Article

A Soil-Dependent Approach for the Design of Novel Negative Stiffness Seismic Protection Devices

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Abstract: Conventional base isolation (BI) techniques require a great reduction in the fundamental frequency of the system in order to mitigate the structural dynamic responses due to earthquake excitations. However, the resulting base displacements are large and can cause utility connection problems, rendering BI inadequate for retrofitting. This paper proposes a vibration control system (VCS) that can be used as a supplement to the conventional BI to increase the effective damping, and thus reduce the required base displacements. A novel passive negative stiffness (NS)-based vibration absorber, based on the KDamper, is implemented in parallel to a BI. The design of the VCS follows a constrained optimization approach that accounts for geometrical and manufacturing limitations. The NS is realized with a realistic displacement-dependent mechanism that generates controlled NS. The VCS is designed for various soil-types in order to determine its effectiveness and soil-structure-interaction (SSI) effects are accounted with respect to the soil-type. The earthquake excitation input is selected according to the EC8 by generating a database of artificial accelerograms for each ground type. Finally, the VCS is compared to a conventional BI, and based on the numerical results obtained, the VCS is an effective alternative to BI and a possible retrofitting option.

Keywords: base isolation; building structures; seismic protection; negative stiffness; damping; vibration control; energy dissipation

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1. Introduction

The fundamental principle of BI is the decoupling of the superstructure from its base. This way the structure fundamental frequency is reduced, and as a result, the earthquake induced forces in the superstructure are significantly mitigated, as it essentially behaves as a rigid body. However, the earthquake induced displacements are decoupled from the structure, resulting in large base displacements. As a result, significant technological demands and implications are imposed on the BI devices to be used, a fact that affects the structural overall performance. In addition, proper utility connections (waterworks, gas fittings) are needed. An adequate separation distance between adjacent buildings is necessary in order to prevent structural pounding/collisions [1]. Furthermore, structural sensitivity in horizontal loads is increased. Perhaps the most important drawback is that BI is essentially prohibitive for retrofitting existing structures. An interesting approach that simultaneously controls base displacements and superstructure accelerations is presented in [2–4], where seismic isolation devices are implemented at various elevations.

These facts have motivated researchers to develop alternative control strategies that can be combined with BI, in order to seismically protect the structure, and at the same time provide possible retrofitting options. Perhaps the most popular and mature approach for vibration control is the Tuned Mass Damper (TMD) system. The TMD consist of an additional mass attached to the primary structure with a positive stiffness element and a linear damper, usually tuned with the fundamental frequency of the primary structure

and placed where the absolute motion of the system is the largest, which for the building structure is at the top [5,6]. TMDs have also been implemented in the bases of structures as a supplement to BI [7–10], as the maximum relative displacements occur at the base level. The placement of the TMD in the base of structures does not result in an increase in the total load of the superstructure. TMDs are known to be effective and reliable, however, their usage encounters significant disadvantages. The TMD properties can be significantly altered by environmental or other external parameters affecting the TMD tuning, and as a result, deteriorate its performance [11]. The essential limitation of the TMD is that in order to achieve significant vibration absorption, the additional oscillating masses need to be high. This fact renders the implementation and realization of TMDs rather difficult.

An indirect approach to increase the effective inertia of the added mass of the TMD in BI systems, is the combination of a conventional BI system with an inerter or a Tuned Inerter Damper (TID) [12–15]. The inerter is a two-terminal device, that aims to increase the overall effective inertia of the structure, without increasing its actual mass [16]. The overall dynamic behavior of the structure is indeed improved, however, marginally superior to that of a high mass TMD. The effectiveness of this system is limited and comparable to a highly damped base isolation system due to the saturation effects observed.

In the last years, the concept of negative stiffness has been introduced in seismic isolation. True negative stiffness is defined as a force that assists the motion instead of opposing it, as in the case of a positive stiffness spring. Negative stiffness devices significantly reduce the stiffness of the isolator and consequently reduce the natural frequency of the system even at almost zero levels, as in Carella et al. [17], being thus called “Quazi Zero Stiffness” (QZS) oscillators. Among others, QZS oscillators find numerous applications in seismic isolation [18–27]. The QZS oscillators are designs of an essentially non-linear stiffness, combining a positive spring in parallel with a non-linear spring, which exhibits a negative stiffness region under nominal load. In this way, QZS oscillators combine a high static stiffness, capable of maintaining the structural load, with a low dynamic stiffness, enabling a low isolation frequency. However, QZS oscillators offer a low damping capacity. In addition, even small load disturbances around the static equilibrium point of the QZS designs can significantly affect their response.

This paper proposes a Vibration Control System (VCS) that can be used as a supplement to conventional BI techniques and combines the beneficial characteristics of the aforementioned control systems, TMDs and QZSs, without their respective drawbacks. More specifically, the internal mechanism of an extended version of KDamper [28,29] is implemented in parallel to a BI. Compared to the TMD, the KDamper has been shown to present extraordinary damping/absorption properties, with relatively small (even minimal) values of the additional mass [30,31]. Although the KDamper incorporates a NS element, it is designed to be statically and dynamically stable, as the nominal system frequency is a freely selectable parameter, overcoming the fundamental limitation of QZS configurations. The VCS is implemented in the bases of new or existing structures for seismic protection, as a supplement to BI approaches. The design of the VCS is realistic as it accounts for the inherent nonlinear behavior of NS, and accounts for different soil types and SSI effects. The essential features and novel aspects of the current paper are:

1. The VCS is implemented as a supplement to BI and provides a novel passive retrofitting option for building structures;
2. The optimal VCS parameters are selected by a constrained optimization procedure that accounts for geometrical and manufacturing limitations;
3. Different set of optimized VCS parameters are obtained with respect to the underlying soil, resulting in a soil-dependent design procedure;
4. The effect of the soil-structure-interaction (SSI) is taken into consideration with the use of nonlinear spring stiffnesses with respect to the considered soil type;
5. A realistic mechanism is proposed for the realization of two-dimensional NS.

