. Therefore,

$$(u, v) \in \underset{\cdot}{\bar{R}} \Rightarrow \underset{\cdot}{\bar{R}} \circ \underset{\cdot}{\bar{R}} \subseteq \underset{\cdot}{\bar{R}} \tag{2}$$

Thus, by (1) and (2),

$\underset{\cdot}{\bar{R}} \circ \underset{\cdot}{\bar{R}} = \underset{\cdot}{\bar{R}}$. □

Theorem 4. *Suppose $\underset{\cdot}{\bar{R}}$ is an IVCI-partial order-FR on an IVCIFS $\underset{\cdot}{\check{E}}$, then the converse relation $\underset{\cdot}{\bar{R}}^c$ of $\underset{\cdot}{\bar{R}}$ is also an IVCI-partial order-FR on $\underset{\cdot}{\check{E}}$.*

Proof. In order to prove the assertion, it is sufficient to show that the converse of a complex intuitionistic partial order fuzzy relation R^c satisfies the three properties of a complex intuitionistic partial order fuzzy relation.

By using the properties of IVCI-partial order-FR $\underset{\cdot}{\bar{R}}$, we prove the statement.

i. It is given that $\underset{\cdot}{\bar{R}}$ is an IVCI-reflexive-FR. Therefore, for any $u \in \mathrm{S}$, $(u, u) \in \underset{\cdot}{\bar{R}} \Rightarrow (u, u) \in \underset{\cdot}{\bar{R}}^c$. Thus, $\underset{\cdot}{\bar{R}}^c$ is an IVCI-reflexive-FR.
ii. Assume that $(u, u) \in \underset{\cdot}{\bar{R}}^c$ and $(v, u) \in \underset{\cdot}{\bar{R}}^c$, then, $(u, v) \in \underset{\cdot}{\bar{R}}$ and $(v, u) \in \underset{\cdot}{\bar{R}}$. However, $\underset{\cdot}{\bar{R}}$ is an IVCI-antisymmetric-FR. Therefore, $(u, v) = (v, u)$. Thus, $\underset{\cdot}{\bar{R}}$ is also an IVCI-antisymmetric-FR.
iii. Suppose that $(u, v) \in \underset{\cdot}{\bar{R}}^c$ and $(v, w) \in \underset{\cdot}{\bar{R}}^c$, then, $(w, v) \in \underset{\cdot}{\bar{R}}$ and $(v, u) \in \underset{\cdot}{\bar{R}}$. However, it is given that $\underset{\cdot}{\bar{R}}$ is an IVCI-transitive-FR. Therefore, $(w, u) \in \underset{\cdot}{\bar{R}} \Rightarrow (u, w) \in \underset{\cdot}{\bar{R}}^c$. Thus, $\underset{\cdot}{\bar{R}}^c$ is also an IVCI-transitive-FR.

From i, ii and iii, the converse relation $\underset{\cdot}{\bar{R}}^c$ of an IVCI-partial order-FR $\underset{\cdot}{\bar{R}}$ is proved to be an IVCI-partial order-FR too. □

Theorem 5. *Suppose $\underset{\cdot}{\bar{R}}$ is an IVCI-equivalence-FR on an IVCIFS $\underset{\cdot}{\check{E}}$, then $(x, y) \in \underset{\cdot}{\bar{R}}$, iff $\underset{\cdot}{\bar{R}}[u] = \underset{\cdot}{\bar{R}}[v]$.*

Proof. Assume that $(u, v) \in \underset{\cdot}{\bar{R}}$ and $w \in \underset{\cdot}{\bar{R}}[u]\, \underset{\cdot}{\bar{R}}(w, u) \in \underset{\cdot}{\bar{R}}$.

Now, by using the fact that an IVCI-equivalence-FR is also an IVCI-transitive-FR, so

$(w, v) \in \underset{\cdot}{\bar{R}} \Rightarrow w \in \underset{\cdot}{\bar{R}}[v]$.

Thus,

$$\underset{\cdot}{\bar{R}}[u] \subseteq R[v] \tag{3}$$

As $(u, v) \in \underset{\cdot}{\bar{R}}$, by using the fact that an IVCI-equivalence-FR is also an IVCI-symmetric-FR, so

$(v, u) \in \underset{\cdot}{\bar{R}}$.

Additionally, assume that $w \in \underset{\cdot}{\bar{R}}[v] \Rightarrow (w, v) \in \underset{\cdot}{\bar{R}}$.

Now, again by using the fact that an IVCI-equivalence-FR is also an IVCI-transitive-FR, so

$(w, u) \in \underset{\cdot}{\bar{R}} \Rightarrow w \in \underset{\cdot}{\bar{R}}[u]$.

Thus,

$$\underset{\cdot}{\bar{R}}[v] \subseteq \underset{\cdot}{\bar{R}}[u] \tag{4}$$

Therefore, (3) and (4) infer that $\underset{\cdot}{\bar{R}}[v] = \underset{\cdot}{\bar{R}}[u]$.

Conversely, assume that $\bar{\underset{.}{\mathrm{R}}}[v] = \bar{\underset{.}{\mathrm{R}}}[u]$, $w \in \bar{\underset{.}{\mathrm{R}}}[u]$ and $w \in \bar{\underset{.}{\mathrm{R}}}[v] \Rightarrow (w, v) \in \bar{\underset{.}{\mathrm{R}}}$ and $(w, u) \in \bar{\underset{.}{\mathrm{R}}}$.

Again, by using the fact that an IVCI-equivalence-FR is also an IVCI-symmetric-FR, so $(w, u) \in \bar{\underset{.}{\mathrm{R}}} \Rightarrow (u, w) \in \bar{\underset{.}{\mathrm{R}}}$.

Now, by the definition of IVCI-transitive-FR,
$(u, w) \in \bar{\underset{.}{\mathrm{R}}}$ and $(w, v) \in \bar{\underset{.}{\mathrm{R}}} \Rightarrow (u, v) \in \bar{\underset{.}{\mathrm{R}}}$,
which completes the proof. □

4. Hasse Diagram for IVCI-Partial Order-FRs

In this section, the Hasse diagram for IVCI-partial order-FR is defined. Moreover, some important ideas related to Hasse diagrams, such as maximum element, minimum element, maximal element, minimal element, supremum, infimum, upper and lower bounds, are defined.

Definition 16. *The graphical representation of an IVCI-partial order-FS is called the Hasse diagram. The diagram is made up of dots and lines, which are called the vertices and edges, respectively. Each vertex represents an element of an IVCI-partial order-FS, while each edge represents a relationship between some pair of these elements. There are certain rules in the construction of a Hasse diagram, that are discussed below:*

i. *The elements are arranged in an ascending order to distinguish the lower ranks and higher ranks. In an ordered pair of an IVCI-partial order-FR, the preceding element is considered to be smaller than the element appearing second in the pair. For example, in the ordered pair (t, x), the element t is smaller than the element x. t will appear higher than x in the diagram.*
ii. *There are no self-loops for IVCI-reflexive-FRs. In a Hasse diagram, the self-relation is not represented by any edge, it is just assumed to be there.*
iii. *There are no directional edges. The directional edges indicate the order of the element in the ordered pair. For example, in the ordered pair (t, x), the relation in normal diagrams, would be represented by a directional edge with an arrow head pointing towards element x. Thus, ranking x higher than t. But in a Hasse diagram, the stepwise ascending arrangement of the elements automatically distinguishes the higher ranked and lower ranked elements.*
iv. There are no redundant edges, for instance the edge for an IVCI-transitive-FRs and IVCI-reflexive-FRs. Consider (t, x) *and then IVCI-transitive-FR* $\bar{\underset{.}{\mathrm{R}}}(t, k)$. In normal diagrams, there would be three edges representing the above three relations. However, in a Hasse diagram, only two edges are constructed, i.e., from *t to x and another from to k. The indirect link between t and k via x is represented by the two edges. The IVCI-transitive-FR is intuitively understood.*

Example 14. *Consider an IVCIFS*

$$\breve{\underset{,}{\mathrm{E}}} = \left\{ \begin{array}{l} \left(u, [0.07, 0.12]e^{i2[0.24,0.29]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right), \\ \left(v, [0.63, 0.71]e^{i2[0.03,0.15]\pi}, [0.11, 0.23]e^{i2[0.73,0.82]\pi}\right), \\ \left(w, [0.37, 0.46]e^{i2[0.15,0.25]\pi}, [0.32, 0.43]e^{i2[0.53,0.68]\pi}\right), \\ \left(x, [0.26, 0.34]e^{i2[0.15,0.25]\pi}, [0.38, 0.49]e^{i2[0.63,0.74]\pi}\right), \\ \left(y, [0.52, 0.58]e^{i2[0.30,0.42]\pi}, [0.18, 0.37]e^{i2[0.06,0.13]\pi}\right), \\ \left(z, [0.28, 0.36]e^{i2[0.43,0.53]\pi}, [0.48, 0.61]e^{i2[0.32,0.42]\pi}\right), \end{array} \right\} \textit{ in a universe } \mathrm{S}.$$

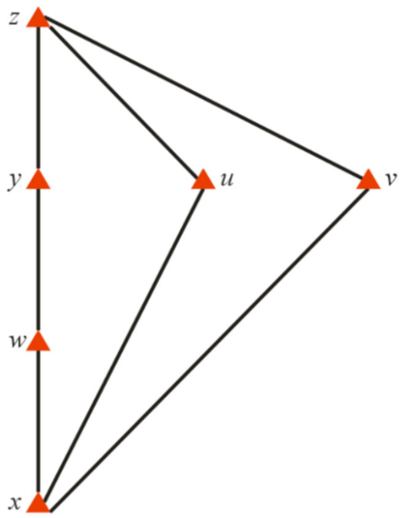
The Cartesian product $\breve{\underset{,}{\mathrm{E}}} \times \breve{\underset{,}{\mathrm{E}}}$ *is*

