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Boundary Deformation Measurement by Mesh-Based Digital Image Correlation Method

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Abstract: Digital image correlation (DIC) is a popular photomechanics method for deformation measurement. The conventional subset-based DIC method obtains the displacement vectors at the subset centers, but cannot calculate the deformation on the specimen edges, which may contain very useful information because specimens usually have greater deformation on edges, especially on curved edges due to stress concentration. In this study, the capability of a mesh-based DIC method using 8-node quadrilateral elements (Q8-mesh-DIC) in boundary deformation measurement was investigated and highlighted for specimens with non-uniform deformation. The results were compared with those obtained by some conventional subset-based DIC methods, and the accuracy of the boundary deformation measurement was verified through simulated and real experiments. The Q8-mesh-DIC appears to be more suitable for the boundary deformation measurement of non-uniform deformation fields.

Keywords: digital image correlation; Q8-DIC; non-uniform deformation field; boundary deformation measurement

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1. Introduction

The digital image correlation (DIC) method [1], also known as the digital speckle correlation method, is a non-contact photomechanics method based on the analysis of speckle images. Conventional subset-based DIC extracts full-field deformation information by matching the corresponding locations in the images recorded before and after the deformation of a specimen. The difference between the coordinates of the center points of each pair of matched subsets is as the measured displacement [2,3]. As a powerful and flexible tool for surface deformation measurement, subset-based DIC has been widely accepted and commonly used in the field of experimental mechanics.

Because only the displacement vectors at the subset centers can be calculated, and the subsets cannot contain invalid pixels from the background, an obvious drawback of conventional subset-based DIC is that the deformation near the specimen edge cannot be measured directly. That is, the deformation in an area along the specimen edge will be excluded, and the width of the missing area depends on the subset size. In general, for high-precision deformation measurement, a contradiction exists between the measurement precision and missing information. The reason is that a large subset size is usually selected to ‘collect’ sufficient information and guarantee high precision [4]; however, a larger subset size will, in turn, cause more missing information near the specimen edge. It is well known that, for many specimens with complex shapes, yielding or damage usually initiates on the curved edges. Thus, the deformation near the curved edges is particularly important in mechanical or failure analyses. For example, for a ring component under diametrical compression, cracks initiate from the hole edge owing to the local stress concentration [5].

From this perspective, the accurate measurement of deformations near or on the curved edges of specimens or structures plays a very important role in practical applications. However, based on conventional subset-based DIC, more information is missed near curved edges than straight ones. To obtain this useful information near the edges, some methods, including extrapolation or changing the subset shape, have been employed in references [6–9]. In the extrapolation method [6], the deformation near the specimen edges that cannot be measured directly is extrapolated from the inner measured displacement field. In principle, the extrapolation method is easy to implement, but the deformation pattern near the edges may substantially differ from that of the inner region owing to factors like high-stress concentration; as a result, the extrapolated result may fail to reflect the true deformation. The half-subset method [8,9] is another method for calculating the boundary deformation based on the conventional subset-based DIC model. In this method, valid and invalid pixels are labeled in the subsets across the edge, and only the valid points are used in the correlation analysis. That is, only parts of the subsets or irregular subsets are used to calculate the displacements near or on the edges, which generally results in insufficient matching information and low measurement accuracy. From the above discussion, it can be concluded that it is difficult to calculate the displacements near the specimen edges accurately using the subset-based DIC methods.

As alternatives to overcome the drawbacks of the subset-based DIC methods in boundary deformation measurement, mesh-based DIC methods [10–12] have their potential prospects. Following the concept of the finite element method (FEM), mesh-based DIC methods discretise the area of interest (AOI) of a speckle image into several elements to ensure continuity of the deformation field within the entire AOI. By placing the element nodes on the edges, boundary deformation can be achieved using such methods. Sun et al. [10] and Besnard et al. [11] established the 4-node-quadrangular mesh-based DIC (Q4-DIC) method for deformation measurement. However, as only two nodes exist on one element edge, only linear interpolation can be realized, and the deformation cannot be measured accurately when the element exhibits complex deformation. More importantly, when the displacement field near or on the curved edges needs to be measured, Q4-DIC cannot cover the AOI very effectively, owing to the straight element edges. To improve on this, Ma et al. [12] introduced 8-node quadrilateral elements into the DIC model (abbreviated as Q8-DIC). High-order shape function interpolation with three nodes on each edge was used in this technique, enabling the Q8 element to be mapped to an element with second-order curved edges. Hence, with this method, quadratic interpolation can be achieved, and the displacement of points close to or even on the curved edges can be measured.

