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Application of a Similarity Measure Using Fuzzy Sets to Select the Optimal Plan for an Air-Assisted Rice Seeder

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Abstract: Air-assisted rice seeders were designed to perform sowing while ensuring the alignment and depth of seeds within the allowable standard range of 5–10 mm. Their performance varies on the basis of different specifications; thus, it is necessary to evaluate them to select the best one. Fuzzy set theory allows us to flexibly handle practical problems, especially applied problems in engineering. In this paper, a new similarity measure using fuzzy sets is proposed, and its advantages were tested using the technique for order preference by similarity to ideal solution (TOPSIS) method to select technical parameters for the sowing machines. The experiments allowed identifying the best results. The correlations between input attributes and decision variables were determined on the basis of their correlation coefficients with technical factors. The influence of technical factors on output results was also examined to determine the technical factors providing superior product quality.

Keywords: sowing machines; air-assisted rice seeder; rice direct seeding; similarity measure of fuzzy sets; optimal plan

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1. Introduction

Rice direct seeding is used in paddy cultivation due to its low labor requirements and reduced production cost [1,2]. Air-assisted direct seeding machines were studied by Chosa et al. [1], Loan et al. [3], and Furuhashi et al. [4], showing many advantages and promising working efficiency. These machines can operate across 32 rows powered by a tractor power take-off PTO. The hardness of the soil surface is a factor affecting the depth of sowing. The accuracy of sowing is controlled by a float installed to increase the area of ground–surface contact. A series of seeding machines, such as those of Iseki, Kubota, and Yanmar, have been marketed to meet the needs of sowing seeds in flooded paddy fields. These sowing machines can sow 6–8 rows at the same time in clusters [5].

The Tokyo University of Agriculture and Technology collaborated with the Vietnam National University of Agriculture to develop an air-assisted rice seeder for flooded paddy fields in Vietnam (Figure 1). This machine can sow 24 rows with an interval spacing of 20–25 cm. The experimental results showed that the seeding depth and seeding area precision were affected by the air-assisted velocity, internal diameter of the injection port, and distance between the injection port and soil surface. Seeding accuracy is one of the most important factors affecting the growth of paddy sprouts. In this study, a similarity measure using fuzzy sets was used to select the optimal plan for the sowing machines.

Figure 1. Image of an air-assisted rice seeder.

Fuzzy logic was first introduced by Zadel in 1965 [6]. Fuzzy set theory allows researchers to flexibly handle practical problems, especially applied problems in engineering [7,8]. For example, the fuzzy multi-criteria decision-making (MCDM) method was used to select machines [9], as well as evaluate reconfigurable machines [10].

In seeding machines, research studies have shown that the seeding depth allows ranges from 3–15 mm, preferably 5–10 mm for Vietnam rice variables, which is a prerequisite for seeds to thrive after sowing [11]. Trapezoidal fuzzy sets are considered the best option for the chosen situation, i.e., to select the optimal plan for sowing machines.

The contributions of this study are as follows: (1) a new similarity measure using fuzzy sets is proposed; (2) this measure is compared with existing similarity measures; (3) the TOPSIS method using the new measure is applied to evaluate the best scenario with respect to specifications of the sowing machine; (4) analysis of the obtained results led to a determination of the correlation coefficient between factors and decision variables in MCDM to interpret/evaluate the identified specifications.

The rest of this paper is organized as follows: Section 2 presents the problem to be solved, i.e., an estimation of the impact of technical parameters on the sowing machine. In this section, some important concepts regarding fuzzy sets and similarity measures of fuzzy sets are reviewed. Furthermore, the new similarity measures are proposed and compared with others from the literature. The TOPSIS model based on our proposed similarity measures is also introduced. In Section 3, the TOPSIS model is applied to evaluate the best scenario with respect to specifications of the sowing machine, and the results are presented. The results are discussed in Section 4, while Section 5 concludes the paper.

2. Materials and Methods

2.1. Sowing Machine Problem

In a sowing machine, the seeding depth and seeding area precision are affected by the air-assisted velocity, internal diameter of the injection port, and distance between the injection port and soil surface. Thus, there is a need to determine the parameters leading to optimal seeding accuracy in terms of a seeding depth of 3–15 mm (preferably 5–10 mm) and a seeding area precision within a diameter of 0–100 mm (preferably 50 mm).

Twenty-five experiments (E) were carried out on seeding machines to examine three variables: air-assisted velocity, internal diameter of the injection port, and distance between the injection port and soil surface (mm). The experimental results are given in Table 1. The expected outcome of this investigation was to determine the best output, suitable in terms of both the technical requirements and the agronomical requirements of rice seeds after sowing.

Table 1. Results of research on seeding accuracy.

