

Article

Vibration Source Signal Separation of Rotating Machinery Equipment and Robot Bearings Based on Low Rank Constraint

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Abstract: With the development of industrial robots and other mechanical equipment to a higher degree of automation, mechanical systems have become increasingly complex. This represents a huge challenge for condition monitoring. The separation of vibration source signals plays an important role in condition monitoring and fault diagnosis. The key to the separation method of the vibration source signal is prior knowledge, such as of the statistical features of the vibration source signal, the number of vibration sources, and so forth. However, effective prior knowledge is difficult to obtain in engineering applications. This study found that low rank is a common feature of rotating machinery vibration source signals. To address the problem of the difficulty obtaining the signal feature of a vibration source, the multi-low-rank constrained vibration source signal separation method was proposed. Its advantages and effectiveness have been verified through simulations and experimental tests. Compared with the blind source separation method of independent component analysis (BSS-ICA) and the ensemble empirical mode decomposition (EEMD) methods, it obtained better clustering results and higher signal-to-signal ratio (SSR) values.

Keywords: fault diagnosis; low rank; vibration source signal; separation; robotic bearings



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1. Introduction

When mechanical equipment is utilized for long periods of time (more than 12 months), the parts in the system (such as the joint bearings) become more likely to sustain damage [1]. Condition monitoring and fault diagnosis can effectively reduce the fault rate of mechanical equipment. Vibration analysis is one of the main methods by which to obtain condition information on key parts of machinery equipment.

The separation of vibration source signals is an interesting and difficult problem in condition monitoring which is based on vibration analysis. The signal collected from the rotating machinery system is a multicomponent signal [2], which contains the environmental noise, the vibration component excited by the internal excitation source of the machine, and the noise of the acquisition system itself. Only the vibration component excited by the internal excitation source of the machine can reflect its condition. In this paper, the vibration component excited by the internal excitation source of the machine is called the vibration source signal. When the vibration signal contains noise, it is difficult to obtain the condition information on key parts of the mechanical equipment. If this information is directly used for condition monitoring and fault diagnosis, an incorrect diagnosis result will be obtained [3,4]. Therefore, separating the vibration source signal of the mechanical equipment can significantly improve the accuracy of condition monitoring.

The existing vibration source signal separation methods for rotating machinery include the denoising method and blind source separation (BSS).

The denoising separation method is widely applied in condition monitoring. The signal features and filters (e.g., time domain, frequency domain, or time-frequency domain filters) are used to separate signals of interest from multiple component signals. First, the mixed vibration signal is decomposed into multiple components. Second, some components are discarded or retained, according to the prior features of the signal. Finally, the retained components are reconstructed as the vibration source signal. For example, Wang et al. [5] used the complex wavelet transform to separate the fault vibration source signal. Wang et al. [6] proposed an enhanced adaptive noise denoising algorithm. In this, the noise feature is used to separate the gear noise component and retain the bearing fault component. M.G.A. et al. [7] proposed a framework using the sailfish optimization (SO) algorithm and Gini index (GI) as a criterion to adaptively select the optimum variational mode decomposition (VMD) parameters for each fault signal. For this method, the bearing fault signal is automatically separated based on maximum GI values. Cui et al. [8] proposed a novel compound fault separation algorithm based on parallel dual-Qfactors and improved maximum correlation kurtosis deconvolution (IMCKD), and separated the inner ring and outer ring bearing fault signals. However, the separation effect of this method is determined by the accuracy of dual-Qfactors. The above studies in the literature reflect that accurate signal prior features are the key to separating the source signal in the denoising separation method.

Since the 21st century [9,10], the BSS method has been applied to mechanical fault diagnosis. Cédric et al. [11] proposed a novel perspective on blind filtering of vibration signals with the purpose of fault detection in rotating machinery, and successfully separated the vibration source signals of the bearing outer ring fault. Zhang et al. [12] proposed an adaptive blind source separation method to solve the non-stationary problem, and applied it to separate wheel fault vibration source signals. W et al. [13] proposed a sparse component analysis (SCA) method based on linear clustering for the purpose of underdetermined blind source separation of vibration signals, and separating rolling bearing fault signals. In order to solve the underdetermined BSS problem, Li et al. proposed a method to separate compound signals in the hyperplane space with variational modal decomposition (VMD), and used it for extracting single-channel fault characteristics of compound signals for rolling bearings [14]. The BSS method plays an important role in the separation of the vibration source signal, but the amplitude and phase of the separated vibration source signal are uncertain. This uncertainty is detrimental to the long-term tracking and monitoring of mechanical conditions. The above studies in the literature reflect that the separation effect of BSS is limited by the accuracy of the number of independent sources.

However, the vibration excitation source is complicated in actual engineering, the prior features of the vibration source signal (or noise) are difficult to know, and the number of independent sources is difficult to determine. The requirements of the denoising separation method are difficult to meet, and the separation effect of the BSS cannot be guaranteed. The difficult-to-obtain prior knowledge is one of the key issues in the separation of vibration source signals.

In order to solve this problem, a low-rank constrained vibration source signal separation method is proposed for rotating machinery (e.g., robot bearings, etc.). Low rank is a common feature of rotating machinery vibration source signals. The low-rank feature is used as the target constraint of the source signal separation in this method. The low-rank noise model and the multi-low-rank noise model are described in this study. This method does not need to obtain prior knowledge of the vibration source signal. The effectiveness and superiority of this method are verified by simulations and experimental tests.

2. Theory and Model

2.1. Low Rank of Vibration Source Signals

Because the working process of rotating machinery is periodic, the vibration signal excited by the internal excitation source has repetitiveness. For example, the vibration signal generated by a bearing outer ring crack fault is a series of repeated impact waveforms.

