Article

Triaxial Compression Performance Research of Steel Slag Concrete on the Unified Strength Theory

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Abstract: To better explore the mechanical properties of steel slag concrete (SSC) under triaxial compression, true triaxial tests were performed on SSC with three replacement ratios (30%, 70%, 100%) by a servo-controlled setup (TAWZ-5000/3000). Through the test, failure modes, peak stress, and corresponding strain of SSC are obtained. Results show that the failure modes of SSC are plate-splitting and slant-shear. Compared with the corresponding uniaxial strength, the triaxial compressive strength of SSC is significantly improved and is influenced by the stress ratio and the replacement ratio. Finally, based on unified strength theory, the strength failure criterion formula of SSC with different replacement rates under triaxial compression is given.

Keywords: steel slag concrete; triaxial strengths; mechanical properties; stress ratio

Citation: Deng, Z.; Zhou, J.; Chen, B.; Wen, X.; Liu, B. Triaxial Compression Performance Research of Steel Slag Concrete on the Unified Strength Theory. *Appl. Sci.* **2021**, *11*, 128. <https://dx.doi.org/10.3390/app11010128>

Received: 14 November 2020

Accepted: 22 December 2020

Published: 24 December 2020

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1. Introduction

Steel slag is a by-product of the steelmaking process and accounts for 15–20% of crude steel output [1]. Every year a large amount of waste steel slag is produced in the steelmaking process. Still, steel slag use is not high [2], especially in underdeveloped countries in Southeast Asia, such as Vietnam and Thailand, where it is less than 10% [3]. At present, a great quantity of waste steel slag can only be stacked and buried as garbage, which causes the grim situation of destroyed land resources and a worsening environment [4]. Meanwhile, urban construction requires huge amounts of building material, leading to a high demand globally for natural coarse aggregates. Using steel slag to replace natural aggregate in concrete can reduce the environmental damage caused by mining natural aggregates and prevent environmental pollution from waste steel slag [5,6].

In the past ten years, the viability of steel slag as a coarse aggregate substitute for gravel in concrete has been extensively investigated [7–13]. Steel slag aggregate is different from the natural aggregate. It has a rougher surface and larger contact area, making the bond between mortar and aggregate tighter [7]. Therefore, concrete with appropriate steel slag has superior compressive properties than ordinary concrete [8,9]. Arabani [10] shows that concrete with appropriate steel slag coarse aggregate is greater than natural aggregate concrete under dynamic load. Yu et al. [11] researched steel slag aggregate concrete-filled steel tubular. The research showed that the expansion of steel slag significantly affects the ultimate bearing capacity and peak strain of confined concrete. Devi [12] shows that concrete durability combined with 21% coarse steel slag replacement is more excellent. Also, steel slag concrete (SSC) has better temperature resistance than plain concrete [13]. When steel slag is added into concrete as coarse aggregate, it also functions as active cement and concrete admixtures [14]. The reason is that many kinds of chemical components in the steel slag, such as C_3S , C_2S , C_4AF , and C_2F , can be fully compatible with cement [15,16]. At the same time, the emission of toxic substances in steel slag is effectively suppressed by concrete [17], which indicates that the SSC can be applied safely to the construction

industry. Presently, the world is advancing sustainable development strategies, and it is of great strategic significance to use waste steel slag.

In reality, concrete is not only subjected to uniaxial forces but may also be subjected to forces in multiple directions. Concrete in applying engineering practice is mostly under complex stress states [18], such as the shear wall, beam-column connection, arch dams, nuclear reactor pressure container, and shell structure [19]. Moreover, many experts and scholars prove that the behavior of concrete under triaxial compression is not the same as uniaxial compression [18–21]. Therefore, it is not comprehensive to only consider uniaxial strength theory in actual engineering. Due to the constraint of lateral stresses, concrete under triaxial compression has advantages in ductility, bearing capacity, and peak strain [22]. The results show that concrete's multiaxial mechanical behaviors are closely related to the type of concrete [23–26]. Many studies have focused on concrete's uniaxial mechanical behaviors; however, few studies have investigated the multiaxial mechanical behaviors of concrete, especially steel slag coarse aggregate (SSCA). It would appear necessary to research the triaxial mechanical properties of SSC.