	C1	C2	C3	C4	C5	C6	C7
E1	6	12	50	100	0	0	26
E2	6	12	100	97	3	0	17
E3	6	12	150	81	19	0	12
E4	6	12	200	75	23	2	6
E5	6	12	250	52	40	8	3
E6	5	15	50	99	1	0	20
E7	5	15	100	91	9	0	12
E8	5	15	150	79	21	0	6
E9	5	15	200	70	28	4	3
E10	5	15	250	43	45	12	1
E11	4	18	50	81	19	0	16
E12	4	18	100	72	22	6	7
E13	4	18	150	58	28	15	3
E14	4	18	200	44	35	21	2
E15	4	18	250	28	43	29	1
E16	6	15	50	98	2	0	25
E17	6	15	100	92	8	0	15
E18	6	15	150	81	19	0	12
E19	6	15	200	73	22	5	5
E20	6	15	250	44	44	12	2
E21	4	15	50	97	3	0	15
E22	4	15	100	89	11	0	11
E23	4	15	150	76	4	0	3
E24	4	15	200	67	27	6	2
E25	4	15	250	41	46	13	1

The assessment of influential factors was considered using an MCDM approach to analyze their correlations with the efficiency of sowing machines.

The considered attribute values are expected to satisfy both technical requirements and practical requirements. For example, the seeding depth (C7) allows a range of 3–15 mm (preferably 5–10 mm). This is a prerequisite for adequate seed growth and alignment after sowing. Thus, seeding area precision should primarily fall within a diameter of 50 mm (C4) (user requirement of 75% or more). No more than 5% of seeds should fall into the C5 or C6 regions. C4 can be considered a benefit criterion, whereas C5 and C6 can be considered damage (or cost) criteria. C1, C3, and C7 were not identified as benefit or cost criteria. In the case of C7, too shallow a depth would not meet the agronomic requirements, whereas too great a depth would lead to seed death, according to its biological characteristics.

Accordingly, we used fuzzy set theory to convert the original problem into a fuzzy MCDM problem taking into account the above requirements, whereby all criteria could be considered as the same type (i.e., as benefit criteria).

2.2. Fuzzy Sets and Similarity Measures

In this subsection, the concepts of fuzzy sets, as well as their operators and similarity measures, are reviewed.

In this paper, it is assumed that $X = \{x_1, x_2, \dots, x_n\}$ is the universal set.

Definition 1 ([6]). A fuzzy set A on the universal set X takes the form

$$A = \{(x_i, \mu_A(x_i)) | x_i \in X\}$$

where $\mu_A : X \rightarrow [0, 1]$ is the membership function of the fuzzy set A , and $\mu_A(x_i)$ is the degree membership of the element x_i in A for all $i = 1, 2, \dots, m$.

For two given fuzzy sets, $A = \{(x_i, \mu_A(x_i)) | x_i \in X\}$ and $B = \{(x_i, \mu_B(x_i)) | x_i \in X\}$ on the universal set X , their operators are as follows:

- + Subset: $A \subseteq B$ if $\mu_A(x_i) \leq \mu_B(x_i)$ for all $x_i \in X$,
- + Equal: $A = B$ if $\mu_A(x_i) = \mu_B(x_i)$ for all $x_i \in X$,
- + Intersection: $A \cap B = \{(x_i, \min(\mu_A(x_i), \mu_B(x_i))) | x_i \in X\}$,
- + Union: $A \cup B = \{(x_i, \max(\mu_A(x_i), \mu_B(x_i))) | x_i \in X\}$,
- + Complement: $A^C = \{(x_i, 1 - \mu_A(x_i)) | x_i \in X\}$,
- + For all real numbers $\lambda > 0$.

$$A^\lambda = \left\{ \left(x_i, (\mu_A(x_i))^\lambda \right) \mid x_i \in X \right\}. \quad (1)$$

$F(X)$ denotes a collection of all fuzzy sets on X .

Definition 2 ([12]). Let $A, B \in F(X)$ be two fuzzy sets on the universal set. A similarity measure of $F(X)$ is the function $S : F(X) \times F(X) \rightarrow [0, 1]$, which satisfies the following conditions:

- (S1) $0 \leq S(A, B) \leq 1$ for all $A, B \in F(X)$,
- (S2) $S(A, B) = S(B, A)$ for all $A, B \in F(X)$,
- (S3) $S(A, B) = 1$ if $A = B$ for all $A, B \in F(X)$,
- (S4) For all $A, B, C \in F(X)$, if $A \subseteq B \subseteq C$, then $S(A, C) \leq \min\{S(A, B), S(B, C)\}$.

2.2.1. New Similarity Measure for Fuzzy Sets

Let $X = \{x_1, x_2, \dots, x_n\}$ be a universal set. In this section, for all $A, B \in F(X)$, the similarity measure in Equation (2) can be considered.