The vibration signal generated by a rotor eccentric fault is a series of repetitive harmonic waveforms. The recurring waveform is called the characteristic waveform in this study. The matrix composed of related data is low rank [15]. Because the repeated characteristic waveforms are correlated, the matrix composed of the characteristic waveforms in the vibration signal from a fault is a low-rank matrix in an ideal state.

In the case of engineering applications, the rank of the matrix obtained from the vibration signal containing noise is higher, or full rank. Low-rank restoration is carried out to eliminate the noise in the vibration signal, making the rank of the matrix lower.

2.2. The Separation Model Based on Low-Rank Constraints

Robust principal component analysis (RPCA) [16,17] is one of the low-rank restoration methods, which, early on, was applied in the field of image processing. It decomposes a matrix D into two matrices: $D = L + E$, where L is a low-rank matrix, and E is a sparse matrix which represents noise.

However, the noise from the vibration signal of mechanical equipment is not sparse. The noise in the vibration signal is mostly Gaussian colored noise and white noise. In order to explore the nature of the noise term in the vibration signal, the Lilliefors method [18] is used for the Gaussian detection of the vibration signal in Section 5. Assume the Gaussian distribution value is 0. If the confidence level is >0.05 , the assumption is true. The test result shows that confidence level is $0.27 > 0.05$. The detection result is that the noise of the mechanical vibration signal has a Gaussian distribution. Therefore, sparsity cannot be used to describe noise in mechanical vibration signals.

Therefore, according to the composition of the mechanical vibration signal, a low-rank constrained vibration source signal separation model is proposed:

$$\begin{aligned} \underset{L,A}{\operatorname{argmin}} \quad & \operatorname{rank}(L) + \beta \times \operatorname{kur}(A) \\ \text{s.t.} \quad & D = L + A \end{aligned} \quad (1)$$

where β is a compromise factor, which can be understood as the weight between the vibration source signal and the noise, and $\operatorname{kur}(\cdot)$ is the kurtosis function. The kurtosis is used to describe the approximate Gaussian degree of the data; the closer the kurtosis of the data is to 0, the higher the Gaussian degree. Because the rank function is non-convex, it is a non-deterministic polynomial problem (NPP). The nuclear norm is a convex approximation of the rank function [19]. Therefore, through convex relaxation, the model can be transformed into:

$$\begin{aligned} \underset{L,A}{\operatorname{argmin}} \quad & \|L\|_* + \beta \times \operatorname{kur}(A) \\ \text{s.t.} \quad & D = L + A \end{aligned} \quad (2)$$

where $\|L\|_*$ is the nuclear norm of the L .

2.3. The Separate Model of Multiple Low-Rank Constraints

The vibration signal obtained by the test generally contains multiple vibration source signals. The separation model of Equation (2) can only be used to separate one vibration source signal. Therefore, the separation requirement cannot be satisfied.

In order to separate multiple vibration source signals at the same time, a multi-low-rank constrained vibration source signal separation model is proposed, as follows:

$$\begin{aligned} \underset{L,A}{\operatorname{argmin}} \quad & \sum_{i=1}^n \lambda_i \|L_i\|_* + \beta \times \operatorname{kur}(N) \\ \text{s.t.} \quad & x = \sum_{i=1}^n L_i + N \end{aligned} \quad (3)$$

where n is the number of vibration source signals, which need to be separated, λ_i is the weight of the i -th low-rank matrix, and N is the noise term. If the length of a one-dimensional vibration signal $x(t)$ is m , the low-rank matrix L_i can be obtained by

$L_i = w(x, o_i, p_i): \mathbb{R}^m \rightarrow \mathbb{R}^{o_i \times p_i}$. w is a block operator, and the calculation method is shown in Figure 1. The block operator w can realize the transformation from one-dimensional vector $x(t)$ to two-dimensional matrix L_i . The vibration source signal is divided into blocks according to the characteristic frequency of the characteristic waveform, where o is the length of a low-rank matrix column, p is the number of low-rank matrix rows, o and p can be calculated by Equations (4) and (5), and l_i is an inverse operation of L_i by block operator.

$$o_i = \frac{f_s}{f_{source\ i}} \quad (4)$$

$$p_i = \frac{m}{o_i} \quad (5)$$

where f_s is the sampling frequency, and $f_{source\ i}$ is the characteristic frequency of the vibration source signal.

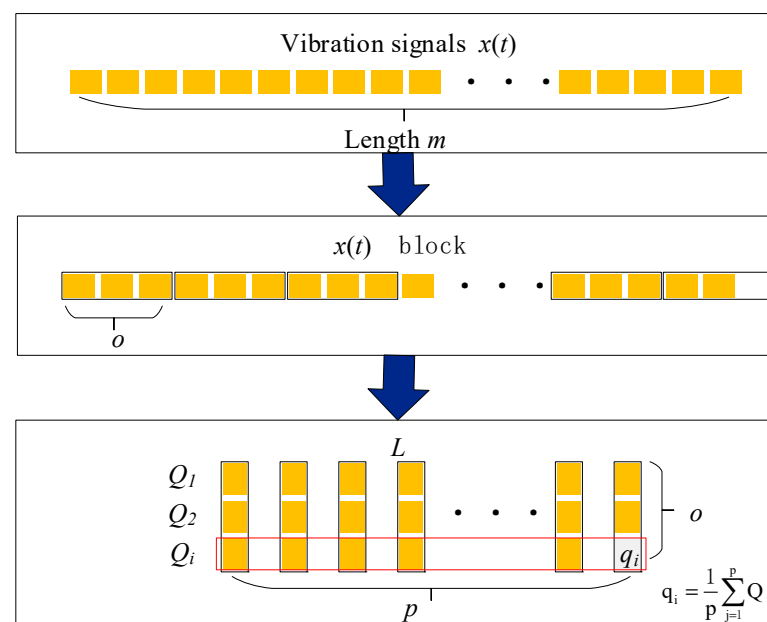


Figure 1. The calculation method of the block operator w .