$$\check{Ȩ} \times \check{Ȩ} = \begin{bmatrix}
\left((u,u), [0.07, 0.12]e^{i2[0.15,0.25]\pi}, [0.56, 0.66]e^{i2[0.63,0.74]\pi}\right), \\
\left((u,v), [0.07, 0.12]e^{i2[0.03,0.15]\pi}, [0.56, 0.66]e^{i2[0.73,0.82]\pi}\right), \\
\left((u,w), [0.07, 0.12]e^{i2[0.15,0.25]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right) \\
\left((u,x), [0.07, 0.12]e^{i2[0.15,0.25]\pi}, [0.56, 0.66]e^{i2[0.63,0.74]\pi}\right), \\
\left((u,y), [0.07, 0.12]e^{i2[0.24,0.29]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right), \\
\left((u,z), [0.07, 0.12]e^{i2[0.24,0.29]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right), \\
\left((v,u), [0.07, 0.12]e^{i2[0.03,0.15]\pi}, [0.56, 0.66]e^{i2[0.73,0.82]\pi}\right), \\
\left((v,v), [0.63, 0.71]e^{i2[0.03,0.15]\pi}, [0.11, 0.23]e^{i2[0.73,0.82]\pi}\right), \\
\left((v,w), [0.37, 0.46]e^{i2[0.03,0.15]\pi}, [0.32, 0.43]e^{i2[0.73,0.82]\pi}\right), \\
\left((v,x), [0.26, 0.34]e^{i2[0.03,0.15]\pi}, [0.38, 0.49]e^{i2[0.73,0.82]\pi}\right), \\
\left((v,y), [0.52, 0.58]e^{i2[0.03,0.15]\pi}, [0.18, 0.37]e^{i2[0.73,0.82]\pi}\right), \\
\left((v,z), [0.28, 0.36]e^{i2[0.03,0.15]\pi}, [0.48, 0.61]e^{i2[0.73,0.82]\pi}\right), \\
\left((w,u), [0.07, 0.12]e^{i2[0.15,0.25]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right), \\
\left((w,v), [0.37, 0.46]e^{i2[0.03,0.15]\pi}, [0.32, 0.43]e^{i2[0.73,0.82]\pi}\right), \\
\left((w,w), [0.37, 0.46]e^{i2[0.15,0.25]\pi}, [0.32, 0.43]e^{i2[0.53,0.68]\pi}\right), \\
\left((w,x), [0.26, 0.34]e^{i2[0.15,0.25]\pi}, [0.38, 0.49]e^{i2[0.63,0.74]\pi}\right), \\
\left((w,y), [0.37, 0.46]e^{i2[0.15,0.25]\pi}, [0.32, 0.43]e^{i2[0.53,0.68]\pi}\right), \\
\left((w,z), [0.28, 0.36]e^{i2[0.15,0.25]\pi}, [0.48, 0.61]e^{i2[0.53,0.68]\pi}\right), \\
\left((x,u), [0.07, 0.12]e^{i2[0.15,0.25]\pi}, [0.56, 0.66]e^{i2[0.63,0.74]\pi}\right), \\
\left((x,v), [0.26, 0.34]e^{i2[0.03,0.15]\pi}, [0.38, 0.49]e^{i2[0.73,0.82]\pi}\right), \\
\left((x,w), [0.26, 0.34]e^{i2[0.15,0.25]\pi}, [0.38, 0.49]e^{i2[0.63,0.74]\pi}\right), \\
\left((x,x), [0.26, 0.34]e^{i2[0.15,0.25]\pi}, [0.38, 0.49]e^{i2[0.63,0.74]\pi}\right), \\
\left((x,y), [0.26, 0.34]e^{i2[0.15,0.25]\pi}, [0.38, 0.49]e^{i2[0.63,0.74]\pi}\right), \\
\left((x,z), [0.26, 0.34]e^{i2[0.15,0.25]\pi}, [0.38, 0.49]e^{i2[0.63,0.74]\pi}\right), \\
\left((y,u), [0.07, 0.12]e^{i2[0.24,0.29]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right), \\
\left((y,v), [0.52, 0.58]e^{i2[0.03,0.15]\pi}, [0.18, 0.37]e^{i2[0.73,0.82]\pi}\right), \\
\left((y,w), [0.37, 0.46]e^{i2[0.15,0.25]\pi}, [0.32, 0.43]e^{i2[0.53,0.68]\pi}\right), \\
\left((y,x), [0.26, 0.34]e^{i2[0.15,0.25]\pi}, [0.38, 0.49]e^{i2[0.63,0.74]\pi}\right), \\
\left((y,y), [0.52, 0.58]e^{i2[0.30,0.42]\pi}, [0.18, 0.37]e^{i2[0.06,0.13]\pi}\right), \\
\left((y,z), [0.26, 0.34]e^{i2[0.43,0.53]\pi}, [0.48, 0.61]e^{i2[0.63,0.74]\pi}\right), \\
\left((z,u), [0.07, 0.12]e^{i2[0.24,0.29]\pi}, [0.56, 0.66]e^{i2[0.57,0.71]\pi}\right), \\
\left((z,v), [0.28, 0.36]e^{i2[0.03,0.15]\pi}, [0.48, 0.61]e^{i2[0.73,0.82]\pi}\right), \\
\left((z,w), [0.28, 0.36]e^{i2[0.15,0.25]\pi}, [0.48, 0.61]e^{i2[0.53,0.68]\pi}\right), \\
\left((z,x), [0.26, 0.34]e^{i2[0.15,0.25]\pi}, [0.38, 0.49]e^{i2[0.63,0.74]\pi}\right), \\
\left((z,y), [0.37, 0.46]e^{i2[0.15,0.25]\pi}, [0.32, 0.43]e^{i2[0.53,0.68]\pi}\right), \\
\left((z,z), [0.28, 0.36]e^{i2[0.43,0.53]\pi}, [0.48, 0.61]e^{i2[0.32,0.42]\pi}\right)
\end{bmatrix}$$

An IVCI-partial order-FR $\bar{\underset{\cdot}{\mathrm{R}}}$ is

$$\bar{\underset{\cdot}{\mathrm{R}}} = \left\{ \begin{array}{c} \left((u,v), [0.07,0.12]e^{i2[0.15,0.25]\pi}, [0.56,0.66]e^{i2[0.63,0.74]\pi}\right), \\ \left((u,y), [0.07,0.12]e^{i2[0.24,0.29]\pi}, [0.56,0.66]e^{i2[0.57,0.71]\pi}\right), \\ \left((u,z), [0.07,0.12]e^{i2[0.24,0.29]\pi}, [0.56,0.66]e^{i2[0.57,0.71]\pi}\right), \\ \left((v,v), [0.63,0.71]e^{i2[0.03,0.15]\pi}, [0.11,0.23]e^{i2[0.73,0.82]\pi}\right), \\ \left((v,z), [0.28,0.36]e^{i2[0.03,0.15]\pi}, [0.48,0.61]e^{i2[0.73,0.82]\pi}\right), \\ \left((w,w), [0.37,0.46]e^{i2[0.15,0.25]\pi}, [0.32,0.43]e^{i2[0.53,0.68]\pi}\right), \\ \left((w,y), [0.37,0.46]e^{i2[0.15,0.25]\pi}, [0.32,0.43]e^{i2[0.53,0.68]\pi}\right), \\ \left((w,z), [0.28,0.36]e^{i2[0.15,0.25]\pi}, [0.48,0.61]e^{i2[0.53,0.68]\pi}\right), \\ \left((x,u), [0.07,0.12]e^{i2[0.15,0.25]\pi}, [0.56,0.66]e^{i2[0.63,0.74]\pi}\right), \\ \left((x,v), [0.26,0.34]e^{i2[0.03,0.15]\pi}, [0.38,0.49]e^{i2[0.73,0.82]\pi}\right), \\ \left((x,w), [0.26,0.34]e^{i2[0.15,0.25]\pi}, [0.38,0.49]e^{i2[0.63,0.74]\pi}\right), \\ \left((x,x), [0.26,0.34]e^{i2[0.15,0.25]\pi}, [0.38,0.49]e^{i2[0.63,0.74]\pi}\right), \\ \left((x,y), [0.26,0.34]e^{i2[0.15,0.25]\pi}, [0.38,0.49]e^{i2[0.63,0.74]\pi}\right), \\ \left((x,z), [0.26,0.34]e^{i2[0.15,0.25]\pi}, [0.38,0.49]e^{i2[0.63,0.74]\pi}\right), \\ \left((y,y), [0.52,0.58]e^{i2[0.30,0.42]\pi}, [0.18,0.37]e^{i2[0.06,0.13]\pi}\right), \\ \left((y,z), [0.26,0.34]e^{i2[0.43,0.53]\pi}, [0.48,0.61]e^{i2[0.63,0.74]\pi}\right), \\ \left((z,z), [0.28,0.36]e^{i2[0.43,0.53]\pi}, [0.48,0.61]e^{i2[0.32,0.42]\pi}\right) \end{array} \right\}$$

Figure 2 depicts the Hasse diagram of the IVCI-partial order-FR $\bar{\underset{\cdot}{\mathrm{R}}}$.



u	$[0.07,0.12]e^{i2[0.24,0.29]\pi}, [0.56,0.66]e^{i2[0.57,0.71]\pi}$
v	$[0.63,0.71]e^{i2[0.03,0.15]\pi}, [0.11,0.23]e^{i2[0.73,0.82]\pi}$
w	$[0.37,0.46]e^{i2[0.15,0.25]\pi}, [0.32,0.43]e^{i2[0.53,0.68]\pi}$
x	$[0.26,0.34]e^{i2[0.15,0.25]\pi}, [0.38,0.49]e^{i2[0.63,0.74]\pi}$
y	$[0.52,0.58]e^{i2[0.30,0.42]\pi}, [0.18,0.37]e^{i2[0.0.06,0.13]\pi}$
z	$[0.28,0.36]e^{i2[0.43,0.53]\pi}, [0.48,0.61]e^{i2[0.32,0.42]\pi}$

Figure 2. Hasse diagram for an IVCI-partial order-FR $\bar{\underset{\cdot}{\mathrm{R}}}$

Definition 17. *Suppose that an IVCI-partial order-FR is represented by a Hasse diagram, then an element is known as:*

1. *The maximal element if it appears at the top of the diagram.*
2. *The minimal if it appears at the bottom of the diagram.*
3. *The maximum or greatest element if every element related to it is smaller than it.*
4. *The minimum or least element if every element related to it is greater than it.*

Definition 18. *Suppose Ḟ be the subset of an IVCI-partial order-FS Ḝ, then an element $x \in$ Ṝ $\subseteq$ Ḝ $\times$ Ḝ is known as the:*

1. *Upper bound of Ḟ if $(v, u) \in$ Ṝ, $\forall v \in$ Ḟ.*
2. *Lower bound of Ḟ if $(u, v) \in$ Ṝ, $\forall v \in$ Ḟ.*
3. *Supremum of Ḟ if it is the least upper bound of Ḟ.*
4. *Infimum of Ḟ if it is the greatest upper bound of Ḟ.*

Example 15. *Assume that Ḝ $= \{f, g, h, k, l, o, p, q, r, s, t, u\}$ is a complex intuitionistic partial order fuzzy set Ḝ. For ease, we close your eyes to the degree of membership and degree of non-membership. Figure 3 is the illustration of the Hasse diagram of set Ḝ.*

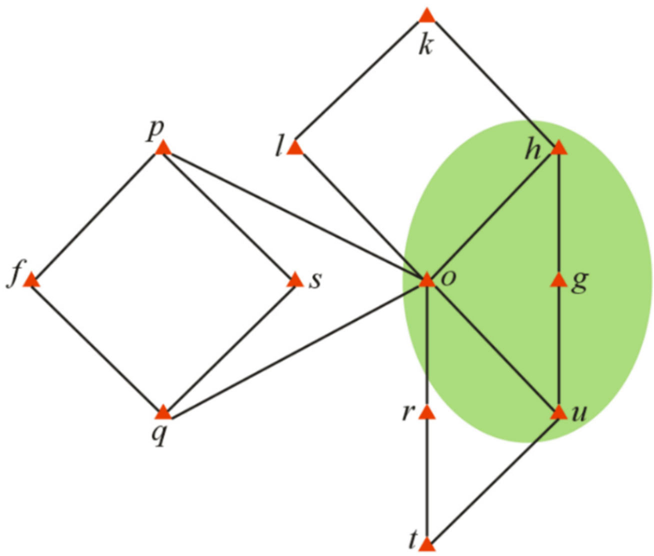


Figure 3. Hasse Diagram for set Ḝ.

In the above diagram,

a. Maximal element is k.
b. Minimal element is t.
c. Maximum elements are k and p.
d. Minimum elements are q and t.

Now considering a subset Ḟ $= \{g, h, o, u\}$ of set Ḝ whose elements are enclosed in the green oval.

e. Upper bounds of Ḟ are h and k.
f. Lower bounds of Ḟ are u and t.
g. Supremum of Ḟ is h.
h. Infimum of Ḟ is u.

5. Application

As the name suggests, in this section the applications of the proposed concepts are presented. We applied the introduced relations and their different types in the fields of computer technology, more precisely: cybersecurity and cybercrime in the industrial control system (ICS).

5.1. Security Measures

Over the past few decades, the automation of industrial systems has been steadily gaining momentum. Business demands continuous improvements in the efficiency of the production process, thus, the depth of IT penetration and system connectivity grows continuously; industrial facilities are connected to corporate networks and are frequently managed remotely over the network. However, along with their benefits, these new technologies have brought new threats into the world of industrial automation, and these new threats came as a surprise. The Industrial Control Systems (ICS) in place today were designed to operate for decades, and many of them were developed without any serious regard to IT security.