Definition 3. For all $A \in F(X)$, the real function $S_H : F(X) \times F(X) \rightarrow [0, 1]$ is defined as follows:

$$S_H(A, B) = \frac{1}{n} \sum_{i=1}^n \frac{2 - 2|\mu_A(x_i) - \mu_B(x_i)|}{2 - |\mu_A(x_i) - \mu_B(x_i)|}. \quad (2)$$

Proof. We consider the real function $g(t) = \frac{2-2t}{2-t}$, $t \in [0, 1]$, to yield $g'(t) = \frac{-2}{(2-t)^2} < 0$ for all $t \in [0, 1]$. Thus, $g(1) = 0 \leq g(t) \leq g(0) = 1$. This implies that $0 \leq \frac{2-2|\mu_A(x_i) - \mu_B(x_i)|}{2 - |\mu_A(x_i) - \mu_B(x_i)|} \leq 1$ for all $x_i \in X$. Accordingly, S_H satisfies condition (S1).

It is easy to see that S_H satisfies conditions (S2) and (S3) in Definition 2.

In the case of condition (S4), the real function $g(t) = \frac{2-2t}{2-t}$ monotonically decreases for all $t \in [0, 1]$. Thus, $g(t_2) = \frac{2-2t_2}{2-t_2} \leq g(t_1) = \frac{2-2t_1}{2-t_1}$ for all $0 \leq t_1 \leq t_2 \leq 1$. Now, assume that $A \subseteq B \subseteq C$ ($A, B, C \in F(X)$). Consider $\mu_A(x_i) \leq \mu_B(x_i) \leq \mu_C(x_i)$ for all $x_i \in X$, such that $0 \leq |\mu_A(x_i) - \mu_B(x_i)| \leq |\mu_A(x_i) - \mu_C(x_i)| \leq 1$ for all $x_i \in X$. This implies that $\frac{2-2|\mu_A(x_i) - \mu_C(x_i)|}{2 - |\mu_A(x_i) - \mu_C(x_i)|} \leq \frac{2-2|\mu_A(x_i) - \mu_B(x_i)|}{2 - |\mu_A(x_i) - \mu_B(x_i)|}$ for all $x_i \in X$. Accordingly, $S_H(A, C) \leq S_H(A, B)$, for all cases of $A, B, C \in F(X)$ that satisfy $A \subseteq B \subseteq C$. In the same way, we obtain $S_H(A, C) \leq S_H(B, C)$ for all cases of $A, B, C \in F(X)$ that satisfy $A \subseteq B \subseteq C$. Hence, for all $A, B, C \in F(X)$, if $A \subseteq B \subseteq C$, then $S(A, C) \leq \min\{S(A, B), S(B, C)\}$. $\square$

2.2.2. Comparison to Other Similarity Measures for Fuzzy Sets

To demonstrate the effectiveness of the newly proposed measure, we compare it with some existing measures through application to pattern recognition problems. First, we review some existing similarity measures for fuzzy sets.

Mondal et al. proposed the following in 2006 [13]:

$$S_{MR}(A, B) = 1 - \left(\frac{\sum_{i=1}^n |\mu_A(x_i) - \mu_B(x_i)|^2}{n} \right)^{\frac{1}{2}}. \quad (3)$$

Pappis and Karacapilidis proposed the following in 1993 [14]:

$$S_{PK1}(A, B) = 1 - \max_{x_i \in X} \{|\mu_A(x_i) - \mu_B(x_i)|\}, \quad (4)$$

$$S_{PK2}(A, B) = 1 - \frac{\sum_{i=1}^n |\mu_A(x_i) - \mu_B(x_i)|}{\sum_{i=1}^n [\mu_A(x_i) + \mu_B(x_i)]}, \quad (5)$$

$$S_{PK3}(A, B) = 1 - \sum_{i=1}^n \frac{|\mu_A(x_i) - \mu_B(x_i)|}{n} \quad (6)$$

$$S_{PK4}(A, B) = \frac{\max_{x_i \in X} (\mu_A(x_i) \times \mu_B(x_i))}{\max \left(\sum_{i=1}^n (\mu_A(x_i))^2, \sum_{i=1}^n (\mu_B(x_i))^2 \right)}, \quad (7)$$

$$S_{PK5}(A, B) = \frac{\sum_{i=1}^n \min(\mu_A(x_i), \mu_B(x_i))}{\sum_{i=1}^n \max(\mu_A(x_i), \mu_B(x_i))}. \quad (8)$$