The matrices and vectors entering Equation (1) are formed as:

$$\mathbf{M}_{(N+3) \times (N+3)} = \begin{bmatrix} \mathbf{M}_N & \mathbf{0}_{N \times 1} & \mathbf{0}_{N \times 1} & \mathbf{0}_{N \times 1} \\ \mathbf{0}_{1 \times N} & m_B & 0 & 0 \\ \mathbf{0}_{1 \times N} & 0 & m_D & 0 \\ \mathbf{0}_{1 \times N} & 0 & 0 & m_F \end{bmatrix} \quad (2)$$

$$\mathbf{K}(u)_{(N+3) \times (N+3)} = \begin{bmatrix} \mathbf{K}_N & \mathbf{0}_{(N-1) \times 1} & \mathbf{0}_{(N-1) \times 1} & \mathbf{0}_{(N-1) \times 1} \\ \mathbf{0}_{1 \times (N-1)} & -k_1 & 0 & 0 \\ \mathbf{0}_{1 \times (N-1)} & 0 & k_1 + k_{NS}(u_{NS}) + k_{BI} & -k_{NS}(u_{NS}) \\ \mathbf{0}_{1 \times (N-1)} & 0 & -k_{NS}(u_{NS}) & k_{NS}(u_{NS}) + k_{PS} \end{bmatrix} \quad (3)$$

$$\mathbf{C}(\dot{u})_{(N+3) \times (N+3)} = \begin{bmatrix} \mathbf{C}_N & \mathbf{0}_{(N) \times 1} & \mathbf{0}_{(N) \times 1} & \mathbf{0}_{(N) \times 1} \\ \mathbf{0}_{1 \times (N)} & c_{NS} + c_{BI} & c_{NS} & -c_{BI} \\ \mathbf{0}_{1 \times (N)} & c_{NS} & c_{NS} + c_{PS} & 0 \\ \mathbf{0}_{1 \times (N)} & -c_{BI} & 0 & c_{BI} + c_{SSI}(\dot{u}_F) \end{bmatrix} \quad (4)$$

$$\mathbf{u}_{(N+3) \times 1} = \begin{bmatrix} \mathbf{u}_{str} & u_B & u_{VCS} & u_F \end{bmatrix}^T \quad (5)$$

where $\mathbf{M}_N$, $\mathbf{K}_N$, and $\mathbf{C}_N$ are the superstructure mass, stiffness and damping matrices, assuming it is mounted on a fixed base, k_1 is the lateral stiffness of the first floor, and $\mathbf{I}$ is the $(N + 3) \times 1$ influence vector associated with the ground motion X_g . The nonlinear NS spring k_{NS} depends on the value of the NS stroke u_{NS} (relative displacement between the terminals of the negative stiffness element). The stiffness of the foundation system (k_{SSI}) that is derived due to the SSI effects depends on the foundation relative displacement u_F . The controlled structure is mounted on a mat foundation, as in the case of a conventional BI structure.

The effect of the SSI is taken into consideration with the use of nonlinear spring stiffnesses (k_{SSI}). The k_{SSI} is calculated assuming a rigid beam subjected to horizontal displacement at the top. The height of this element is measured from the bottom of the mat foundation to the bottom end of the implemented bearing (Figure 1). As a consequence, the calculated stiffness (k_{SSI}), is a resultant of both the rotational and horizontal displacement of the soil footing and is expressed at the top of the rigid beam element.

The soil stratum is considered to be isotropic/homogeneous. The interface between foundation–soil is considered to be ‘fully bonded’ with the foundation elements considered attached to the ground. A Matlab-based code is incorporated to model the original superstructure and the proposed VCS as a series of masses, linear dampers, and linear and non-linear positive and negative stiffness elements.

2.2. VCS Dynamic Properties and Nonlinear Behavior

The fundamental advantage of the VCS is that in order to be effective, it does not require significant reduction in the fundamental frequency of the system, as in the case of BI systems, due to the increase of the effective damping that the KDamper configuration offers. Thus, the nominal system frequency f_0 is a freely selectable parameter, the value of which can be selected with respect to the structural system to be controlled in its initial undeformed (static) state.

$$(2\pi f_0)^2 m_{total} = k_0 = k_R + \frac{k_{PS}k_{NS}(u_{NS} = 0)}{k_{PS} + k_{NS}(u_{NS} = 0)} \quad (6a)$$

$$k_R = \frac{k_{SSI}(u_F = 0)k_{BI}}{k_{SSI}(u_F = 0) + k_{BI}} \quad (6b)$$

Although theoretically the values of k_{NS} , k_{PS} , and k_R are selected according to Equation (6a) to ensure the static stability, due to various reasons, such as temperature variations, manufacturing tolerances, or non-linear behavior, k_{NS} , k_{PS} , and k_R may present variations in practice, since almost all NS designs result from unstable nonlinear systems. An increase of the absolute value of k_{NS} and/or a decrease of the values of the positive stiffness elements k_{PS} and k_R by a factor ε_{NS} , ε_{PS} , and ε_R , respectively, may result in the system being unstable. This happens when the determinant of the stiffness matrix (structure-VCS) equals zero:

$$\det \mathbf{K} = |\mathbf{K}| = 0 \Rightarrow (1 - \varepsilon_R) \times k_R + \frac{(1 - \varepsilon_{PS}) \times k_{PS} \times (1 + \varepsilon_{NS}) \times k_{NS}(u_{NS} = 0)}{(1 - \varepsilon_{PS}) \times k_{PS} + (1 + \varepsilon_{NS}) \times k_{NS}(u_{NS} = 0)} = 0 \quad (7)$$

