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Review

A Review of Thermal Error Modeling Methods for Machine Tools

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Abstract: Thermal error caused by thermal deformation is one of the most significant factors influencing the accuracy of the machine tool. Compensation is a practical and efficient method to reduce the thermal error. Among all the thermal error compensation processes, thermal error modeling is the premise and basis because the effectiveness of the compensation is directly determined by the accuracy and robustness of modeling. In this paper, an overview of the thermal error modeling methods that have been researched and applied in the past ten years is presented. First, the modeling principle and compensation methods of machine tools are introduced. Then, the methods are classified and summarized in detail. Finally, the future research trend of thermal error modeling is forecasted.

Keywords: precision machine tool; machine tool thermal error; thermal error compensation; thermal error modeling method

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1. Introduction

As a symbol of the level of modern machinery manufacturing, precision machine tools are becoming more and more important. The requirements for the accuracy of machine tools are also increasing with the continuous development of industrial technology. There are many factors that affect the accuracy of machine tools such as geometric and kinematic errors, thermal errors, cutting-force induced errors, and other errors such as the tool wear and the errors induced by assembling and chattering [1–3]. According to Bryan's research published in 1990, the error caused by heat accounted for 40% to 70% of the total machining error of the machine tool [4]. In Ramesh et al.'s [5] paper published in 2000, it was found that thermal error had a very significant effect on the machining accuracy, which basically reached 60~70% of the total error. Therefore, it is essential to investigate the method to remove the effect of the thermal error [6].

Thermal error refers to the machining error caused by the thermal expansion of the machine components that leads to the relative displacement between the work piece and tool [7]. Generally, there are two ways to reduce the thermal error at present: error avoidance and error compensation [8]. The error avoidance is an attempt to eliminate or reduce possible sources of thermal errors by improving design and manufacturing methods [9]. For example, changing traditional metal bearings to ceramic bearings to reduce the heat generated by friction during bearing operation. In addition, using carbon fiber reinforced plastics (CFRP) or other materials with a low thermal expansion coefficient to construct machine tool parts helps to prevent thermal deformation when the temperature of the machine tool rises. Moreover, the thermally symmetric design of machine structure, the separation of heat sources, room temperature-controlled workshop [10], etc. are other methods to reduce the thermal error [11,12]. However, the use of these methods will increase the cost so that it is hard to be applied in practice. The error compensation is mainly to master the influence of the key heat source temperature of the machine tool on the thermal error through analysis, statistics, and induction. By studying the functional relationship between the thermal error and the key heat source temperature, the thermal error model is obtained. Based on this model, the thermal error is predicted and then compensated [13]. Compared with the error avoidance method, it is a more convenient and cost-efficient way to reduce the thermal error by compensation [14,15]. The thermal error compensation principle diagram of a CNC machine tool spindle is shown in Figure 1. By inserting the compensation signal generated from the thermal error model into the feedback loop of the servo system, the error compensation is achieved.

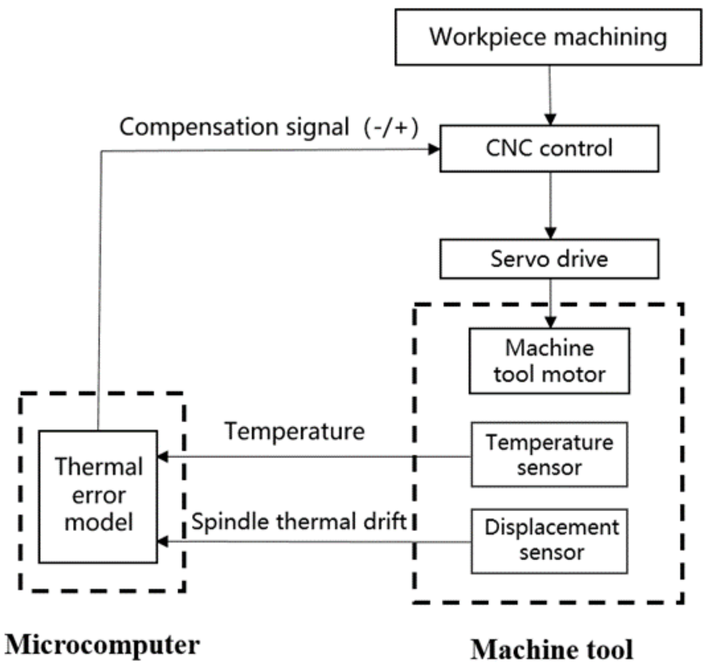

Figure 1. Thermal error compensation principle diagram.

In this paper, the existing thermal error modeling methods for machine tools that have been researched and applied in the past ten years are introduced, classified, and summarized. The classified modeling methods are cited in the article according to the time of publication. The advantages and disadvantages of these modeling methods are compared in Section 3, and the future research trend is forecasted in Section 4.

2. Thermal Error Modeling Method for Machine Tools

As early as the mid- to late-1980s, the research on thermal error modeling of machine tools has emerged. In the research of Moriwaki [16] in Japan, a tool-work piece relative displacement compensation control system was established to compensate the thermal deformation error of the spindle caused by the heat generated from the hydrostatic oil bearing during the rotation process. In the 1990s, Indonesian scholars Chen and Hsu [17] proposed a time-variant volumetric error model to comprehensively compensate the geometric and thermal errors of the machining center. Based on this model, the accuracy of the machining center was improved by an order of magnitude. Since the 21st century, a variety of mathematical modeling methods such as multiple regression, artificial neural network, time series modeling, etc., have been applied to develop the thermal error model of machine tools. According to the principles of the different mathematical methods, the modeling methods including the least square method, multivariable regression analysis, Grey system, neural network, support vector machine, hybrid model, etc., will be introduced in order.

