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Radiation and Multiple Slip Effects on Magnetohydrodynamic Bioconvection Flow of Micropolar Based Nanofluid over a Stretching Surface

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Citation: Abdal, S.; Alhumade, H.; Siddique, I.; Alam, M.M.; Ahmad, I.; Hussain, S. Radiation and Multiple Slip Effects on Magnetohydrodynamic Bioconvection Flow of Micropolar Based Nanofluid over a Stretching Surface. *Appl. Sci.* **2021**, *11*, 5136. <https://doi.org/10.3390/app11115136>

Academic Editor: Giuseppe Lacidogna

Received: 11 April 2021

Accepted: 25 May 2021

Published: 31 May 2021

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Abstract: Our aim in this article is to study the radiation and multiple slip effects on magnetohydrodynamic bioconvection flow of micropolar based nanofluid over a stretching surface. In addition, a steering mechanism of making improvements to the Brownian motion and thermophoresis motion of nanoparticles is integrated. The numerical solution of 2-dimensional laminar bioconvective boundary layer flow of micropolar based nanofluids is presented. The basic formulation as partial differential equations is transmuted into ordinary differential equations with the help of suitable similarity transformations. Which are then solved by using the Runge–Kutta method of fourth-order with shooting technique. Some important and relevant characteristics of physical quantities are evaluated via inclusive numerical computations. The influence of vital parameters such as buoyancy parameter λ , bioconvection Rayleigh number Rb , the material parameter K are examined. This investigation showed that with the increment in material parameter, micro rotation and velocity profile increases. In addition, the temperature rises due to the enhancement in Nb (Brownian motion) and Nt (thermophoresis parameter).

Keywords: bioconvection; nanofluid; micropolar fluid; magnetohydrodynamic; multi slips; radiation; Runge-Kutta shooting scheme

1. Introduction

There are many applications of bio-convection, such as oil model, enhance oil recovery (EOR). This is the reason some analysts have observed the processes of bio-convection. The EOR is a new process of technology that is used for the recovery of oil and gas. It contains the injection of some tiny organisms which can not be seen with the human eye into cartons and the remaining oil is decreased by situ amplification. The self-incite micrograms which are in motion increase the volume of fluid outflow in a specific path to generate bio-convection. Bioconvection is known as the gradual progress of a form in solutions of tiny organisms which cannot be observed with the common eyes such as bacteria and algae. During swimming in the vertical direction, the volume of the main fluid is raised with the propulsion of these organisms by their selves. Nayak et al. [1] investigated the impact of

3-D bioconvection with multislip effects of Casson nanofluid along with motile gyrotactic microorganisms. Kezzar et al. [2] solved the non-linear problem for nano bioconvective flow between parallel plates by using adomian decomposition method. Balla et al. [3] investigated the combined impacts of bioconvective flow and chemical reaction in a square cavity. Khan et al. [4] used the Homotopy analysis method for entropy generation and gyrotactic microorganisms. Liaqat et al. [5] explored the unsteady case for multi slips effects on bioconvective micropolar nanofluids using the stretching sheet. Ayodeji with his coworkers [6] discussed the Nb and Nt effects for magnetohydrodynamic bioconvective flow with multi slips. Nanofluid flow with self-propelled microorganisms for a nonlinear stretching sheet was discussed by Mondal and Pal [7]. Bhatti et al. [8] examined the activation energy of nanoparticles along with gyrotactic microorganisms over a stretchable sheet. Ansari et al. [9] studied the gyrotactic microorganism effects and the effects of nano-particles in magnetohydrodynamic Casson fluid. Magagula with his co-workers [10] studied double dispersion with Casson fluid as a base fluid in the presence of gyrotactic microorganism. Khaled et al. [11] analyzed the application of bioconvection over a moving surface by using a homotopy analysis scheme.

Microfluid has gained wide concentration because of its many usages in various fields of industry, construction and engineering. Compared to Newtonian liquids, micropolar liquids are more resistant to the motion of outflow. This process also suggests that the outflow of viscous fluid is higher for the greater micropolar value of the parameter. It has also been analyzed that in the process of laminar flow, micropolar fluids can be very effective fluid media. The idea of micropolar fluids studies the effect of micro-rotation in hydrodynamics, which contains rotational microcomponents. Ali et al. [12] investigated the impact of ferromagnetic and ferrimagnetic past over a stretching sheet. They used Ethylene glycol and water as base fluids with the magnetic dipole. Liaqat and his co-workers [13] analyzed the boundary layer flow of ferrite nanoparticles. They used Paramagnetic, Diamagnetic, and (Ferromagnetic) as ferrites and water and ethylene glycol as a base fluid. Abdal et al. [14] examined the radiation and dissipation effects due to stretching surface. Sadiq et al. [15] analyzed micropolar fluid's outflow along with the boundary layer having different properties. Aslani et al. [16] discussed the micropolar Couette fluid flow with magnetic fields. Mishra et al. [17] worked on the effect of magnetohydrodynamic outflow by using the micropolar fluids, keeping the medium porous. Nadeem et al. [18] investigated heat flow of 3-D micropolar fluids with Riga plate. Aslani et al. [19] analyzed radiation effects of a micropolar fluid with mass transpiration. Janardhana et al. [20] worked on the micropolar fluids and solved it numerically under the effect of transfer of heat as well as the radiations. He used a cylinder for this, which was empty and in the upright direction. He also applied the idea of Bejan's function of heat. Ramadevi et al. [21] numerically studied the mixed convection micropolar fluid flow. Ismail et al. [22] discussed that micropolar fluid beneath the consequence of convinced magnetic fields. Similar work was being done by [23,24].

A fluid consists of very small flecks whose size can be measured in a nanometer known as nanofluid. Such type of fluid plays an important role in the colloidal interruption of fluid. The tiny particles that are involved in the manufacturing of nanofluids, mostly made of carbon, oxides, carbides and oil-based nanofluids [25]. Nanofluids can be regarded as the prospect of the transfer of heat. Due to the presence of suspended nanoparticles with high thermal conductivity, they are expected to have better thermal properties than conventional fluids [26,27]. Recently, several studies have shown improvements in the thermal conductivity of nanofluid. The use of nanofluids can significantly increase the heat transfer rate. To expand the application of nanofluids, it is important to further study the basis of heat transfer and friction factors in the case of nanofluids. In all types of fluids, nanofluids are the best option to discuss the accomplishment of the transfer of heat. Liaqat et al. [28] investigated the multi-slips effects of magnetohydrodynamics Casson nanofluid past over a shrinking sheet. Sohaib et al. [29] numerically investigated the unsteady case for multislip effects on micropolar nanofluid with heat source and

radiation. Bagh with his co-workers [30] studied the unsteady case for axisymmetric nanofluid. Bagh et al. [31] studied Stefan blowing effect on thermal radiation for nanofluid flow on the leading edge. Yang et al. [32] analyzed transfer of heat and flow optimization of nanofluids. It is obvious that typical generator oils use approximately less thermal conductivity and transfer of heat characteristics. Yang with his co-workers [33] analyzed it by using the nanofluids. Karvelas et al. [34] discussed the aggregation of nanoparticles. Abdal et al. [35] studied the analytical solution of Casson nanofluid. Ji et al. [36] worked on the improvement of thermal conductivity by using the nanofluids in water. They observed the effect of temperature. Moraveji et al. [37] discussed the flow of nanofluid by making tiny pathways with the help of dissipation of heat and also observed its property of hydraulic. Mousavi et al. [38] studied the experimental comparison between hybrid nanofluid ZnO and MoS_2 . Abbas et al. [39] discussed boundary layer flow of nanofluid. Husnain et al. [40] investigated the buoyancy effects of MHD nanofluid over a vertical plate. Similarly, many researchers done research in this field [41,42].