Based on the background, we have investigated the triaxial mechanical properties of SSC. The failure mode, strength, peak strain, and stress-strain curve of SSC are analyzed. Draw on the unified strength theory Yu [27] put forward, a modified strength failure criterion for SSC is given. This paper's experimental data and theoretical models can provide a reference for SSC, which has great significance for reusing waste steel slag and environmental protection.

2. Materials and Experimental Procedures

2.1. Specimen Production

There are two types of coarse aggregates used in this true triaxial tests: Natural Coarse Aggregate (NCA) and Steel Slag Aggregate (SSA). NCA was natural gravel produced in Nanning, China. SSA was by-products from Guangxi Liuzhou Iron and Steel Group. As shown in Figure 1, NCA and SSA's diameter is in the range of 5–20 mm. Table 1 shows the material performance of the coarse aggregate. Steel slag concrete was made of P. O 42.5 Ordinary Portland cement, and the fine aggregate was the natural river sand with a fineness modulus of 2.5.

Figure 1. The graining curves for SSA and NCA.

Table 1. The material performance of coarse aggregates.

Classification	Gradation (mm)	Bulk Density (kg/m ³)	Apparent Density (kg/m ³)	Crush Index (%)	Water Absorption (%)
SSA	5–20	2280	3230	6.10	1.36
NCA	5–20	1480	2760	10.40	0.65

Table 2 shows the mixing ratio of the mixture and the main parameters of SSC. The concrete mix ratio was designed according to the Specification for Mix Proportion Design of Ordinary Concrete [28]. Three kinds of SSA replacement ratios were designed: 30%, 70%, 100% by mass of NCA. SSC-30 means that the SSA with a replacement ratio of 30%.

Table 2. Mix proportions and major parameters of concrete (kg/m³).

No.	W/C ¹	Water	Cement	Sand	SSA ²	NCA ³
SSC-30	0.56	200	357	777	350	816
SSC-70	0.56	200	357	777	816	350
SSC-100	0.56	200	357	777	1166	0

¹ Water/Cement; ² steel slag coarse aggregate; ³ natural coarse aggregate.

After mixing, the concrete was cast in a mold and then transported to a vibrating table for vibrating to remove air bubbles in the specimen. After 24 h cured in the room, the specimens were put into a standard curing room with a temperature of 20 ± 2 °C and relative humidity of 95% for 28 days. Ten stress ratios ($\sigma_1 : \sigma_2 : \sigma_3 = 0.00:0.00:1, 0.00:1.00:1, 0.05:0.25:1, 0.05:0.5:1, 0.05:0.75:1, 0.05:1:1, 0.1:0.25:1, 0.1:0.5:1, 0.1:0.75:1, \text{ and } 0.1:1:1$) were designed for each mix of proportions. The test specimens in this test were cubes with a side length of 100 mm, and there were three directions of vertical stress on the surface of the specimens, as shown in Figure 2. Three specimens were tested for each mixture, so a total of 90 specimens were tested. The experimental results took the average of each data set after deleting the discrete ones in this study.

Figure 2. Triaxial loading setup: (a) the exterior of the setup; (b) the interior of the setup; (c) the loading schematic of the specimen; (d) the loading direction of the specimen.