In this study, the capability of Q8-DIC with spatial continuity in boundary deformation measurement was explored and highlighted for specimens with non-uniform deformation. The measurement results were compared with those from the extrapolation and half-subset methods, both of which are based on conventional subset-based DIC. The method was verified using simulated tests and effectively applied to actual experiments. The results demonstrated that the calculation accuracy of Q8-DIC in deformation measurement near curved edges was much higher than those of the extrapolation and half-subset methods. The remainder of this paper is organized as follows. In Section 2, the basic principles of the conventional subset-based methods and their deficiencies in terms of boundary deformation measurement are briefly explained. Section 3 compares the accuracy of the three methods in measuring the deformation near curved edges using simulated speckle images. In Section 4, the application of the three methods in a real experiment is presented. Finally, several concluding remarks are provided.

2. Basic Principles

2.1. Conventional Methods and Their Deficiencies in Boundary Deformation Measurement

The basic idea of the traditional subset-based DIC method is to match (or track) the same physical points located in the reference (or source) image and deformed (or target) image. Relevant work in this area has been discussed in detail in articles [3,13]. Although

it is simple in principle and implementation, the existing DIC technique exhibits certain deficiencies. For example, to measure the displacement of a point located near the edge of a specimen, the subset may contain unwanted or foreign pixels outside of the specimen, as illustrated in Figure 1. In such cases, errors will be induced in the correlation and interpolation calculation. That is, the boundary deformation cannot be calculated, and the displacements in a strip along the specimen edge will be missing. Therefore, a smaller subset size is usually selected to obtain more deformation information near the specimen edges. However, a smaller subset will lead to a larger standard deviation (SD) in the displacements. A conflict always exists between the standard deviation and measuring area loss near the edges. A numerical experiment can be used to understand and explain these two conflicting aspects more intuitively. In this experiment, a speckle image with a size of 500×500 pixels and 256 gray levels was used as the reference image (Figure 1). The speckle pattern was obtained by randomly spraying black and white paints onto the surface of a flat specimen. To form the deformed image, random Gaussian noise with a gray-scale intensity variance of 4 was added to the reference image. Based on these two images, the displacement field was calculated with subset sizes ranging from 11×11 to 77×77 pixels, with an interval of 6 pixels. As no actual deformation occurred, the SD error of the measured displacement field can be expressed as

$$\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^N d_i^2} \quad (1)$$

where N is the total number of measuring points and d_i represents the displacement component (u or v) of point i . The SD error reflects the deviations of the measured displacements from their mean values. Figure 2 illustrates the SD error of the u -displacement and measuring area loss versus the subset size. It is clear that, with an increase in the subset size, the SD error decreased, but the area loss increased. To solve this contradiction, i.e., to obtain the inner displacement field precisely with a relatively larger subset, as well as the boundary deformation, conventional techniques such as the extrapolation and half-subset methods can be used. The basic principles of these two methods in boundary deformation measurement are briefly introduced next.

Figure 1. Speckle image with subset located near the specimen edge.

Extrapolation is an interpolation method whereby the displacements near the specimen boundary are obtained using those calculated by the conventional subset-based DIC method inside the specimen (Figure 3a). However, the extrapolated boundary deformation may not be accurate if the deformation pattern near the specimen edge differs from that inside the specimen. For example, owing to the high-stress concentration, the material near

the edge may already yield, while it remains elastic in other regions. In this case, a reliable result cannot be always assured.

Figure 2. Standard deviation (SD) error of u-displacement and area loss near specimen edge vs. subset size.

Figure 3. Conventional methods for boundary displacement calculation: (a) Extrapolation method and (b) half-subset method.