Motivated by the above literature, authors have shown deep interest in investigating the magnetohydrodynamic fluid flow of micropolar based nanofluid over a stretching surface. The improved heat transfer via nanofluid motion is augmented with thermal radiation and multi-slip conditions. Another aspect of innovation is bioconvection of micro-organism to provide stability by mixing and thus it may avoid aglow migration of nano entities. In view of such examination, partial differential equations are transformed into ordinary differential equations with the help of suitable transformations and then solved by using Runge–Kutta fourth-order method with shooting technique. In addition, in this study, the flow velocity behavior along with temperature, concentration and motile gyrotatic microorganism due to the impact of several physical parameters have been assimilated through graphs and tables.

2. Physical Model and Mathematical Formulation

Choose x -axis and y -axis in such a way that y -axis is orthogonal to x -axis, shown in Figure 1. Consider $\tilde{U}(x, t) = \tilde{a}\tilde{x}/(1 - \tilde{\lambda}\tilde{t})$ is the non-uniform velocity with moving sheet, where $\tilde{a}$ represent the stretching/shrinking rate in the direction of x -axis and $\tilde{\lambda}\tilde{t} < 1$. $\tilde{B}(x) = \tilde{B}_0\tilde{x}^{-1/2}$ is the magnetic field along y -direction, where $\tilde{B}_0 \neq 0$ is the magnetic field strength. $\tilde{T}_\infty$ represents free stream temperature, $\tilde{C}_\infty$ is the nano-particle concentration, $\tilde{n}_\infty$ is the microorganism concentration. $\tilde{T}_w(\tilde{x}, \tilde{t})$ is the temperature of the sheet such that ([29]):

$$\tilde{T}_w - \tilde{T}_0 \left(\frac{\tilde{a}\tilde{x}}{2\tilde{\nu}(1 - \tilde{\lambda}\tilde{t})^2} \right) = \tilde{T}_\infty.$$

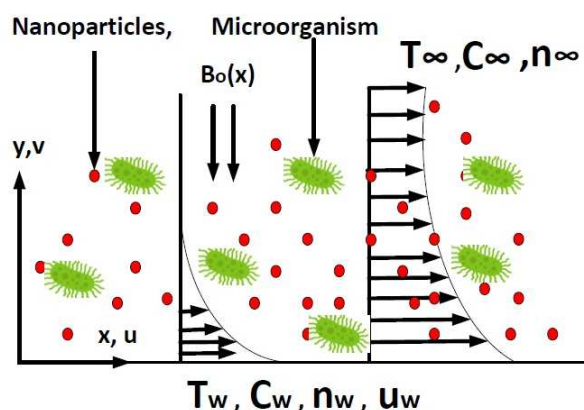


Figure 1. Flow geometry.

Similarly $\tilde{C}_w(\tilde{x}, \tilde{t})$ and $\tilde{n}_w(\tilde{x}, \tilde{t})$ are the nano-particles concentrations and microorganism concentrations defined as

$$\begin{aligned}\tilde{C}_w - \tilde{C}_0 \left(\frac{\tilde{a}\tilde{x}}{2\tilde{v}(1-\tilde{\lambda}\tilde{t})^2} \right) &= \tilde{C}_\infty, \\ \tilde{n}_w - \tilde{n}_0 \left(\frac{\tilde{a}\tilde{x}}{2\tilde{v}(1-\tilde{\lambda}\tilde{t})^2} \right) &= \tilde{n}_\infty,\end{aligned}$$

where reference temperature represented by $\tilde{T}_0$, reference nano particle concentration is $\tilde{C}_0$ and $\tilde{n}_0$ is the reference concentration of microorganism, respectively.

Using aforementioned assumptions, the governing flow equations are given below [28,29,43]:

$$\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0, \quad (1)$$

$$\begin{aligned}\frac{\partial \tilde{u}}{\partial \tilde{t}} + \tilde{u} \frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v} \frac{\partial \tilde{u}}{\partial \tilde{y}} &= \left(\frac{\tilde{\mu} + \tilde{k}}{\tilde{\rho}} \right) \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2} + \frac{\tilde{k}}{\tilde{\rho}} \frac{\partial \tilde{N}}{\partial \tilde{y}} - \frac{\tilde{\sigma} \tilde{B}^2(\tilde{x}) \tilde{u}}{\tilde{\rho}} - \frac{\tilde{v}}{\tilde{k}} \tilde{u} + \\ \frac{1}{\tilde{\rho}} \left[\tilde{\rho} \tilde{\beta} \tilde{g}(\tilde{T} - \tilde{T}_\infty)(1 - \tilde{C}_\infty) - \tilde{g}(\tilde{C} - \tilde{C}_f)(\tilde{\rho}_p - \tilde{\rho}) - \tilde{\gamma}^* \tilde{g}(\tilde{\rho}_m - \tilde{\rho})(\tilde{n} - \tilde{n}_\infty) \right],\end{aligned} \quad (2)$$

$$\tilde{\rho} \tilde{j} \left(\frac{\partial \tilde{N}}{\partial \tilde{t}} + \tilde{u} \frac{\partial \tilde{N}}{\partial \tilde{x}} + \tilde{v} \frac{\partial \tilde{N}}{\partial \tilde{y}} \right) = \tilde{\gamma} \frac{\partial^2 \tilde{N}}{\partial \tilde{y}^2} - \tilde{k} \left(2\tilde{N} + \frac{\partial \tilde{u}}{\partial \tilde{y}} \right), \quad (3)$$

$$\frac{\partial \tilde{T}}{\partial \tilde{t}} + \tilde{u} \frac{\partial \tilde{T}}{\partial \tilde{x}} + \tilde{v} \frac{\partial \tilde{T}}{\partial \tilde{y}} = \tilde{\alpha} \frac{\partial^2 \tilde{T}}{\partial \tilde{y}^2} - \frac{\partial \tilde{q}_r}{\partial \tilde{y}} + \tilde{\tau} \left(\tilde{D}_B \frac{\partial \tilde{T}}{\partial \tilde{y}} \frac{\partial \tilde{C}}{\partial \tilde{y}} + \tilde{T}_\infty \left(\frac{\partial \tilde{T}}{\partial \tilde{y}} \right)^2 \right), \quad (4)$$

$$\frac{\partial \tilde{C}}{\partial \tilde{t}} + \tilde{u} \frac{\partial \tilde{C}}{\partial \tilde{x}} + \tilde{v} \frac{\partial \tilde{C}}{\partial \tilde{y}} = \tilde{D}_B \frac{\partial^2 \tilde{C}}{\partial \tilde{y}^2} + \frac{\tilde{D}_T}{\tilde{T}_\infty} \frac{\partial^2 \tilde{T}}{\partial \tilde{y}^2}, \quad (5)$$

$$\frac{\partial \tilde{n}}{\partial \tilde{t}} + \tilde{u} \frac{\partial \tilde{n}}{\partial \tilde{x}} + \tilde{v} \frac{\partial \tilde{n}}{\partial \tilde{y}} + \tilde{b} \tilde{W}_c \frac{\partial}{\partial \tilde{y}} \left(\frac{\tilde{n}}{\Delta \tilde{C}} \frac{\partial \tilde{C}}{\partial \tilde{y}} \right) = \tilde{D}_m \frac{\partial^2 \tilde{n}}{\partial \tilde{y}^2}. \quad (6)$$