The model is a can convex optimization problem, and it can be solved by the augmented Lagrange multiplier method. First, we constructed the augmented Lagrange function La :

$$La(l_i, N, Y, \mu_k) = \sum_{i=1}^n \lambda_i \|L_i\|_* + \beta \times kur(S) + \left\langle Y, x - \sum_{i=1}^k l_i - N \right\rangle + \frac{\mu_k}{2} \left\| x - \sum_{i=1}^k l_i - N \right\|_F^2 \quad (6)$$

In Equation (6), La is the augmented Lagrange function, Y is the Lagrange multiplier, μ is the iteration step size, and $\langle \rangle$ is the point product computation. The constrained objective optimization problem is transformed into the minimum value problem of the augmented Lagrange function La , and the expression of the optimal solution is as follows:

$$(l_i^*, N^*) = \underset{l_i, N}{argmin} (La(l_i, N, Y, \mu_k)), i = 1, 2, \dots, n \quad (7)$$

where (l_i^*, N^*) is the optimal solution of (l_i, N) .

The optimization problem is solved by the alternating direction multiplier method. Each step of the iterative process can be decomposed into the optimization problem for

finding the minimum nuclear norm of each low-rank matrix L_i and the minimum kurtosis of the noise term. The iteration process is as follows:

$$l_{i,k+1}^* = \arg_{l_i} \left\{ \frac{\lambda_i}{\mu_k} \|L_i\|_* + \frac{1}{2} \left\| x - \sum_{j=1}^{i-1} l_{j,k} - l_i - \sum_{j=i+1}^n l_{j,k} - N_k + \frac{1}{\mu_k} Y_k \right\|_F^2 \right\} = \arg_{l_i} \left\{ \frac{\lambda_i}{\mu_k} \|L_i\|_* + \frac{1}{2} \|l_i - T_{i,k}\|_F^2 \right\} \quad (8)$$

$$N_{k+1}^* = \arg_n \left\{ \frac{\beta}{\mu_k} \text{kur}(N) + \frac{1}{2} \left\| x - \sum_{j=1}^q l_{j,k} - N + \frac{1}{\mu_k} Y_k \right\|_F^2 \right\} \quad (9)$$

The suboptimization problem of Equation (8) can be solved by singular value decomposition (SVD) and soft threshold operations:

$$[U_i, S_i, V_i] = \text{SVD}(w(T_{i,k}, o_i, p_i)) \quad (10)$$

$$l_{i,k+1} = \text{reshape} \left(U_i S_{\lambda_i/\mu_k} [S_i] V_i^T, 1, o_i \times p_i \right) \quad (11)$$

where $\text{SVD}()$ is the singular value decomposition operator, and $S_{\lambda_i/\mu_k}[]$ is the soft threshold shrinkage operator for the positive parameter λ_i/μ_k .

For the suboptimization problem in Equation (9), the penalty term $\left\| x - \sum_{j=1}^q l_{j,k} - N + \frac{1}{\mu_k} Y_k \right\|_F^2$ can be transformed into $x + \frac{1}{\mu_k} Y_k = \sum_{j=1}^n l_{j,k} + N$ to obtain the equivalent optimization subproblem, as follows:

$$\min_n \text{kur}(N), \text{ s.t. } x + \frac{1}{\mu_k} Y_k = \sum_{j=1}^n l_{j,k} + N \quad (12)$$

The problem of Equation (12) can also be solved by singular value decomposition and soft threshold operations, as follows:

$$[U_N, S_N, V_N] = \text{SVD} \left(w \left(x + \frac{1}{\mu_k} Y_k, o_1, p_1 \right) \right) \quad (13)$$

$$N_{k+1} = N_k - \text{reshape} \left(U_N S_{\beta} [S_N] V_N^T, 1, o_i \times p_i \right) \quad (14)$$

where $S_{\beta}[]$ is the soft threshold shrinkage operator of the positive parameter β .

Equations (10), (11), (13), and (14) can be used to solve Equations (8) and (9) by updating $l_{i,k+1}$. The Lagrange operator Y_{k+1} and threshold α_{k+1} need to be updated by:

$$Y_{k+1} = Y_k + \alpha_k \left(x - \sum_{i=1}^q l_{i,k+1} - n_{k+1} \right) \quad (15)$$

$$\alpha_{k+1} = \rho \alpha_k, \quad \rho = 1.5 \quad (16)$$

3. The Separation Method Based on Multi-Low-Rank Constrained

According to the multi-low-rank model and its solving method in Section 2.3, the multi-low-rank constrained vibration source signal separation method is proposed. The separation method is mainly composed of two parts: determining the vibration source and separating the vibration source signal. The schematic diagram of the separation method is shown in Figure 2.