The half-subset method proposed by Pan et al. [9] is another approach for tackling the boundary deformation problem. In this method, for a subset centered at a point close to or on the AOI boundary (Figure 3b), a binary mask is established to identify the valid points within the AOI and the invalid points outside the AOI but within the subset. During the correlation analysis, only the labeled valid points are used, while the invalid points are excluded. Further details regarding the half-subset method can be found in reference [9]. Although the half-subset method can extend the deformation measurement area towards the AOI boundary, the accuracy and precision of the calculated displacements with half subsets are generally lower than those calculated using full subsets, owing to the lower amount of pixels used for correlation coefficient optimization. Theoretically, the number of valid points contained in a subset should be substantially greater than the number of unknown parameters to be determined; otherwise, the displacements at the centre point may not be obtained.

2.2. Basic Principle of Q8-DIC in Boundary Deformation Measurement

Using the concept of the FEM, Q8-DIC divides the AOI into several elements connected through nodes, thereby ensuring the continuity of the displacement field within the entire AOI. The boundary deformation can be calculated directly when the element nodes are placed on the edges (Figure 4). Instead of matching the subsets individually to obtain the displacements of each point, an objective function combining the reference and deformed

images was constructed to resolve the displacements of all searched points in Q8-DIC. A brief introduction to Q8-DIC is provided below.

Figure 4. Original and deformed meshes in 8-node quadrilateral elements (Q8-DIC).

An 8-node isoparametric quadrangular element in the global coordinate system xoy can be mapped onto a square in the local coordinate system $\xi\eta$ (Figure 5). The relationship between the two coordinate systems is expressed as follows:

$$\begin{aligned} x &= \sum_{i=1}^8 N_i(\xi, \eta) x_i \\ y &= \sum_{i=1}^8 N_i(\xi, \eta) y_i \end{aligned} \quad (2)$$

where ξ and η are the horizontal and vertical coordinates in the local coordinate system, x and y are the corresponding coordinates in the global coordinate system and $N_i(\xi, \eta)$ is the shape function of the i th node of the element, which has the same form as the shape function in the FEM.

Figure 5. Relationship of global (a) and local (b) coordinates in the Q8-element.

The displacements of a point in the global coordinate system can be obtained by the corresponding shape function and displacements of the elemental nodes in the local coordinate system:

$$\begin{aligned} u_{\Omega_m}(x, y) &= \sum_{i=1}^8 N_i(\xi, \eta) u_{mi} \\ v_{\Omega_m}(x, y) &= \sum_{i=1}^8 N_i(\xi, \eta) v_{mi} \end{aligned} \quad (3)$$

where Ω_m represents the m th element, u_{Ω_m} and v_{Ω_m} are the displacement components of the point with the coordinate (x, y) in Ω_m , and u_{mi} and v_{mi} are the displacement components of the i th node in Ω_m . The displacements of each node are essentially the parameters to be optimized. By selecting an appropriate objective function, the solution of the displacement

field can finally be transformed into an optimization problem. The objective function can be expressed as follows:

$$loss = \sum_{m=1}^M \sum_{\Omega_m} (f(x, y) - g(x + u_{\Omega_m}(x, y), y + v_{\Omega_m}(x, y)))^2 \quad (4)$$

where f and g represent the gray intensity of the AOI in the reference and deformed images, respectively. An iterative algorithm (such as the Newton–Raphson algorithm [13], Levenberg–Marquardt (LM) algorithm [14], genetic algorithm [15], and the inverse compositional Gauss–Newton algorithm [16]) can be used to minimize the objective function. The displacement vectors near or even on the edges can be obtained by Equation (4) once the solution is complete.

3. Verification of Q8-DIC in Boundary Deformation Measurement

To verify the accuracy and efficiency of the three methods described in Section 2 for deformation measurement near curved edges, a simple simulated test was conducted in the form of a square plate under open-hole compression (Figure 6a). The reference image, with a size of 1001×1001 pixels and hole radius of $a = 240$ pixels, was cropped from a real speckle image. Distributed loading was applied on the top and bottom with a load density of q . Based on the solution of the displacement field of the problem, the deformed image was obtained from the reference image by cubic interpolation. The pixel intensities of the deformed image were encoded into 8-bit integers with values ranging from 0 to 255.

Figure 6. The geometry of open-hole specimen (a) and meshing scheme in Q8-DIC (b).