2.1. Least Square Method (LS)

The least square method is used to seek the best function matching of data by minimizing the sum of squares of errors. Its general form is shown in Equation (1).

$$y = a_1 f_1(x) + \cdots + a_k f_k(x) \tag{1}$$

By searching for the reasonable parameters $a_1 \ldots a_k$, the sum of squares of the error between calculated value y and measured value x is minimized [18]. The fitting modeling based on the LS is an earlier way to build the thermal error model of machine tools. Because of its simple principle, it is easier to applied for compensation in practical.

With the use of Chebyshev polynomial-based orthogonal least squares regression, the thermal drift error curve of the spindle was fitted [19] in 2015. The obtained curve was in good agreement with the measurement error curve. In detail, the maximum modeling residuals in the X, Y, and Z directions were 0.8–1 μm, 0.6–0.9 μm, and 1.2–1 μm, respectively. In 2016, Christian et al. [20] determined the best position of temperature sensor by judging the covariance characteristic value of the least square. After thermal error compensation, the maximum approximate error of the tool center point was reduced up to 25%. In 2020, Liu and his colleagues [21] used the least square method to establish a thermal error model of the machine tool spindle in the axial and radial direction which was used for compensation afterwards. The experiment showed that the contour accuracy of the work piece was increased by 78.4% after compensation.

2.2. Multivariable Regression Analysis (MRA)

Multiple regression is another common method to establish a linear or non-linear mathematical model between one dependent variable and one or more independent variables through statistical analysis of the experimental sample data [22,23]. Compared with the least square method, the multiple regression model has more independent variables and is more coordinate to the actual situation. Therefore, the thermal error model established by this method has higher accuracy and robustness.

The basic model of linear multivariate regression thermal error [24] is shown in Equation (2),

$$Y = \beta X + \varepsilon \tag{2}$$

where Y represents the thermal error; X is the temperature; β is the coefficient; and ε is the residual. Multivariate regression modeling is used to find the right coefficient β to ensure that the residual is equal at each data point and the objective function value $\sum_{i=1}^{n} (y_i - {x_i}^T \beta)^2$ is minimized.

In 2011, Pajor and Zapłata [25] established an analytical model for the spindle based on the multiple regression model. When the model was applied for thermal error compensation of the spindle, the thermal error of reduced from 73 μm to 13 μm. By replacing the original objective function with the new objective function $\sum_{i=1}^{n} w_i (y_i - {x_i}^T \beta)^2$, an improved linear multiple regression model was proposed by Jian et al. [26,27] from 2011 to 2012. With this method, the thermal error model of the spindle was established and used to reduce the maximum residual error of the spindle in the Z and Y directions from 25 μm and 7 μm to 1.8 μm and 0.7 μm. In addition, in the paper of Yuan et al. [28] published in 2013, a position-independent thermal error model of multiple linear regression for the linear axis was constructed. After compensation based on the model, the maximum fitting residuals of the three errors of Y axis Sy, Sz, and $R\alpha$ were reduced to 9.6 μm, 2.4 μm, and 7.1 μrad, respectively. Similarly, Zhang et al. [29], Shi et al. [30], and Liu et al. [31] also established the linear axis thermal error models based on regression analysis. Among them, based on the homogeneous transformation, Liu et al. [31] proposed a data-driven thermal error compensation method of high-speed and precision five-axis machine tools. The thermal error of the liner axis was expressed as the polynomial function of the time and the position.

In this model, the thermal error and geometric error were separated. Compared with the uncompensated and traditional error compensation situations, the machining errors were reduced 85% and 37%, respectively. The fitting performance of the model is shown in Figure 2.