Contrary to the KDamper concept, which foresees variation only in the NS element k_{NS} , in the proposed VCS, a variation in all stiffness elements is taken into account. Assuming that the nominal VCS frequency f_0 , and the value of the negative stiffness element k_{NS} are known, using Equations (6a) and (7), the rest of the stiffness elements result as:

$$\frac{k_R}{k_0} = \frac{-b - \sqrt{b^2 - 4 \times a \times c}}{2a} \quad (8)$$

$$\frac{k_{PS}}{k_0} = \frac{(k_{NS}(u_{NS} = 0)/k_0) - (k_R/k_0)(k_{NS}(u_{NS} = 0)/k_0)}{(k_R/k_0) + (k_{NS}(u_{NS} = 0)/k_0) - 1} \quad (9)$$

where parameters a , b , and c are defined as:

$$a = (1 - \varepsilon_R)[(1 - \varepsilon_{PS}) - (1 + \varepsilon_{NS})] \quad (10)$$

$$b = \frac{k_{NS}(u_{NS} = 0)}{k_0}(1 + \varepsilon_{NS})[(1 - \varepsilon_P) - (1 - \varepsilon_R)] + (1 - \varepsilon_R)[(1 + \varepsilon_{NS}) - (1 - \varepsilon_P)] \quad (11)$$

$$c = -(1 - \varepsilon_P)(1 + \varepsilon_N) \frac{k_{NS}(u_{NS} = 0)}{k_0} \quad (12)$$

The NS element of the VCS configuration is realized with a displacement-dependent mechanism, proposed in [29] as illustrated in Figure 2. A vertical conventional positive stiffness element (coil spring) is attached to the additional mass m_D , and is connected to the base floor with an articulated mechanism.

Figure 2. Proposed mechanism for the realization of the NS element.

The generated NS is calculated according to the following procedure. Initially, the potential energy due to the deformation of the vertical spring is given, and consequently, the value of the elastic nonlinear force and the equivalent generated NS are calculated:

$$U_{NS}(u_{NS}) = \frac{1}{2} k_V (l_V(u_{NS}) - l_{V1})^2 \quad (13)$$

$$l_V(u_{NS}) = b - \sqrt{(a^2 - u_{NS}^2)} \quad (14)$$

$$f_{NS}(u_{NS}) = \frac{\partial U_{NS}}{\partial u_{NS}} = -k_V \left(1 + \frac{l_{VI} - b}{\sqrt{a^2 - u_{NS}^2}} \right) u_{NS} = -k_V \left(1 + \frac{c_I}{\sqrt{1 - u_{NS}^2/a^2}} \right) u_{NS} \quad (15)$$

$$u_{NS} = u_B - u_D \quad (16)$$

$$c_I = (l_{VI} - b)/a \quad (17)$$

$$k_{NS}(u_{NS}) = \frac{\partial f_{NS}}{\partial u_{NS}} = -k_V \left(1 + \frac{c_I}{\sqrt{1 - u_{NS}^2/a^2}^3} \right) \quad (18)$$

where k_V is the value of the positive vertical stiffness element, l_{VI} and l_V is the length of the vertical spring in its initial and deformed state, and c_I is a non-dimensional design variable that correlates the constant geometric properties of the configuration, i.e., the length a of the rod, the total distance b between the masses and the l_{VI} . The value of c_I takes values in the range $[-0.1, -0.01]$ in order to achieve a near to linear behavior. Having selected a value for c_I , and the variation of the NS element value ε_{NS} , Equation (7), the parameters a and k_V are obtained by solving the system of equations presented below:

$$k_{NS}(u_{NS} = 0) \times (1 - \varepsilon_{NS}) = -k_V(1 + c_I) \quad (19)$$

$$k_{NS}(|u_{NS}| = \max) \times (1 + \varepsilon_{NS}) = -k_V \left(1 + \frac{c_I}{\sqrt{1 - \max(u_{NS})^2/a^2}^3} \right) \quad (20)$$

3. Soil-Dependent Design of Vibration Control System

3.1. Earthquake Ground Motion Representation

The structure's displacements and accelerations are within specified limits for the given fundamental structure period and damping ratio, according to the seismic design codes. The parameters that mainly control these limits are the ground conditions (soil type) and the expected seismic intensity. However, the selection of the VCS parameters is not possible by following such approaches due to complexity of the VCS and the geometrical and manufacturing limitation imposed for a realistic design.

Therefore, analysis in the time-domain is necessary. Ground motions can be generated, among others, from: synthetic (seismological models), real earthquakes (not all soil types, not smoothed spectra), and artificial accelerograms, compatible with a specific design response spectrum [32,33]. In this paper, a database of design response spectrum (EC8) compatible ground acceleration excitations for various soil types will be generated using the SeismoArtif Software [34]. More specifically, 10 artificial accelerograms are generated for each of the following soil types: (1) firm (ground type A), (2) medium (ground type C), and soft (ground type D). For each soil type, the following seismic properties are assumed: spectral acceleration 0.36 g, spectrum type I and importance class II. Figure 3 presents the mean acceleration response spectrum of all the artificial accelerograms and soil types, as compared to the respective EC8 design response spectrum. An accurate match is observed in all cases.

Figure 3. Mean acceleration response spectrum of all the artificial accelerograms of the database for: (a) firm soil (ground type A); (b) medium soil (ground type C); (c) soft soil (ground type D).