2.2. Testing Method

Specimens of multiaxial mechanical properties were tested using a servo-controlled setup (TAWZ-5000/3000) (Changchun City Chaoyang Testing Instrument Co. LTD,

Changchun, China), which was performed at the State Key Laboratory of Structural Engineering at Guangxi University. The test machine consisted of six independent hydraulic actuators and can simultaneously apply complex stress in three directions. In the Z-direction of the test machine, the machine's maximum load can reach 5000 kN, and the other two directions can reach 3000 kN. The built-in load sensor collected load information, and displacement information was measured by the linear displacement sensors (LVDTs). The test setup and installation of the specimen and the LVDTs are shown in Figure 2. Figure 2a shows the exterior of the setup, and Figure 2b shows the setup's interior. The loading schematic of the specimen is shown in Figure 2c and the specimen's loading direction is shown in Figure 2d. To eliminate the restraint on the loading surfaces, before installing the fixture, the friction-reducing pads which were using three layers of polyethylene film with glycerin were attached to the six sides of the specimens for all tests; an attrition machine polished the surfaces of the specimens to eliminate the adverse effects of uneven surfaces. After pre-loading 5 kN, the specimens started loading. The loading rate in the principal stress direction was 0.3 MPa/s. At the same time, in the other two directions, the loading rate was determined by the stress ratio during the loading process. During the test, the deformation, strength, and stress-strain curves of the specimens can be obtained from the data acquisition system. The principal stresses were expressed as $\sigma_1 \geq \sigma_2 \geq \sigma_3$. Compression is expressed as negative, and tension is expressed as positive. The stress-strain curve of SSC generated by the data acquisition system can be seen in Figure 3.

Figure 3. Cont.

Figure 3. The stress-strain curves for SSC under triaxial compression: (a) $\sigma_1 : \sigma_3 = 0.05:1$, SSC-30; (b) $\sigma_1 : \sigma_3 = 0.05:1$, SSC-70; (c) $\sigma_1 : \sigma_3 = 0.05:1$, SSC-100; (d) $\sigma_1 : \sigma_3 = 0.1:1$, SSC-30; (e) $\sigma_1 : \sigma_3 = 0.1:1$, SSC-70; (f) $\sigma_1 : \sigma_3 = 0.1:1$, SSC-100.

3. Test Results and Discussion

3.1. Failure Modes

Under the triaxial compression stage, the typical failure modes of SSC are shown in Figure 4, they were failure modes of the single-splitting, slant-shear, respectively. Generally speaking, the effect of replacement ratios on SSC had not to change the failure modes. However, SSC's brittleness was more significant with the increase of the SSA replacement ratio. The number of cracks on the specimen's surface increases with the SSA replacement ratio, which indicated that the SSC with a high replacement ratio was more seriously damaged. The failure mode of SSC was significantly affected by the stress ratio. When the stress ratio σ_2/σ_3 was relatively small (e.g., $\sigma_2 : \sigma_3 = 0.25:1$), the failure mode that appeared in SSC was plate-splitting. Because the third principal stress σ_3 was much greater than the others, the tensile strain appeared in the vertical direction of the surfaces of σ_1 and σ_2 . However, due to the constraint of σ_2 , the tensile strain that appeared in the normal direction of the surfaces of σ_2 was prevented. With the increase of the load, microcracks appeared on the surfaces of σ_3 and σ_2 and gradually extended throughout the specimens. The typical plate-splitting of SSC is shown in Figure 4a–c. With the increase of stress ratio σ_2/σ_3 , the tensile strain was prevented due to the constraint of σ_1 , and the slant-shear cracks occurred on the surface of σ_2 due to the large shear stress $(\sigma_3 - \sigma_1)/2$. It can be found in Figure 4d–i. There are the single-shear and double-shear cracks on the surface of σ_2 , and the angles between slant-shear cracks and the direction σ_3 are about 20° – 30° . In addition, the stress ratio $\sigma_1 : \sigma_3$ also affects the failure mode of the specimen. When the stress ratio σ_2/σ_3 was constant, the number of cracks generated by stress ratio $\sigma_1 : \sigma_3 = 0.1:1$ was less than that of by $\sigma_1 : \sigma_3 = 0.05:1$.