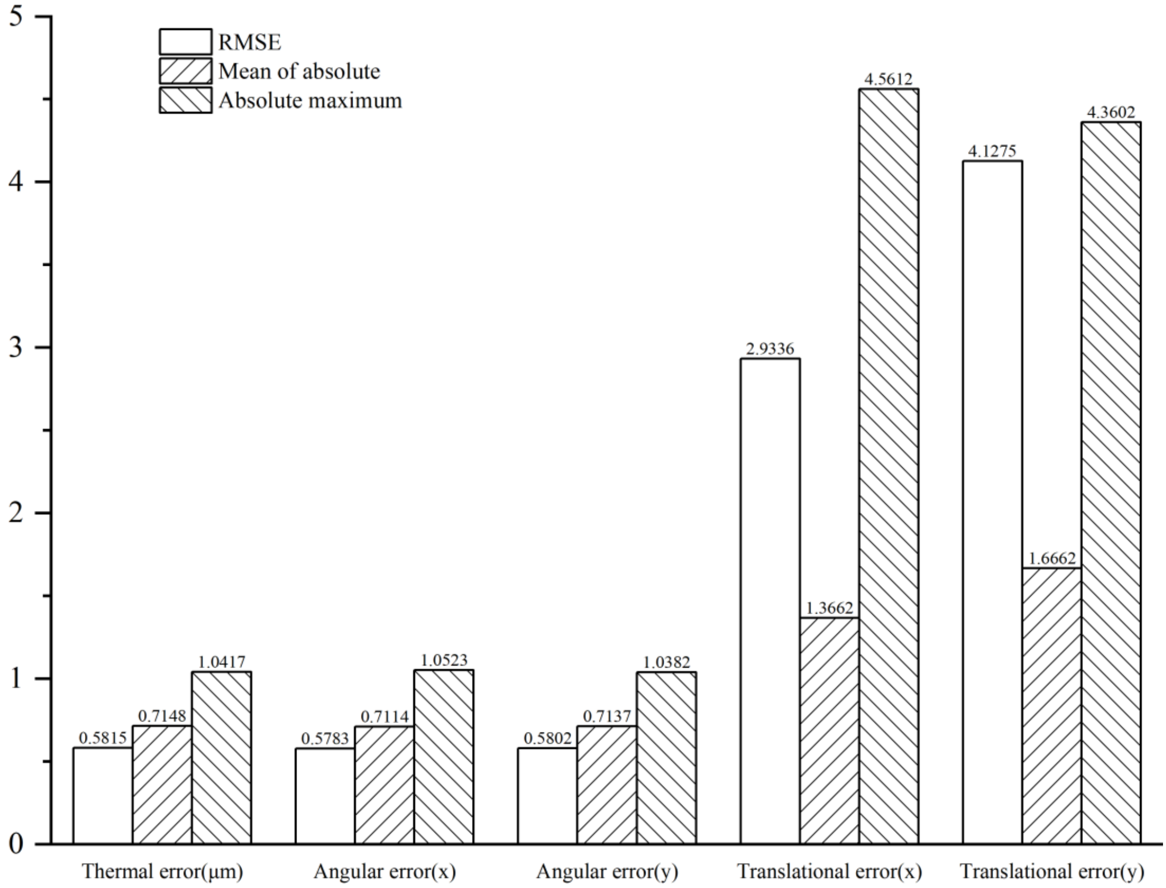

Figure 2. Fitting performance of the MRA model.

Moreover, based on the linear regression analysis, Lin et al. [32] presented a thermal deformation compensation model for lathe machining. The experimental results showed that the thermal errors were reduced to 19μ9 when temperature was rising and to 15 μm when it reached to a stable state.

However, it is worth noting that when there are more variables, the calculation time of MRA is too long. Moreover, the correlation between the thermal error and the temperature is only considered by the MRA, which will lead to the temperature variable coupling and reducing the accuracy of the model [33–35].

2.3. Grey System

In 1982, Professor Deng, a Chinese scholar, established the theory of "grey system". It is a mathematical method that can extract useful information from data with limited quantity and accuracy to obtain the characteristics of the research object [36]. The general grey system model (GM(n,m)) [37], is shown in Equation (3) below.

$$\frac{d^n X_1{}^{(\xi)}(k)}{dt^n} + a_1 \frac{d^{n-1} X_1{}^{(\xi)}(k)}{dt^{n-1}} + \cdots + a_n X_1{}^{(\xi)}(k) = b_1 X_2{}^{(\xi)}(k) + b_2 X_3{}^{(\xi)}(k) + \cdots + b_{m-1} X_m{}^{(\xi)}(k) \quad (3)$$

It is an n-order gray model with m variables where X_1 is the thermal error; $X_2 \ldots X_m$ are the temperature data; N is the order of the differential equation; M is the number of data types; and ζ is the number of transformations. Notably, by changing the number of variables and order of equation, different GM models will be obtained. Among these models, the first-order univariate model GM(1,1) is the most commonly used model.

Modeling based on the grey system theory is simpler, more convenient, and does not depend on massive and complete data information [38]. Based on the standard gray system model GM(1,1), a genetic algorithm was used by Jiang and Yang [39] to optimize the dimensions and variable weights of the new gray system model to minimize the residual value of the optimized model. This optimized model can not only reflect the systematic trend of thermal errors, but can also weaken the influence of the random change of thermal error, thereby the prediction accuracy of the thermal error model was improved. However, considering that the solution of the existing gray prediction model GM(1,N) was not accurate enough, Tien [40] used gray control parameters to develop an improved model GMC(1,N) on the basis of GM(1,N). The improved model was more accurate because its grey differential equation was linear and its solution was based on the superposition principle. In order to further improve the prediction accuracy of grey GMC(1,N) model, Wang et al. [41] proposed a new gray GMC(1,N) machine tool thermal error prediction method based on the CS algorithm named as CS-GMC(1,N) model. This model was compared with the gray model based on particle swarm optimization (PSO) named as the PSO-GMC(1,N) model. It was found that the prediction accuracy of the CS-GMC(1,N) model was higher than that of the PSO-GMC(1,N) model.

2.4. Neural Network (NN)

Neural network is a hot research spot that has emerged from the field of artificial intelligence since the 1980s. In the neural network model, multiple variables are used as input and output. There are three layers (input, output, and hidden) in classical NN model. The input layer accepts external signals and data. The output layer realizes the output of the system processing results, and the hidden layer is a unit that lies between the input and output layers and cannot be observed from the outside of the system. As shown in Figure 3, it is a kind of structure of thermal error compensation model based on a BP neural network with five inputs.