3.2. Constrained Optimization for the Selection of VCS Parameters

The fixed parameters of the VCS are the additional mass m_D and the stability factors ε_{NS} , ε_{PS} , and ε_R . The independent design variables sought in the optimization problem are: (1) the nominal VCS frequency f_0 , (2) the value of the NS element k_{NS} , and (3, 4) the value of the artificial dampers c_{NS} and c_{PS} . The optimization problem is formed for the mean of the maximum dynamic responses for the 10 artificial records of the database of the respective soil type. The objective function and the geometrical and manufacturing constraints are selected from the geometrically non-linear time domain responses, as described below:

1. Assign values to the fixed parameters. According to previous work of KDamper [31], an additional mass of 5% is efficient. In this paper, in an effort to drastically reduce the additional mass, m_D is selected as 0.1%. The stability factors of the positive stiffness elements k_{PS} , k_R and of the NS element k_{NS} are selected as 5%;
2. Selection of soil type and thus the respective artificial accelerogram database;
3. Set the objective function (OF) as the minimization of the mean of the maximum relative to the ground base displacements of the structure;

$$\min : \text{mean}[\max[\text{abs}(u_B)]] \quad (\text{OF}) \quad (21)$$

4. Set an acceleration filter (AF) as a constraint for the structure absolute acceleration, expressed as a percentage of the mean PGA of the artificial accelerograms of the database of the selected soil type;

$$\text{mean}[\max[\text{abs}(a_i)]] \leq \text{AF} \times \text{mean}(\text{PGA}) \quad (i = 1, \dots, 10) \quad (\text{Constraint 1}) \quad (22)$$

5. The NS element value and stroke are set as constraints, indicated from the proposed design of the NS configuration presented in [29] (50% lower compared to [28,35]);

$$|k_{NS}| \leq 50 \text{ kN/m/tm} \quad (\text{Constraint 2}) \quad (23)$$

$$\text{mean}[\max[\text{abs}(u_B - u_D)]] \leq 0.10 \text{ m} \quad (\text{Constraint 3}) \quad (24)$$

6. Set an upper limit for c_{NS} , c_{PS} with respect to the superstructure mass. This constraint is based on previous works [29] as well as on manufacturing restrictions;

$$c_{NS} \leq 1000 \text{ kNs/m} \quad (\text{Constraint 4}) \quad (25)$$

$$c_{PS} \leq 1000 \text{ kNs/m} \quad (\text{Constraint 5}) \quad (26)$$

7. The nominal SBA frequency f_0 varies in the range $[0.15 \text{ } 1.5]$ (Hz) (Constraint 6);

The VCS design process is schematically represented with a flowchart in Figure 4. For the optimization process, the harmony search algorithm (HS) [36] is adopted. After the optimal system parameters are obtained, the proposed NS mechanism is designed, and consequently, nonlinear dynamic analysis is performed with nonlinear NS, in order to verify the effectiveness and realistic design of the proposed VCS. The conversion criteria mentioned in the outer loop of the flowchart presented in Figure 4 are:

- $\text{mean}[\max[\text{abs}(a_i)]] \leq AF \times \text{mean}(PGA)$ ($i = 1, \dots, 10$) (Considering nonlinear NS)
- $\text{mean}[\max[\text{abs}(u_B - u_D)]] \leq 0.10 \text{ m}$ (Considering nonlinear NS)

The first one ensures the effectiveness of the VCS in mitigating the seismic effects on the superstructure at the desired level (AF). The second one safeguards the realistic design of the proposed NS mechanism with conventional structural elements.

Figure 4. Flowchart of the VCS design implemented in the bases of structures as a supplement to conventional base isolation approaches (BI).

4. Numerical Results and Discussion

The examined original structure (OS) is a three-story residential RC building (Figure 5a). The elastic modulus of reinforced concrete (assuming long-term cracked conditions) is equal to $E = 26 \text{ GPa}$. The concentrated mass of each floor is $m_i = 80 \text{ tn}$. Rayleigh damping is assumed with $\zeta_i = 2\%$. A $15 \times 15 \text{ m}$ floor plan-view (Figure 5b) has been considered with $0.3 \times 0.3 \text{ m}$ square columns. The resulting natural periods of the OS are $T_i [\text{sec}] = [0.495, 0.177, 0.122]$. The mass of the base level and the mat foundation are equal to $m_B = 60 \text{ tn}$ and $m_F = 80 \text{ tn}$, resulting in a total superstructure mass of $m_{\text{total}} = 300 \text{ tn}$. The VCS mass is selected as 0.1% of the m_{total} and is $m_D = 300 \text{ kg}$.

The foundation system is designed based on the properties and approximate loading of a typical residential building. On the basis of empirical correlations, the initial, small

strain, elastic modulus (E_0) of the soil was considered equal to $1800 S_u$. Based on the provisions of the EC8, the undrained shear strength S_u varies with respect to the ground type. For ground type A (firm soil), the underlying soil is a rock-like formation and can be thus considered as a fixed base. For ground type C (medium soil), S_u varies in the range [70 250] kPa, and for the analysis, is selected as the average, $S_u = 160$ kPa. For ground type D (soft soil), S_u is equal or less than 70 kPa, and this value is adopted for this paper. The soil stratum is considered to be isotropic/homogeneous and the same is applied for the mat foundation which is, however, considered to be elastic (Reinforced Concrete). The boundaries of the model have been strategically selected sufficiently far to avoid spurious boundary-effects on the system response. Static pushover analyses are subsequently undertaken in order to derive the non-linear soil-foundation stiffness relationships (k_{SSI}).