The stable operation of today's industrial networks can be disrupted, not only by a failure at a production unit or an operator's error but also by a software error, an accidental infection of workstations with malware or a deliberate cybercriminal attack.

Some security measures are given below. Figure 4 portrays the flowchart for the process being followed in the application.

I. **Default-deny as a standard policy:** In default-deny mode, the ICS works in a protected environment that only allows programs to run that are required for the technological process to function. All unknown and unwanted applications, including malicious programs, are blocked. Thus, a secure running environment is created with minimum load on system resources.

II. **Proactive protection** against unknown malicious programs and automatic protection against exploits. The technology scans executable programs, assessing the security of each application by monitoring its activities when in operation.

III. **Device Control technology** helps to manage removable devices (USB storages, GPRS modems, smartphones, USB network cards) and creates limited lists of permitted devices and the users who can access them.

IV. **All-in-one IT security console** helps to monitor and control all solutions to ensure IT security. With the single management console, admins can install, configure and manage security, and access reports.

V. **Integration with SIEM** (using special connectors) allows admins to export information about security incidents at protected nodes of the technological network into the corporate SIEM system.

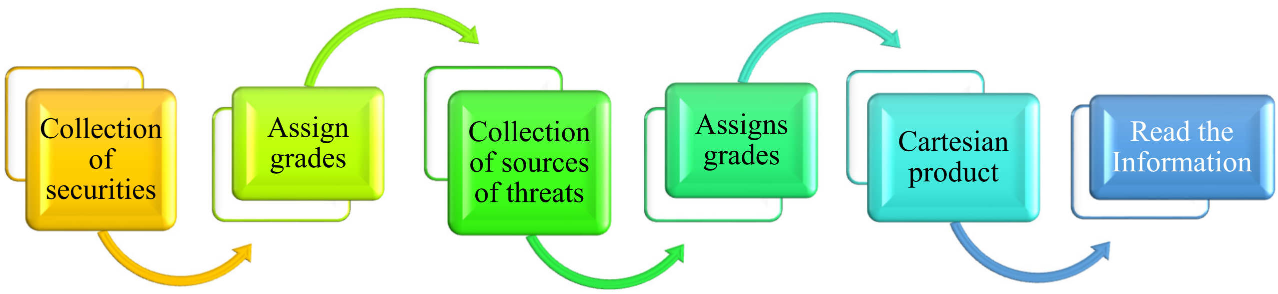


Figure 4. Flowchart for the process being followed.

The above security measures are abbreviated and assigned degrees of membership and degrees of non-membership values. Table 1 contains the details of their abbreviations and values of degrees.

Table 1. Details of security measures.

Security Measures	Abbreviations	Membership	Non-Membership
Default-deny as a standard policy	DD	$[0.550, 0.600]e^{i16[0.500,0.750]\pi}$	$[0.300, 0.400]e^{i16[0.125,0.313]\pi}$
Proactive protection	PP	$[0.475, 0.525]e^{i16[0.375,0.438]\pi}$	$[0.250, 0.375]e^{i16[0.125,0.250]\pi}$
Device Control technology	DCT	$[0.375, 0.525]e^{i16[0.438,0.563]\pi}$	$[0.200, 0.350]e^{i16[0.188,0.250]\pi}$
All-in-one IT Security Console	$ITSC$	$[0.400, 0.500]e^{i16[0.313,0.438]\pi}$	$[0.250, 0.350]e^{i16[0.250,0.313]\pi}$
Integration with SIEM	$SIEM$	$[0.300, 0.375]e^{i16[0.563,0.613]\pi}$	$[0.275, 0.325]e^{i16[0.188,0.313]\pi}$

5.2. Sources of Code Penetration

In a computer system there are certain weaknesses. Cybercriminals are always looking for opportunities. They inject their codes using media that are sources of code penetration. Following are the eight sources of code penetration:

I. Mobile Devices
II. Via USB Ports
III. Via Remote Access
IV. Wi-Fi
V. HMI Interface
VI. Internet Connections
VII. Outside Contractors
VIII. Via Corporate Networks

Table 2 contains the details of the above sources such as the abbreviations used in calculations and the degree of membership and degree of non-membership.

Table 2. Details of code penetration sources.

Source	Abbreviation	Membership	Non-Membership
Mobile Devices	MD	$[0.275, 0.375]e^{i16[0.375,0.500]\pi}$	$[0.350, 0.400]e^{i16[0.250,0.313]\pi}$
USB Ports	USB	$[0.375, 0.450]e^{i16[0.250,0.375]\pi}$	$[0.275, 0.325]e^{i16[0.125,0.250]\pi}$
Remote Access	RA	$[0.350, 0.400]e^{i16[0.375,0.438]\pi}$	$[0.325, 0.350]e^{i16[0.188,0.313]\pi}$
Wi-Fi	$WiFi$	$[0.450, 0.550]e^{i16[0.313,0.500]\pi}$	$[0.325, 0.400]e^{i16[0.188,0.250]\pi}$
HMI Interface	HMI	$[0.300, 0.375]e^{i16[0.438,0.563]\pi}$	$[0.200, 0.300]e^{i16[0.063,0.188]\pi}$
Internet Connections	IC	$[0.475, 0.575]e^{i16[0.500,0.613]\pi}$	$[0.250, 0.350]e^{i16[0.125,0.313]\pi}$
Outside Contractors	OC	$[0.500, 0.550]e^{i16[0.375,0.500]\pi}$	$[0.225, 0.300]e^{i16[0.063,0.250]\pi}$
Corporate Networks	CN	$[0.200, 0.475]e^{i16[0.313,0.613]\pi}$	$[0.250, 0.375]e^{i16[0.125,0.188]\pi}$

5.3. Calculations

Next, the relationships are analyzed, i.e., the effectiveness and ineffectiveness of each cybersecurity method against every cybercrime. We carry out the following mathematics.

We present the following two IVCIFSs Ę̌ and Ḟ representing the set of securities and the set of sources of threats, respectively.

$$\check{Ę} = \left\{ \begin{array}{c} \left(DD, [0.550, 0.600]e^{i16[0.500,0.750]\pi}, [0.300, 0.400]e^{i16[0.125,0.313]\pi}\right), \\ \left(PP, [0.475, 0.525]e^{i16[0.375,0.438]\pi}, [0.250, 0.375]e^{i16[0.125,0.250]\pi}\right), \\ \left(DCT, [0.375, 0.525]e^{i16[0.438,0.563]\pi}, [0.200, 0.350]e^{i16[0.188,0.250]\pi}\right), \\ \left(ITSC, [0.400, 0.500]e^{i16[0.313,0.438]\pi}, [0.250, 0.350]e^{i16[0.250,0.313]\pi}\right), \\ \left(SIEM, [0.300, 0.375]e^{i16[0.563,0.613]\pi}, [0.275, 0.325]e^{i16[0.188,0.313]\pi}\right) \end{array} \right\}$$

$$\dot{F} = \left\{ \begin{array}{c} \left(MD, [0.275, 0.375]e^{i16[0.375,0.500]\pi}, [0.350, 0.400]e^{i16[0.250,0.313]\pi}\right), \\ \left(USB, [0.375, 0.450]e^{i16[0.250,0.375]\pi}, [0.275, 0.325]e^{i16[0.125,0.250]\pi}\right), \\ \left(RA, [0.350, 0.400]e^{i16[0.375,0.438]\pi}, [0.325, 0.350]e^{i16[0.188,0.313]\pi}\right), \\ \left(WiFi, [0.450, 0.550]e^{i16[0.313,0.500]\pi}, [0.325, 0.400]e^{i16[0.188,0.250]\pi}\right), \\ \left(HMI, [0.300, 0.375]e^{i16[0.438,0.563]\pi}, [0.200, 0.300]e^{i16[0.063,0.188]\pi}\right), \\ \left(IC, [0.475, 0.575]e^{i16[0.500,0.613]\pi}, [0.250, 0.350]e^{i16[0.125,0.313]\pi}\right), \\ \left(OC, [0.500, 0.550]e^{i16[0.375,0.500]\pi}, [0.225, 0.300]e^{i16[0.063,0.250]\pi}\right), \\ \left(CN, [0.200, 0.475]e^{i16[0.313,0.613]\pi}, [0.250, 0.375]e^{i16[0.125,0.188]\pi}\right) \end{array} \right\}$$

To find out the efficacy of certain security measures against particular sources of code penetration, we use the Cartesian product. Thus, finding the Cartesian product between the IVCIFSs Ḝ and Ḟ

$$
\bar{\underset{\cdot}{\mathrm{R}}} = \breve{\text{Ȩ}} \times \dot{\mathrm{F}}
$$