The theoretical elastic solution of the displacement field of the open-hole compression problem can be expressed by Equation (5):

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} u_r \\ u_\theta \end{bmatrix} \quad (5)$$

where

$$u_r = \frac{(1+\nu)qr}{2E} \left\{ \frac{1-\nu}{1+\nu} + \frac{a^2}{r^2} + \left[1 + \frac{4}{1+\nu} \frac{a^2}{r^2} - \frac{a^4}{r^4} \right] \right\} \cos 2\theta \quad (6)$$

$$u_\theta = -\frac{(1+\nu)qr}{2E} \left[1 + \frac{2-2\nu}{1+\nu} \frac{a^2}{r^2} + \frac{a^4}{r^4} \right] \sin 2\theta \quad (7)$$

in which r and θ are the polar coordinates with the origin at the hole center, and a represents the hole radius. In this simulated test, the load density $q = 1.4 \times 10^7$ N/pixel, Young's modulus $E = 10$ GPa, and Poisson's ratio $\nu = 0.33$.

Based on the reference and deformed images, the v displacement field was first solved by the subset-based DIC. The commercial image correlation software VIC-2D was used with a subset size of 47 pixels to assess the displacement distribution. The displacements near the specimen edges that could not be directly calculated by the subset-based DIC were obtained using cubic extrapolation and the half-subset method. The displacement field was also calculated by Q8-DIC with the meshing scheme illustrated in Figure 6b. In total, there were 8 elements and 40 nodes. The LM optimization algorithm was employed, and the optimization iteration was stopped when the change of *loss* in Equation (4) was less than 1×10^{-6} or the variations in the parameters to be optimized were less than 1×10^{-3} .

It can be observed from Figure 7 that the displacement field calculated by Q8-DIC was smoother and more consistent with the theoretical elastic solution than those obtained by the extrapolation and half-subset methods. This is because the continuity of the entire displacement field was considered in the Q8-DIC method. For a clearer comparison of the accuracy of the three methods in the boundary deformation measurement, the displacement distribution around different circles near the hole can be plotted. Figure 8a,b presents the vertical displacements around two circles with radii of 241 and 255 pixels, respectively. It can be observed that even on the hole edge ($r = 241$ pixels), Q8-DIC could yield a sufficiently accurate result, but the extrapolation method exhibited significant deviation from the theoretical solution, while the half-subset method provided completely unreasonable displacements. The accuracy of the calculated displacements improved with an increasing distance from the hole edge. When $r = 255$ pixels, the displacement distributions already agreed strongly with the theoretical solution, except for the relatively large error near the top ($\theta = 90^\circ$) and bottom ($\theta = 270^\circ$) of the circle in the extrapolation method. Figure 8c illustrates the absolute errors of the three methods compared with the theoretical solution; the non-smoothness of the displacement field obtained by the half-subset method can be clearly observed.

Figure 7. Displacement fields obtained by different methods.

Figure 9 presents the relative errors of the displacements measured by the three methods along a radius ($\theta = 90^\circ$). The relative error is defined as the difference between the measured and theoretical values, divided by the theoretical value. The Q8-DIC method could provide accurate displacements along the entire radius, starting from the hole edge. The extrapolation method exhibited a relative error within 30%, which decreased along the radius, and became negligible when the radius was larger than 264 pixels at the junction of the subset DIC and extrapolation regions. Reasonable displacement could not be obtained near the hole edge by the half-subset method, but the accuracy improved more significantly along the radius compared to the extrapolation method.

(a) $r = 241$ pixels (b) $r = 255$ pixels (c) $r = 255$ pixels

Figure 8. Displacements and measurement errors near the hole edge: (a) $r = 241$ pixels, (b) $r = 255$ pixels, (c) measurement errors of displacements ($r = 255$ pixels).

Figure 9. Relative errors along radius at $\theta = 90^\circ$.