The mean PGAs of the artificial accelerograms of the database are 0.421, 0.545, and 0.614 (g) for the firm, medium, and soft soil, respectively. The acceleration filter (AF) is set to 80% of the respective mean PGA of the database, and the optimal VCS parameters are obtained following the procedure described in Section 3.2 and are presented in Table 1.

Figure 5. Typical 3-story residential RC building examined in this paper: (a) sketch; (b) floor plan.

Table 1. Optimum VCS parameters for each soil type.

Soil Type	f_0 (Hz)	m_D (tn)	k_{NS} (kN/m)	k_{PS} (kN/m)	k_R (kN/m)	c_{NS} (kNs/m)	c_{PS} (kNs/m)
firm	0.838	0.3	−14,914	27,799	40,499	997	914
medium	0.548	0.3	−12,681	34,450	23,630	982	840
soft	0.527	0.3	−13,455	41,238	23,260	892.8	388.8

In order to confirm the effectiveness of the proposed VCS, a comparison is made with a conventionally base isolated structure (BI) with a base nominal frequency of $f_B = 0.4$ Hz ($T_B = 2.5$ s) and a BI damping ratio of $\zeta_B = 5\%$. The system main dynamic responses, considering the max values of the dynamic responses for all the Artificial Accelerograms in the database, mean of 10 maxes for each soil category, of the original structure (OS), the conventionally base isolated one (BI), and the controlled structure with the proposed vibration control system (VCS) are collected and presented in the bar plots of Figures 6 and 7.

Figure 6. Top floor absolute acceleration $a_{S,top}$ of the OS, BI, and VCS systems for all soil types.

Figure 7. First floor drift (critical) of the OS, BI, and VCS systems for all soil types.

Figures 6 and 7 present the structural top floor acceleration and first floor drift expressed as a percentage of the OS values. It is observed that the BI mitigates the superstructure dynamic responses by 80–90%, while the VCS reductions are in the range of 70–80%. Figure 8 presents the base relative displacements expressed as a percentage of the BI values. The VCS system manages to retain the u_B in the order of a few centimetres, 70–90% lower as compared to the BI. In Tables 2–4 are the main dynamic responses of the OS, BI and VCS for all the artificial accelerograms of the database for all the soil types examined. The reductions that refer to the superstructure dynamic performance (floor accelerations and interstory drifts) are with respect to the OR, and the reductions of the base displacements are with respect to the BI system.

Figure 8. Base relative displacement u_B of the BI and VCS systems for all soil types.

Table 2. Main dynamic responses of the OS, the BI, and the VCS for the firm soil type (class A).

	OS		BI			VCS		
	Max(a_S) (g)	Max(drift) (cm)	Max(a_S) (g)	Max(drift) (cm)	Max(u_B) (cm)	Max(a_S) (g)	Max(drift) (cm)	Max(u_B) (cm)
acc #1	0.66	1.46	0.05	0.16	6.91	0.19	0.43	1.06
acc #2	0.78	1.95	0.05	0.16	6.68	0.20	0.55	1.21
acc #3	0.67	1.76	0.06	0.17	6.65	0.22	0.59	1.07
acc #4	0.66	1.46	0.05	0.16	6.91	0.19	0.43	1.06
acc #5	1.24	3.24	0.09	0.28	10.91	0.49	1.03	2.30
acc #6	1.68	4.19	0.12	0.35	14.68	0.43	1.24	2.62
acc #7	1.27	3.36	0.12	0.35	14.38	0.42	1.15	2.56
acc #8	1.37	3.13	0.16	0.46	19.42	0.47	1.07	2.02
acc #9	1.34	3.33	0.11	0.34	14.03	0.42	0.96	2.20
acc #10	1.34	2.73	0.12	0.33	14.31	0.44	0.99	2.33
Average	1.10	2.66	0.09	0.28	11.49	0.35	0.84	1.84
Reduction (%)	-	-	91.8	89.5	-	68.2	68.4	84

Table 3. Main dynamic responses of the OS, the BI, and the VCS for the medium soil type (class C).

	OS		BI			VCS		
	Max(a_S) (g)	Max(dr) (cm)	Max(a_S) (g)	Max(dr) (cm)	Max(u_b) (cm)	Max(a_S) (g)	Max(dr) (cm)	Max(u_B) (cm)
acc #1	1.50	4.44	0.13	0.41	16.96	0.49	1.22	5.10
acc #2	1.43	4.32	0.18	0.54	21.68	0.53	1.08	5.23
acc #3	1.98	5.23	0.17	0.48	19.04	0.39	1.22	5.57
acc #4	1.71	4.42	0.19	0.55	22.60	0.39	0.93	5.22
acc #5	1.39	4.05	0.17	0.53	21.82	0.44	1.02	4.52
acc #6	2.27	6.28	0.18	0.55	23.23	0.49	1.14	5.16
acc #7	1.47	4.49	0.17	0.53	21.87	0.37	1.05	5.02
acc #8	1.56	3.73	0.15	0.51	21.72	0.36	0.96	4.45
acc #9	1.70	4.32	0.16	0.49	20.34	0.58	1.14	5.36
acc #10	1.83	4.80	0.17	0.52	21.01	0.47	1.26	5.33
Average	1.68	4.61	0.17	0.51	21.03	0.45	1.10	5.09
Reduction (%)	-	-	89.9	88.9	-	73.2	76.1	75.8

Table 4. Main dynamic responses of the OS, the BI, and the VCS for the soft soil type (class D).