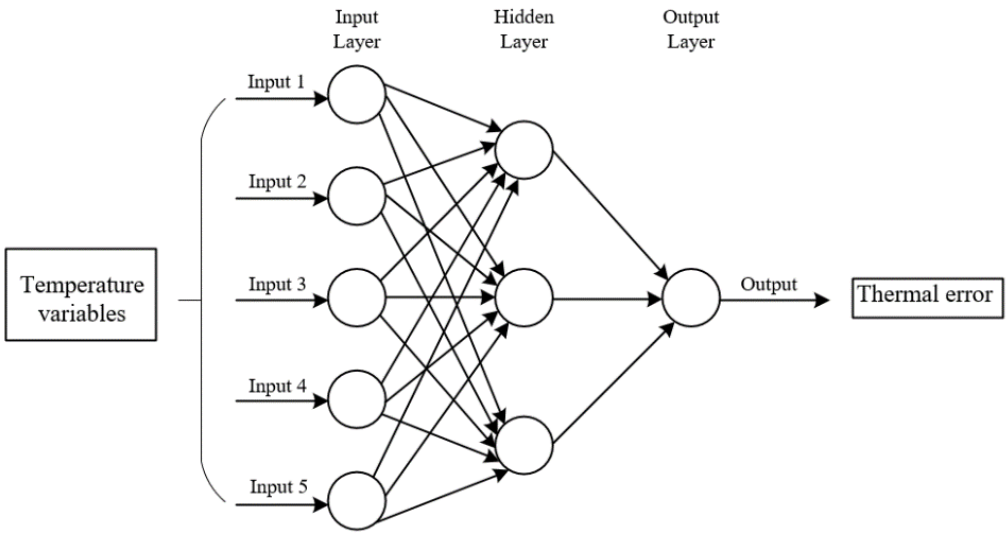

Figure 3. The structure of thermal error compensation based on BP neural network.

Compared with the regression model, the artificial neural network model has good nonlinear fitting performance and better prediction accuracy. As early as the end of the last century, artificial neural networks have been used for the thermal error modeling of machine tools [42]. Usually, the temperature variables tested by multiple sensors are taken as the input and the thermal errors of machine tools are the output of NN model. After the network learning and training, the thermal errors of machine tools in multiple directions can be accurately fitted and predicted [43–45]. Back propagation (BP) and radial basis function (RBF) are two kinds of the most common used neural network methods.

By using a BP neural network, complex nonlinear relations can be fully approximated. BP neural networks have high robustness and fault tolerance ability, so it is widely used for thermal error modeling. However, the convergence speed of the BP neural network is slow and it is easy to fall into value-extremum [46]. In order to solve these problems, Ma and Jiang [47] used a hybrid particle swarm algorithm to optimize the structure of the BP neural network and to avoid unstable prediction performance. Similarly, Chen et al. [48], Yuan et al. [49], Huang et al. [50], and Li et al. [51] used the variable inertia factor particle swarm algorithm to optimize the weights and thresholds of the BP neural network, and build the thermal error model of the machine tool. The improved BP neural network model could better overcome the limitations of the traditional BP algorithm, and the accuracy was relatively high. Among them, the prediction accuracy of Huang's model was 93.155%, and that of Li's model reached to 96.5%. In order to solve the problem that the BP neural network cannot adjust independent parameters of new data in thermal error modeling, Liu et al. [52] proposed an improved BP neural network model based on the overall adjustment strategy. By achieving the minimum value of the square error of the overall data, the prediction accuracy of this improved thermal error model was increased 50%. In the paper of Tan et al. [53], multiple BP neural network models were integrated to enhance the predictive performance. With the improved model on a horizontal machining center THM6380, the root mean squared error (RMSE) of the integrated model was 5.6522 μm. To improve the robustness of the BP neural network, Ren et al. [54] modified the BP network into a dynamic feedback network. With this method, the residual error after thermal error compensation was smaller. The ability dealing with dynamic information was also improved and the training time was greatly reduced. Furthermore, based on the research of the relationship between the thermal error and the actual motion error of machine tool parts, Xin et al. [55] used the "two-step method" to establish the thermal error model of a ball screw. In this method, BP neural network was first used to model the thermal deformation of the screw. Then, the relationship between the axial thermal error of the table and the actual thermal deformation of the screw was obtained based on the polynomial fitting method. However, this method has not been applied for complex machine tools.

The RBF neural network is a feedforward neural network that has good approximation and global optimal performance. However, its key feature functions are more difficult to be extracted than the BP neural network, and its generalization performance is worse [56]. Su et al. [57] used the RBF neural network to build a more accurate thermal error model. But it took a long time to determine the number of neurons in the hidden layer in the algorithm. In order to improve the prediction accuracy of the RBF model, Zhang et al. [58] improved the RBF neural network and constructed the thermal error prediction model by applying the PSO algorithm to optimize the important parameters of the RBF neural network. Lu et al. [59] of Hunan University devoted to solve the problem of low modeling efficiency of the existing thermal error modeling methods and unsatisfactory model prediction accuracy. They proposed a generalized RBF neural network modeling method and applied it to the thermal error modeling of the spindle box of a NC guide rail grinder. Compared with the conventional RBF neural network modeling method, the generalized RBF neural network model had the advantages of high prediction accuracy and generalization ability. Li and Wu [60] developed a fuzzy RBF neural network prediction model for the milling machine spindle. The experimental results showed that the maximum errors of the Y-axis and Z-axis predictions were reduced to 3.5 μm and 2.9 μm after compensation. Zhang et al. [61] of Hua Zhong University of Science and Technology used the fuzzy clustering method and grey relational analysis method to optimize the temperature measurement points, and then established a radial basis function neural network prediction model based on genetic algorithm. Compared with the traditional RBF neural network method, the improved model had higher precision and stronger robustness and the forecasting model residual error ranged from 4.88 to 3.80 μm down to 2.48–2.14 μm.