4. Application in Real Experiment

To compare the three methods in terms of real deformation measurements near curved edges, a diametrical compression test was performed using a ring made of aluminum alloy 6061. Figure 10a and Table 1 present the ring specimen dimensions and the mechanical properties of the aluminum alloy 6061. An irregular speckle pattern was created on the specimen beforehand by spraying black and white paints on the front surface. The test was conducted on a universal WDW-50E testing machine (Figure 11). Displacement control was applied with a loading rate of 0.05 mm/min. During the loading process, an imaging system consisting of a charge coupled device (CCD) camera with a spatial resolution of 1628×1236 pixels and a 50 mm fixed-focus lens was used to capture images of the specimen front surface at a speed of 1 fps. Figure 11 presents the undeformed image with an inner radius of 243 pixels and an outer radius of 486 pixels for the ring. The collected deformed images were processed using the VIC-2D software with a subset size of 47 pixels, the extrapolation method, the half-subset method, and Q8-DIC to obtain the displacement field of the front surface. The meshing scheme illustrated in Figure 10b, with a total of 42 elements and 154 nodes, was used in the Q8-DIC, and the rules for meshing in Q8-DIC referred to the papers [12,17].

Figure 10. Diametrical compression test diagram of the aluminum ring (a) and meshing scheme in Q8-DIC (b).

Figure 11. Experimental setup of diametrical compression test.

Table 1. Specimen dimensions and mechanical properties of aluminum alloy 6061.

d (mm)	D (mm)	b (mm)	Modulus (GPa)	Poisson's Ratio
25	50	15	68.9	0.33

A 2-dimensional FE model (Figure 12) of the diametrical compression test was created by using the commercial software ABAQUS. The loading and supporting platens were modeled as analytical rigid bodies, and the frictionless contact option was used between the ring and platens. A concentrated force was applied at the reference point of the loading plate. The ring was discretized into 6000 4-node plane stress elements (CPS4R in ABAQUS notation), with 30 elements in the radial direction and 200 elements in the circumferential direction. Isotropic properties were assigned to the elements, as detailed in Table 1.

Figure 12. Finite element model of diametrical compression test.

Figure 13 presents the load–displacement curve of the indenter during the diametrical compression test. For comparison with the elastic solution obtained by the finite element analysis, the image captured at a very early loading stage (point A in Figure 13) was selected to calculate the displacement field. Figure 14 illustrates the calculated displacement fields in the vertical direction using different methods. In general, all measured displacement distributions agreed strongly with the simulated result. However, as the Q8-DIC method considered the continuity constraint for the entire field, it produced a smoother displacement field than the other two methods.

Figure 13. Load–displacement curve of indenter in diametrical compression test.

Figure 14. Displacement fields using different methods.

To provide an intuitive comparison of the three methods' accuracy in the deformation measurement near curved edges, the displacement distributions around 2 circles with radii of 244 and 259 pixels were plotted in Figure 15a,b, respectively. It can be observed that the Q8-DIC calculation result was always in strong agreement with that predicted by the FE model, even on the hole edge ($r = 244$ pixels). The extrapolation method could also indirectly produce the displacements on the hole edge, but with relatively large deviations from the FEM result. The half-subset method even failed to provide a reasonable displacement distribution around the hole. However, the measurement accuracy of the half-subset method improved with increasing distance from the hole edge. On the circle with a radius of 259 pixels, the measured displacements from the half-subset method were already almost the same as the Q8-DIC result. Figure 15c illustrates the absolute errors of the three methods compared to the FEM result. It was clearly indicated that, although the absolute errors were at the same level for the half-subset and Q8-DIC methods, the latter provided a substantially smoother deformation field. This was certainly owing to the continuity constraint over the entire displacement field in the Q8-DIC method.

(a) $r = 244$ pixels (b) $r = 259$ pixels (c) $r = 259$ pixels

Figure 15. Displacements and measurement errors near ring edge: (a) $r = 244$ pixels, (b) $r = 259$ pixels, and (c) measurement errors of displacements ($r = 259$ pixels).

5. Conclusions

Despite its wide application, the conventional subset-based DIC method exhibits a fundamental defect in boundary deformation calculation. Boundary deformation information is of paramount importance in many mechanical problems because damage or yielding generally initiates from the boundary, particularly on curved edges as a result of stress concentration. To resolve this issue, the capability of the novel Q8-DIC in deformation near curved edges was explored and highlighted in this work. The effectiveness and efficiency of Q8-DIC in the boundary deformation calculation were compared with those of two conventional methods, namely the extrapolation and half-subset methods, by means of simulated and real experiments. The results revealed that, owing to the continuity constraint over the entire displacement field, Q8-DIC provided significantly higher accuracy in boundary deformation measurement and a substantially smoother displacement field than the other two methods. A precise whole field (including the boundary) deformation is very useful for material property characterization or failure prediction. For example, mesh-based DIC and finite element model updating methods can be combined to inversely obtain the damage evolution law or yielding criterion of material under complex loading. Further studies will be performed in these fields.