$$
= \begin{bmatrix}
\left((DD, MD), [0.275, 0.375]e^{i16[0.375,0.500]\pi}, [0.350, 0.400]e^{i16[0.250,0313]\pi}\right), \\
\left((DD, USB), [0.375, 0.450]e^{i16[0.2500.375]\pi}, [0.300, 0.400]e^{i16[0.125,031:3]\pi}\right), \\
\left((DD, RA), [0.350, 0.400]e^{i16[0.375,0.438]\pi}, [0.325, 0.400]e^{i16[0.188,0.313]\pi}\right), \\
\left((DD, WiFi), [0.450, 0.550]e^{i16[0.313,0.500]\pi}, [0.325, 0.400]e^{i16[0.188,0.313]\pi}\right), \\
\left((DD, HMI), [0.300, 0.375]e^{i16[0.438,0.563]\pi}, [0.300, 0.400]e^{i16[0.125,0.313]\pi}\right), \\
\left((DD, IC), [0.475, 0.575]e^{i16[0.500,0.613]\pi}, [0.350, 0.400]e^{i16[0.125,0.313]\pi}\right), \\
\left((DD, OC), [0.500, 0.550]e^{i16[0.375,0.500]\pi}, [0.300, 0.400]e^{i16[0.125,0.313]\pi}\right), \\
\left((DD, CN), [0.200, 0.475]e^{i16[0.313,0.613]\pi}, [0.300, 0.400]e^{i16[0.125,0.313]\pi}\right), \\
\left((PP, MD), [0.275, 0.375]e^{i16[0.375,0.438]\pi}, [0.350, 0.400]e^{i16[0.250,0.313]\pi}\right), \\
\left((PP, USB), [0.375, 0.450]e^{i16[0.250,0.375]\pi}, [0.275, 0.375]e^{i16[0.125,0.250]\pi}\right), \\
\left((PP, RA), [0.350, 0.400]e^{i16[0.375,0.438]\pi}, [0.325, 0.375]e^{i16[0.188,0.313]\pi}\right), \\
\left((PP, WiFi), [0.450, 0.525]e^{i16[0.313,0.438]\pi}, [0.325, 0.400]e^{i16[0.188,0.250]\pi}\right), \\
\left((PP, HMI), [0.300, 0.375]e^{i16[0.375,0.438]\pi}, [0.250, 0.375]e^{i16[0.125,0.250]\pi}\right), \\
\left((PP, IC), [0.475, 0.525]e^{i16[0.375,0.438]\pi}, [0.250, 0.375]e^{i16[0.125,0.313]\pi}\right), \\
\left((PP, OC), [0.475, 0.525]e^{i16[0.375,0.438]\pi}, [0.250, 0.375]e^{i16[0.125,0250]\pi}\right), \\
\left((PP, CN), [0.200, 0.475]e^{i16[0.313,0.438]\pi}, [0.250, 0.375]e^{i16[0.125,0.188]\pi}\right), \\
\left((DCT, MD), [0.275, 0.375]e^{i16[0.375,0.500]\pi}, [0.350, 0.400]e^{i16[0.250,0.313]\pi}\right), \\
\left((DCT, USB), [0.375, 0.450]e^{i16[0.250,0.375]\pi}, [0.275, 0.350]e^{i16[0.188,0.250]\pi}\right), \\
\left((DCT, RA), [0.350, 0.400]e^{i16[0.375,0.438]\pi}, [0.325, 0.350]e^{i16[0.188,0.313]\pi}\right), \\
\left((DCT, WiFi), [0.375, 0.525]e^{i16[0.313,0.500]\pi}, [0.325, 0.400]e^{i16[0.188,0.250]\pi}\right), \\
\left((DCT, HMI), [0.300, 0.375]e^{i16[0.438,0.563]\pi}, [0.200, 0.350]e^{i16[0.188,0.250]\pi}\right), \\
\left((DCT, IC), [0.375, 0.525]e^{i16[0.438,0.563]\pi}, [0.250, 0.350]e^{i16[0.125,0.313]\pi}\right), \\
\left((DCT, OC), [0.375, 0.525]e^{i16[0.375,0.500]\pi}, [0.225, 0.350]e^{i16[0.188,0.250]\pi}\right), \\
\left((DCT, CN), [0.200, 0.475]e^{i16[0.313,0.563]\pi}, [0.250, 0.375]e^{i16[0.188,0.250]\pi}\right), \\
\left((ITSC, MD), [0.275, 0.375]e^{i16[0.313,0.438]\pi}, [0.350, 0.400]e^{i16[0.250,0.313]\pi}\right), \\
\left((ITSC, USB), [0.375, 0.450]e^{i16[0.250,0.375]\pi}, [0.275, 0.350]e^{i16[0.250,0.313]\pi}\right), \\
\left((ITSC, RA), [0.350, 0.400]e^{i16[0.313,0.438]\pi}, [0.325, 0.350]e^{i16[0.250,0.313]\pi}\right), \\
\left((ITSC, WiFi), [0.400, 0.500]e^{i16[0.313,0.438]\pi}, [0.325, 0.400]e^{i16[0.250,0.313]\pi}\right), \\
\left((ITSC, HMI), [0.300, 0.375]e^{i16[0.313,0.438]\pi}, [0.250, 0.350]e^{i16[0.250,0.313]\pi}\right), \\
\left((ITSC, IC), [0.400, 0.500]e^{i16[0.313,0.438]\pi}, [0.250, 0.350]e^{i16[0.250,0.313]\pi}\right), \\
\left((ITSC, OC), [0.400, 0.500]e^{i16[0.313,0.438]\pi}, [0.250, 0.350]e^{i16[0.250,0.313]\pi}\right), \\
\left((ITSC, CN), [0.200, 0.475]e^{i16[0.313,0.438]\pi}, [0.250, 0.375]e^{i16[0.250,0.313]\pi}\right), \\
\left((SIEM, MD), [0.275, 0.375]e^{i16[0.375,0.500]\pi}, [0.350, 0.400]e^{i16[0.250,0.313]\pi}\right), \\
\left((SIEM, USB), [0.300, 0.375]e^{i16[0.250,0.375]\pi}, [0.275, 0.325]e^{i16[0.188,0.313]\pi}\right), \\
\left((SIEM, RA), [0.300, 0.375]e^{i16[0.375,0.438]\pi}, [0.325, 0.350]e^{i16[0.188,0.313]\pi}\right), \\
\left((SIEM, WiFi), [0.300, 0.375]e^{i16[0.313,0.500]\pi}, [0.325, 0.400]e^{i16[0.188,0.313]\pi}\right), \\
\left((SIEM, HMI), [0.300, 0.375]e^{i16[0.438,0.563]\pi}, [0.275, 0.3250]e^{i16[0.188,0.313]\pi}\right), \\
\left((SIEM, IC), [0.300, 0.375]e^{i16[0.500,0.613]\pi}, [0.275, 0.350]e^{i16[0.188,0.313]\pi}\right), \\
\left((SIEM, OC), [0.300, 0.375]e^{i16[0.375,0.500]\pi}, [0.275, 0.325]e^{i16[0.188,0.313]\pi}\right), \\
\left((SIEM, CN), [0.200, 0.375]e^{i16[0.313,0.613]\pi}, [0.275, 0.375]e^{i16[0.188,0.313]\pi}\right)
\end{bmatrix}
$$

Every member of Ḝ × Ḟ is an order pair, which characterizes the connection between that pair, i.e., the influences and impacts of the first parameter on the second, in an ordered pair. The degrees of membership indicate the efficacy of a security measure to overcome a particular source of vulnerability with respect to time. In contrast, the degrees of non-membership give the inefficiency or ineptness of a certain security measure against a specific source of threat injection. For instance, the ordered pair $\left((DD, IC), [0.475, 0.575]e^{i16[0.500,0.613]\pi}, [0.350, 0.400]e^{i16[0.125,0.313]\pi}\right)$ expresses that the default-deny mode can successfully tackle the risks incoming through internet connections. Further, the numbers explain that the level of inefficiency is low. More specifically, the degree values are translated as: the grade of security that a default-deny mode provides against the vulnerabilities injected through internet connections is 47.5% to 57.5%, with respect to 8 to 10 time units and the chances of a cyberattack via internet connections bypassing the default-deny mode is 35% to 40%, with respect to 2 to 5 time units. As far as the security is concerned, the longer duration of time in the degree of membership is considered better, while the smaller time frame in the degree of non-membership is better.

5.4. Cyber-Security Techniques and Practices

An industry's digital system faces threats of all shapes and sizes. Thus, they should be ready to defend, identify and respond to a full range of attacks. The security strategy needs to be able to address the various methods these cybercriminals might employ. The process of the method is depicted in Figure 5.

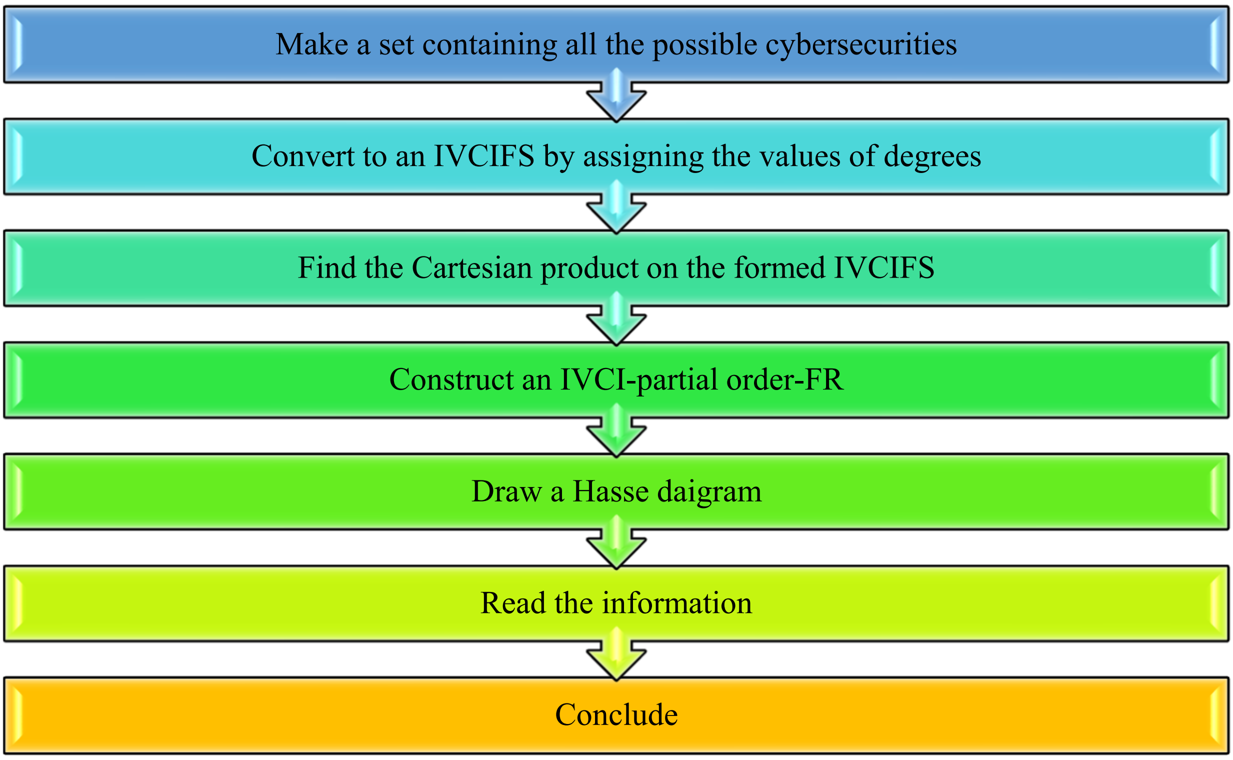


Figure 5. Flowchart of the process for selecting the best cyber-security.

Following are fourteen different security practices and techniques:

I. **Access control (*AC*)** If the cyberattacker is unable to access your industrial network, then they will do very limited harm. *AC* limits user access according to their responsibilities, which increases the security, and especially, internal breaches are restricted.

$$\left(AC, [0.455, 0.575]e^{i[0.305,0.540]\pi}, [0.100, 0.180]e^{i[0.009,0.270]\pi}\right)$$

II. **Anti-malware software** (***AMS***) Viruses, Trojans, worms, key-loggers and spyware are all malwares, which are used to infect digital systems. Anti-malware software identifies risky programs and files, then prevents them from spreading.

$$\left(AMS, [0.560, 0.610]e^{i[0.350,0.460]\pi}, [0.350, 0.390]e^{i[0.300,0.450]\pi}\right)$$

III. **Anomaly detection (*AD*)** Identifying anomalies is a difficult task. Henceforth, anomaly detection engines (ADE) are designed that allow the analysis of an industrial network. This alerts the authorities when breaches occur so that they can respond at the right times.

$$\left(AD, [0.295, 0.385]e^{i[0.390,0.480]\pi}, [0.270, 0.350]e^{i[0.250,0.330]\pi}\right)$$

IV. **Application security (*AS*)** An AS establishes security parameters for any applications that are relevant to industrial security.

$$\left(AS, [0.250, 0.340]e^{i[0.300,0.375]\pi}, [0.280, 0.400]e^{i[0.220,0.300]\pi}\right)$$

V. **Data loss prevention (DLP)** DLP equipment and strategies protect employees and users from ill-use, such as giving away sensitive data.

$$\left(DLP, [0.360, 0.425]e^{i[0.290,0.380]\pi}, [0.310, 0.385]e^{i[0.200,0.300]\pi}\right)$$

VI. **Email security (*ES*)** An ES system basically identifies risky emails. These phishing emails are usually very convincing because their target is to trick people. Further, this system also stops cyberattacks and prevents the sharing of important data.

$$\left(ES, [0.405, 0.505]e^{i[0.650,0.780]\pi}, [0.050, 0.100]e^{i[0.060,0.100]\pi}\right)$$

VII. **Endpoint security (*EPS*)** These days, the difference between personal and business devices is nearly non-existent. Unfortunately, personal devices are targeted to attack businesses. Endpoint security is a defensive layer between business networks and such remote devices.

$$\left(EPS, [0.375, 0.440]e^{i[0.350,0.450]\pi}, [0.450, 0.560]e^{i[0.350,0.450]\pi}\right)$$

VIII. **Firewalls (*FW*)** FW act as gateways in a network, used to secure the borders between the internet and local networks. They are used to manage network traffic by allowing approved traffic and blocking non-authorized traffic.

$$\left(FW, [0.450, 0.530]e^{i[0.500,0.580]\pi}, [0.220, 0.290]e^{i[0.130,0.210]\pi}\right)$$