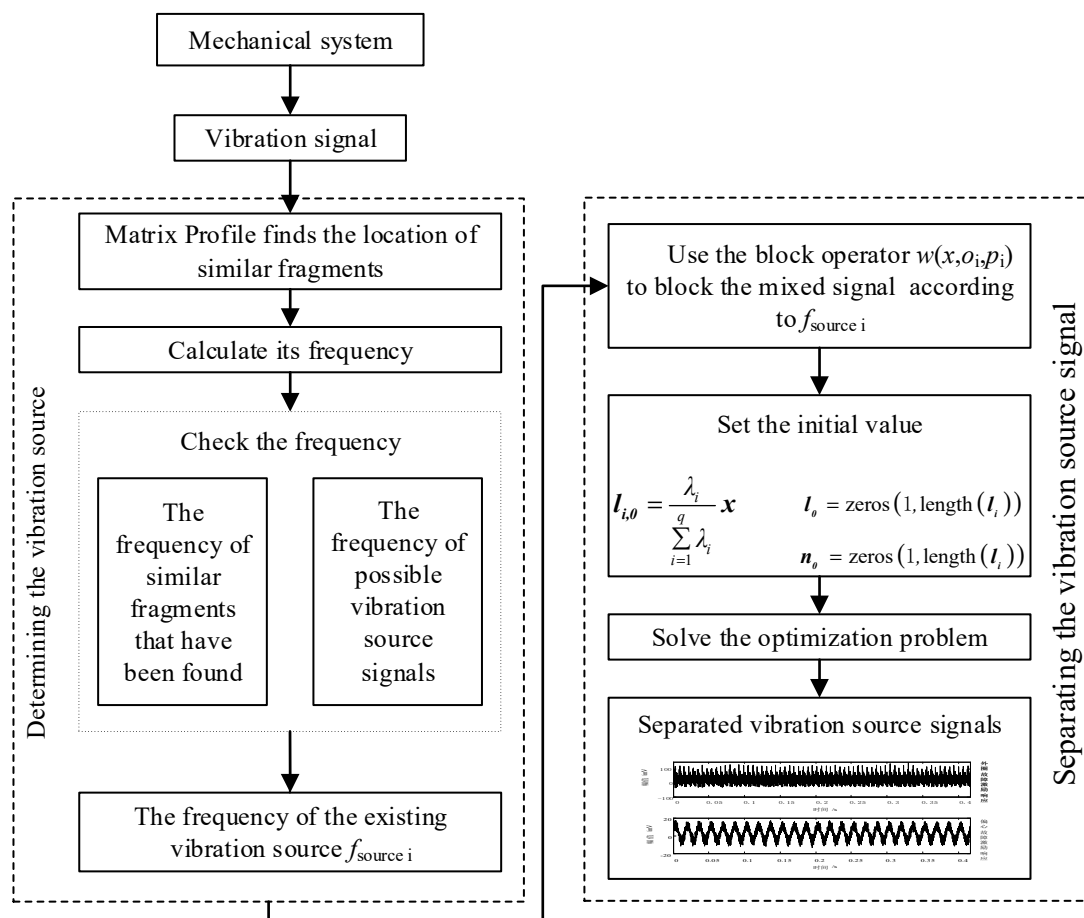


Figure 2. The process of the vibration source signal separation method with multiple-low-rank constraints.

The purpose of the determination vibration source part is to find the vibration source signal in the collected vibration signal. To determine the vibration source, firstly, the Matrix Profile algorithm [20] is used to find some similar fragments in the vibration signal. The Matrix Profile algorithm is an algorithm for finding similar fragments in a time series, which can quickly find the most similar subsequence pair (motif) in the time series. Then, the frequency of similar fragments that have been found and the frequency of possible vibration source signals are checked. If the frequencies are equal, it means that the vibration source signal exists in the mixed signal.

In the separating vibration source signal part, the vibration source signal can be separated from the collected vibration signal by the multi-low-rank model. When this method is used to separate the vibration source signals, λ_i is the regularization parameter for balancing, and is fixed to $1/\sqrt{\max(o_i, p_i)}$. β is usually set to 1.

4. Simulation Analysis

In order to explore the separation effect of the multi-low-rank constrained vibration source separation method, the simulation signals of the bearing inner ring and outer ring crack faults were constructed. The impact caused by a bearing crack fault can be modeled by the single-degree-of-freedom mass-spring-damper impact response [21,22].

$$s(t) = Ae^{-\alpha t} \sin \omega_r(t)u(t) \quad (17)$$

The fault-bearing vibration signal model can be expressed as [23,24]:

$$x_b(t) = \sum_{m=1}^M A_m e^{-\alpha(t-mT_p)} \sin(2\pi\omega_r(t-mT_p))u(t-mT_p) \quad (18)$$

where A_m is the amplitude of the m -th fault impulse, and $u(t)$ is the unit step function. T_p is the spacing of adjacent impulses caused by bearing fault, α is the structural damping factor, ω_r is the resonance frequency induced by the bearing fault.

The simulation signals of the bearing inner ring and outer ring crack faults can be defined as:

$$x(t) = x_{bo}(t) + x_{bi}(t) + ns(t) \quad (19)$$

where $ns(t)$ is white noise, and f_s is the sampling frequency.

The parameters of the simulation signal are listed in Table 1. The characteristic frequency of the crack fault of the bearing outer ring f_o is 79.2 Hz, and the characteristic frequency of the crack fault of the bearing outer ring f_i is 138.9 Hz from Table 1.

Table 1. Simulation parameters.

Parameter	Value	Parameter	Value
A_{mo}	1.5 V	A_{mi}	1 V
α_o	1200	α_i	800
T_{Po}	1/79.2 s	T_{Pi}	1/138.9 s
ω_r	2800 Hz	f_s	24 kHz
f_o	79.2 Hz	f_i	138.9 Hz

The simulation signal is shown in Figure 3. The impulse waveform caused by the inner ring and outer ring faults was not observed in the mixed signal from Figure 3.