Grey system theory has the advantage of dealing with an uncertain system with incomplete information and inaccurate data. It can extract useful information from small samples to study the characteristics of the system. In 2012, Zhang et al. [62] combined the gray theory and neural network to establish a parallel gray neural network model (PGNN). After applying it to the spindle on a five-axis machining center, the maximum prediction thermal error was reduced to 3.65 μm. By using the PSO algorithm to obtain the optimal weight threshold of the gray neural network (GNN), Feng and Wang [63] built the spindle thermal error model and the relative error of the prediction was 1.55%. Similarly, Zheng et al. [64] used genetic algorithm to optimize the weight threshold of the grey neural network, and took the average relative error as the evaluation standard of the prediction model, which effectively avoided the problem that the grey neural network fell into the local optimal solution. Considering the influence of cutting conditions on the thermal error of the machine tool, a genetic algorithm gray neural network (GAGN) was used by Chen et al. [65] to optimize the thermal error model of grey neural network. The maximum relative error after compensation was 7.02 μm. However, the training parameters of the thermal error model established by it need to be determined according to the cutting conditions. When the cutting conditions changed, the original model was no longer be applicable, so its versatility was poor.

In addition, there are other improved method based on neural networks such as Fourier neural network, multiple-input multiple-output (MIMO), long short-term memory (LSTM), etc. Wang et al. [66] established a comprehensive mathematical model of milling machine geometric error and thermal error using a Fourier neural network. After compensation, the maximum positioning error was reduced from 20.2 μm to 4.4 μm. Yang et al. [67] established a MIMO artificial neural network model of the axial thermal elongation and radial thermal drift of the coordinate boring machine spindle, and the model prediction accuracy reached 86%. Zhong et al. [68] from Chongqing University used fuzzy C-mean clustering to select temperature variable, and used probabilistic neural networks to establish a compensation model for the relationship between the radial thermal deformation error M and temperature of a worm wheel gear grinding machine, then proposed a thermal error compensation method based on a probabilistic neural network. In l most previous studies, the correlation between the historical accumulated temperatures and the thermal error of machine tool were ignored. Based on a long short-term memory (LSTM) cyclic neural network, Tan et al. [69] proposed a method that effectively used the temperature rise data of the current and historical moments of the machine tool. By using this method, the thermal error could be characterized more in line with the actual situation. However, this method had poor predictive performance for the thermal error under unknown conditions. Liu et al. [70] also considered the actual working conditions, but they focused the relationship between the radial thermal error of the spindle and the inner diameter of the work piece. The measurement data of the inner diameter of the work piece on the production line was used as input, and the multi-layer perceptron neural network (MLP) was used to build the thermal error models of the machine tool under actual processing conditions. Du et al. [71] introduced the genetic algorithm and backpropagation algorithm in the model training process to train the fuzzy neural network parameters. It improved the situation that the robustness of the thermal error model was reduced due to the randomness of the membership function value, and improved modeling and prediction capabilities for complex systems. Shi et al. [72] proposed a new thermal error modeling method based on a Bayesian neural network. By applying it into the feed drive system compensation, the maximum thermal error was reduced from 18.2 μm to 5.14 μm. Tu et al. [73] constructed a thermal evaluation model of a machine tool spindle based on particle swarm optimization weighted naive Bayesian to obtain the global optimal solution of the weights, and the model prediction accuracy was 94.1%. In 2021, in the research of Zheng et al. [74], an improved adaptive genetic algorithm (IAGA) incorporated with a back-propagation neural network (BPNN) was presented. Compared with the experimental results, accuracy of the improved model forward went beyond that of the traditional modeling methods such as MRA, genetic algorithm with BPNN (GA-BPNN), and PSO with BPNN (PSO-BPNN) models.

2.5. Support Vector Machine (SVM)

SVM is a classifier developed from the generalized portrait algorithm in pattern recognition. Its early work was published in 1963 by the former Soviet scholars Vladimir N. Vapnik and Alexander Y. Lerner [75]. Based on the principle of structural risk minimization, the upper limit of functional error is minimized by SVM. It has strong nonlinear function fitting ability [76], which makes it solve the practical problems better [77–82]. Although SVM had faster convergence speed and higher prediction accuracy, it is still a new attempt to apply the support vector machine regression modeling theory for the thermal error modeling of machine tools.

In 2013, Miao et al. [78] established the spindle thermal error models based on SVM and multiple regression respectively for the leaderway-V450 machining center. By analyzing the accuracy of the cross-quarter experimental data, it was verified that the SVM model used for thermal error compensation not only had high accuracy, but also good robustness. Besides, it was found out that when there were less data for modeling, the prediction accuracy and robustness of multiple regression model were poor. However, the SVM model still had higher prediction accuracy even when the working conditions changed [79]. In 2017, Zhang et al. [81] combined the grid search method to optimize the parameters (the penalty parameters and kernel parameters) of the SVM thermal error model to improve the performance of the thermal error model. After applying the improved model on the X and Z axes of the CNC platform, the X-axis positioning error and the Z-axis positioning error were reduced 89.55% and 85.67%, respectively. The compensation performance was better than that of the BP neural network. Similarly, in the research of Zhang et al. [80] in 2020, the PSO algorithm was used to optimize the parameters of the SVM, and the prediction model PSO-SVM was proposed to predict thermal error. Compared with the traditional SVM model, the new model had higher prediction accuracy. More importantly, it could avoid falling into local optimization and reduce the training time effectively. Furthermore, in the research of Li et al. [82], based on least squares support vector machine (LSSVM, a mature algorithm improved on the SVM), a more effective model (GA-LSSVM), which was combined with genetic algorithm (GA) was proposed. Compared with the LSSVM model and the traditional BP model, the GA-LSSVM model had the following superiority: (1) the absolute residual and mean square error were the smallest; (2) the model determination coefficient R_2 was the largest; and (3) performance was the best. Based on the synthetic temperature information (STI), the whale algorithm was used to optimize the thermal error model of support vector regression by Li et al. [83] in 2021. Compared with the S-GA-SVR thermal error model optimized by genetic algorithm, the RMSE of thermal error in the Z direction of the spindle was reduced 46.6%.