IX. **Intrusion prevention systems (*IPS*)** Different types of cyberattacks are identified and then rapidly responded to by IPS after scanning and analyzing the traffic. These systems use databases of well-known cyberattack approaches; therefore, they immediately recognize threats.

$$\left(IPS, [0.275, 0.375]e^{i[0.375,0.500]\pi}, [0.350, 0.400]e^{i[0.250,0.350]\pi}\right)$$

X. **Network segmentation (*NS*)** NS restricts the traffic from suspicious sources that carries risky threats, and allows the authorized and right traffic.

$$\left(NS, [0.500, 0.580]e^{i[0.400,0.500]\pi}, [0.200, 0.280]e^{i[0.200,0.300]\pi}\right)$$

XI. Security information and event management (SIEM) $SIEM$ is a field of cyber-security that provides instantaneous analysis of security warnings spawned by applications and network hardware.

$$\left(SIEM, [0.500, 0.600]e^{i[0.650,0.700]\pi}, [0.040, 0.080]e^{i[0.050,0.120]\pi}\right)$$

XII. Virtual private network (VPN) The communication between secure networks and an endpoint device is authenticated by using VPN tools. They block other parties from spying by creating an encrypted line.

$$\left(VPN, [0.320, 0.450]e^{i[0.200,0.300]\pi}, [0.350, 0.410]e^{i[0.375,0.500]\pi}\right)$$

XIII. Web security (*WS*) WS is an extensive word that describes the security measures taken by businesses to ensure a harmless web experience when connected to an internal network. This prevents web-based cyberthreats from using browsers as access points to get into the network.

$$\left(WS, [0.325, 0.375]e^{i[0.365,0.490]\pi}, [0.335, 0.400]e^{i[0.275,0.395]\pi}\right)$$

XIV. Wireless security (*WiS*) Traditional networks are generally more secure than wireless networks. Thus, severe types of WiS measures are essential to certify that cybercriminals are not gaining access.

$$\left(WiS, [0.450, 0.560]e^{i[0.330,0.460]\pi}, [0.285, 0.390]e^{i[0.230,0.315]\pi}\right)$$

XV. Encryption (*E*) The encryption of data is an exceptionally effective security technique. The term "encryption" means the transformation of data to a code language or cipher text that cannot be read by a human. Special keys are used to decrypt these codes and cipher text back to a readable format. The complex encryption algorithms keep the information safe. Some of these algorithms are; Twofish algorithm, Rivest–Shamir–Adleman (RSA) algorithm and triple data encryption algorithm. Figure 6 illustrates the process of data encryption.

$$\left(E, [0.650, 0.810]e^{i[0.660,0.790]\pi}, [0.085, 0.190]e^{i[0.040,0.130]\pi}\right)$$

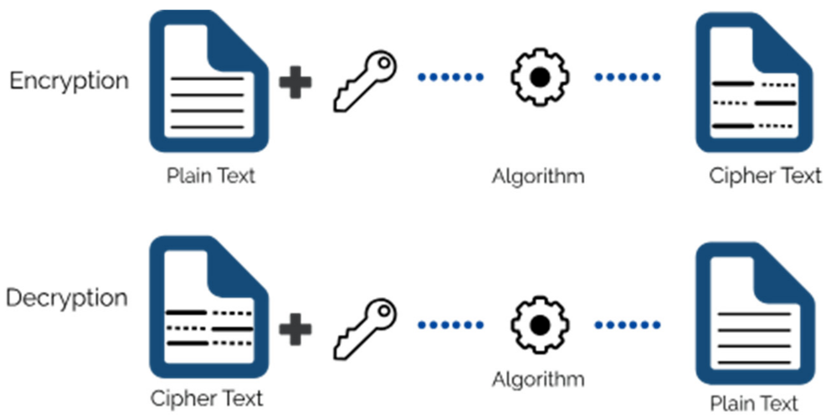


Figure 6. Data Encryption.

Let us assign the degree of membership and the degree of non-membership to each of the security measures and construct an IVCIFS $\breve{Ȩ}$

$$\breve{Ȩ} = \left\{ \begin{array}{c} \Big(AC, [0.455, 0.575]e^{i[0.305,0.540]\pi}, [0.100, 0.180]e^{i[0.009,0.270]\pi}\Big), \\ \Big(AMS, [0.560, 0.610]e^{i[0.350,0.460]\pi}, [0.350, 0.390]e^{i[0.300,0.450]\pi}\Big), \\ \Big(AD, [0.295, 0.385]e^{i[0.390,0.480]\pi}, [0.270, 0.350]e^{i[0.250,0.330]\pi}\Big), \\ \Big(AS, [0.250, 0.340]e^{i[0.300,0.375]\pi}, [0.280, 0.400]e^{i[0.220,0.300]\pi}\Big), \\ \Big(DLP, [0.360, 0.425]e^{i[0.290,0.380]\pi}, [0.310, 0.385]e^{i[0.200,0.300]\pi}\Big), \\ \Big(E, [0.650, 0.810]e^{i[0.660,0.790]\pi}, [0.085, 0.190]e^{i[0.040,0.130]\pi}\Big), \\ \Big(ES, [0.405, 0.505]e^{i[0.650,0.780]\pi}, [0.050, 0.100]e^{i[0.060,0.100]\pi}\Big), \\ \Big(EPS, [0.375, 0.440]e^{i[0.350,0.450]\pi}, [0.450, 0.560]e^{i[0.350,0.450]\pi}\Big), \\ \Big(FW, [0.450, 0.530]e^{i[0.500,0.580]\pi}, [0.220, 0.290]e^{i[0.130,0.210]\pi}\Big), \\ \Big(IPS, [0.275, 0.375]e^{i[0.375,0.500]\pi}, [0.350, 0.400]e^{i[0.250,0.350]\pi}\Big), \\ \Big(NS, [0.500, 0.580]e^{i[0.400,0.500]\pi}, [0.200, 0.280]e^{i[0.200,0.300]\pi}\Big), \\ \Big(SIEM, [0.500, 0.600]e^{i[0.650,0.700]\pi}, [0.040, 0.080]e^{i[0.050,0.120]\pi}\Big), \\ \Big(VPN, [0.320, 0.450]e^{i[0.200,0.300]\pi}, [0.350, 0.410]e^{i[0.375,0.500]\pi}\Big), \\ \Big(WS, [0.325, 0.375]e^{i[0.365,0.490]\pi}, [0.335, 0.400]e^{i[0.275,0.395]\pi}\Big), \\ \Big(WiS, [0.450, 0.560]e^{i[0.330,0.460]\pi}, [0.285, 0.390]e^{i[0.230,0.315]\pi}\Big), \end{array} \right\}$$

Table 3 contains the descriptions of an interval-valued complex intuitionistic partial order-FR $\bar{Ṛ}$ that is obtained from the CP $\breve{Ȩ} \times \breve{Ȩ}$.

Table 3. IVCI-partial order-FR $\bar{Ṛ} \subseteq \breve{Ȩ} \times \breve{Ȩ}$.

Ordered Pairs ($\bar{Ṛ}$=$\breve{Ȩ}$×$\breve{Ȩ}$)	**Degrees of Membership ($\acute{m}_{\mathbb{C}}$)**	**DegreesofNon-Membership($\underline{n}_{\mathbb{C}}$)**
(AC, AC)	$[0.455, 0.575]e^{i[0.305,0.540]\pi}$	$[0.100, 0.180]e^{i[0.009,0.270]\pi}$
$(AC,\ SIEM)$	$[0.455, 0.575]e^{i[0.305,0.540]\pi}$	$[0.100, 0.180]e^{i[0.050,0.270]\pi}$
(AMS, AMS)	$[0.560, 0.610]e^{i[0.350,0.460]\pi}$	$[0.350, 0.390]e^{i[0.300,0.450]\pi}$
(AMS, E)	$[0.560, 0.610]e^{i[0.350,0.460]\pi}$	$[0.350, 0.390]e^{i[0.300,0.450]\pi}$
$(AMS, SIEM)$	$[0.500, 0.600]e^{i[0.350,0.460]\pi}$	$[0.350, 0.390]e^{i[0.300,0.450]\pi}$
(AD, AD)	$[0.295, 0.385]e^{i[0.390,0.480]\pi}$	$[0.270, 0.350]e^{i[0.250,0.330]\pi}$
(AD, AMS)	$[0.295, 0.385]e^{i[0.350,0.460]\pi}$	$[0.350, 0.390]e^{i[0.300,0.450]\pi}$
(AD, E)	$[0.295, 0.385]e^{i[0.390,0.480]\pi}$	$[0.270, 0.350]e^{i[0.250,0.330]\pi}$
$(AD, SIEM)$	$[0.295, 0.385]e^{i[0.390,0.480]\pi}$	$[0.270, 0.350]e^{i[0.250,0.330]\pi}$
(AS, AMS)	$[0.250, 0.340]e^{i[0.300,0.375]\pi}$	$[0.350, 0.400]e^{i[0.300,0.450]\pi}$
(AS, AS)	$[0.250, 0.340]e^{i[0.300,0.375]\pi}$	$[0.280, 0.400]e^{i[0.220,0.300]\pi}$
(AS, E)	$[0.650, 0.810]e^{i[0.660,0.790]\pi}$	$[0.280, 0.400]e^{i[0.220,0.300]\pi}$
$(AS,\ SIEM)$	$[0.250, 0.340]e^{i[0.300,0.375]\pi}$	$[0.280, 0.400]e^{i[0.220,0.300]\pi}$
(DLP, AC)	$[0.360, 0.425]e^{i[0.290,0.380]\pi}$	$[0.310, 0.385]e^{i[0.200,0.300]\pi}$
(DLP, DLP)	$[0.360, 0.425]e^{i[0.290,0.380]\pi}$	$[0.310, 0.385]e^{i[0.200,0.300]\pi}$
(DLP, EPS)	$[0.360, 0.425]e^{i[0.290,0.380]\pi}$	$[0.450, 0.560]e^{i[0.350,0.450]\pi}$
$(DLP.SIEM)$	$[0.360, 0.425]e^{i[0.290,0.380]\pi}$	$[0.310, 0.385]e^{i[0.200,0.300]\pi}$
(E, E)	$[0.360, 0.425]e^{i[0.290,0.380]\pi}$	$[0.085, 0.190]e^{i[0.040,0.130]\pi}$
$(E, SIEM)$	$[0.500, 0.600]e^{i[0.650,0.700]\pi}$	$[0.085, 0.190]e^{i[0.050,0.130]\pi}$
(ES, AC)	$[0.405, 0.505]e^{i[0.305,0.540]\pi}$	$[0.100, 0.180]e^{i[0.060,0.270]\pi}$
(ES, DLP)	$[0.360, 0.425]e^{i[0.290,0.380]\pi}$	$[0.310, 0.385]e^{i[0.200,0.300]\pi}$
(ES, ES)	$[0.405, 0.505]e^{i[0.650,0.780]\pi}$	$[0.050, 0.100]e^{i[0.060,0.100]\pi}$
(ES, EPS)	$[0.375, 0.440]e^{i[0.350,0.450]\pi}$	$[0.450, 0.560]e^{i[0.350,0.450]\pi}$
$(ES, SIEM)$	$[0.405, 0.505]e^{i[0.650,0.700]\pi}$	$[0.050, 0.100]e^{i[0.060,0.120]\pi}$