Raha et al. proposed the following in 2002 [15]:

$$S_{RP1}(A, B) = 1 - \frac{\max_{x_i \in X} \min(\mu_A(x_i), \mu_B(x_i))}{\max_{x_i \in X} \max(\mu_A(x_i), \mu_B(x_i))}, \quad (9)$$

$$S_{RP2}(A, B) = \min \left\{ \sqrt{\frac{\sum_{i=1}^n \mu_A(x_i) \mu_B(x_i)}{\sum_{i=1}^n \max(\mu_A(x_i), \mu_B(x_i))^2}}, \sqrt{\frac{\sum_{i=1}^n (1 - \mu_A(x_i))(1 - \mu_B(x_i))}{\sum_{i=1}^n \max(1 - \mu_A(x_i), 1 - \mu_B(x_i))^2}} \right\}. \quad (10)$$

Wang proposed the following in 1997 [16]:

$$S_{W1}(A, B) = \sum_{i=1}^n \frac{\min(\mu_A(x_i), \mu_B(x_i))}{\max(\mu_A(x_i), \mu_B(x_i))}, \quad (11)$$

$$S_{W2}(A, B) = \frac{\sum_{i=1}^n [\mu_A(x_i) \mu_B(x_i) + (1 - \mu_A(x_i))(1 - \mu_B(x_i))]}{\sqrt{\sum_{i=1}^n [(\mu_A(x_i))^2 + (1 - \mu_A(x_i))^2]} \sqrt{\sum_{i=1}^n [(\mu_B(x_i))^2 + (1 - \mu_B(x_i))^2]}} \quad (12)$$

Lee-Kwang et al. proposed the following in 1994 [17]:

$$S_{LS}(A, B) = \max_{x_i \in X} \min(\mu_A(x_i), \mu_B(x_i)). \quad (13)$$

Pal et al. proposed the following in 2014 [18]:

$$S_{PM1}(A, B) = \frac{\sum_{i=1}^n \min(\mu_A(x_i), \mu_B(x_i))}{\max \left\{ \sum_{i=1}^n \mu_A(x_i), \sum_{i=1}^n \mu_B(x_i) \right\}}, \quad (14)$$

$$S_{PM2}(A, B) = \frac{\sum_{i=1}^n \min(\mu_A(x_i), \mu_B(x_i))}{\sum_{i=1}^n \mu_A(x_i)}, \quad (15)$$

$$S_{PM3}(A, B) = \frac{\sum_{i=1}^n \min(\mu_A(x_i), \mu_B(x_i))}{\sum_{i=1}^n \mu_B(x_i)}, \quad (16)$$

$$S_{PM4}(A, B) = \min \left\{ \frac{\sum_{i=1}^n \min(\mu_A(x_i), \mu_B(x_i))}{\sum_{i=1}^n \mu_A(x_i)}, \frac{\sum_{i=1}^n \min(\mu_A(x_i), \mu_B(x_i))}{\sum_{i=1}^n \mu_B(x_i)} \right\}. \quad (17)$$

In pattern recognition problems, the similarity measures are used to classify the sample B into a class of given patterns $A = \{A_1, A_2, \dots, A_m\}$. Given a set of m patterns $A = \{A_1, A_2, \dots, A_m\}$, where A_i is a fuzzy set on the universal set X , a new sample B also belongs to a fuzzy set on the universal set X .

Question: What class does B belong to?

To solve this problem, it is possible to use the similarity measures according to the two steps in Algorithm 1.

Algorithm 1 Solving pattern recognition problems using similarity measures.

Step 1. Calculate all similarity measures $s(B, A_i)$ from sample B to patterns A_i for all $i = 1, 2, \dots, m$.

Step 2. Classify sample B into pattern A_{i^*} with $s(A_{i^*}, B) = \max_{i=1,2,\dots,m} \{s(A_i, B)\}$.

Below, we consider two examples to compare the similarity measures.

Example 1. Given a set of three patterns $A = \{A_1, A_2, A_3\}$, A_i ($i = 1, 2, 3$) is a fuzzy set on the universal set $X = \{x_1, x_2, x_3, x_4\}$: $A_1 = \{(x_1, 0.2), (x_2, 0.3), (x_3, 0.1), (x_4, 0.5)\}$, $A_2 = \{(x_1, 0.25), (x_2, 0.4), (x_3, 0.1), (x_4, 0.6)\}$, and $A_3 = \{(x_1, 0.5), (x_2, 0.3), (x_3, 0.5), (x_4, 0.4)\}$.

New sample $B = \{(x_1, 0.2), (x_2, 0.3), (x_3, 0.5), (x_4, 0.6)\}$ also belongs to a fuzzy set on the universal set X .

Question: What class does B belong to?

Result: Using Algorithm 1, the results shown in Table 2 can be obtained. In this case, it can be seen that our proposed similarity measure (Equation (2)), as well as 10 other similarity measures, assigned sample B to pattern A_3 . On the other hand, three measures (Equations (7), (9) and (13)) assigned sample B to pattern A_2 , whereas Equation (15) assigned sample B to pattern A_1 . Lastly, measure S_{PK3} (Equation (6)) failed to classify sample B .