2.6. Hybrid Model

Due to the complex structure and varying operating conditions of the machine tool, its thermal characteristics are nonlinear and change in real time. Therefore, it is difficult to establish a thermal error prediction model with good versatility, high accuracy, and strong robustness based on a single data fitting method. In order to solve this problem, some scholars have tried to establish a hybrid prediction model or a comprehensive prediction model of thermal error, which could combine the advantages of the separated two modeling methods [84].

In 2010, Lin and Fu [85] proposed a support vector machine-neural network hybrid model. In order to verify the superiority of the hybrid model over the models based on the other two single method (SVM and NN), all three models were compared. The results are shown in Figure 4.

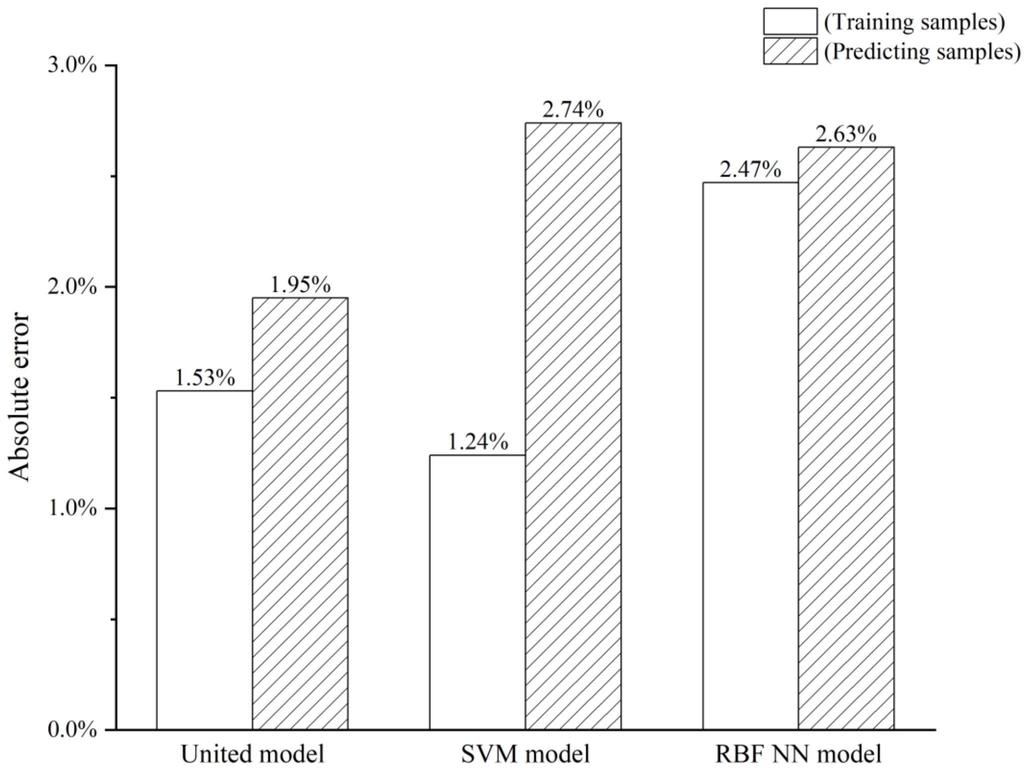

Figure 4. Comparison of SVM, RBF NN, and hybrid model for modeling and predicting.

When the spindle speed was 2000 rpm, the errors based on these three models were all within 0.5 μm. However, as seen in Figure 4, the absolute error percentage of the hybrid model was only 1.95%, while the absolute error percentages of the other two single models were respectively 2.74% and 2.63%. This indicated that its forecast performance was better than any a single forecasting method. In 2012, Zhang et al. [62] proposed a gray neural network model that combined the gray system with the neural network model. When applied the model on the five-axis machining center spindle, it showed good fitting accuracy and prediction accuracy. In 2015, by combining the GM(1,n) model with the LSSVM model, Yao et al. [86] established a hybrid prediction model, in which the prediction validity algorithm was used to adjust the model weighting coefficient. The optimal effective degree composite forecasting model (OE-CM) was established to obtain the best forecasting effect. Similarly, in the research of Li et al. [87], a hybrid model of spindle thermal error based on time series and neural network was mentioned. The model was applied to compensate the thermal of the VM850 vertical machining center. By using this hybrid model, the number of temperature sensors was reduced and the error prediction of the machine tool was more comprehensive and stable. It is worth noting that the model still had high prediction accuracy in a complex processing environment. In 2018, Li et al. [88] applied a combined algorithm (BP neural network based on the BAT algorithm) to solve the thermal error modeling problem. The experimental results showed that the RMSE of the prediction error and the measurement data was 3.80011 μm. In addition, in Jin's [89] paper published in 2020, based on a hybrid model of COMSOL and PLS, a thermal error modeling method for electric spindles was applied to avoid the shortcomings of poor generalization of a single model. The overall model prediction performance of the hybrid model was better, and it could predict the thermal deformation of the electric spindle well under different working conditions.