Table 3. *Cont.*

Ordered Pairs (Ṝ=Ḝ×Ḝ)	Degrees of Membership ($\acute{m}_{\mathbb{C}}$)	DegreesofNon-Membership($\underline{n}_{\mathbb{C}}$)
(EPS, EPS)	$[0.375, 0.440]e^{i[0.350,0.450]\pi}$	$[0.450, 0.560]e^{i[0.350,0.450]\pi}$
$(EPS, SIEM)$	$[0.375, 0.440]e^{i[0.350,0.450]\pi}$	$[0.450, 0.560]e^{i[0.350,0.450]\pi}$
(FW, AMS)	$[0.450, 0.530]e^{i[0.350,0.460]\pi}$	$[0.350, 0.390]e^{i[0.300,0.450]\pi}$
(FW, AD)	$[0.295, 0.385]e^{i[0.390,0.480]\pi}$	$[0.270, 0.350]e^{i[0.250,0.330]\pi}$
(FW, E)	$[0.450, 0.530]e^{i[0.500,0.580]\pi}$	$[0.220, 0.290]e^{i[0.130,0.210]\pi}$
(FW, FW)	$[0.450, 0.530]e^{i[0.500,0.580]\pi}$	$[0.220, 0.290]e^{i[0.130,0.210]\pi}$
$(FW, SIEM)$	$[0.450, 0.530]e^{i[0.500,0.580]\pi}$	$[0.220, 0.290]e^{i[0.130,0.210]\pi}$
(IPS, AMS)	$[0.275, 0.375]e^{i[0.375,0.500]\pi}$	$[0.350, 0.400]e^{i[0.300,0.450]\pi}$
(IPS, AD)	$[0.275, 0.375]e^{i[0.375,0.480]\pi}$	$[0.350, 0.400]e^{i[0.250,0.350]\pi}$
(IPS, AS)	$[0.250, 0.340]e^{i[0.300,0.375]\pi}$	$[0.350, 0.400]e^{i[0.250,0.350]\pi}$
(IPS, E)	$[0.275, 0.375]e^{i[0.375,0.500]\pi}$	$[0.350, 0.400]e^{i[0.250,0.350]\pi}$
(IPS, FW)	$[0.275, 0.375]e^{i[0.375,0.500]\pi}$	$[0.350, 0.400]e^{i[0.250,0.350]\pi}$
(IPS, IPS)	$[0.275, 0.375]e^{i[0.375,0.500]\pi}$	$[0.350, 0.400]e^{i[0.250,0.350]\pi}$
$(IPS, SIEM)$	$[0.275, 0.375]e^{i[0.375,0.500]\pi}$	$[0.350, 0.400]e^{i[0.250,0.350]\pi}$
(NS, AC)	$[0.455, 0.575]e^{i[0.305,0.500]\pi}$	$[0.200, 0.280]e^{i[0.200,0.300]\pi}$
(NS, AMS)	$[0.500, 0.580]e^{i[0.350,0.460]\pi}$	$[0.350, 0.390]e^{i[0.300,0.450]\pi}$
(NS, AD)	$[0.295, 0.385]e^{i[0.390,0.480]\pi}$	$[0.270, 0.350]e^{i[0.250,0.330]\pi}$
(NS, AS)	$[0.250, 0.340]e^{i[0.300,0.375]\pi}$	$[0.280, 0.400]e^{i[0.220,0.300]\pi}$
(NS, DLP)	$[0.360, 0.425]e^{i[0.290,0.380]\pi}$	$[0.310, 0.385]e^{i[0.200,0.300]\pi}$
(NS, E)	$[0.500, 0.580]e^{i[0.400,0.500]\pi}$	$[0.200, 0.280]e^{i[0.200,0.300]\pi}$
(NS, ES)	$[0.405, 0.505]e^{i[0.400,0.500]\pi}$	$[0.200, 0.280]e^{i[0.200,0.300]\pi}$
(NS, EPS)	$[0.375, 0.440]e^{i[0.350,0.450]\pi}$	$[0.450, 0.560]e^{i[0.350,0.450]\pi}$
(NS, FW)	$[0.450, 0.530]e^{i[0.400,0.500]\pi}$	$[0.220, 0.290]e^{i[0.200,0.300]\pi}$
(NS, IPS)	$[0.275, 0.375]e^{i[0.375,0.500]\pi}$	$[0.350, 0.400]e^{i[0.250,0.350]\pi}$
(NS, NS)	$[0.500, 0.580]e^{i[0.400,0.500]\pi}$	$[0.200, 0.280]e^{i[0.200,0.300]\pi}$
$(NS, SIEM)$	$[0.500, 0.580]e^{i[0.400,0.500]\pi}$	$[0.200, 0.280]e^{i[0.200,0.300]\pi}$
$(SIEM, SIEM)$	$[0.500, 0.600]e^{i[0.650,0.700]\pi}$	$[0.040, 0.080]e^{i[0.050,0.120]\pi}$
(VPN, AC)	$[0.320, 0.450]e^{i[0.200,0.300]\pi}$	$[0.350, 0.410]e^{i[0.375,0.500]\pi}$
(VPN, AMS)	$[0.320, 0.450]e^{i[0.200,0.300]\pi}$	$[0.350, 0.410]e^{i[0.375,0.500]\pi}$
(VPN, AD)	$[0.295, 0.385]e^{i[0.200,0.300]\pi}$	$[0.350, 0.410]e^{i[0.375,0.500]\pi}$
(VPN, AS)	$[0.250, 0.340]e^{i[0.200,0.300]\pi}$	$[0.350, 0.410]e^{i[0.375,0.500]\pi}$
(VPN, DLP)	$[0.320, 0.425]e^{i[0.200,0.300]\pi}$	$[0.350, 0.410]e^{i[0.375,0.500]\pi}$
(VPN, E)	$[0.320, 0.450]e^{i[0.200,0.300]\pi}$	$[0.350, 0.410]e^{i[0.375,0.500]\pi}$
(VPN, ES)	$[0.320, 0.450]e^{i[0.200,0.300]\pi}$	$[0.350, 0.410]e^{i[0.375,0.500]\pi}$
(VPN, EPS)	$[0.320, 0.450]e^{i[0.200,0.300]\pi}$	$[0.450, 0.560]e^{i[0.375,0.500]\pi}$
(VPN, FW)	$[0.320, 0.450]e^{i[0.200,0.300]\pi}$	$[0.350, 0.410]e^{i[0.375,0.500]\pi}$
(VPN, IPS)	$[0.275, 0.375]e^{i[0.200,0.300]\pi}$	$[0.350, 0.410]e^{i[0.375,0.500]\pi}$
(VPN, NS)	$[0.320, 0.450]e^{i[0.200,0.300]\pi}$	$[0.350, 0.410]e^{i[0.375,0.500]\pi}$
$(VPN, SIEM)$	$[0.320, 0.450]e^{i[0.200,0.300]\pi}$	$[0.350, 0.410]e^{i[0.375,0.500]\pi}$
(VPN, VPN)	$[0.320, 0.450]e^{i[0.200,0.300]\pi}$	$[0.350, 0.410]e^{i[0.375,0.500]\pi}$
(VPN, WiS)	$[0.320, 0.450]e^{i[0.200,0.300]\pi}$	$[0.350, 0.410]e^{i[0.375,0.500]\pi}$
(WS, AC)	$[0.325, 0.375]e^{i[0.305,0.490]\pi}$	$[0.335, 0.400]e^{i[0.275,0.395]\pi}$
(WS, AMS)	$[0.325, 0.375]e^{i[0.350,0.460]\pi}$	$[0.350, 0.400]e^{i[0.300,0.450]\pi}$
(WS, AD)	$[0.295, 0.385]e^{i[0.390,0.480]\pi}$	$[0.335, 0.400]e^{i[0.275,0.395]\pi}$
(WS, AS)	$[0.250, 0.340]e^{i[0.300,0.375]\pi}$	$[0.335, 0.400]e^{i[0.275,0.395]\pi}$
(WS, DLP)	$[0.325, 0.375]e^{i[0.290,0.380]\pi}$	$[0.335, 0.400]e^{i[0.275,0.395]\pi}$
(WS, E)	$[0.325, 0.375]e^{i[0.365,0.490]\pi}$	$[0.335, 0.400]e^{i[0.275,0.395]\pi}$
(WS, ES)	$[0.325, 0.375]e^{i[0.365,0.490]\pi}$	$[0.335, 0.400]e^{i[0.275,0.395]\pi}$
(WS, EPS)	$[0.325, 0.375]e^{i[0.350,0.450]\pi}$	$[0.450, 0.560]e^{i[0.350,0.450]\pi}$
(WS, FW)	$[0.325, 0.375]e^{i[0.365,0.490]\pi}$	$[0.335, 0.400]e^{i[0.275,0.395]\pi}$
(WS, IPS)	$[0.275, 0.375]e^{i[0.365,0.490]\pi}$	$[0.350, 0.400]e^{i[0.275,0.395]\pi}$
(WS, NS)	$[0.325, 0.375]e^{i[0.365,0.490]\pi}$	$[0.335, 0.400]e^{i[0.275,0.395]\pi}$
$(WS, SIEM)$	$[0.325, 0.375]e^{i[0.365,0.490]\pi}$	$[0.335, 0.400]e^{i[0.275,0.395]\pi}$

Table 3. *Cont.*

Ordered Pairs ($\bar{\mathrm{R}}=\breve{\mathrm{E}}\times\breve{\mathrm{E}}$)	Degrees of Membership ($\acute{m}_{\mathbb{C}}$)	DegreesofNon-Membership($\underline{n}_{\mathbb{C}}$)
(WS, WS)	$[0.325, 0.375]e^{i[0.365,0.490]\pi}$	$[0.335, 0.400]e^{i[0.275,0.395]\pi}$
(WS, WiS)	$[0.325, 0.375]e^{i[0.330,0.460]\pi}$	$[0.335, 0.400]e^{i[0.275,0.395]\pi}$
(WiS, AC)	$[0.450, 0.560]e^{i[0.305,0.460]\pi}$	$[0.285, 0.390]e^{i[0.230,0.315]\pi}$
(WiS, AMS)	$[0.450, 0.560]e^{i[0.330,0.460]\pi}$	$[0.350, 0.390]e^{i[0.200,0.450]\pi}$
(WiS, AD)	$[0.295, 0.385]e^{i[0.330,0.460]\pi}$	$[0.285, 0.390]e^{i[0.250,0.330]\pi}$
(WiS, AS)	$[0.250, 0.340]e^{i[0.300,0.375]\pi}$	$[0.280, 0.400]e^{i[0.220,0.300]\pi}$
(WiS, DLP)	$[0.360, 0.425]e^{i[0.290,0.380]\pi}$	$[0.310, 0.385]e^{i[0.230,0.315]\pi}$
(WiS, WiS)	$[0.450, 0.560]e^{i[0.330,0.460]\pi}$	$[0.285, 0.390]e^{i[0.230,0.315]\pi}$
(WiS, ES)	$[0.405, 0.505]e^{i[0.330,0.460]\pi}$	$[0.285, 0.390]e^{i[0.230,0.315]\pi}$
(WiS, EPS)	$[0.375, 0.440]e^{i[0.330,0.460]\pi}$	$[0.450, 0.560]e^{i[0.350,0.450]\pi}$
(WiS, FW)	$[0.450, 0.530]e^{i[0.330,0.460]\pi}$	$[0.285, 0.390]e^{i[0.230,0.315]\pi}$
(WiS, IPS)	$[0.275, 0.375]e^{i[0.330,0.460]\pi}$	$[0.350, 0.400]e^{i[0.250,0.350]\pi}$
(WiS, NS)	$[0.450, 0.560]e^{i[0.330,0.460]\pi}$	$[0.285, 0.390]e^{i[0.230,0.315]\pi}$
$(WiS, SIEM)$	$[0.450, 0.560]e^{i[0.330,0.460]\pi}$	$[0.285, 0.390]e^{i[0.230,0.315]\pi}$
(WiS, WiS)	$[0.450, 0.560]e^{i[0.330,0.460]\pi}$	$[0.285, 0.390]e^{i[0.230,0.315]\pi}$

Figure 7 portrays the Hasse diagram for the above interval-valued complex intuitionistic partial order fuzzy relation. For ease, the degrees of membership and non-membership are kept hidden in the following diagram of $\bar{\mathrm{R}}$. In Figure 7, one can clearly tell that the best security technique of these fourteen tools is the $SIEM$ because it is the maximum as well as the maximal element. On the contrary side, the VPN and WS are considered to provide the least protection as they appear at the bottom of the diagram.