Table 2. Classification results for Example 1.

Similarity Measures	The Similarity Measures for Classifying Sample B in Patterns A_i			Classification Results
	$s(A_1, B)$	$s(A_2, B)$	$s(A_3, B)$	
Equation (3)	0.9150	0.9138	0.9350	A_3
Equation (4)	0.6000	0.6000	0.7000	A_3
Equation (5)	0.8148	0.8136	0.8485	A_3
Equation (6)	0.8750	0.8625	0.8750	Null
Equation (7)	0.4054	0.4865	0.3333	A_2
Equation (8)	0.6875	0.6857	0.7368	A_3
Equation (9)	0.8333	1	0.8333	A_2
Equation (10)	0.6486	0.6967	0.7158	A_3
Equation (11)	0.7583	0.6875	0.7667	A_3
Equation (12)	0.9318	0.9285	0.9414	A_3
Equation (13)	0.5000	0.6000	0.5000	A_2
Equation (14)	0.6875	0.7500	0.8235	A_3
Equation (15)	1	0.8889	0.8235	A_1
Equation (16)	0.6875	0.7500	0.8750	A_3
Equation (17)	0.6875	0.7500	0.8235	A_3
Equation (2) (proposed measure)	0.9243	0.9179	0.9281	A_3

The maximum similarity measures are indicated in bold.

In Example 1, we can see that several measures were feasible, with most assigning sample B to pattern A_3 . In Example 2, several measures failed to classify sample B, whereas the majority (including the proposed measure) assigned sample B to pattern A_2 .

Example 2. Given a set of three patterns $A = \{A_1, A_2, A_3\}$, A_i ($i = 1, 2, 3$) is a fuzzy set on the universal set $X = \{x_1, x_2, x_3, x_4\}$: $A_1 = \{(x_1, 0.2), (x_2, 0.3), (x_3, 0.1), (x_4, 0.5)\}$, $A_2 = \{(x_1, 0.5), (x_2, 0.4), (x_3, 0.4), (x_4, 0.5)\}$, and $A_3 = \{(x_1, 0.5), (x_2, 0.3), (x_3, 0.5), (x_4, 0.4)\}$.

New sample $B = \{(x_1, 0.5), (x_2, 0.5), (x_3, 0.8), (x_4, 0.9)\}$ also belongs to a fuzzy set on the universal set X .

Question: What class does B belong to?

Result: Using Algorithm 1, the results shown in Table 3 can be obtained. In this case, it can be seen that the measures S_{PK4} (Equation (7)), S_{RP1} (Equation (9)), S_{LS} (Equation (13)), and S_{PM2} (Equation (15)) failed to classify sample B. However, the proposed similarity measure (Equation (2)), as well as eleven other similarity measures, assigned sample B to pattern A_2 .

Examples 1 and 2 demonstrate the feasibility of our proposed similarity measure through application to classification problems. It was able to match or surpass the performance of the other measures identified in the literature when applied to recognition problems (Examples 1 and 2).

Table 3. Classification results for Example 2.

Similarity Measures	The Similarity Measures for Classifying Sample B in Patterns A_i			Classification Results
	$s(A_1, B)$	$s(A_2, B)$	$s(A_3, B)$	
Equation (3)	0.6100	0.8000	0.7727	A_2
Equation (4)	0.3000	0.6000	0.5000	A_2
Equation (5)	0.5789	0.8000	0.7727	A_2
Equation (6)	0.6000	0.7750	0.7500	A_2
Equation (7)	0.2308	0.2308	0.2051	Null
Equation (8)	0.4074	0.6667	0.6296	A_2
Equation (9)	0.5556	0.5556	0.5556	Null
Equation (10)	0.4000	0.5902	0.5630	A_2
Equation (11)	0.4201	0.7138	0.6674	A_2
Equation (12)	0.6930	0.8590	0.8380	A_2
Equation (13)	0.5000	0.5000	0.5000	Null
Equation (14)	0.4074	0.6667	0.6296	A_2
Equation (15)	1	1	1	Null
Equation (16)	0.4074	0.6667	0.6296	A_2
Equation (17)	0.4074	0.6667	0.6296	A_2
Equation (9) (proposed measure)	0.731	0.8618	0.8448	A_2

The maximum similarity measures are indicated in bold.