The boundary conditions for the given problem is given below [28]:

$$\begin{cases} \tilde{u} = \tilde{U}(x, t) + \tilde{U}_{slip}, \tilde{v} = \tilde{v}_w, \tilde{N} = -\tilde{m} \frac{\partial \tilde{u}}{\partial \tilde{y}} + \tilde{T} - \tilde{T}_w(x, t) - \tilde{T}_{slip} = 0, \\ \tilde{C} - \tilde{C}_w(x, t) - \tilde{C}_{slip} = 0, \tilde{n} - \tilde{n}_w = 0, \text{ as } \tilde{y} = 0, \\ \tilde{u} \rightarrow 0, \tilde{N} \rightarrow 0, \tilde{T} - \tilde{T}_\infty \rightarrow 0, \tilde{C} - \tilde{C}_\infty \rightarrow 0, \tilde{n} - \tilde{n}_\infty \rightarrow 0 \text{ as } \tilde{y} \rightarrow \infty. \end{cases} \quad (7)$$

Here, $\tilde{u}$ and $\tilde{v}$ are the component of velocity along $\tilde{x}$ and $\tilde{y}$, respectively, $\tilde{j}$ is micro-inertia, $\tilde{m}$ is constant ($0 \leq m \leq 1$), dynamic viscosity is denoted by $\tilde{\mu}$, $\tilde{N}$ represents micro-rotation vector, vortex viscosity is $\tilde{k}$, $\tilde{\rho}$ represents fluid density, electrical conductivity is denoted by $\tilde{\sigma}$, thermal diffusivity is $\tilde{\alpha}$, spin gradient viscosity is $\tilde{\gamma}$, $\tilde{g}$ is gravity acceleration, $\tilde{\rho}_m$ is microorganism density, $\tilde{\rho}_p$ is nano particle density, $\tilde{D}_T$ refers as thermal diffusivity, $\tilde{D}_B$ refers as Brownian diffusivity, $\tilde{D}_m$ refers as molecular diffusivity, $\tilde{W}_c$ is the speed of cell swimming, $\tilde{T}$ represents temperature, $\tilde{C}$ represents nanoparticle concentration, $\tilde{n}$ refers to the motile density of microorganisms, $\tilde{b}$ refers to chemotaxis constant.

Furthermore, Rosseland approximation $\tilde{q}_r$ in Equation (4) is defined as, $\tilde{q}_r = \frac{-4\tilde{\sigma}^*}{3\tilde{k}^*} \frac{\partial \tilde{T}^4}{\partial \tilde{y}}$, where $\tilde{\sigma}^*$ is refer as Stefan–Boltzmann constant, $\tilde{k}^*$ refer as mean absorption coefficient (see [12,44]). On expanding $\tilde{T}^4$ by Taylor series, we get $\tilde{T}^4 = 4\tilde{T}\tilde{T}_\infty^3 - 3\tilde{T}_\infty^4$, neglecting the higher order so, $\frac{\partial \tilde{q}_r}{\partial \tilde{y}} = \frac{16\tilde{T}_\infty^3 \tilde{\sigma}^*}{3\tilde{k}^* \tilde{\kappa}} \frac{\partial^2 \tilde{T}}{\partial \tilde{y}^2}$. Generally, ψ (stream function) is defined as $\frac{\partial \psi}{\partial \tilde{y}} = u$ and $-\frac{\partial \psi}{\partial \tilde{x}} = v$. With the help of given similarity transformations, convert the Equations (1)–(6) into ordinary differential equations [28,29].

$$\begin{cases} \eta = \left(\frac{a}{\tilde{v}(1-\tilde{\lambda}\tilde{t})} \right)^{1/2} y, \psi = \left(\frac{a\tilde{v}}{(1-\tilde{\lambda}\tilde{t})} \right)^{1/2} x f(\eta), \tilde{N} = \left(\frac{a^3}{\tilde{v}(1-\tilde{\lambda}\tilde{t})^3} \right)^{1/2} x h(\eta), \\ \theta(\eta) - \frac{\tilde{T} - \tilde{T}_\infty}{\tilde{T}_w - \tilde{T}_\infty} = 0, \phi(\eta) - \frac{\tilde{C} - \tilde{C}_\infty}{\tilde{C}_w - \tilde{C}_\infty} = 0, \zeta(\eta) - \frac{\tilde{n} - \tilde{n}_\infty}{\tilde{n}_w - \tilde{n}_\infty} = 0. \end{cases} \quad (8)$$

After transformation above equations we get:

$$(1+K)\frac{d^3f}{d\eta^3} + f\frac{d^2f}{d\eta^2} - \left(\frac{df}{d\eta}\right)^2 - \delta\left(\frac{\eta}{2}\frac{d^2f}{d\eta^2} + \frac{df}{d\eta}\right) + K\frac{dg}{d\eta} - M\frac{df}{d\eta} - k_p\frac{df}{d\eta} + \lambda(\theta(\eta) + Nr\phi(\eta) - Rb\zeta(\eta)), \quad (9)$$

$$\left(1 + \frac{K}{2}\right)\frac{dg^2}{d\eta^2} + f\frac{dg}{d\eta} - \frac{df}{d\eta}g - \delta\left(\frac{\eta}{2}\frac{d^2f}{d\eta^2} + \frac{3}{2}\frac{df}{d\eta}\right) - K\left(2g + \frac{d^2f}{d\eta^2}\right) = 0, \quad (10)$$

$$(1+R)\frac{1}{Pr}\frac{d^2\theta}{d\eta^2} - \frac{df}{d\eta}\theta + f\frac{d\theta}{d\eta} - \delta\left(\frac{\eta}{2}\frac{d\theta}{d\eta} + 2\theta\right) + Nb\frac{d\phi}{d\eta}\frac{d\theta}{d\eta} + Nt\left(\frac{d\theta}{d\eta}\right)^2 = 0, \quad (11)$$

$$\frac{d^2\phi}{d\eta^2} - Ln\left[\frac{df}{d\eta}\phi - f\frac{d\phi}{d\eta} + \delta\left(\frac{\eta}{2}\frac{d\phi}{d\eta} + 2\phi\right)\right] + \frac{Nt}{Nb}\frac{d^2\theta}{d\eta^2} = 0, \quad (12)$$

$$\frac{d^2\zeta}{d\eta^2} - Lb\left(\zeta\frac{df}{d\eta} - f\frac{d\zeta}{d\eta}\right) - Pe\left(\frac{d^2\phi}{d\eta^2}(\zeta + \Omega) + \frac{d\zeta}{d\eta}\frac{d\phi}{d\eta}\right) - \delta\left(\frac{\eta}{2}\frac{d\zeta}{d\eta} + 2\zeta\right)Lb = 0, \quad (13)$$

$$\begin{cases} f(0) = f_w, \frac{df(0)}{d\eta} = 1 + S_f\frac{d^2f(0)}{d\eta^2}, g(0) = -m\frac{d^2f(0)}{d\eta^2}, \theta(0) = 1 + S_\theta\frac{d\theta(0)}{d\eta}, \\ \phi(0) = 1 + S_\phi\frac{d\phi(0)}{d\eta}, \zeta(0) = 1 - B, \quad \text{at } \eta = 0, \\ \frac{df(\infty)}{d\eta} \rightarrow 0, g(\infty) \rightarrow 0, \theta(\infty) \rightarrow 0, \phi(\infty) \rightarrow 0, \zeta(\infty) \rightarrow 0. \quad \text{at } \eta \rightarrow \infty. \end{cases} \quad (14)$$

the parameters in Equation (9)–(14) are defined as:

$M = \frac{\tilde{\sigma}(1-\tilde{\lambda}\tilde{f})}{\tilde{\rho}a\tilde{x}}\tilde{B}_0^2$ is magnetic parameter, $\delta = \frac{\tilde{\lambda}}{a}$ is unsteadiness parameter, $\kappa = \frac{\tilde{k}}{\tilde{\mu}}$ is material parameter, $\lambda = \frac{Gr}{R_{\tilde{x}}^2}$ is buoyancy parameter, $Nr = \frac{(\tilde{\rho}_p - \tilde{\rho})(\tilde{C}_w - \tilde{C}_\infty)}{\tilde{\beta}\tilde{\rho}(1-\tilde{C}_\infty)(\tilde{T}_w - \tilde{T}_\infty)}$ is Bouyancy ratio parameter, $Rb = \tilde{\gamma}^* \frac{(\tilde{\rho}_m - \tilde{\rho})(\tilde{n}_w - \tilde{n}_\infty)}{\tilde{\beta}\tilde{\rho}(1-\tilde{C}_\infty)(\tilde{T}_w - \tilde{T}_\infty)}$ is Rayleigh number, $R = \frac{16\tilde{\sigma}^*\tilde{T}_\infty^3}{3\tilde{k}^*\tilde{K}}$ is thermal radiation, $Nt = \frac{\tilde{\tau}\tilde{D}_T(\tilde{T}_w - \tilde{T}_\infty)}{\tilde{\nu}\tilde{T}_\infty}$ is the thermophoresis parameter, $Nb = \frac{\tilde{\tau}\tilde{D}_B(\tilde{C}_w - \tilde{C}_\infty)}{\tilde{\nu}}$ is the Brownian motion parameter, $Ln = \frac{\tilde{\nu}}{\tilde{D}_B}$ is the Lewis number, $Lb = \frac{\tilde{\nu}}{\tilde{D}_m}$ is the bioconvection Lewis number, $Pe = \frac{\tilde{b}\tilde{W}_c}{\tilde{D}_m}$ bioconvection Peclet number, $Pr = \frac{\tilde{\nu}}{\tilde{\kappa}}$ is Prandtl number, $\Omega = \frac{\tilde{n}_\infty}{\tilde{n}_w - \tilde{n}_\infty}$ is micro-organism concentration difference, $f_w = -\frac{\tilde{v}_0}{\sqrt{x}}\left(\frac{(1-\tilde{\lambda}\tilde{f})}{\tilde{\nu}a}\right)^{1/2}$ is the parameter of suction/injection.

3. Mathematical Scheme

Ordinary differential Equations (9)–(13) are numerically solved with the help of shooting technique and Runge–Kutta method. This scheme is more capable and easy to apply as compared with other methods such as, homotopy perturbation method (HPM), finite difference method (FDM). For this, new variables are introduced:

$$\begin{aligned} w'_1 &= w_2, \\ w'_2 &= w_3, \\ w'_3 &= \frac{-1}{1+K}[w_1w_3 - w_2^2 + Kw_5 - (M + Kp)w_2 - \delta(\frac{\eta}{2}w_3 + w_2) + \lambda_1(w_6 - Nr w_8 + Rbw_{10})], \\ w'_4 &= w_5, \\ w'_5 &= \frac{-1}{(1+\frac{K}{2})}[w_1w_5 - w_2w_4 - K(2w_4 + w_3) - \delta(\frac{\eta}{2}w_5 + \frac{3}{2}w_4)], \\ w'_6 &= w_7, \\ w'_7 &= \frac{-Pr}{1+R}[w_1w_7 - w_2w_6 - \delta(\frac{\eta}{2}w_7 + 2w_6) + Nb w_7 w_9 + Nt w_7^2], \\ w'_8 &= w_9, \\ w'_9 &= [Ln(w_2w_8 - w_1w_9 + \delta(\frac{\eta}{2}w_9 + 2w_8)) - \frac{Nt}{Nb}w_7^2], \\ w'_{10} &= w_{11}, \\ w'_{11} &= Lb(w_2w_{10} - w_1w_{11} + \delta((\frac{\eta}{2})w_{11} + 2w_{10})) + Pe[w_9w_{11} + (\Omega + w_{10})w'_9], \end{aligned}$$

The relations in Equation (14) are:

$$\eta = 0 : w_1 = f_w, w_2 = 1 + S_f w_3, w_4 = -m w_3, w_6 = 1 + S_\theta w_5, w_8 = 1 + S_\phi w_7, w_{10} - 1 - B = 0,$$

$$\eta \rightarrow \infty : w_2 \rightarrow 0, w_4 \rightarrow 0, w_6 \rightarrow 0, w_8 \rightarrow 0, w_{10} \rightarrow 0.$$

it is required to induce eleven guess and five unknown conditions, let $w_3(0) = a$, $w_5(0) = b$, $w_7(0) = c$, $w_9(0) = d$, $w_{11}(0) = e$. These conditions are satisfied when $\eta \rightarrow \infty$.

4. Results and Discussion

The main purpose of this study is to investigate the study of radiation and multiple slip effects on magnetohydrodynamic bioconvection flow of micropolar based nanofluid over a stretching surface. In Table 1, the comparison of skin friction coefficient with already published papers is made to justify the validation of the current structure. A strong correlation is observed between the results. Table 2 shows the comparison of Pr and M for skin friction with already published papers. In Table 3, comparison of $-\theta'(0)$ with different values of Pr is shown. It is obvious that with the boosting values of Pr there is an increment in the $-\theta'(0)$. For the confirmation of the accuracy of the present numerical structure, a comparison of the current result for Nusselt number with different values of Pr, M and R are made when all other parameters are zero, with already published results shown in Table 4. It is found that there is an outstanding correlation arise between the results.

Table 1. Comparison of skin friction coefficient for various values of M .

M	Shahid et al. [45] $\beta = 0$	Gireesha et al. [46]	Sohaib et al. [29]	Our Results
0.0	1.0000080	1.000	1.0000130	1.0000110
0.2	1.0954458	1.095	1.0954463	1.0954453
0.5	1.2247446	1.224	1.2247454	1.2247434
1.0	1.4142132	1.414	1.4142180	1.4142160
1.2	1.4832393	1.483	1.4832402	1.4832396
1.5	1.5811384	1.581	1.5811396	1.5811382
2.0	1.7320504	1.732	1.7320516	1.7320512

Table 2. Skin friction and Prandtl number comparison for various values of M and Pr .