	OS		BI			VCS		
	Max(a_S) (g)	Max(dr) (cm)	Max(a_S) (g)	Max(dr) (cm)	Max(u_B) (cm)	Max(a_S) (g)	Max(dr) (cm)	Max(u_B) (cm)
acc #1	1.93	5.08	0.25	0.76	31.20	0.54	1.44	7.89
acc #2	2.11	5.82	0.27	0.80	33.27	0.46	1.27	11.31
acc #3	2.11	5.82	0.27	0.80	33.27	0.46	1.27	11.31
acc #4	1.97	5.15	0.19	0.61	24.85	0.56	1.38	8.51
acc #5	1.97	5.15	0.19	0.61	24.85	0.56	1.38	8.51
acc #6	1.80	5.31	0.23	0.67	27.34	0.47	1.24	7.18
acc #7	2.48	6.36	0.23	0.72	30.11	0.46	1.24	6.54
acc #8	1.78	4.93	0.23	0.71	29.04	0.47	1.26	6.60
acc #9	1.78	4.93	0.23	0.71	29.04	0.47	1.26	6.60
acc #10	2.36	5.86	0.25	0.77	32.41	0.51	1.35	8.65
Average	2.03	5.44	0.23	0.72	29.54	0.50	1.31	8.31
Reduction (%)	-	-	88.7	86.8	-	75.4	75.9	71.9

The effectiveness of the VCS is examined also with real earthquakes. Two real ground motions are selected for each soil type. Chi-Chi and Niigata fall into the soft soil category, JMA and Northridge fall into the medium soil category, and Kocaeli and Tabas fall into the firm soil category. The categorization is based on the average shear-wave velocity for the upper 30-m depth of the station. The acceleration time histories are presented in Figure 9, and the details of the ground motions are presented in Table 5.

Figure 9. Two selected real earthquake excitations for each soil category. Chi–Chi and Niigata for soft soil, JMA and Northridge for medium soil, and Kocaeli and Tabas for firm soil.

Figures 10 and 11 and Table 6 present the controlled system main dynamic responses of the OS, BI, and VCS for the real earthquake records. The reductions presented in Table 6 that refer to the superstructure dynamic performance (floor accelerations and interstory drifts) are with respect to the OR, and the reductions of the base displacements are with respect to the BI system. It is observed that the reductions in the superstructure dynamic responses regarding both seismic protection approaches, BI and VCS, are in the same order of magnitude as in the case of the artificial accelerograms. For all the considered ground

motions, artificial and real, the VCS manages to seismically protect the superstructure (more than 50% improvement) and at the same time retain the required base displacements in reasonable ranges, in the order of a few centimeters.

Table 5. Information of the considered real earthquake ground motions.

Soil Type	Earthquake	Year	Station	Mw	PGA (g)	Rjb (km)	Dur (s)	v_s (m/s)
Soft	Chi-Chi Niigata	1999	CHY012	7.62	0.0626	59.04	42.8	198.4
		2004	FKS020	6.63	0.043	101.78	22.5	133.05
Medium	JMA	1995	Amagasaki	6.9	0.276	11.34	50	256.0
	Northridge	1994	N Hollywood	6.69	0.309	7.89	20	326.47
Firm	Kocaeli	1999	Izmit	7.51	0.1651	3.62	8.2	811.0
	Tabas	1978	Tabas	7.35	0.854	1.79	8.3	766.77

Table 6. Dynamic responses of the OS, the BI, and the VCS for the two real earthquake records for each soil category.

Soil Type	System	OS		BI		VCS (Soft Soil)			
	Dynamic Response	Max(a_s) (g)	Max(dr) (cm)	Max(a_s) (g)	Max(dr) (cm)	Max(u_B) (cm)	Max(a_s) (g)	Max(dr) (cm)	Max(u_B) (cm)
Soft	Chi-Chi	0.26	0.71	0.05	0.18	7.82	0.07	0.20	1.73
	Reduction (%)	-	-	79.58	74.20	-	73.00	71.53	77.88
	Niigata	0.14	0.44	0.02	0.07	2.94	0.04	0.10	0.77
	Reduction (%)	-	-	85.62	83.98	-	70.30	76.14	73.89
Medium	Kobe	1.52	3.97	0.16	0.53	21.74	0.34	1.08	5.79
	Reduction (%)	-	-	89.74	86.76	-	77.90	72.73	73.37
	Northridge	0.64	1.64	0.10	0.34	14.49	0.34	0.46	2.42
	Reduction (%)	-	-	84.07	79.18	-	46.98	72.00	83.32
Firm	Kocaeli	0.66	2.04	0.08	0.24	10.16	0.25	0.67	1.39
	Reduction (%)	-	-	87.99	88.02	-	61.81	67.16	86.35
	Tabas	2.73	6.28	0.34	0.97	40.61	1.33	2.91	6.45
	Reduction (%)	-	-	87.57	84.60	-	51.16	53.62	84.11

Figure 10. Cont.

Figure 10. Cont.

Figure 10. Top floor absolute acceleration $a_{s,top}$ (g) of the OS, BI, and VCS systems for the two selected real earthquakes for each soil type. Chi–Chi and Niigata for soft soil, JMA and Northridge for medium soil, and Kocaeli and Tabas for firm soil.

Figure 11. *Cont.*

Figure 11. Base relative displacements (cm) of the OS, BI, and VCS systems for the two selected real earthquakes for each soil type. Chi–Chi and Niigata for soft soil, JMA and Northridge for medium soil, and Kocaeli and Tabas for firm soil.