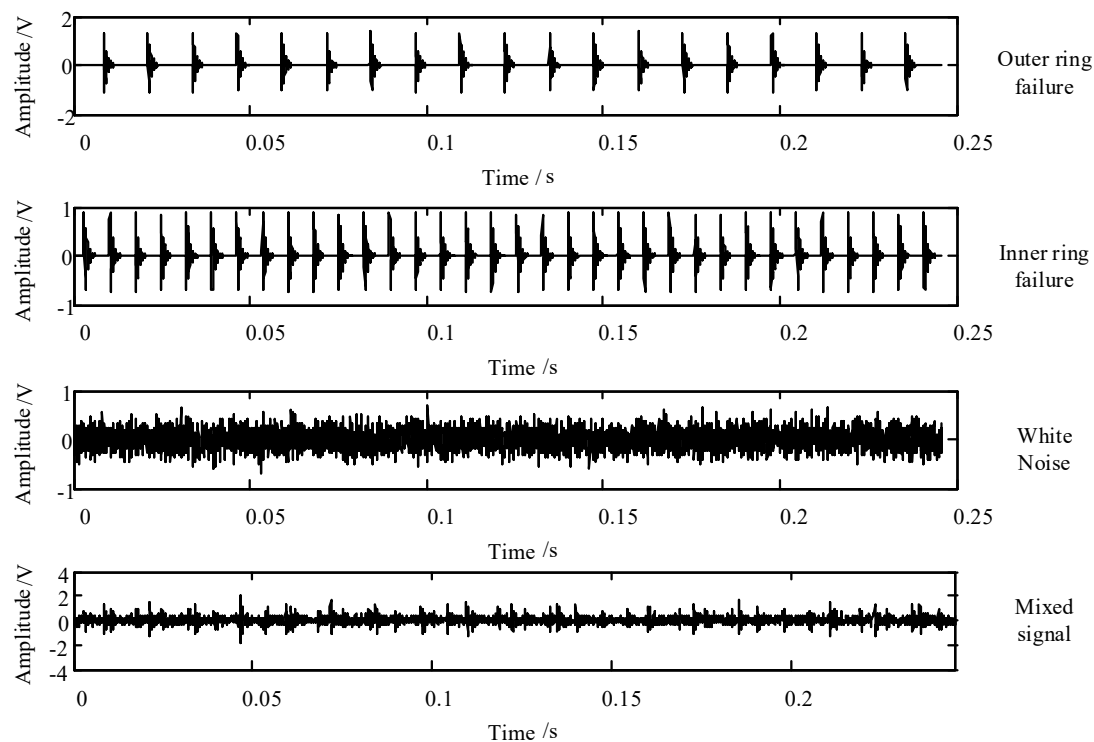


Figure 3. Time domain waveforms of the simulated signal.

The vibration source signal separation method with low-rank constraint was used to separate the bearing fault vibration source signal in the simulation signal. The separation result is shown in Figure 4. The crack fault signals of the inner ring and outer ring are separated, and the pulse waveform can be clearly observed from Figure 4.

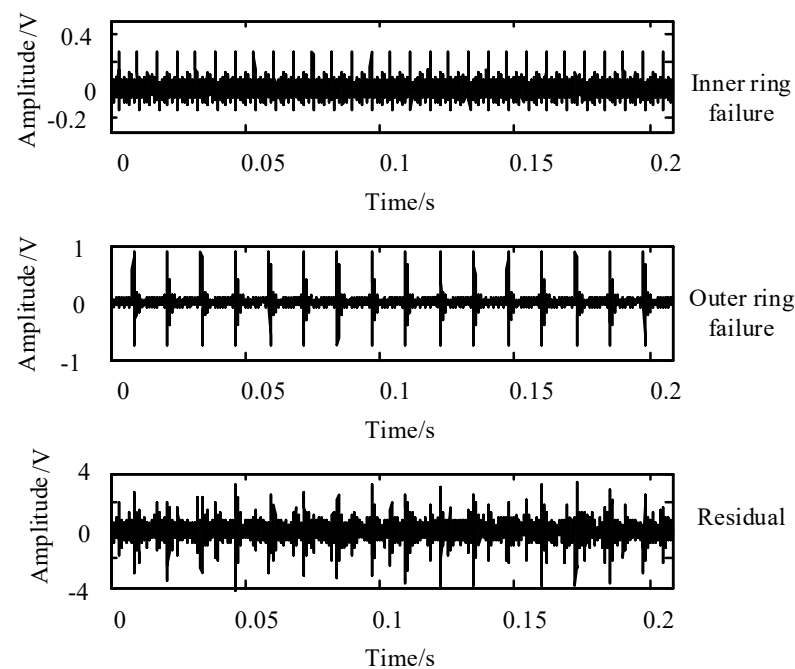


Figure 4. The separated time domain waveform.

In order to observe the components of the separated inner and outer ring crack fault signals, their envelope spectrum was calculated separately, as shown in Figure 5. There were peaks at the first harmonic and the second harmonic of the fault characteristic frequency in the envelope spectrum, as can be seen in Figure 5. This verified that this method can effectively separate the inner ring and outer ring crack fault signals.

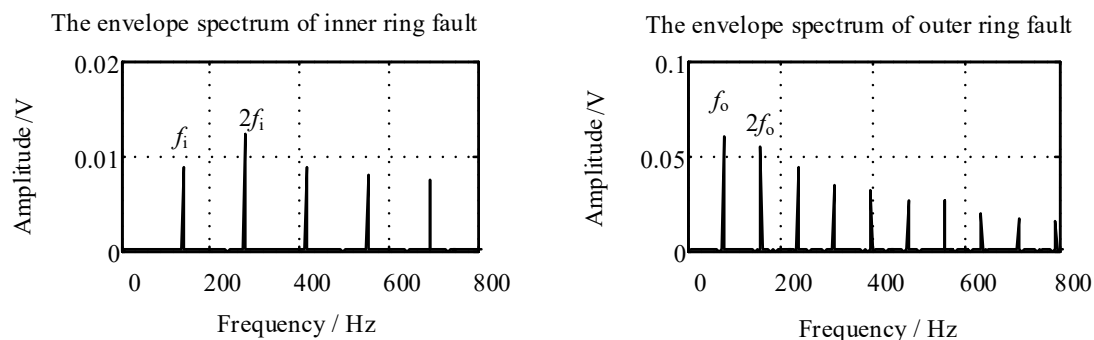


Figure 5. The envelope spectrum of inner and outer ring faults.