3.2. Triaxial Strength

According to Table 3, the peak stress and peak strain of this experiment have been summarized. The coefficient of variation for each group in Table 3 was not greater than 15% (most of that were not more than 10%). As shown in Table 3, compared with uniaxial strength, SSC's strength under triaxial compression has been greatly improved. The constraints in other directions have an obvious effect on the improvement of σ_3 . The value of stress σ_3 can improve to 108–189 Mpa due to the constraint of σ_2 and σ_1 . The relative value σ_3/f_m (triaxial strength to uniaxial strength) is between 2.93 and 5.05, which means that SSC's triaxial compressive strength is obviously enhanced.

It can be seen that the influence of stress ratio on the value of σ_3/f_m in Figure 5. When the stress ratio σ_1/σ_3 is constant, σ_3/f_m increases first and decreases with the increase of σ_2/σ_3 . Most of (except for the SSC-30 at a stress ratio of $\sigma_1/\sigma_3 = 0.1$) the maximal value of

σ_3/f_m was obtained at $\sigma_2/\sigma_3 = 0.5$. It can be seen that the influence of the first principal stress σ_1 on σ_3/f_m is visible. When the stress ratio σ_2/σ_3 is constant, the values of σ_3/f_m at $\sigma_1/\sigma_3 = 0.1$ is larger than that at $\sigma_1/\sigma_3 = 0.05$, which proves that the stress ratio σ_1/σ_3 also can influence the triaxial strength. The reason is that the first principal stress σ_1 constrains the lateral expansion of the concrete in the σ_1 direction and effectively slows the crack extension. The greater stress σ_1 is, the more obvious the constraint is. Using SSC-100 as an example, when stress ratio $\sigma_2 : \sigma_3 = 0.75:1$, σ_3/f_m growth can reach 159% from $\sigma_1/\sigma_3 = 0.05$ to $\sigma_1/\sigma_3 = 0.1$.

On the other hand, the SSA replacement ratio also has an effect on triaxial strength. When the stress ratio $\sigma_1/\sigma_3 = 0.05$, the value of σ_3/f_m is very close at different replacement ratios, and the maximal value of σ_3/f_m is SSA replacement ratio of 100%. When the stress ratio $\sigma_1/\sigma_3 = 0.1$, the maximal value of σ_3/f_m was obtained at the replacement ratio of 100%.

Figure 4. Schematic diagram of failure form under triaxial compression: (a) Plate-splitting cracks, $\sigma_1 : \sigma_2 : \sigma_3 = 0.05:0.25:1$, SSC-70; (b) 0.05:0.25:1, SSC-100; (c) 0.1:0.25:1, SSC-70; (d) Single-shear cracks, 0.05:0.5:1, SSC-70; (e) 0.05:0.75:1, SSC-100; (f) 0.1:0.75:1, SSC-100; (g) Double-shear cracks, 0.05: 0.75:1, SSC-70; (h) 0.05:0.5:1, SSC-100; (i) 0.1:1:1, SSC-100.

Table 3. Test results of SSC under triaxial compression.