2.7. Other Modeling Methods

In the paper of Zhang et al. [90] published in 2011, a thermal error modeling method based on the optimal selection of the time series model was proposed. This model integrated the dynamic and historical information of the machine tool and had higher modeling accuracy than the general least square method. It also had stronger anti-interference ability against random factors, which indicated that the time series model can further improve the prediction accuracy of the thermal error model. In the research of Xiao et al. [91], a machine tool thermal error modeling method based on the state space (SS) model was mentioned. With the use of the SS model for compensation, the thermal error of the lathe was reduced from 19.40 µm to 4.20 µm. In 2015, Du et al. [92] proposed a natural exponential model method to symmetrically predict the thermal error of the spindle. Based on the model, the constant of thermal equilibrium time and steady-state thermal error at random speeds were calculated according to the finite element analysis results. Compensation tests have been carried out on the HDBS-63 horizontal machining center. It was found that the maximum axial error was reduced from 55 µm to 16 µm, and the radial error was reduced from 15 µm to 6 µm after compensation. In 2018, Zhao and his colleagues [93] proposed a new method for machine tool error modeling based on the Kalman filter. It regarded the regression coefficient of the statistical model as the state vector and the statistical model as the observation equation. The modeling accuracy of the Kalman filter method was 10.5% and 1.8% higher than that of the least square method and LSSVM method, respectively. Besides, the modeling time was 0.9% and 6.8% less than that of least square method and LSSVM method, respectively. In the paper of Mares et al. [94] in 2020, an improved method based on transfer functions was presented. Its modelling and calculation speed were suitable for real-time applications. When the method was applied on the 5-axis machine tool, it was found out that the thermal error reduction in the X, Y and Z directions were 62%, 56%, and 73%, respectively, after compensation. Early in 2021, a new dynamic linear modeling method of machine tool thermal error based on data driven control theory was proposed by Liu et al. [95]. The prediction error of the new thermal error model of the CNC machine tool was less than 4%. Moreover, fuzzy clustering combined with principal component regression (PCR) was applied for thermal error modeling by Li et al. [96]. The prediction accuracy of the thermal error model was about 86%. In the paper of Yang et al. [97], based on the bond graph method, a thermal characteristic model was developed. The maximum error of thermal deformation predicted by the model was 1.45 µm, which indicated that this method has great advantages in real-time prediction of spindle thermal behavior, and can be used in industrial applications. In recent years, deep learning has been performing well in various fields. It takes the advantage of large datasets and efficient training algorithms to outperform other approaches in various machine learning [98]. The principle of machine learning is shown in Figure 5.

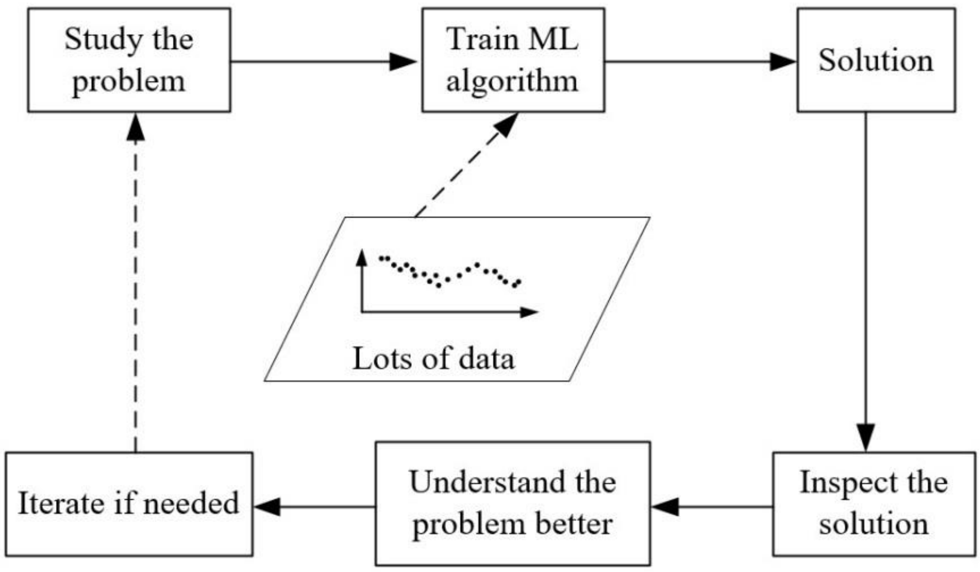

Figure 5. The principle of machine learning.

Particularly, deep learning has also become an important means to realize thermal error compensation of machine tools. For example, based on bidirectional short-term memory (BILSTM) deep learning, Liu et al. [99] completed thermal error modeling and applied it on the machine tool, reducing the thermal error by 85%. In the paper of Tian et al. [100], a thermal prediction model based on a self-organizing deep neural network (DNN) was developed to facilitate accurate-based training for thermal error modeling of heavy-duty machine tool-foundation systems. With the model, feature extraction capabilities were improved, test errors was reduced, and convergence speed was improved. Real-time modeling is a dynamic modeling method, which is more flexible and accurate than static modeling. In 2015, Chen et al. [13] proposed a real-time compensation method for thermal errors of the milling machine. The proposed thermal error model was applied to the milling machine, and the displacement variations on the x- and y-axes and the position error at the tool center were controlled within 20 µm when the compensation system was activated.

3. Discussion

In this section, the advantages and disadvantages of the models are summarized in the following table (Table 1).