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References

- Peters, W.H.; Ranson, W.F. Digital imaging techniques in experimental stress analysis. *Opt. Eng.* **1982**, *21*, 427–431. [\[CrossRef\]](#)
- Sutton, M.A. Digital image correlation for shape and deformation measurements. In *Springer Handbook of Experimental Solid Mechanics, Part C*; Sharpe, W.N., Ed.; Springer: New York, NY, USA, 2008.
- Pan, B.; Qian, K.M.; Xie, H.M.; Asundi, A. Two-dimensional digital image correlation for in-plane displacement and strain measurement: A review. *Meas. Sci. Technol.* **2009**, *20*, 062001. [\[CrossRef\]](#)
- Pan, B.; Xie, H.M.; Wang, Z.Y.; Qian, K.M.; Wang, Z.Y. Study on subset size selection in digital image correlation for speckle patterns. *Opt. Express* **2008**, *46*, 033601. [\[CrossRef\]](#) [\[PubMed\]](#)
- Pour-Ghaz, M.; Poursaeed, A.; Spragg, R.; Weiss, W.J. Experimental methods to detect and quantify damage in restrained concrete ring specimens. *J. Adv. Concr. Technol.* **2011**, *9*, 251–260. [\[CrossRef\]](#)
- Wang, P.; Pierron, F.; Thomsen, O.T. Identification of material parameters of PVC foams using digital image correlation and the virtual fields method. *Exp. Mech.* **2013**, *53*, 1001–1015. [\[CrossRef\]](#)
- Zhou, Y.; Sun, C.; Chen, J. Adaptive subset offset for systematic error reduction in incremental digital image correlation. *Opt. Lasers Eng.* **2014**, *55*, 5–11. [\[CrossRef\]](#)
- Pan, B.; Xie, H.M.; Xu, B.Q.; Dai, F.L. Full-field deformation measurement of specimen with discontinuity using digital image correlation. *J. Exp. Mech.* **2007**, *22*, 379–384.
- Pan, B.; Wang, Z.Y.; Lu, Z.X. Genuine full-field deformation measurement of an object with complex shape using reliability-guided digital image correlation. *Opt. Express* **2010**, *18*, 1011–1023. [\[CrossRef\]](#) [\[PubMed\]](#)
- Sun, Y.F.; Pang, H.L.; Wong, C.K.; Su, F. Finite element formulation for a digital image correlation method. *App. Opt.* **2005**, *44*, 7357–7363. [\[CrossRef\]](#) [\[PubMed\]](#)
- Besnard, G.; Hild, F.; Roux, S. Finite-element displacement fields analysis from digital images: Application to Portevin–LeChatelier bands. *Exp. Mech.* **2006**, *46*, 789–803. [\[CrossRef\]](#)
- Ma, S.; Zhao, Z.; Wang, X. Mesh-based digital image correlation method using higher order isoparametric elements. *J. Strain Anal. Eng. Des.* **2012**, *47*, 163–175. [\[CrossRef\]](#)
- Bruck, H.A.; McNeill, S.R.; Sutton, M.A.; Peter, W.H. Digital image correlation using Newton–Raphson method of partial differential correction. *Exp. Mech.* **1989**, *29*, 261–267. [\[CrossRef\]](#)
- More, J.J. The Levenberg–Marquardt algorithm: Implementation and theory. *Lect. Notes Math.* **1978**, *630*, 105–116.
- Haupt, R.L.; Haupt, S.E. *Practical Genetic Algorithms*, 2nd ed.; Wiley: Hoboken, NJ, USA, 2004.

-
16. Shao, X.; Dai, X.; He, X. Noise robustness and parallel computation of the inverse compositional Gauss-Newton algorithm in digital image correlation. *Opt. Lasers Eng.* **2015**, *71*, 9–19. [[CrossRef](#)]
 17. Wang, X.; Ma, S.P. Mesh-based digital image correlation method using non-uniform elements for measuring displacement fields with high gradient. *Exp. Mech.* **2014**, *54*, 1545–1554. [[CrossRef](#)]