5. Conclusions

In this paper, a negative stiffness-based VCS is employed, as a supplement to a conventional BI, in the base of a multi-story residential RC building structure for seismic protection. The methodology for the implementation of the VCS is presented and the nonlinear dynamic model of the controlled structure is formed. A realistic displacement-dependent mechanism that generates controlled nonlinear negative stiffness is proposed, and the SSI effects are accounted with the use of nonlinear springs, with respect to the considered soil category, and are coupled in series with the stiffness of the conventional BI (k_{BI}) providing the total stiffness of the base level (k_R). The fixed VCS parameters are selected and the independent design variables are presented. The optimal design of the VCS follows a constrained optimization procedure to be as realistic as possible. Various VCS set of optimized parameters are obtained for different soil types in order to determine the effectiveness of the proposed vibration control strategy. Based on a comparison with a BI and the numerical results obtained, the following conclusive remarks can be made:

1. The proposed dynamic model of the controlled structure accounts for the nonlinear dynamic behavior of the NS element and the effect of SSI;
2. The VCS design is realistic, employing small added masses and realistic constraints;
3. The VCS is effective in seismically protecting the superstructure for all the considered soil types (firm, medium, soft), and retaining at the same time the base displacements significantly lower as compared to a BI;
4. The superstructure dynamic behavior is greatly improved with the proposed VCS. More specifically, the floor accelerations and interstory drift are reduced 60–80% as compared to the original multistory structure;
5. The required base displacements are retained in reasonable ranges, in the order of a few centimeters (2–8 cm), and render the VCS a possible retrofiting option. As compared to a conventionally BI structure, reductions of 70–90% are observed.

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References

1. Anagnostopoulos, S.A.; Spiliopoulos, K.V. An Investigation of Earthquake Induced Pounding between Adjacent Buildings. *Earthq. Eng. Struct. Dyn.* **1992**, *21*, 289–302. [\[CrossRef\]](#)
2. Charmpis, D.C.; Komodromos, P.; Phocas, M.C. Optimized Earthquake Response of Multi-Storey Buildings with Seismic Isolation at Various Elevations. *Earthq. Eng. Struct. Dyn.* **2012**, *41*, 2289–2310. [\[CrossRef\]](#)
3. Daniel, Y.; Lavan, O. Gradient Based Optimal Seismic Retrofitting of 3D Irregular Buildings Using Multiple Tuned Mass Dampers. *Comput. Struct.* **2014**, *139*, 84–97. [\[CrossRef\]](#)
4. Charmpis, D.C.; Phocas, M.C.; Komodromos, P. Optimized Retrofit of Multi-Storey Buildings Using Seismic Isolation at Various Elevations: Assessment for Several Earthquake Excitations. *Bull. Earthq. Eng.* **2015**, *13*, 2745–2768. [\[CrossRef\]](#)
5. Shih, M.-H.; Sung, W.-P. Seismic Resistance and Parametric Study of Building under Control of Impulsive Semi-Active Mass Damper. *Appl. Sci.* **2021**, *11*, 2468. [\[CrossRef\]](#)
6. Zucca, M.; Longarini, N.; Simoncelli, M.; Aly, A.M. Tuned Mass Damper Design for Slender Masonry Structures: A Framework for Linear and Nonlinear Analysis. *Appl. Sci.* **2021**, *11*, 3425. [\[CrossRef\]](#)
7. Tsai, H.C. The Effect of Tuned-Mass Dampers on the Seismic Response of Base-Isolated Structures. *Int. J. Solids Struct.* **1995**, *32*, 1195–1210. [\[CrossRef\]](#)
8. Palazzo, B.; Petti, L.; de Ligio, M. Response of Base Isolated Systems Equipped with Tuned Mass Dampers to Random Excitations. *J. Struct. Control.* **1997**, *4*, 9–22. [\[CrossRef\]](#)
9. Taniguchi, T.; Der Kiureghian, A.; Melkumyan, M. Effect of Tuned Mass Damper on Displacement Demand of Base-Isolated Structures. *Eng. Struct.* **2008**, *30*, 3478–3488. [\[CrossRef\]](#)
10. Hashimoto, T.; Fujita, K.; Tsuji, M.; Takewaki, I. Innovative Base-Isolated Building with Large Mass-Ratio TMD at Basement for Greater Earthquake Resilience. *Future Cities Environ.* **2015**, *1*, 1–19. [\[CrossRef\]](#)
11. Weber, B.; Feltrin, G. Assessment of Long-Term Behavior of Tuned Mass Dampers by System Identification. *Eng. Struct.* **2010**, *32*, 3670–3682. [\[CrossRef\]](#)
12. Lazar, I.F.; Neild, S.A.; Wagg, D.J. Using an Inerter-Based Device for Structural Vibration Suppression. *Earthq. Eng. Struct. Dyn.* **2014**, *43*, 1129–1147. [\[CrossRef\]](#)
13. Giaralis, A.; Taflanidis, A.A. Optimal Tuned Mass-Damper-Inerter (TMDI) Design for Seismically Excited MDOF Structures with Model Uncertainties Based on Reliability Criteria. *Struct. Control Health Monit.* **2018**, *25*, e2082. [\[CrossRef\]](#)
14. De Domenico, D.; Ricciardi, G. An Enhanced Base Isolation System Equipped with Optimal Tuned Mass Damper Inerter (TMDI). *Earthq. Eng. Struct. Dyn.* **2018**, *47*, 1169–1192. [\[CrossRef\]](#)