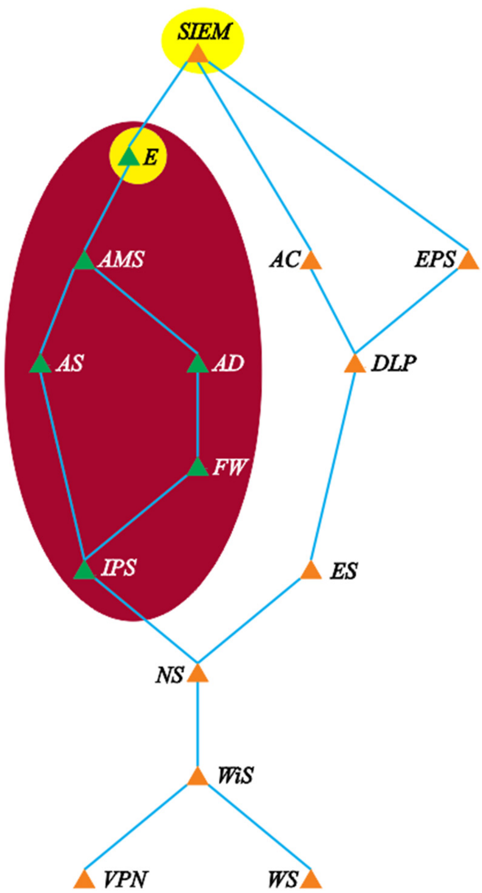


Figure 7. Hasse diagram for IVC-partial order-FR $\bar{\mathrm{R}}$

If we consider that an industry has to choose from among the following shortlisted security techniques because of some restrictions, these nominated security techniques and practices are listed in the following subset Ḟ.

$$\dot{F} = \left\{ \begin{array}{c} \left(AMS, [0.560, 0.610]e^{i[0.350,0.460]\pi}, [0.350, 0.390]e^{i[0.300,0.450]\pi}\right), \\ \left(AD, [0.295, 0.385]e^{i[0.390,0.480]\pi}, [0.270, 0.350]e^{i[0.250,0.330]\pi}\right), \\ \left(AS, [0.250, 0.340]e^{i[0.300,0.375]\pi}, [0.280, 0.400]e^{i[0.220,0.300]\pi}\right), \\ \left(E, [0.650, 0.810]e^{i[0.660,0.790]\pi}, [0.085, 0.190]e^{i[0.040,0.130]\pi}\right), \\ \left(FW, [0.450, 0.530]e^{i[0.500,0.580]\pi}, [0.220, 0.290]e^{i[0.130,0.210]\pi}\right), \\ \left(IPS, [0.275, 0.375]e^{i[0.375,0.500]\pi}, [0.350, 0.400]e^{i[0.250,0.350]\pi}\right) \end{array} \right\}$$

The business seeks to select the best security technique from among the shortlisted techniques in set Ḟ. In order to do so, they must look out for the upper bounds and supremum. In the following diagram, the elements of Ḟ are enclosed in the dark red circle. Here, the upper bounds are $\{E, SIEM\}$. The supremum is the least upper bound, therefore, encryption is the supremum in this case. Amongst the members of the subset Ḟ, encryption (E) is the best choice for coping with the cybersecurity reasons.

6. Comparative Analysis

In this section, the reliability of the proposed framework of IVCIFRs is verified through comparing it with the pre-existing structures such as CFRs or IFRs and IVIFRs.

6.1. Comparison with FRs, CFRs, IVFRs and IVCFRs

The leading limitation of FR, CFR, IVFRs and IVCFR as compared to IVCIFRs is that these notions discuss the degrees of membership only and do not talk about the degrees of non-membership, while an IVCIFR argues about both of the degrees. Thus, an IVCIFR has the power to analyze both the strength and the weakness of any relationship.

Moreover, the complex structure of IVCIFRs can model multivariable problems, whereas the frameworks of FRs and IFRs are unable to model such problems. As an IVCFR is the greatest structure among the aforementioned contestants, so a detailed comparison between IVCIFRs and IVCFRs is given below.

We investigate the problem discussed in Section 5.3 by using IVCFRs and thinking of the following two IVCFSs Ḝ and Ḟ representing the set of securities and the set of sources of threats, respectively. In order to minimize the amount of calculation and to conclude the comparative analysis, some of the securities and the sources of threats are omitted.

$$\breve{Ȩ} = \left\{ \begin{array}{c} \left(DD, [0.550, 0.600]e^{i16[0.500,0.750]\pi}\right), \left(PP, [0.475, 0.525]e^{i16[0.375,0.438]\pi}\right) \\ \left(SIEM, [0.300, 0.375]e^{i16[0.563,0.613]\pi}\right) \end{array} \right\}$$

$$\dot{F} = \left\{ \begin{array}{c} \left(USB, [0.375, 0.450]e^{i16[0.250,0.375]\pi}\right), \left(RA,\ [0.350, 0.400]e^{i16[0.375,0.438]\pi}\right), \\ \left(IC, [0.475, 0.575]e^{i16[0.500,0.613]\pi}\right), \left(OC, [0.500, 0.550]e^{i16[0.375,0.500]\pi}\right) \end{array} \right\}$$

Table 4 contains the details of abbreviations used in the above sets.

Table 4. Abbreviations.

Abbreviations	Full Names
DD	Default-Deny as a standard policy
PP	Proactive Protection
$SIEM$	Integration with SIEM
USB	USB Ports
RA	Remote Access
IC	Internet Connections
OC	Outside Contractors

The IVCFR $\bar{\text{Ṛ}}$ between $\breve{\text{Ȩ}}$ and $\dot{\text{F}}$ is

$$\bar{\text{Ṛ}} = \left\{ \begin{array}{c} \left((DD, USB), [0.375, 0.450]e^{i16[0.250,0.375]\pi}\right), \\ \left((DD, RA), [0.350, 0.400]e^{i16[0.375,0.438]\pi}\right), \\ \left((DD, IC), [0.475, 0.575]e^{i16[0.500,0.613]\pi}\right), \\ \left((DD, OC), [0.500, 0.550]e^{i16[0.375,0.500]\pi}\right), \\ \left((PP, USB), [0.375, 0.450]e^{i16[0.250,0.375]\pi}\right), \\ \left((PP, RA), [0.350, 0.400]e^{i16[0.375,0.438]\pi}\right), \\ \left((PP, IC), [0.475, 0.525]e^{i16[0.375,0.438]\pi}\right), \\ \left((PP, OC), [0.475, 0.525]e^{i16[0.375,0.438]\pi}\right), \\ \left((SIEM, USB), [0.300, 0.375]e^{i16[0.250,0.375]\pi}\right), \\ \left((SIEM, RA), [0.300, 0.375]e^{i16[0.375,0.438]\pi}\right), \\ \left((SIEM, IC), [0.300, 0.375]e^{i16[0.500,0.613]\pi}\right), \\ \left((SIEM, OC), [0.300, 0.375]e^{i16[0.375,0.500]\pi}\right) \end{array} \right\}$$

It can be seen that the above IVCFR $\bar{\text{Ṛ}}$ only gives the information for the degree of membership. That is, it only gives the effectiveness of a cybersecurity technique against a source of penetration and fails to reveal the ineffectiveness of these securities versus the source of penetration's relations because the structure of an IVCFR does not have any degree of non-membership. Hence, these structures have certain limitations and, thus, they give limited information.

6.2. Comparison with IFRs, CIFRs and IVIFRs

An IVCIFR argues about both the degree of membership and non-membership, as do the structures of IFRs, CIFRs and IVIFRs. However, IFRs and IVIFRs involve only real numbers, which limits them to single variable problems. They cannot express the problems involving time (periodic) or with phase changes.

The structure of CIFRs is based on complex numbers and, thus, consists of amplitude and phase terms. However, the advantage of an interval-valued structure of IVCIFRs over the crisp valued structure of CIFRs is that the interval represents a set, thus covering the uncertainties, including mistakes made by the person, experimental errors or computer approximations that lead to fuzziness. A detailed comparison between IVCIFRs and IVIFRs is given below.

We look to solve the problem discussed in Section 5.3 by using IVIFRs. We present the following two IVIFSs $\breve{\text{Ȩ}}$ and $\dot{\text{F}}$ representing the set of securities and the set of sources of threats, respectively. We omit some of the securities and the sources in order to minimize the amount of calculation. The details of abbreviations used in the following sets are in Table 4.

$$\breve{\text{Ȩ}} = \left\{ \begin{array}{c} (DD, [0.550, 0.600], [0.300, 0.400]), (PP, [0.475, 0.525], [0.250, 0.375]), \\ (SIEM, [0.300, 0.375], [0.275, 0.325]) \end{array} \right\}$$

$$\dot{\text{F}} = \left\{ \begin{array}{c} (USB, [0.375, 0.450], [0.275, 0.325]), (RA, \ [0.350, 0.400], [0.325, 0.350]), \\ (IC, [0.475, 0.575], [0.250, 0.350]), (OC, [0.500, 0.550], [0.225, 0.300]) \end{array} \right\}$$

The IVIFR $\underset{\cdot}{\bar{R}}$ between $\breve{\underset{,}{E}}$ and $\dot{F}$ is

$$\underset{\cdot}{\bar{R}} = \left\{ \begin{array}{c} ((DD, USB), [0.375, 0.450], [0.300, 0.400]), \\ ((DD, RA), [0.350, 0.400], [0.325, 0.400]), \\ ((DD, IC), [0.475, 0.575], [0.350, 0.400]), \\ ((DD, OC), [0.500, 0.575], [0.300, 0.400]), \\ ((PP, USB), [0.375, 0.450], [0.275, 0.375]), \\ ((PP, RA), [0.350, 0.400], [0.325, 0.375]), \\ ((PP, IC), [0.475, 0.525], [0.250, 0.375]), \\ ((PP, OC), [0.475, 0.525], [0.250, 0.375]), \\ ((SIEM, USB), [0.300, 0.375], [0.275, 0.325]), \\ ((SIEM, RA), [0.300, 0.375], [0.325, 0.350]), \\ ((SIEM, IC), [0.300, 0.375], [0.275, 0.350]), \\ ((SIEM, OC), [0.300, 0.375], [0.275, 0.325]) \end{array} \right\}$$

From the above IVCFR $\underset{\cdot}{\bar{R}}$, it is observed that it explains the effectiveness and ineffectiveness of securities against the sources of penetration. However, unlike IVCIFR, it does not involve the time frame. We are also interested in the time durations for which certain security can successfully handle the vulnerability. Therefore, the complex valued degrees must be involved to achieve the needed information.

6.3. Cons of Alternative Methods

I. The structure of FRs, IVFRs, CFRs and IVCFRs lack the degree of non-membership.

II. The IFR, CIFR and IVIFR methods discuss the degree of membership as well as the degree of no-membership, but they have certain limitations.

- IFR, with its single valued degrees, does not cope with uncertainty as efficiently as interval-valued structures. Moreover, it cannot model multivariable problems.
- Though CIFR is capable of modeling multivariable problems, it lags behind in handling uncertainty due to its single valued degrees.
- An IVIFR can grip the uncertainty quite well with its interval-valued structure, but it is only limited to one-dimensional problems.