5. Experimental Analysis

5.1. Experimental Verification

The measured signal was collected by the ST5000A rolling bearing vibration test bench, as shown in Figure 6. The rolling bearing vibration test bench was installed with SKF-6000 deep groove ball bearings. The KSWL-3806-20 encoder and the tachometer were installed at the end of the rotating shaft in the test bench. The TDGC2-0.5k VA contact voltage regulator was used to control the speed of the AC motor. Finally, the CA-YD-1181 acceleration sensor, YE6231 data acquisition card, and matching acquisition software were used to collect the signals synchronously.

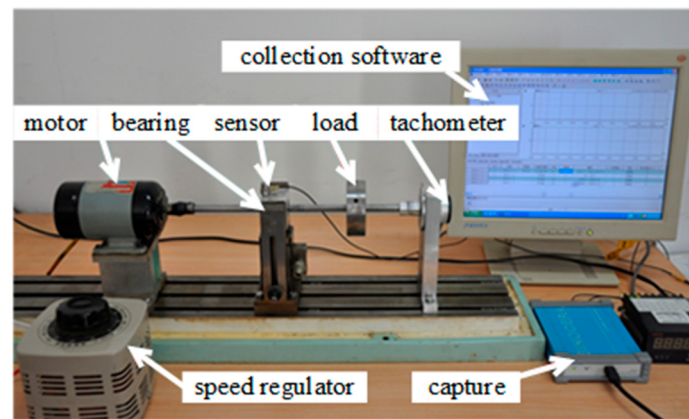


Figure 6. The bearing experimental bench ST500A.

The measured vibration signals mainly included outer ring faults and eccentric fault. The environmental noise and noise from the acquisition system itself are mixed into the vibration signal during the acquisition process. The outer ring crack of the bearing was artificially faulted by electrical discharge machining (EDM). The load was an eccentric flywheel. The sampling frequency is 24 kHz, and the rotation speed of the bearing f_r is 3600 rpm, and the geometric parameters of the bearing are shown in Table 2. The characteristic frequency of the outer ring fault f_o was 152.9 Hz, and the characteristic frequency of outer ring fault f_e was 60 Hz.

Table 2. The bearing structure parameters.

Parameter Name	Value
Type	6000
Number of Rolling Elements	7
Contact Angle	0°
Pitch Diameter D/mm	17.65
Ball Diameter d/mm	4.8
Characteristic Frequency of Outer Ring Fault f_o /Hz	$2.548f_r$
Characteristic Frequency of eccentric Fault f_e /Hz	$1f_r$

The time-domain waveform of the collected vibration signal is shown in Figure 7. The characteristic waveforms of the outer ring fault (impact-waveform) and the eccentric fault (sine-waveform) cannot be distinguished from Figure 7.

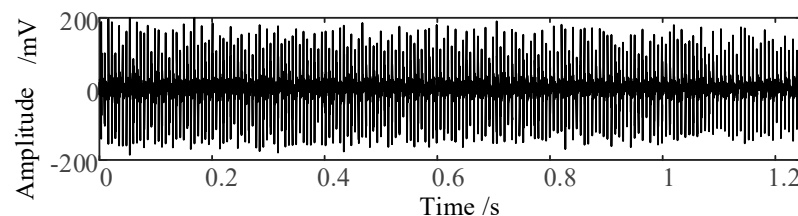


Figure 7. The time domain waveform of the collected vibration signal.

The test signal was separated by the method proposed in this article, and the separation result is shown (Figure 8). The waveform of the signal corresponding to the outer ring fault is a series of impact-waveforms, and the waveform of the eccentric fault signal is a continuous sine-waveform from Figure 8. The waveforms of these two signals are consistent with the actual situation.

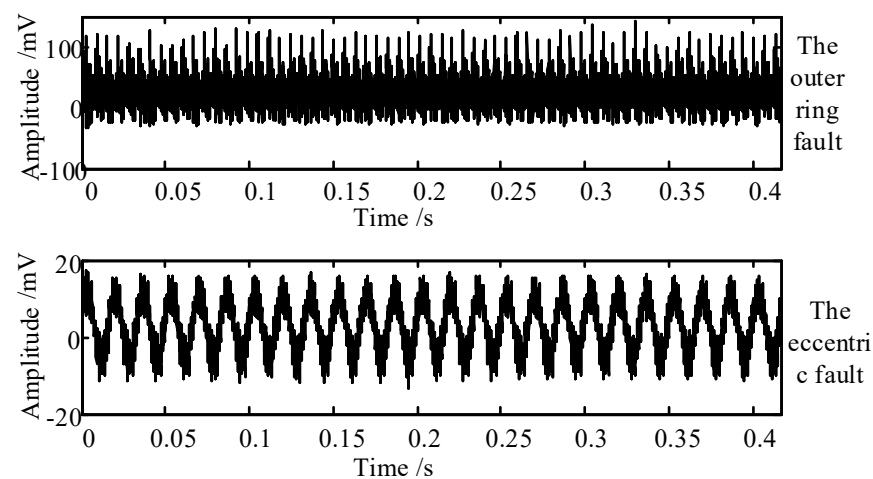


Figure 8. The separation results.