2.3. Fuzzy Similarity-Based MCDM

The MCDM problem involves selecting the optimal option from a set of m alternatives, $P = \{P_1, P_2, \dots, P_m\}$, according to n criteria $C = \{C_1, C_2, \dots, C_n\}$, in which $P_i = \{(C_j, y_{ij}) | C_j \in C\}$, $y_{ij} \in [0, 1]$ for all $i = 1, 2, \dots, m$; $j = 1, \dots, n$. The relationship between m alternatives and n criteria can be represented in the form of a fuzzy decision matrix.

$$D = \begin{matrix} & C_1 & C_2 & C_n \\ \begin{matrix} P_1 \\ P_2 \\ \vdots \\ P_m \end{matrix} & \begin{bmatrix} y_{11} & y_{12} & y_{1n} \\ y_{21} & y_{22} & y_{2n} \\ \vdots & \vdots & \vdots \\ y_{m1} & y_{m2} & y_{mn} \end{bmatrix} \end{matrix}$$

Next, TOPSIS was applied using the new similarity measure to solve this MCDM problem, as described in Algorithm 2.

Algorithm 2 The fuzzy similarity-based TOPSIS model

Step 1: Normalize the data table according to each criterion to obtain a fuzzy decision matrix.

Step 2: Determine weight of each criterion.

Step 3: Define the best option $P^+ = \left\{ (C_j, y_{ij}^+ = \max_{i=1, \dots, m} y_{ij}) \mid C_j \in C \right\}$ and worst option

$$P^- = \left\{ (C_j, y_{ij}^- = \min_{i=1, \dots, m} y_{ij}) \mid C_j \in C \right\}.$$

Step 4: Use Equation (2) to compute the similarity measures S_i^+ and S_i^- for each alternative P_i ($i = 1, 2, \dots, m$) to identify P^+ and P^- , respectively.

Step 5: Determine the close coefficient $CC_i = \frac{S_i^+}{S_i^+ + S_i^-}$ ($i = 1, 2, \dots, m$), whereby

$CC = \{(P_i, CC_i) | P_i \in P\}$ can be considered the decision set of alternatives.

Step 6. Rank $P_i \prec P_k$ if $CC_i \leq CC_k$ for all $i, k = 1, 2, \dots, m$.

2.4. Correlation Coefficient

To evaluate the strength of two independent variables, the correlation coefficient plays an important role in statistical analysis. Suppose that $X = \{X_1, X_2, \dots, X_m\}$ and

$Y = \{Y_1, Y_2, \dots, Y_m\}$ are two independent observations; then, the correlation coefficient of variables X and Y is expressed as follows:

$$r(X, Y) = \frac{\sum_{i=1}^m (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^m (X_i - \bar{X})^2} \sqrt{\sum_{i=1}^m (Y_i - \bar{Y})^2}}, \quad (18)$$

where $\bar{X} = \frac{1}{m} \sum_{i=1}^m X_i$, $\bar{Y} = \frac{1}{m} \sum_{i=1}^m Y_i$, and $-1 \leq r(X, Y) \leq 1$. If $r(X, Y) \geq 0$, then variables X, Y increase (decrease) simultaneously in the same direction. If $r(X, Y) \leq 0$, then variables X, Y increase (decrease) simultaneously in the opposite direction.

3. Results

Algorithm 2 was applied to solve the sowing machine problem, so as to select the parameters yielding the optimal seeding accuracy of the air-assisted rice seeder.

Step 1: The data table was normalized according to each criterion to obtain a fuzzy decision matrix.

+ Air-assisted velocity

$$y_{C_1}(x) = \begin{cases} \frac{x-3.5}{1.5} & \text{if } 3.5 \leq x \leq 5 \\ \frac{7-x}{2} & \text{if } 5 \leq x \leq 7 \\ 0 & \text{if others} \end{cases}$$

+ Internal diameter of injection port

$$y_{C_2}(x) = \begin{cases} \frac{x-10}{5} & \text{if } 10 \leq x \leq 15 \\ \frac{20-x}{5} & \text{if } 5 \leq x \leq 20 \\ 0 & \text{if others} \end{cases}$$

+ Distant between injection port and soil surface

$$y_{C_3}(x) = \begin{cases} \frac{x-70}{80} & \text{if } 70 \leq x \leq 150 \\ 1 & \text{if } 150 \leq x \leq 200 \\ \frac{250-x}{50} & \text{if } 200 \leq x \leq 250 \\ 0 & \text{if others} \end{cases}$$

+ Seeding area precision (diameter 50 mm)

$$y_{C_4}(x) = \begin{cases} 0 & \text{if } 0 \leq x \leq 60 \\ \frac{x-60}{20} & \text{if } 60 \leq x \leq 80 \\ 1 & \text{if } 80 \leq x \leq 100 \end{cases}$$

+ Seeding area precision (diameter 100 mm)

$$y_{C_5}(x) = \frac{100-x}{100}$$

+ Seeding area precision (diameter 150 mm)

$$y_{C_6}(x) = \frac{100-x}{100}$$

+ Seeding depth

$$y_{C_7}(x) = \begin{cases} \frac{x-3}{2} & \text{if } 3 \leq x \leq 5 \\ 1 & \text{if } 5 \leq x \leq 10 \\ \frac{15-x}{5} & \text{if } 10 \leq x \leq 15 \\ 0 & \text{if others} \end{cases}$$

Then, the normalized data shown in Table A1 (Appendix A) were obtained.