15. De Domenico, D.; Impollonia, N.; Ricciardi, G. Soil-Dependent Optimum Design of a New Passive Vibration Control System Combining Seismic Base Isolation with Tuned Inerter Damper. *Soil Dyn. Earthq. Eng.* **2018**, *105*, 37–53. [CrossRef]
16. Smith, M.C. Synthesis of Mechanical Networks: The Inerter. *IEEE Trans. Automat. Contr.* **2002**, *47*, 1648–1662. [CrossRef]
17. Carrella, A.; Brennan, M.J.; Waters, T.P. Static Analysis of a Passive Vibration Isolator with Quasi-Zero-Stiffness Characteristic. *J. Sound Vib.* **2007**, *301*, 678–689. [CrossRef]
18. Nagarajaiah, S.; Pasala, D.T.R.; Reinhorn, A.; Constantinou, M.; Sirilis, A.A.; Taylor, D. Adaptive Negative Stiffness: A New Structural Modification Approach for Seismic Protection. *Adv. Mater. Res.* **2013**, *639–640*, 54–66. [CrossRef]
19. DeSalvo, R. Passive, Nonlinear, Mechanical Structures for Seismic Attenuation. *J. Comput. Nonlinear Dyn.* **2007**, *2*, 290. [CrossRef]
20. Iemura, H.; Pradono, M.H. Advances in the Development of Pseudo-Negative-Stiffness Dampers for Seismic Response Control. *Struct. Control Health Monit.* **2009**, *16*, 784–799. [CrossRef]
21. Pasala, D.T.R.; Sarlis, A.A.; Nagarajaiah, S.; Reinhorn, A.M.; Constantinou, M.C.; Taylor, D. Negative Stiffness Device for Seismic Response Control of Multi-Story Buildings. In Proceedings of the 20th Analysis and Computation Specialty Conference, Chicago, IL, USA, 29–31 March 2012; pp. 83–96.
22. Sarlis, A.A.; Pasala, D.T.R.; Constantinou, M.C.; Reinhorn, A.M.; Nagarajaiah, S.; Taylor, D.P. Negative Stiffness Device for Seismic Protection of Structures. *J. Struct. Eng.* **2012**, *139*, 1124–1133. [CrossRef]
23. Pasala, D.T.R.; Sarlis, A.A.; Nagarajaiah, S.; Reinhorn, A.M.; Constantinou, M.C.; Taylor, D. Adaptive Negative Stiffness: New Structural Modification Approach for Seismic Protection. *J. Struct. Eng.* **2013**, *139*, 1112–1123. [CrossRef]
24. Sarlis, A.A.; Pasala, D.T.R.; Constantinou, M.C.; Reinhorn, A.M.; Nagarajaiah, S.; Taylor, D.P. Negative Stiffness Device for Seismic Protection of Structures: Shake Table Testing of a Seismically Isolated Structure. *J. Struct. Eng.* **2016**, *142*, 04016005. [CrossRef]
25. Sun, T.; Lai, Z.; Nagarajaiah, S.; Li, H.N. Negative Stiffness Device for Seismic Protection of Smart Base Isolated Benchmark Building. *Struct. Control Health Monit.* **2017**, *24*, e1968. [CrossRef]
26. Wang, M.; Sun, F.; Jin, H. Performance Evaluation of Existing Isolated Buildings with Supplemental Passive Pseudo-Negative Stiffness Devices. *Eng. Struct.* **2018**, *177*, 30–46. [CrossRef]
27. Shen, Y.; Peng, H.; Li, X.; Yang, S. Analytically Optimal Parameters of Dynamic Vibration Absorber with Negative Stiffness. *Mech. Syst. Signal Process.* **2017**, *85*, 193–203. [CrossRef]
28. Antoniadis, I.A.; Kanarachos, S.A.; Gryllias, K.; Sapountzakis, I.E. KDamping: A Stiffness Based Vibration Absorption Concept. *J. Vib. Control* **2018**, *24*, 588–606. [CrossRef]
29. Kapasakalis, K.A.; Antoniadis, I.A.; Sapountzakis, E.J. Constrained Optimal Design of Seismic Base Absorbers Based on an Extended KDamping Concept. *Eng. Struct.* **2021**, *226*, 111312. [CrossRef]
30. Kapasakalis, K.A.; Sapountzakis, E.I.; Antoniadis, I.A. Implementation of the KDamping Concept to Wind Turbine Towers. In Proceedings of the COMPDYN 2017—6th International Conference on Computational Methods in Structural Dynamics and Earthquake Engineering, Rhodes, Greece, 15–17 June 2017.
31. Kapasakalis, K.A.; Antoniadis, I.A.; Sapountzakis, E.J. Performance Assessment of the KDamping as a Seismic Absorption Base. *Struct. Control Health Monit.* **2020**, *27*, e2482. [CrossRef]
32. Giaralis, A.; Spanos, P.D. Derivation of Response Spectrum Compatible Non-Stationary Stochastic Processes Relying on Monte Carlo-Based Peak Factor Estimation. *Earthq. Struct.* **2012**, *3*, 581–609. [CrossRef]
33. Cacciola, P.; D’Amico, L. Response-Spectrum-Compatible Ground Motion Processes. In *Encyclopedia of Earthquake Engineering*; Springer: Berlin/Heidelberg, Germany, 2015; pp. 1–27.
34. Seismosoft. SeismoArtif—A Computer Program for Generating Artificial Earthquake Accelerograms Matched to a Specific Target Response Spectrum. 2018. Available online: <http://www.seismosoft.com> (accessed on 7 July 2018).
35. Kapasakalis, K.; Antoniadis, I.; Sapountzakis, E. Novel Vibration Absorption Systems with Negative Stiffness Elements for the Seismic Protection of Structures. In Proceedings of the 4th National Conference on Earthquake Engineering and Engineering Seismology (Hellenic Association for Earthquake Engineering—HAEE/ETAM), Athens, Greece, 5–7 September 2019.
36. Geem, Z.W.; Kim, J.H.; Loganathan, G.V. A New Heuristic Optimization Algorithm: Harmony Search. *Simulation* **2001**, *76*, 60–68. [CrossRef]