No	Stress Ratio	σ_1 (MPa)	σ_2 (MPa)	σ_3 (MPa)	σ_3/f_m	$\varepsilon_1/10^{-3}$	$\varepsilon_2/10^{-3}$	$\varepsilon_3/10^{-3}$
SSC-30	0.00: 0.00:1	0.00	0.00	−38.32	1.00	-	-	−2.11
	0.00:1.00:1	0.00	−50.86	−50.86	1.33	-	−2.27	−2.27
	0.05:0.25:1	−5.66	−28.29	−113.17	2.95	4.02	−2.12	−10.87
	0.05:0.50:1	−6.00	−60.04	−120.07	3.13	7.92	−4.14	−12.04
	0.05:0.75:1	−5.84	−87.59	−116.79	3.05	10.74	−8.31	−11.14
	0.05:1.00:1	−5.73	−114.53	−114.53	2.99	14.32	−10.36	−10.36
	0.1:0.25:1	−15.44	−38.61	−154.42	4.03	3.61	−0.64	−17.50
	0.1:0.50:1	−17.33	−86.67	−173.33	4.52	6.12	−5.75	−18.48
	0.1:0.75:1	−17.91	−134.36	−179.14	4.68	7.61	−10.96	−17.69
	0.1:1.00:1	−16.78	−167.75	−167.75	4.38	8.15	−15.92	−15.92
SSC-70	0.00:0.00:1	0.00	0.00	−43.86	1.00	-	-	−2.29
	0.00:1.00:1	0.00	−58.83	−58.83	1.34	-	−2.42	−2.42
	0.05:0.25:1	−6.43	−32.13	−128.53	2.93	4.58	−2.59	−11.68
	0.05:0.50:1	−6.91	−69.09	−138.17	3.15	8.12	−4.91	−12.34
	0.05:0.75:1	−6.72	−100.83	−134.44	3.07	11.03	−8.62	−11.56
	0.05:1.00:1	−6.58	−131.61	−131.61	3.00	15.14	−10.92	−10.92
	0.1:0.25:1	−16.51	−41.29	−165.14	3.77	4.03	−0.78	−18.95
	0.1:0.50:1	−18.97	−94.85	−189.69	4.32	6.73	−5.98	−20.49
	0.1:0.75:1	−18.84	−141.29	−188.38	4.29	8.07	−11.94	−18.47
	0.1:1.00:1	−18.01	−180.12	−180.12	4.11	8.56	−16.68	−16.68
SSC-100	0.00: 0.00:1	0.00	0.00	−36.17	1.00	—	—	−1.92
	0.00:1.00:1	0.00	−54.04	−54.04	1.49	—	−2.07	−2.07
	0.05:0.25:1	−5.43	−27.15	−108.59	3.00	3.71	−1.87	−10.11
	0.05:0.50:1	−5.85	−58.49	−116.95	3.23	7.32	−3.64	−10.98
	0.05:0.75:1	−5.64	−84.63	−112.84	3.12	9.92	−7.56	−10.32
	0.05:1.00:1	−5.55	−111.07	−111.07	3.07	13.17	−9.76	−9.76
	0.1:0.25:1	−15.99	−39.98	−159.92	4.42	3.44	0.25	−16.92
	0.1:0.50:1	−18.26	−91.28	−182.55	5.05	5.96	−5.45	−18.24
	0.1:0.75:1	−17.94	−134.56	−179.41	4.96	7.36	−10.25	−17.58
	0.1:1.00:1	−16.92	−169.15	−169.15	4.68	7.73	−15.83	−15.83

3.3. Deformation Characteristic

Table 3 shows the peak strain of SSC under triaxial compression. We can see that under triaxial compression, the strains in the three directions are not equal ($\varepsilon_1 \neq \varepsilon_2 \neq \varepsilon_3$) and strain ε 's value depends on the stress ratio. The influence of stress ratio σ_2/σ_3 on the peak strain is shown in Figure 6. In the σ_3 direction, compare with strain ε_m under uniaxial stress, the peak strain ε_3 under triaxial compression is much higher, and increases first and then decreases as σ_2/σ_3 increases. The maximal value of ε_3 is obtained at the stress ratio $\sigma_2/\sigma_3 = 0.5$. In the σ_2 direction, the peak strain ε_2 increases approximately linearly with the increase of σ_2/σ_3 and reaches the maximum compressive strain at $\sigma_2 = \sigma_3$, where $\varepsilon_2 = \varepsilon_3$. When $\sigma_1/\sigma_3 = 0.05$, all of the strain ε_2 are compressive strain, but when $\sigma_1/\sigma_3 = 0.1$, the ε_2 is positive at the stress ratio $\sigma_2/\sigma_3 = 0.25$, and changes to negative at $\sigma_2/\sigma_3 = 0.5$. When intermediate stress σ_2 is small, tensile stress occurs in the σ_2 direction because of the σ_3 direction's extrusion pressure. In the σ_1 direction, the value of ε_1 depends on the stress ratios σ_1/σ_3 and σ_2/σ_3 , and increases as σ_2/σ_3 increases but decreases as σ_1/σ_3 increases. When the stress ratio σ_1/σ_3 is small, the growth rate of ε_1 is faster. As the stress ratio σ_1/σ_3 increases, the growth rate of ε_1 slows down. The strain ε of SSC was also affected by the replacement ratio. It can be seen that ε_3 , ε_2 , and ε_1 increased first and then decreased with the increase in the replacement ratio. When the stress ratio is constant, the general trends of ε_3 , ε_2 , and ε_1 with replacement ratio were SSC-70 > SSC-30 > SSC-100, which means that the strain of ε_3 , ε_2 , and ε_1 reach the maximal value at a replacement ratio of 70%.