Table 1. Comparison of different thermal error modeling methods.

Modeling Method		Advantages	Disadvantages	References
LS		Mature theory. Simple model structure. Widely used.	Less independent variables. Low predictive ability under complex conditions.	[18–21]
MRA		Simple model structure. Reliable in performance.	Calculation time is too long with more variables. Temperature variable coupling.	[22–35]
Grey system		Simpler modeling. Do not rely on massive and complete data information.	When changing the input, the model will be very different.	[36–42]
NN	BP	High prediction accuracy.	Slow convergence speed. Easy to fall into a local minimum The initial value is very difficult to determine.	[47–56]
	RBF	Simple structure design. Faster training speed.	The key feature functions are more difficult to extract. Worse generalization performance.	[57–62]
SVM		Strong nonlinear function fitting ability. The best theory for small sample statistics and predictive learning.	Not easy to select parameters. A lot of computing resources. slow convergence speed.	[75–83]
Hybrid model		Combine the advantages of the separated two modeling methods.Good versatility. High accuracy. Strong robustness.	Increase the difficulty of modeling.	[84–90]

Specifically, LS is an earlier way to compensate the thermal errors of machine tools with mature theory and simple model structure, so it is widely used. However, it has less independent variables and is not quite in line with the actual situation and its predictive ability is limited under complex conditions. The modeling process of the MRA is not only simple in structure, but also reliable in performance. The MRA model for a machine tool with simple structure and working conditions has higher accuracy. However, when there are a lot of variables, the calculation time of MRA is long. However, the correlation between the thermal error and the temperature is only considered by the MRA, which will lead to the temperature variable coupling and reduce the accuracy of the model. Modeling based on the grey system theory is simpler, more convenient, and does not depend on massive and complete data information. However, it is a model that predicts its own development based on its own original data. So taking different data in the original data sequence, the model will be very different. Generally, neural networks have higher prediction accuracy compared to LS and MRA. Among them, the BP neural network had the high accuracy, but it had a slow convergence speed and it was easy to fall into a local minimum. The initial value was also very difficult to determine. Because most of the parameters can be given better initial values during initialization according to actual problems, the structure design of RBF neural network is simple and the training speed is faster. However, its key feature functions are more difficult to extract than the BP neural network, and its prediction performance is worse. The SVM has strong nonlinear function fitting ability, which makes it able to solve the practical problems better. It is considered as the best theory for small sample statistics and predictive learning. The biggest problem in using SVM for thermal error modeling is the optimal selection of parameters. Another problem with SVM is that the solution process requires a lot of computing resources, and the speed is slow. A hybrid model could combine the advantages of two separated modeling methods. It is easy to establish a thermal error prediction model with good versatility, high accuracy, and strong robustness based on the hybrid method. However, the hybrid model will increase the difficulty of modeling, and a reasonable determination of the weight of a single model in the hybrid model is the key to determine the prediction accuracy of the model.

4. Conclusions

In the past ten years, more and more attention has been given to the research field of machine tool thermal errors. Some modeling methods such as LSSVM and neural network have performed well, but these type of models need many training samples and large number of calculations. It is easy to be invalidate in the process of adjusting weights, and they require a lot of time training samples, even with today's highly developed computer hardware technology. Therefore, there is not a blowout development of the research on neural networks or SVM. However, with the continuous development of computer hardware technology, neural networks and other modeling methods will surely show more possibilities. Traditional modeling methods such as least squares or multiple linear regression, etc. will not disappear completely for now. Because their mathematical principles are simple and the prediction performances are remarkable, models based on some improved algorithms still show strong vitality and have a place in engineering applications. Based on the discussion in the last section, the advantages and disadvantages should be considered for the use of thermal error modeling methods under different conditions. When the structure of the machine tool and working conditions are simple, LS and MRA should be considered first, and the prediction accuracy of MRA is generally higher. In contrast, NN and SVM are more suitable for complex machine tools and working conditions. If a single model cannot meet the requirement of accuracy and robustness, a hybrid model should be considered to solve this problem.

In the future, research on the thermal error modeling technology of machine tools could be conducted in the following aspects.

(1) On the premise of ensuring the prediction accuracy, the robustness of the thermal error model needs to be further improved. The established thermal error model under the same machine tool, specific working conditions, and environment has high accuracy. However, when the external environment, working conditions, instrument measurement accuracy, and other factors change, the prediction accuracy of the thermal error model will begin to deteriorate.

(2) The current research on the thermal error of machine tools, from the selection of temperature measurement points, the establishment of the model, to the actual application of the compensation model, was all carried out on the same type of machine tool. However, when the model was transferred to other types of machine tools, the accuracy of the model was greatly compromised, or even completely invalid. Future research should consider whether the thermal error model can be successfully applied to different machine tools.

(3) Besides the error compensation method, there is also the error prevention method to reduce the thermal error of machine tool. Considering that both of them have their own advantages and disadvantages in practical applications, how to combine the two reasonably to complement each other should become the content of future scholars' in-depth thinking and research.

(4) The hybrid forecasting model is established based on two different mathematical modeling methods. By adjusting the weights of two methods, its performance can be improved. Compared with the single model, the hybrid forecasting model has a more comprehensive and stable error prediction performance for the machine tool. Under complex working conditions, it still has higher prediction accuracy. However, there are few studies on hybrid forecasting models, and the types of mathematical principles used in the hybrid model are limited to only two. In the future, a thermal error hybrid forecasting model based on three or more mathematical modeling methods may achieve unexpected performance.

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