M	Shahid et al. [45]	Fazle [47]	Our Results	Pr	Shahid et al. [45]	Fazle [47]	Our Results
0	−1.0000082	−1.0000084	−1.0000081	-	-	-	-
1	1.41421353	1.41421356	1.41421354	-	-	-	-
5	2.44948963	2.44948974	2.44948968	0.72	0.8088	0.8088	0.8086
10	3.31662463	3.31662479	3.31662475	1	1.0000	1.0000	1.0000
50	7.14142839	7.14142843	7.14142841	3	1.9237	1.9237	1.9245
100	10.0498751	10.0498756	10.0498754	10	3.7207	3.7207	3.7212
500	22.3830283	22.3830293	22.3830298	-	-	-	-
1000	31.6385833	31.6385840	31.6385838	-	-	-	-

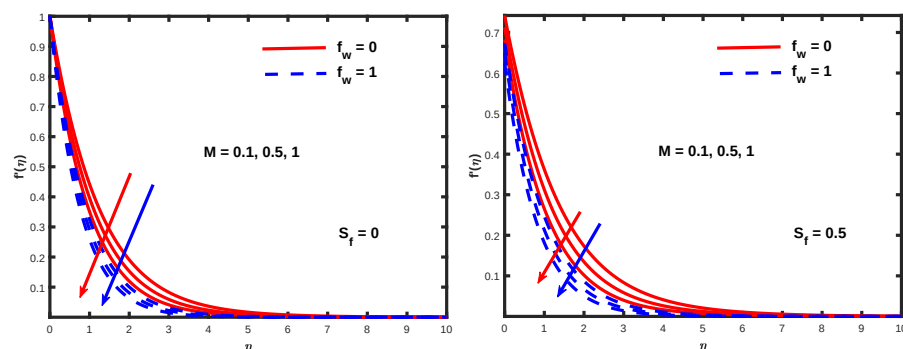
Table 3. Nusselt number comparison for various values of Prandtl number.

Pr	Liaqat et al. [5]	Shahid et al. [45]	Bagh et al. [48]	Haile et al. [49]	Our Results
0.72	0.8086	0.808634	0.808634	-	0.808633
1.00	1.0000	1.000008	1.000001	1.0004	1.000008
3.00	1.9236	1.923678	1.923683	1.9234	1.923677
10.0	3.7206	3.720668	3.720674	3.7205	3.720658
100	12.2946	-	-	12.2962	12.29405

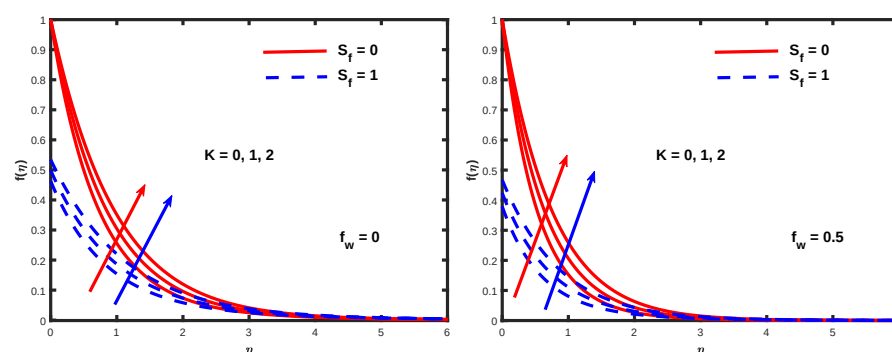
Table 4. Comparison of R , Pr and M .

R	Pr	M	Majeed et al. [50]	Ishak [51]	Mabood et al. [52]	Mukhopadhyay [53]	Our Results
0	1	0	0.954783	0.9548	0.95478	0.9547	0.9546
1	1	0	0.531730	-	0.53121	0.5312	0.5310
0.5	2	0	1.073519	1.0735	1.07352	1.0734	1.0738
1	1	1	0.450571	0.450571	0.450571	-	0.450571
1	0	1	-	0.5312	0.5312	0.5312	0.5311

Now, we see the influence of different parameters graphically on velocity $f(\eta)$, temperature $\theta(\eta)$, volume fraction of nanoparticles $\phi(\eta)$, density of motile microorganisms $\zeta(\eta)$ and angular velocity $g(\eta)$ profiles along with fixing the remaining parameters $M = 1$, $K = 0.5$, $K_p = 0.2$, $Rb = 0.1$, $Nr = 0.1$, $\lambda = 0.2$, $\delta = 0.2$, $R = 0.5$, $Pr = 1$, $B = Nb = Nt = 0.1$, $\Omega = 0.2$, $Lb = 1.2$, $Pe = 1.2$ and $Ln = 5$, $f_w = 1$, $m = 0.5$. The effect of magnetic parameter M and hydrodynamic slip is shown in Figure 2. Decrement behavior shown for velocity with rising the values of M . The decrement of velocity profile is caused due to the increment of resistive Lorentz force. It is also observed that velocity profile decreases with suction (f_w) and boundary layer thickness decreases when hydrodynamic slip is in tact.

**Figure 2.** Influence of magnetic and suction/injection parameter on velocity.

It is clear from the Figure 3, with the increasing values of material parameter K , an increment is seen in velocity profile but velocity profile decreases with the increment in hydrodynamic slip S_f . Physically, material constant K is the combination of the vortex and dynamic viscosity. The larger value of K means smaller dynamic viscosity μ and hence the speed of flow becomes faster.

**Figure 3.** Influence of material and suction/injection parameter on velocity.

The effect of permeability parameter k_p and hydrodynamic slip on velocity profile is shown in Figure 4. Decrement behavior of velocity with the rising values of k_p and S_f is noticed. It is also observed that the velocity profile decreases when f_w increases also boundary layer thickness decreases. The basic reason for this phenomenon is that in the porous medium the resistance becomes higher, due to this the momentum development for

the flow regime declines. In Figure 5, the effect of Rayleigh number Rb and f_w is discussed on the velocity profile. Due to this effect velocity profile shows a decrement under the action of hydrodynamic slip S_f . Physically, the decrement occurs because buoyancy force produces resistance against the movement in the horizontal direction.

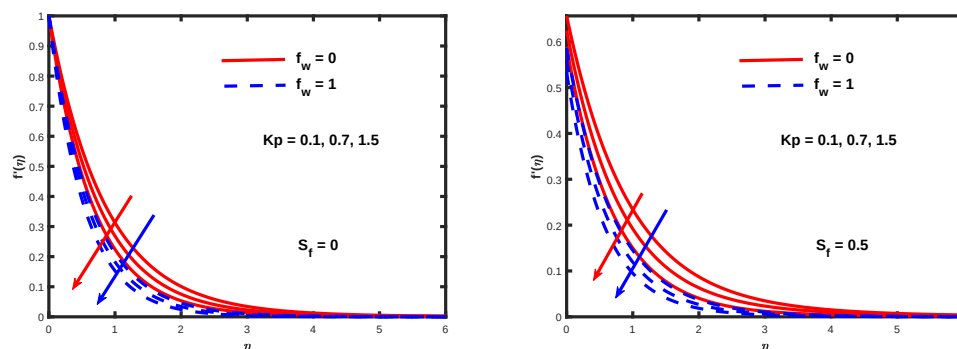


Figure 4. Influence of permeability and suction/injection parameter on velocity.