In order to analyze the components of the separated two fault vibration sources, their envelope spectrums were calculated, as shown in Figure 9. f_o is the characteristic frequency of the outer ring fault, and f_e is the characteristic frequency of the eccentric fault. From Figure 9, there are peaks at the first harmonic and the second harmonic of fault characteristic frequency in the envelope spectrums. The method presented in this article proved to be effective in separating the eccentric and outer ring crack fault signals.

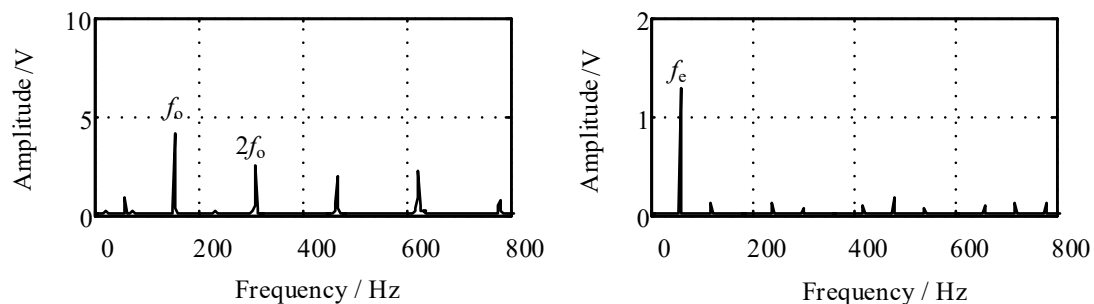


Figure 9. The envelope spectrum of the outer ring fault and eccentric fault.

5.2. Comparative Analysis

In order to verify the advantages of this method, the blind source separation method of independent component analysis (BSS-ICA) [25] and ensemble empirical mode decomposition (EEMD) [26] were used for comparison experiments. The separation effect was compared by considering four aspects: time-domain waveforms, envelope spectrum, signal-to-signal ratio (SSR), and fault classification.

(1) The time-domain waveforms

The experimental results of the BSS-ICA and the EEMD are shown in Figure 10. From Figure 10, the waveform of the signal corresponding to the outer ring fault is a series of impact-waveforms, but the waveform of the eccentric fault signal is a consecutive sine-waveform according to the EEMD. The waveform of the signal corresponding to the outer ring fault is a series of impact-waveforms, but the waveform of the eccentric fault signal is not a continuous sine-waveform according to the BSS-ICA. However, this assumption is not consistent with reality.

The amplitude of the separated signal is 400 mV, which is much larger than the original signal. From Figure 10, the amplitude of the separated signal according to the BSS-ICA is 5 mV, which is much smaller than the original signal. The separated time-domain waveform has distortion problems by the EEMD and BSS-ICA.

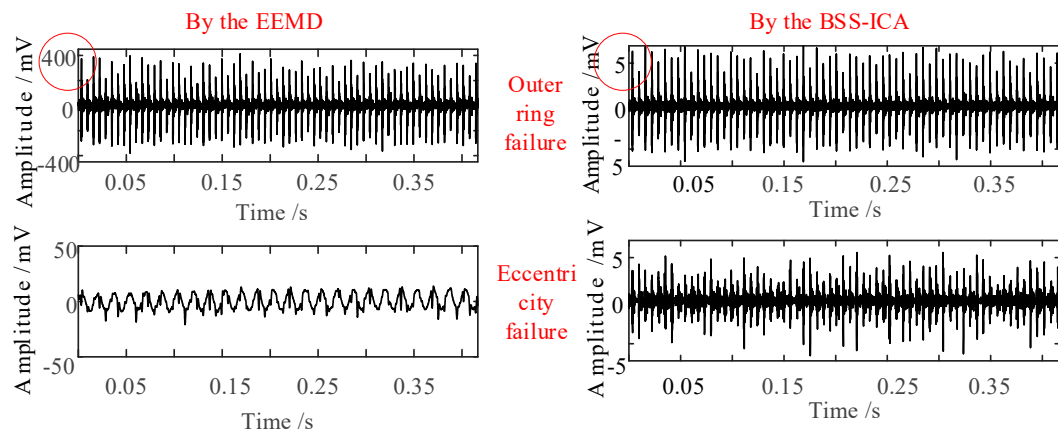


Figure 10. The separated time domain waveforms by the EEMD and BSS-ICA.

(2) Envelope spectrum

The corresponding envelope spectra are shown in Figure 11. f_o can be obtained in the envelope spectrum of the outer ring fault, and f_e can be obtained in the envelope spectrum of the eccentric fault by EEMD. This shows that EEMD can separate the two vibration source signals corresponding to two faults. At the same time, f_o and f_e can be obtained in the envelope spectrum of the outer ring fault and the eccentric fault by BSS-ICA. This shows that BSS-ICA cannot separate the two vibration source signals corresponding to two faults.