Figure 5. Influence of stress ratio on σ_3/f_m : (a) $\sigma_1:\sigma_3 = 0.05:1$; (b) $\sigma_1:\sigma_3 = 0.1:1$.

Figure 6. Variation of peak strain with stress ratio under triaxial compression: (a) $\sigma_1:\sigma_3 = 0.05:1$; (b) $\sigma_1:\sigma_3 = 0.1:1$.

3.4. Failure Criterion

The unified strength theory established by Professor “Yu” [27] can be applied to concrete under the triaxial compression. Its expression is as follows:

$$(1+b)(3+\beta)\frac{\rho}{\sqrt{2}}\cos\theta + \sqrt{3}(1-\beta)(1-b)\frac{\rho}{\sqrt{2}}\sin\theta + f_1(\xi) = 0 \quad 0^\circ \leq \theta \leq \theta_b, \quad (1a)$$

$$(3+\beta-2b\beta)\frac{\rho}{\sqrt{2}}\cos\theta + \sqrt{3}(1-\beta+2b)\frac{\rho}{\sqrt{2}}\sin\theta + f_2(\xi) = 0 \quad \theta_b \leq \theta \leq 60^\circ, \quad (1b)$$

The relationship between the cylindrical coordinates (ξ, ρ, θ) and the primary stresses $(\sigma_1, \sigma_2, \sigma_3)$ are shown below:

$$\xi = \frac{1}{\sqrt{3}}(\sigma_1 + \sigma_2 + \sigma_3), \quad \rho = \frac{1}{\sqrt{3}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \cos\theta = \frac{2\sigma_1 - \sigma_2 - \sigma_3}{\sqrt{6}\rho}$$

$f_1(\xi), f_2(\xi)$ are the quadratic function of the hydrostatic stress axis, and the expressions be calculated from:

$$f_1(\xi) = -2\sqrt{3}C + 2[\beta(1+b) + A_1]\xi + \frac{2}{\sqrt{3}}B_1\xi^2, \quad (2a)$$

$$f_2(\xi) = -2\sqrt{3}C + 2[\beta(1+b) + A_2]\xi + \frac{2}{\sqrt{3}}B_2\xi^2, \quad (2b)$$

The tension meridian and the compress meridian expression as follows:

$$\rho_t = m_0 + m_1\xi + m_2\xi^2 \quad \theta = 0^\circ, \quad (3a)$$

$$\rho_c = n_0 + n_1\zeta + n_2\zeta^2 \quad \theta = 60^\circ, \quad (3b)$$

The parameters m_0, m_1, m_2, n_0, n_1 , and n_2 in Equation (3a,b) can be determined by the test data and the characteristics of the destruction envelope surface. When the hydrostatic pressure increases gradually, the partial plane is approximately circular, and $m_2 = n_2$. In the triaxial experiments, the triaxial tensile stress point ($\sigma_1 = \sigma_2 = \sigma_3 = f_{ttt} > 0$) is difficult to test, we used the point $(0, \xi_0)$ to replace it. In the cylindrical coordinates (ζ, ρ, θ) , the boundary conditions are uniaxial compressive stress $(\sqrt{6}\alpha/3, \sqrt{6}/3, 60^\circ)$, uniaxial tensile stress $(\sqrt{3}\alpha/3, \sqrt{6}\alpha/3, 0)$, biaxial compressive stress $(-2\sqrt{3}\alpha/3, \sqrt{6}\alpha/3, 0)$, a point in the tensile meridian (ζ, ρ, θ) , and a point in the compression meridian $(0, \xi_0, 60^\circ)$. By simplifying the boundary conditions, the coefficients of Equation (3a,b) can be expressed by Equation (4a–f).

$$m_2 = \frac{3\left\{\alpha(\sqrt{3}\alpha - 3\zeta)(\sqrt{2}\zeta - \rho) + \bar{\alpha}[3\sqrt{2}\alpha^2 - 2\sqrt{3}\alpha(2\sqrt{2}\zeta + \rho) + 3\zeta(\sqrt{2}\zeta + \rho)]\right\}}{(2\bar{\alpha} + \alpha)(\alpha - \sqrt{3}\zeta)[2\bar{\alpha}(\sqrt{3}\alpha - 3\zeta) + 3\zeta(\alpha - \sqrt{3}\zeta)]}, \quad (4a)$$

$$m_1 = \frac{(2\bar{\alpha} - \alpha)}{\sqrt{3}}m_2 + \frac{3\alpha}{\sqrt{2}(2\bar{\alpha} + \alpha)} - \frac{\sqrt{2}}{2}, \quad (4b)$$

$$m_0 = -\frac{2\alpha\bar{\alpha}}{3}m_2 - \frac{2\sqrt{6}\bar{\alpha}^2}{2\bar{\alpha} + \alpha} + \sqrt{6}\bar{\alpha}, \quad (4c)$$

$$\xi_0 = \frac{-m_1 - \sqrt{m_1^2 - 4m_0m_2}}{2m_2}, \quad (4d)$$

$$n_1 = \frac{-\sqrt{6} + n_2 - 3\xi_0^2n_2}{\sqrt{3} + 3\xi_0}, \quad (4e)$$

$$n_0 = -\frac{\xi_0(-\sqrt{6} + n_2 + \sqrt{3}n_2\xi_0)}{\sqrt{3} + 3\xi_0}, \quad (4f)$$

where $\alpha = f_t/f_c = 0.1$, $\bar{\alpha} = f_{cc}/f_c$, f_t represents tensile strength, f_c represents compressive strength, and f_{cc} represents biaxial compressive strength. The results are shown in Table 4:

Table 4. Values of m and n for SSC under triaxial compression.

No	m_0	m_1	m_2	n_0	n_1	n_2
SSC-30	0.119101	−0.647975	−0.0121243	0.19794	−1.07837	−0.0121243
SSC-70	0.119322	−0.651706	−0.0139721	0.197478	−1.08024	−0.0139721
SSC-100	0.119839	−0.660695	−0.0133628	0.196045	−1.08237	−0.0133628

For Equation (4a–f), there is $\frac{n_0}{m_0} = \frac{3-\beta}{3+\beta}$, so:

$$\beta = \frac{3(n_0 - m_0)}{n_0 + m_0}, \quad (5)$$

The ultimate strength expression of the failure surface can be as follows:

$$\frac{\rho}{f_c} = \frac{(1+b)(3+\beta)\rho_t}{[(1+b)(3+\beta)\cos\theta + \sqrt{3}(1-\beta)(1-b)\sin\theta]} \quad 0^\circ \leq \theta \leq \theta_b, \quad (6a)$$

$$\frac{\rho}{f_c} = \frac{(1+b)(3-\beta)\rho_c}{[(3+\beta-2b\beta)\cos\theta + \sqrt{3}(1-\beta+2b)\sin\theta]} \quad \theta_b \leq \theta \leq 60^\circ, \quad (6b)$$

$$\theta_b = \arctan \frac{(3+\beta)[\rho_t(3+\beta-2b\beta) - \rho_c(1+b)(3-\beta)]}{\sqrt{3}[\rho_c(3-\beta)(1-b)(1-\beta) - \rho_t(3+\beta)(1+2b-\beta)]}, \quad (6c)$$

Substituting β into Equation (6a–c) the limit expression of the failure surface can be obtained.