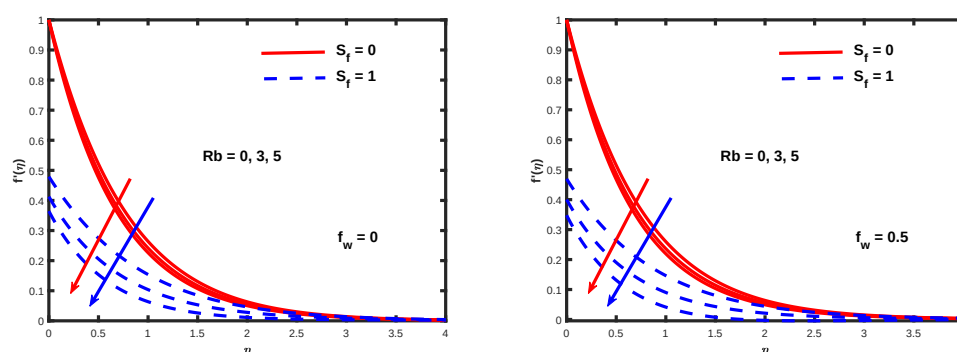


Figure 5. Influence of Rayleigh number and hydrodynamic slip on velocity.

From Figure 6, the effect of mixed convection parameter λ along with hydrodynamics slip S_f on angular velocity is observed. It is seen that angular velocity increases with the rising values of λ and S_f . However, angular velocity profile decreases for the rising values of f_w (suction). Figure 7 describes the rise of material parameter K which causes a gradual increase to the angular velocity $g(\eta)$. From these figures, it is cleared that the angular velocity is promoted along with the expanding values of K and f_w . Because the increment in K means larger vortex viscosity and hence stronger micro-rotation. However, with the increment in the hydrodynamic slip S_f , the profile of $g(\eta)$ decreases. Opposite behavior of $g(\eta)$ is shown in Figure 8, for unsteady parameter δ . It is because the larger lays after the stretch in the sheet show the overall motion of the fluid. The temperature profile is plotted in Figure 9 with the values of magnetic number M and thermal slip S_θ . The temperature $\theta(\eta)$ is gradually increasing directly with the increment in the inputs of these parameters. In addition, it is observed that the thermal profile decreases with the increment in f_w . Figure 10 shows clearly that the effect of Brownian motion parameter Nb along with thermal slip S_θ . It is observed that temperature rises with the rising values of Nb . According to the concept of Brownian motion, nanoparticles are directly related to temperature. With the increment in temperature, an enhancement yields in the kinetic energy of these particles.

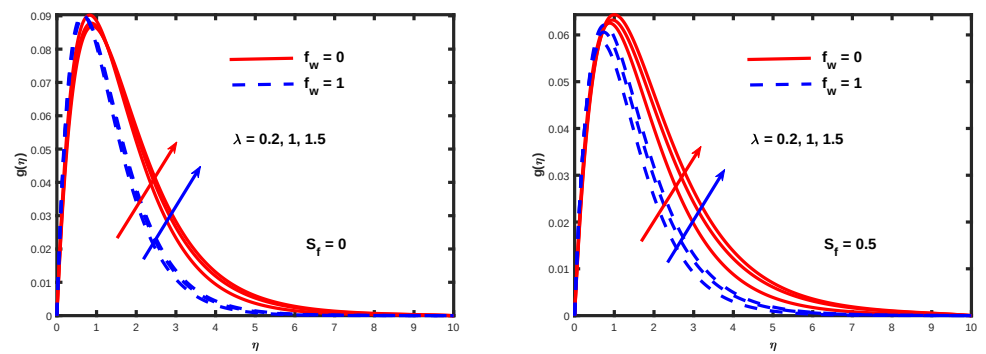


Figure 6. Influence of buoyancy and suction/injection parameter on g .

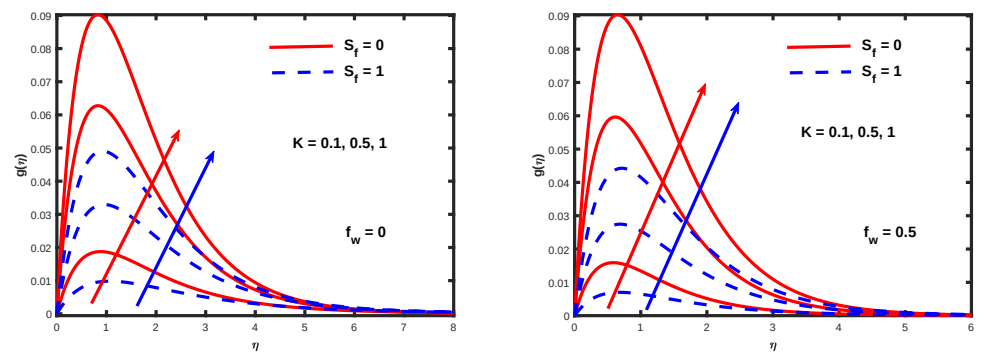


Figure 7. Influence of material and suction/injection parameter on g .

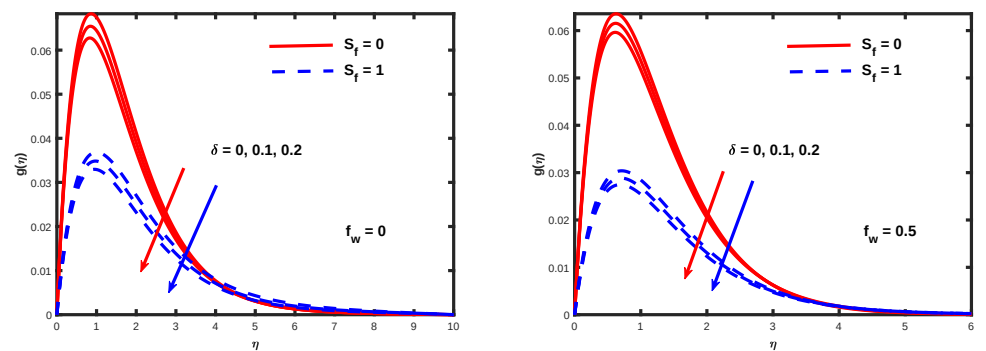


Figure 8. Influence of unsteady parameter and hydrodynamic slip on g .

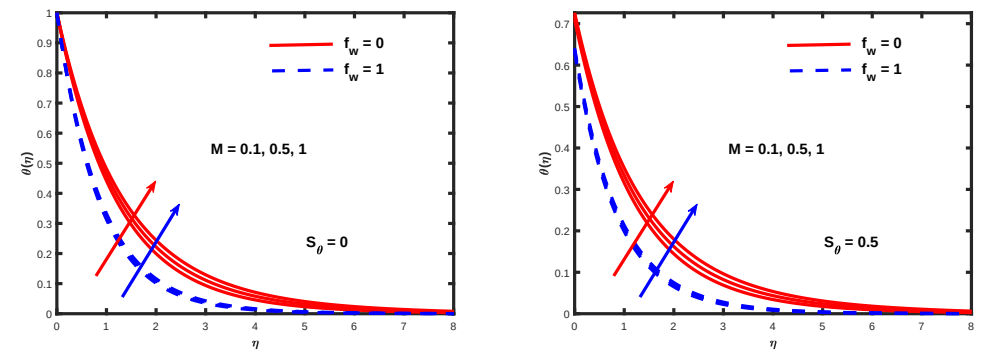


Figure 9. Influence of material and suction/injection parameter on θ .