6.4. Pros of IVCIFR

I. The structure is composed of the degrees of membership and non-membership.

II. Interval values cover the mistakes and errors made by the expert or that occur during the survey or experiments.

III. Complex valued memberships and non-memberships can be used to cope with multidimensional variables.

Table 5 provides a summary of the characteristics of eight different structures in fuzzy set and logic theory. From Table 5, the grand structure of IVCIFRs is verified as it ticks all four characteristics, while the rest of the competitors have limitations in their structures.

Table 5. Comparison on the basis of structural properties.

Structure	Membership	Non-Membership	Multidimensional Variables	Interval-Values
FR	✓	×	×	×
CFR	✓	×	✓	×
IVFR	✓	×	×	✓
IVCFR	✓	×	✓	✓
IFR	✓	✓	×	×
CIFR	✓	✓	✓	×
IVIFR	✓	✓	×	✓
IVCIFR	✓	✓	✓	✓

7. Conclusions

This article introduced the innovative concepts of interval-valued complex intuitionistic fuzzy relation (IVCIFR) and the Cartesian product (CP) between two interval-valued complex intuitionistic fuzzy sets (IVCIFSs). Further, various types of IVCIFRs are also defined, including the interval-valued complex intuitionistic equivalence fuzzy relation (IVCI-equivalence-FR), IVCI-partial order-FR, IVCI-total order-FR, IVCI-composite-FR and many more. Moreover, the Hasse diagram has been introduced for the IVCI-partial order-FR and IVCI-partial order-FS. The concepts and ideas related to the Hasse diagram have also been defined. Suitable examples are given for each of the definitions and some results are proved for the different types of IVCIFRs. Furthermore, the proposed ideas are utilized to investigate the relationships between different types of cybersecurity and cybercrimes and their sources. The section titled *comparative analysis* verifies the omnipotence of IVCIFRs by its head-to-head comparison with other alternative mathematical techniques. In addition, it also sums up the vast structure of IVCIFRs and the limitations of pre-existing frameworks. The flaws of IVCIFRs include the absence of a degree of abstinence as well as the restrictions and constraints on the sum of the degrees of membership and non-membership. In the future, these concepts can be extended to the generalizations of fuzzy sets, which will give rise to many interesting structures with a vast range of applications.

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References

1. Zadeh, L.A. Fuzzy sets. *Inf. Control.* **1965**, *8*, 338–353. [CrossRef]
2. Klir, G.J.; Folger, T.A. *Fuzzy Sets, Uncertainty, and Information*; Prentice Hall: Englewood Cliffs, NJ, USA, 1988.
3. Mendel, J.M. Fuzzy logic systems for engineering: A tutorial. *Proc. IEEE* **1995**, *83*, 345–377. [CrossRef]
4. Zadeh, L.A. The concept of a linguistic variable and its application to approximate reasoning—I. *Inf. Sci.* **1975**, *8*, 199–249. [CrossRef]
5. Bustince, H.; Burillo, P. Mathematical analysis of interval-valued fuzzy relations: Application to approximate reasoning. *Fuzzy Sets Syst.* **2000**, *113*, 205–219. [CrossRef]
6. Goguen, J.A., Jr. Concept representation in natural and artificial languages: Axioms, extensions and applications for fuzzy sets. *Int. J. Man-Mach. Stud.* **1974**, *6*, 513–561. [CrossRef]
7. Żywica, P. Modelling medical uncertainties with use of fuzzy sets and their extensions. In Proceedings of the International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems, Cádiz, Spain, 18 May 2018; Springer: Cham, Germany, 2018; pp. 369–380. [CrossRef]
8. Román-Flores, H.; Barros, L.C.; Bassanezi, R.C. A note on Zadeh's extensions. *Fuzzy Sets Syst.* **2001**, *117*, 327–331. [CrossRef]
9. Dubois, D.; Prade, H. Gradualness, uncertainty and bipolarity: Making sense of fuzzy sets. *Fuzzy Sets Syst.* **2012**, *192*, 3–24. [CrossRef]
10. Gehrke, M.; Walker, C.; Walker, E. Some comments on interval valued fuzzy sets! *Structure* **1996**, *1*, 2. [CrossRef]
11. Bustince, H. Indicator of inclusion grade for interval-valued fuzzy sets. Application to approximate reasoning based on interval-valued fuzzy sets. *Int. J. Approx. Reason.* **2000**, *23*, 137–209. [CrossRef]

12. Turksen, I.B. Interval-valued fuzzy sets and 'compensatory AND'. *Fuzzy Sets Syst.* **1992**, *51*, 295–307. [CrossRef]
13. Ramot, D.; Milo, R.; Friedman, M.; Kandel, A. Complex fuzzy sets. *IEEE Trans. Fuzzy Syst.* **2002**, *10*, 171–186. [CrossRef]
14. Ramot, D.; Friedman, M.; Langholz, G.; Kandel, A. Complex fuzzy logic. *IEEE Trans. Fuzzy Syst.* **2003**, *11*, 450–461. [CrossRef]
15. Greenfield, S.; Chiclana, F.; Dick, S. Interval-valued complex fuzzy logic. In Proceedings of the 2016 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), Vancouver, BC, Canada, 24–29 July 2016; pp. 2014–2019.
16. Nasir, A.; Jan, N.; Gumaei, A.; Khan, S.U. Medical diagnosis and life span of sufferer using interval valued complex fuzzy relations. *IEEE Access* **2021**, *9*, 93764–93780. [CrossRef]
17. Chen, Z.; Aghakhani, S.; Man, J.; Dick, S. ANCFIS: A neurofuzzy architecture employing complex fuzzy sets. *IEEE Trans. Fuzzy Syst.* **2010**, *19*, 305–322. [CrossRef]
18. Yazdanbakhsh, O.; Dick, S. A systematic review of complex fuzzy sets and logic. *Fuzzy Sets Syst.* **2018**, *338*, 1–22. [CrossRef]
19. Tamir, D.E.; Rishe, N.D.; Kandel, A. Complex fuzzy sets and complex fuzzy logic an overview of theory and applications. *Fifty Years Fuzzy Log. Its Appl.* **2015**, *326*, 661–681.
20. Dai, S.; Bi, L.; Hu, B. Distance measures between the interval-valued complex fuzzy sets. *Mathematics* **2019**, *7*, 549. [CrossRef]
21. Greenfield, S.; Chiclana, F.; Dick, S. Join and meet operations for interval-valued complex fuzzy logic. In Proceedings of the 2016 Annual Conference of the North American Fuzzy Information Processing Society (NAFIPS), El Paso, TX, USA, 31 October–4 November 2016; pp. 1–5.
22. Atanassov, K.T. Intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **1986**, *20*, 87–96. [CrossRef]
23. Burillo, P.; Bustince, H. Intuitionistic fuzzy relations (Part I). *Mathw. Soft Comput.* **2016**, *2*, 5–38.
24. Atanassov, K.T. Interval valued intuitionistic fuzzy sets. In *Intuitionistic Fuzzy Sets*; Physica: Heidelberg, Germany, 1999; pp. 139–177.
25. Alkouri, A.S.; Salleh, A.R. Complex intuitionistic fuzzy sets. *AIP Conf. Proc.* **2012**, *1482*, 464.
26. Garg, H.; Rani, D. Complex interval-valued intuitionistic fuzzy sets and their aggregation operators. *Fundam. Inform.* **2019**, *164*, 61–101. [CrossRef]
27. Li, D.F. Multiattribute decision making models and methods using intuitionistic fuzzy sets. *J. Comput. Syst. Sci.* **2005**, *70*, 73–85. [CrossRef]
28. De, S.K.; Biswas, R.; Roy, A.R. An application of intuitionistic fuzzy sets in medical diagnosis. *Fuzzy Sets Syst.* **2001**, *117*, 209–213. [CrossRef]
29. Vlachos, I.K.; Sergiadis, G.D. Intuitionistic fuzzy information–applications to pattern recognition. *Pattern Recognit. Lett.* **2007**, *28*, 197–206. [CrossRef]
30. Lee, K.M.; LEE, K.M.; CIOS, K.J. Comparison of interval-valued fuzzy sets, intuitionistic fuzzy sets, and bipolar-valued fuzzy sets. In *Computing and Information Technologies: Exploring Emerging Technologies*; World Scientific: Hackensack, NJ, USA, 2001; pp. 433–439. [CrossRef]
31. Grzegorzewski, P. Distances between intuitionistic fuzzy sets and/or interval-valued fuzzy sets based on the Hausdorff metric. *Fuzzy Sets Syst.* **2004**, *148*, 319–328. [CrossRef]
32. Nasir, A.; Jan, N.; Yang, M.-S.; Khan, S.U. Complex T-spherical fuzzy relations with their applications in economic relationships and international trades. *IEEE Access* **2021**, *9*, 66115–66131. [CrossRef]
33. Khan, S.U.; Nasir, A.; Jan, N.; Ma, Z.H. Graphical Analysis of Covering and Paired Domination in the Environment of Neutrosophic Information. *Math. Probl. Eng.* **2021**, *2021*. [CrossRef]
34. Nasir, A.; Jan, N.; Gumaei, A.; Khan, S.U.; Al-Rakhami, M. Evaluation of the Economic Relationships on the Basis of Statistical Decision-Making in Complex Neutrosophic Environment. *Complexity* **2021**, *2021*. [CrossRef]
35. Jan, N.; Rehman, S.U.; Nasir, A.; Aydi, H.; Khan, S.U. Analysis of Economic Relationship Using the Concept of Complex Pythagorean Fuzzy Information. *Secu. Comm. Nets.* **2021**, *2021*. [CrossRef]
36. Ali, M.; Tamir, D.E.; Rishe, N.D.; Kandel, A. Complex intuitionistic fuzzy classes. In Proceedings of the 2016 IEEE International Conference on Fuzzy Systems (FUZZ-IEEE), Vancouver, BC, Canada, 24–29 July 2016; pp. 2027–2034.
37. Liu, Y.; Jiang, W. A new distance measure of interval-valued intuitionistic fuzzy sets and its application in decision making. *Soft Comput.* **2019**, *24*, 1–17. [CrossRef]
38. Bustince, H.; Burillo, P. Correlation of interval-valued intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **1995**, *74*, 237–244. [CrossRef]
39. Nayagam, V.L.G.; Sivaraman, G. Ranking of interval-valued intuitionistic fuzzy sets. *Appl. Soft Comput.* **2011**, *11*, 3368–3372. [CrossRef]
40. Otero, A.R.; Tejay, G.; Otero, L.D.; Ruiz-Torres, A.J. A fuzzy logic-based information security control assessment for organizations. In Proceedings of the 2012 IEEE Conference on Open Systems, Kuala Lumpur, Malaysia, 21–24 October 2012; pp. 1–6.
41. Tariq, M.I.; Ahmed, S.; Memon, N.A.; Tayyaba, S.; Ashraf, M.W.; Nazir, M.; Hussain, A.; Balas, V.E.; Balas, M.M. Prioritization of information security controls through fuzzy AHP for cloud computing networks and wireless sensor networks. *Sensors* **2020**, *20*, 1310. [CrossRef] [PubMed]
42. Tariq, M.I.; Tayyaba, S.; Ali Mian, N.; Sarfraz, M.S.; De-la-Hoz-Franco, E.; Butt, S.A.; Santarcangelo, V.; Rad, D.V. Combination of AHP and TOPSIS methods for the ranking of information security controls to overcome its obstructions under fuzzy environment. *J. Intell. Fuzzy Sys.* **2020**, *38*, 6075–6088. [CrossRef]
43. Mokhtari, S.M.; Alinejad-Rokny, H.; Jalalifar, H. Selection of the best well control system by using fuzzy multiple-attribute decision-making methods. *J. Appl. Stat.* **2014**, *41*, 1105–1121. [CrossRef]