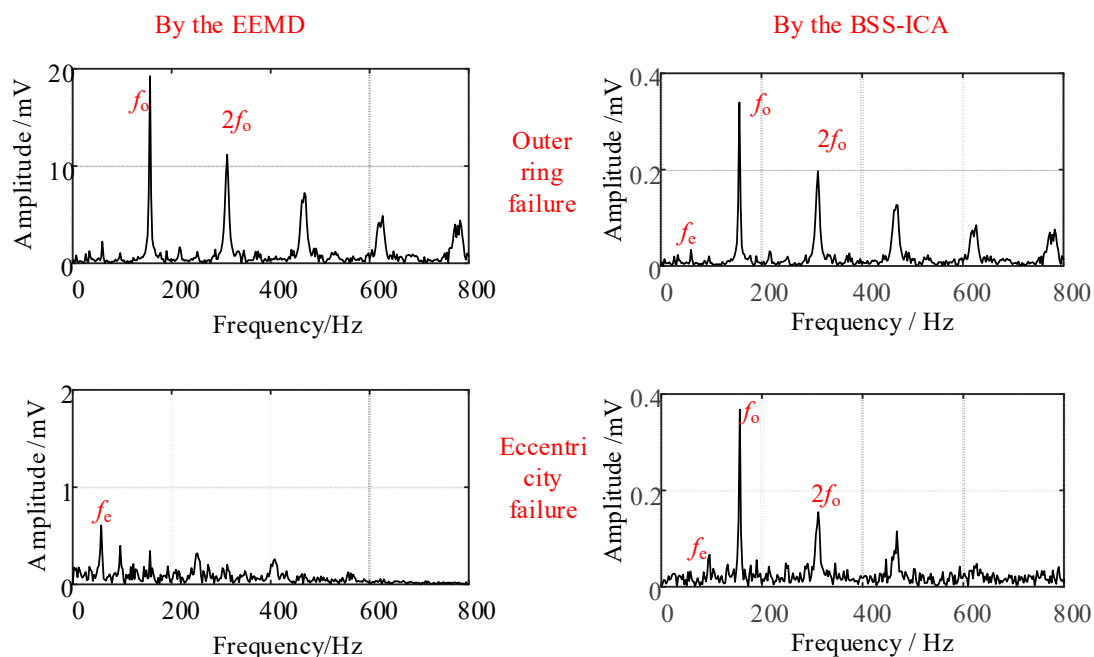


Figure 11. The envelope spectrums by EEMD and BSS-ICA.

(3) The signal-to-signal ratio (SSR)

In order to quantitatively describe the separation results, the signal-to-signal ratio (SSR) method [27] was used to evaluate the quality of the separated signal. The mathematical expression of the SSR is as follows:

$$SSR_i = \min_{1 \leq j \leq q, j \neq n-i} \left(10 \log \frac{PSD_i}{PSD_j} \right) \quad (20)$$

where PSD_i and PSD_j are the power spectral densities of signals i and j in the “home” spectrum of signal i , respectively [27].

The SSR values of the two separated vibration source signals obtained by the method of this paper were 5.6 and 3.2. The SSR values by EEMD were 3.2 and 2.2. The SSR values by BSS-ICA were 1.1 and 0.1. Therefore, the SSR value obtained by the separation method in this study is higher than that obtained by the EEMD method. The SSR value obtained by the EEMD was higher than that obtained by the BSS-ICA method.

(4) The fault classification

In order to compare the effects of the two methods on the fault diagnosis results, a self-organizing map (SOM) was used to cluster the separated vibration source signals. The clustering results are shown in Figure 12. Type 1 represents the vibration source signal of the outer ring fault, and type 2 represents the vibration source signal of the eccentric fault. From Figure 12, the two types of fault vibration source signals could be grouped into two types by the method in this paper. There is a certain amount of aliasing between the two types of faults by EEMD, which has a negative effect on fault diagnosis. However, the two types of faults could not be distinguished by BSS-ICA. Therefore, the method in this paper achieved a better clustering result, and is more advantageous than EEMD and BSS-ICA for fault diagnosis.

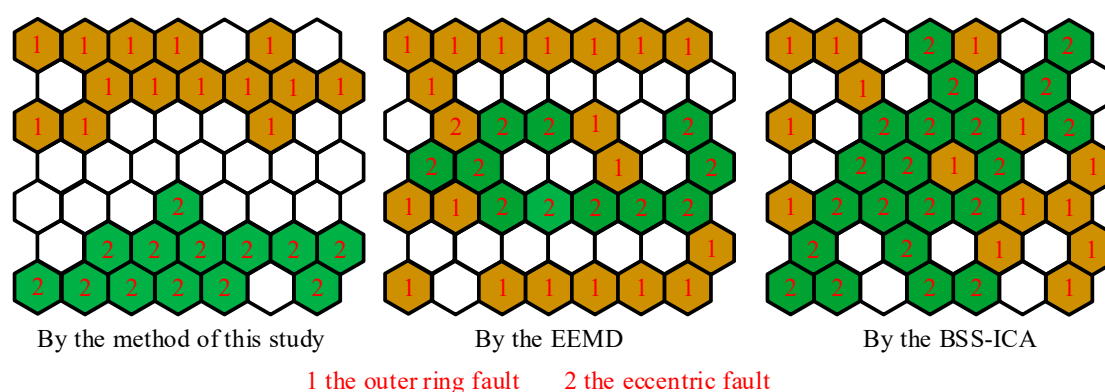


Figure 12. SOM clustering results of separated vibration source signal by the methods of this paper, EEMD and BSS-ICA.

6. Conclusions

As it is difficult to obtain the signal characteristics of the vibration source, the multi-low-rank constrained vibration source signal separation method was proposed and studied in this paper. The low-rank nature can be used as a common feature for the separation of vibration source signals. In this method, only the characteristic frequency of the vibration source signal needs to be given. The characteristic frequency can be calculated from the geometrical dimensions of the mechanical parts. There is no need to obtain the prior knowledge of the vibration source signal in advance. This method provides a reference for fault diagnosis and condition monitoring of rotating machinery and robot bearings.

The advantages and effectiveness of this method were verified through simulation and experimental tests. Compared with the EEMD and BSS-ICA methods, the method of this study obtained better clustering results, higher fidelity, and higher SSR values.

Author Contributions: All the authors contributed to this paper in different ways. Z.H. proposed a separation model, performed experimental verification, and wrote the original draft. W.C. proposed the original concept and provided suggestions for the paper’s improvement. J.X., W.W., and M.L. edited and reviewed the paper. All authors have read and agreed to the published version of the manuscript.

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