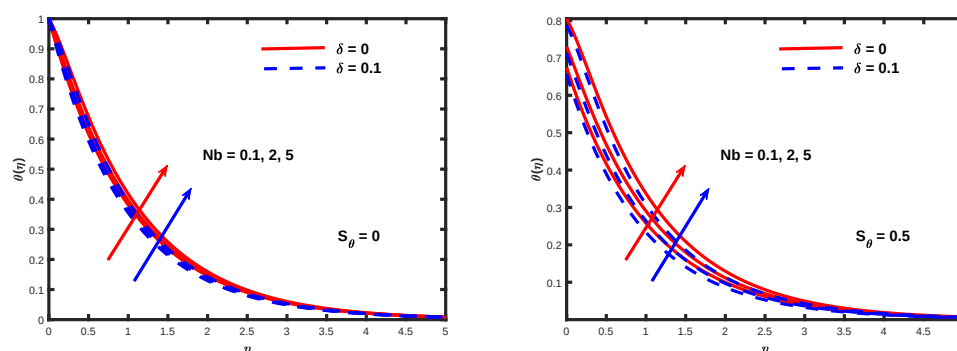


Figure 10. Influence of unsteady and Brownian motion parameter on θ .

Figure 11 showed the rise in temperature profile $\theta(\eta)$ under the action of thermophoresis parameter Nt with thermal slip S_θ . The particles which have high temperature are pushed away from the hotter region to a colder one. Due to this reason, the fluid's temperature gets changed. Therefore, temperature increases with the increment in thermophoresis parameter Nt but temperature profile decreases with the small increment in the unsteady parameter δ . With the growing values of buoyancy parameter and hydrodynamic slip, decreasing behavior of temperature is seen in Figure 12. In Figure 13, the impact of Lewis number Ln with concentration slip S_ϕ in the presence of suction parameter on the volume fraction of nanoparticles $\phi(\eta)$ profile can be seen. It is observe that $\phi(\eta)$ decreases with the increment in Ln with f_w . Basically, Ln is the ratio of momentum diffusivity to Brownian diffusivity. The reason behind the decrement in Ln is that molecules collide randomly with the increment in the Brownian diffusivity parameter. Figure 14 is sketched to discuss the rise of Brownian motion parameter Nb on $\phi(\eta)$. The inclination in Nb results a decrease in volume fraction of nanoparticles $\phi(\eta)$. Moreover, the concentration profile decreases with the increment in f_w . The opposite behavior of $\phi(\eta)$ is noticed for thermophoresis parameter Nt in Figure 15. Motile density $\zeta(\eta)$ is a decreasing function for bioconvection Lewis number Lb , as seen in Figure 16. With the increment in Lb , a decrement is generated in its diffusivity. Because mass diffusivity shows accession with the increment in Lb which causes a decrement in the density of motile organism. In addition motile density profile $\zeta(\eta)$ decreased with the increment in unsteady parameter δ . Figure 17 expressed the effect of microorganisms difference parameter Ω on motile density $\zeta(\eta)$. It is very clear from the figure that by increasing the value of Ω and B , the decrement is observed for motile density $\zeta(\eta)$. In addition, motile density profile $\zeta(\eta)$ decreased with unsteady parameter δ . In Figure 18, the increasing values of Peclet number Pe mean smaller mass diffusivity of microorganism and hence the distribution of motile microorganisms is declined.

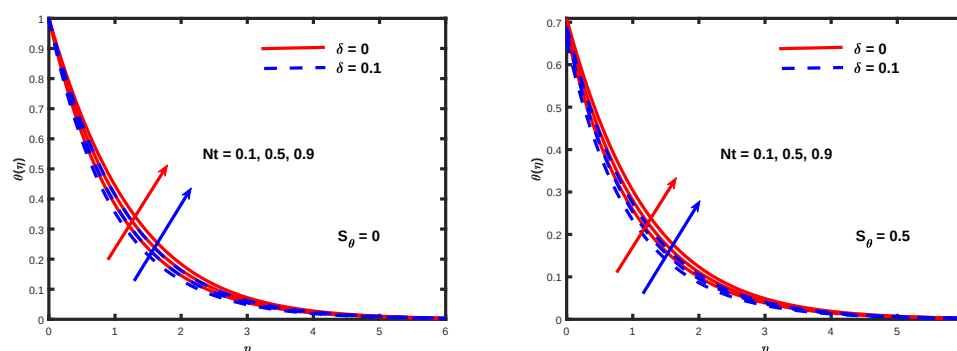


Figure 11. Influence of unsteady and thermophoresis parameter on θ .

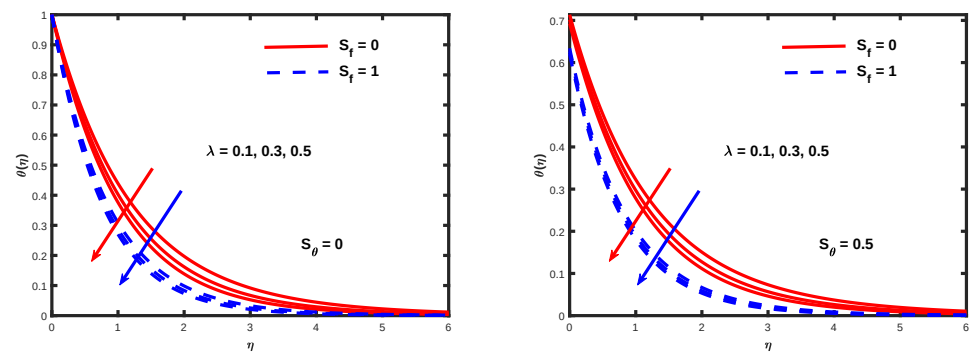


Figure 12. Influence of buoyancy parameter and hydrodynamic slip on θ .

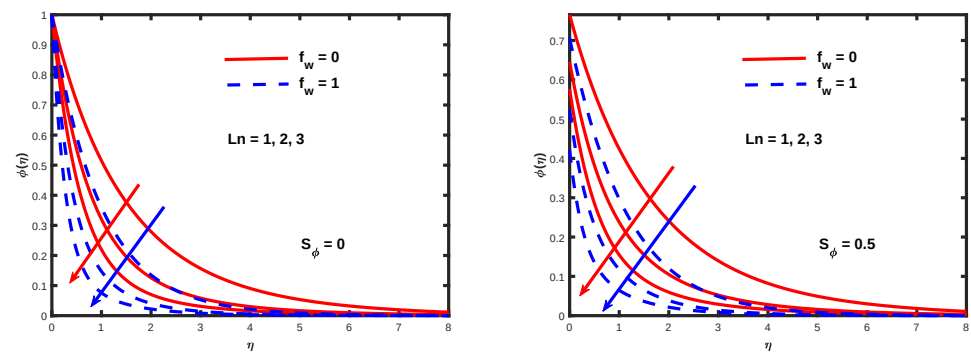


Figure 13. Influence of Lewis number and suction/injection parameter on ϕ .