Step 2. The weight vector $w = (0.025, 0.025, 0.025, 0.2, 0.025, 0.6)$ was chosen. Due to the requirements of the problem, the weight determined in the criterion of seeding depth was greatest to meet the criterion of accuracy within 50 mm.

Step 3: The best option and the worst option were determined, as shown in Appendix A (Table A1).

Step 4. Equation (2) was used to compute the similarity measures S_i^+ and S_i^- for each alternative P_i ($i = 1, 2, \dots, m$) to identify P^+ and P^- , respectively. The results are shown in Table A2 (Appendix B).

Step 5. The close coefficient $CC_i = \frac{S_i^+}{S_i^+ + S_i^-}$ ($i = 1, 2, \dots, m$) was determined as shown in Table A2 (Appendix B) and Figure 2.

Figure 2. Values of CC using newly proposed similarity measure.

Step 6. Accordingly, the top five results could be ranked as $E8 \succ E4 \succ E19 \succ E12 \succ E22$.

4. Discussion

To further validate the results, the results were compared with other methods, namely, the Hamming distance measure (Figure 3), the Euclidean distance measure (Figure 4), and the fuzzy simple additional weighting (fuzzy SAW) method [19] (Figure 5). It was shown that the Hamming distance method and fuzzy SAW ranked the top five results identically to our proposed method ($E8 \succ E4 \succ E19 \succ E12 \succ E22$). On the other hand, the Euclidean distance measure ranked them slightly differently ($E8 \succ E4 \succ E19 \succ E22 \succ E12$). However, experiments E12 and E22 led to poor results, whereby E12 had 6% dispersion outside the 150 mm diameter, while the seed depth in experiment E22 exceeded the standard threshold of 5–10 mm, which would cause the seeds to rot. Therefore, in terms of practical application techniques, the top three results were E8, E4, and E19. The specifications and outputs of these options are shown in Table 4.

Figure 3. Values of CC using Hamming distance.

Figure 4. Values of CC using Euclidean distance.

Figure 5. Values of CC using fuzzy SAW method.

Table 4. Selection results with respect to optimal seeding accuracy.

Selections	1st	2nd	3rd	4th	5th
Air-assisted velocity (m/s)	5	6	6	4	4
Distant between injection port and soil surface (mm)	150	200	200	100	100
Diameter of injection port (mm)	15	12	15	18	15
Seeding area precision, diameter 50 mm (%)	79	75	73	72	89
Seeding area precision, diameter 100 mm (%)	21	23	22	22	11
Seeding area precision, diameter 150 mm (%)	0	2	5	6	0
Seeding depth (mm)	6	6	5	7	11

As can be seen in Table 4, when comparing seeding depth, the top four choices all met the agronomic requirements, whereas option 5 exceeded the optimal range. The seeding area precision of option 5 was highest; however, the seed depth in this case could lead to rotting paddy sprouts, thereby decreasing the germination rate. Therefore, option 5 is not recommended. On the other hand, option 1 presented the best seeding area precision at a diameter of 50 mm among the options with appropriate seeding depth.

The distance between the injection port and soil surface is also an influential factor. Too great a distance would lead to the seeding accuracy being easily affected by wind during sowing, whereas too small a distance would affect the working mode of the machine in the case of uneven fields.

The above analysis validates the selection order identified using Algorithm 2. The chosen weight vectors were varied to further confirm the above conclusion.

The correlation of each variable with the CC decision variable was also determined, corresponding to the above weight vectors, to determine their significance (i.e., hstqi denotes the correlation coefficient corresponding to weight w_i ($i = 1, 2, 3, 4, 5$, and 6)) (See Table 5). This allows determining the factors contributing to product quality. It can be seen that, if the weight of the variable “seeding depth” is lower than 50%, the correlation coefficient of this variable becomes significantly worse; otherwise, its correlation coefficient exceeds 90% (see Figure 6 and Table 6). This is completely consistent with the actual requirements of the sowing machine applied in agriculture. The correlation coefficients of C1 and C3 were higher than those of C2, indicating their greater influence on the final result.

Figure 6. Correlation coefficients between variables and the CC decision variable.

Table 5. The top five results with differences in weight vectors.