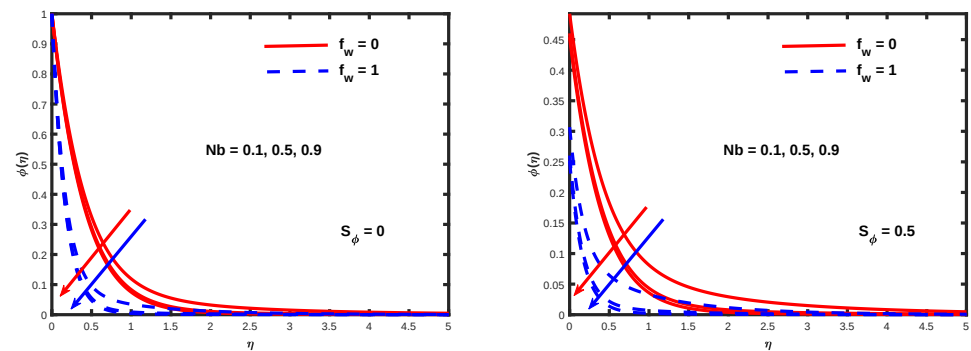


Figure 14. Influence of suction/injection and Brownian motion parameter on ϕ .

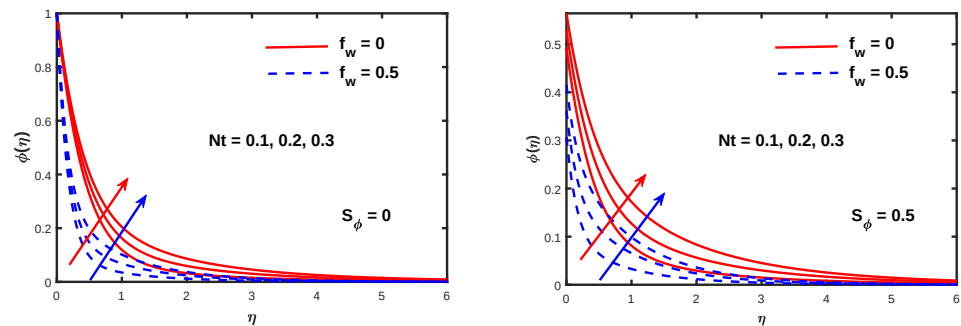


Figure 15. Influence of suction/injection and thermophoresis parameter on ϕ .

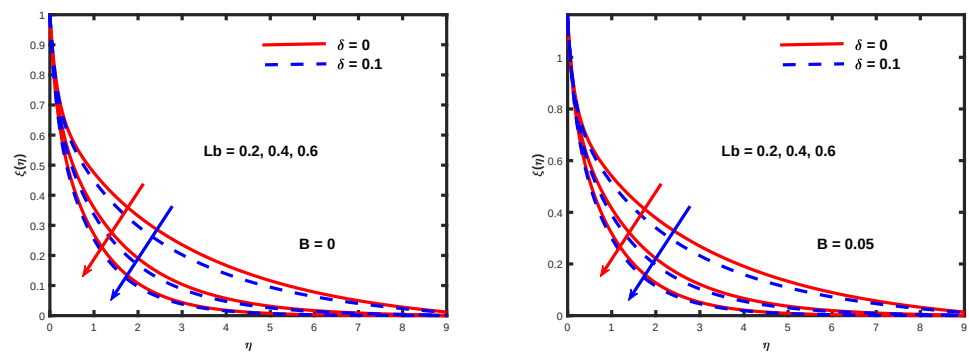


Figure 16. Influence of unsteady parameter and microorganism concentration difference on ξ .

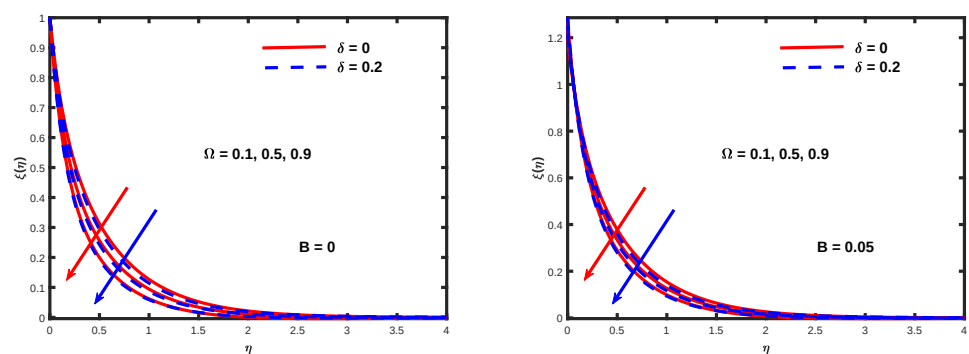


Figure 17. Influence of unsteady parameter and microorganism concentration difference on ξ .

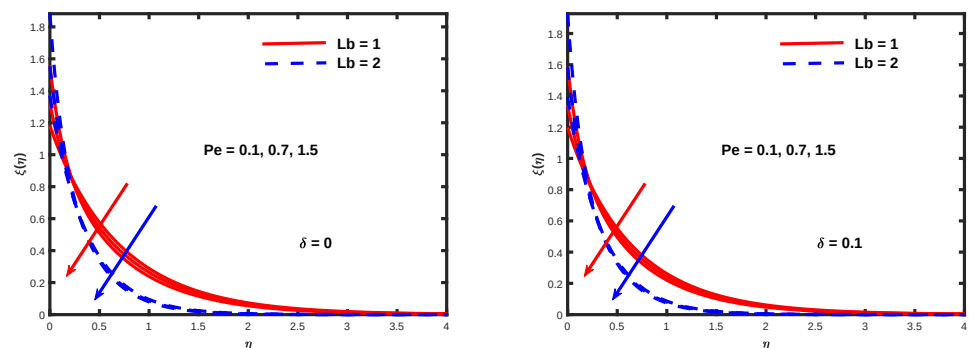


Figure 18. Influence of bioconvection Peclet numeral and bioconvection Lewis number on ξ .

5. Conclusions

In this article, the numerical study of magnetohydrodynamic stratified micropolar bioconvective fluid containing gyrotactic microorganisms and nanoparticles with radiation effects is investigated. With the help of similarity transformation, partial differential equations are converted into ordinary differential equations then solved with the Runge–Kutta method. Moreover, the impact of different physical parameters is analyzed. The main findings of this articles are given below:

- The fluid velocity decreases with the increasing values of M , K_p and Rb while the opposite behavior is noticed for K and λ .
- With the increasing values of M , micro-rotation profile decreases while it increases with the increment in K .
- Temperature increases with the increment in the values of Nb , Nt , and M while opposite behavior shows for λ .
- A decrement in the concentration profile is seen when the values of Ln and Nb rises while the profile rises with the rise in Nt .

- With the increment in the values of Lb , Ω and Pe the motile micro-organism density profile decreases.

Author Contributions: S.A. and S.H. modeled the problem and wrote the manuscript. I.S. thoroughly checked the mathematical modeling. S.A. solved the problem using MATLAB software. S.H. and I.S. contributed to the results and discussions. H.A., M.M.A. and I.A. reviewed and edited the manuscript. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Informed consent was obtained from all subjects involved in the study.

Data Availability Statement: Not applicable.

Acknowledgments: Authors are thankful to the Deanship of Scientific Research, King Khalid University, Abha, Saudi Arabia, for financially supporting this work through the General Research Project under Grant no. R.G.P.2/7/42.

Conflicts of Interest: The authors declare no conflict of interest.

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