Weight Vectors	The Top Three Ranking	Meaning	Correlation Coefficients
w1 = (0.025, 0.025, 0.025, 0.2, 0.1, 0.025, 0.6)	$E8 \succ E4 \succ E19$	Good	hstq1
w2 = (0.05, 0.05, 0.05, 0.2, 0.1, 0.05, 0.5)	$E8 \succ E4 \succ E19$	Good	hstq2
w3 = (0.05, 0.05, 0.05, 0.2, 0.05, 0.05, 0.55)	$E8 \succ E4 \succ E19$	Good	hstq2
w4 = (0.05, 0.05, 0.05, 0.2, 0.1, 0.5, 0.05)	$E8 \succ E7 \succ E18$	Bad	hstq4
w5 = (0.05, 0.05, 0.05, 0.5, 0.1, 0.05, 0.2)	$E8 \succ E22 \succ E7$	Bad	hstq5
w6 = (0.05, 0.05, 0.15, 0.25, 0.05, 0.05, 0.4)	$E8 \succ E4 \succ E19$	Pretty good	hstq6

Table 6. Correlation coefficients between variables and the CC decision variable.

	hstq1	hstq2	hstq3	hstq 4	hstq 5	hstq 6
C1	0.1757	0.2044	0.1994	0.2786	0.2178	0.1988
C2	0.0906	0.1265	0.1183	0.2671	0.2017	0.1389
C3	0.427	0.4526	0.4583	0.2887	0.2563	0.567
C4	0.6191	0.6522	0.6214	0.9532	0.954	0.6837
C5	0.4018	0.4373	0.4014	0.8214	0.8019	0.4765
C6	0.5628	0.597	0.57	0.8945	0.853	0.6241
C7	0.9455	0.9252	0.9398	0.5056	0.5859	0.8735

5. Conclusions

In this paper, a new similarity measure using fuzzy sets was proposed. It was proven to be superior to some existing metrics through numerical examples. Then, a TOPSIS model was constructed using this newly proposed measure to solve a multi-criteria decision-making problem, i.e., to evaluate the quality of a sowing machine's specifications. The results obtained using the new measure and those obtained using classical measures were also compared. In the future, intuitionistic fuzzy sets [20] will be used to evaluate the quality of technical parameters. Furthermore, the results will be put into practice to evaluate sowing machine quality.

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Appendix A

Table A1. The fuzzy decision matrix, highlighting the best option and the worst option.

	C1	C2	C3	C4	C5	C6
E1	0.5	0	1.00	1.00	1.00	0
E2	0.5	0.375	1.00	0.97	1.00	0
E3	0.5	1	1.00	0.81	1.00	0.6
E4	0.5	1	0.75	0.77	0.98	1
E5	0.5	0	0.00	0.60	0.92	0
E6	1	0	1.00	0.99	1.00	0
E7	1	0.375	1.00	0.91	1.00	0.6
E8	1	1	0.95	0.79	1.00	1
E9	1	1	0.50	0.74	0.96	0
E10	1	0	0.00	0.55	0.88	0
E11	0.3333	0	1.00	0.81	1.00	0
E12	0.3333	0.375	0.60	0.78	0.94	1
E13	0.3333	1	0.00	0.73	0.85	0
E14	0.3333	1	0.00	0.65	0.79	0
E15	0.3333	0	0.00	0.57	0.71	0
E16	0.5	0	1.00	0.98	1.00	0
E17	0.5	0.375	1.00	0.92	1.00	0
E18	0.5	1	1.00	0.81	1.00	0.6
E19	0.5	1	0.65	0.78	0.95	1
E20	0.5	0	0.00	0.56	0.88	0
E21	0.3333	0	1.00	0.97	1.00	0
E22	0.3333	0.375	1.00	0.89	1.00	0.8
E23	0.3333	1	0.80	0.80	0.96	0
E24	0.3333	1	0.35	0.73	0.94	0
E25	0.3333	0	0.00	0.54	0.87	0
P−	0.3333	0	0	0.54	0.71	0
P+	1	1	1	1	1	1

Appendix B

Table A2. The values of S+, S−, and CC metrics.

	S+	S−	CC
E1	0.4333	0.672	0.392
E2	0.4599	0.6617	0.41
E3	0.8531	0.4149	0.6728
E4	0.9335	0.2512	0.788
E5	0.1188	0.988	0.1073
E6	0.4498	0.652	0.4082
E7	0.8499	0.4293	0.6644
E8	0.9865	0.1379	0.8774
E9	0.3915	0.8123	0.3252
E10	0.1323	0.9701	0.12
E11	0.4198	0.6837	0.3804
E12	0.8696	0.3466	0.715
E13	0.1631	0.941	0.1477
E14	0.1585	0.945	0.1436
E15	0.1028	0.9992	0.0933
E16	0.4328	0.6729	0.3914
E17	0.4585	0.6637	0.4086
E18	0.8531	0.4149	0.6728
E19	0.9122	0.2874	0.7604
E20	0.116	0.9903	0.1049
E21	0.4242	0.6778	0.3849
E22	0.8938	0.336	0.7268
E23	0.4351	0.7354	0.3717
E24	0.3212	0.8746	0.2686
E25	0.1066	1	0.0963

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