As shown in Figure 7, the tensile strength, meridian, and strength envelopes in the deviatoric plane of SSC fit well with the experimental data. In Figure 7, ξ stands for hydrostatic stress, ρ stands for deviatoric stress. When the σ_2 increases, the shear strength also increases, and the trend of the meridian curve for SSC is approximately the same. Steel slag as coarse aggregate is well combined with concrete, but the steel slag replacement rate of 100% has an attenuating effect on the meridian.

It can be seen from Table 5 that the theoretical value ρ' calculated by the unified theory is close to the experimental value ρ^{exp} . The relative error E reflects the difference between the computation data and the experimental data and was computed using $(\rho' - \rho^{exp})/\rho^{exp}$, which is within 6%. Except for the $\sigma_1 : \sigma_2 : \sigma_3 = 0.1:0.25:1$, the error E reaches 12%. It shows that the simplified meridian model can accurately predict the triaxial compressive strength of SSC.

Figure 7. The meridians of SSC and the strength envelopes in deviatoric plane: (a) The meridians of SSC; (b) The deviatoric plane of SSC-30; (c) SSC-70; (d) SSC-100.

Table 5. The validation of SSC.

Stress Ratio	SSC-30			SSC-70			SSC-100		
	$\rho^{exp 1}$ (MPa)	$\rho' ^2$ (MPa)	$E ^3$ (%)	ρ^{exp} (MPa)	ρ' (MPa)	E (%)	ρ^{exp} (MPa)	ρ' (MPa)	E (%)
0.05:0.25:1	2.09	2.16	3.32%	2.08	2.14	3.27%	2.13	2.22	4.27%
0.05:0.50:1	2.11	2.11	0.35%	2.12	2.12	0.02%	2.17	2.20	1.20%
0.05:0.75:1	2.12	2.10	1.17%	2.13	2.10	1.58%	2.17	2.17	0.18%
0.05:1.00:1	2.32	2.26	2.53%	2.33	2.26	3.00%	2.38	2.34	1.57%
0.1:0.25:1	2.75	3.09	12.34%	2.57	2.26	12.07%	3.01	3.38	12.14%
0.1:0.50:1	2.88	3.06	6.16%	2.76	2.93	6.08%	3.22	3.40	5.62%
0.1:0.75:1	3.07	3.13	2.06%	2.82	2.89	2.43%	3.26	3.33	2.17%
0.1:1.00:1	3.22	3.22	0.00%	3.02	3.02	0.00%	3.44	3.44	0.00%

¹ ρ^{exp} is measured experimentally. ² ρ' is computed using Equation 6(a–c). ³ E is the relative error.

4. Conclusions

1. The failure modes of SSC are plate-splitting and slant-shear. When the stress ratio $\sigma_2 : \sigma_3 = 0.25 : 1$, the failure mode of SSC is plate-splitting. At other stress ratios, the failure mode is slant-shear.
2. Compared with uniaxial compressive strength, the strength of SSC under triaxial compression is greater at the same replacement ratio, and the influence of stress ratio on the triaxial compressive strength is obvious. When the stress ratio σ_1 / σ_3 is constant, σ_3 / f_m increases first and then decreases with the increase of σ_2 / σ_3 . When the stress ratio σ_2 / σ_3 is constant, The value of σ_3 / f_m increases with the increase of stress ratio σ_1 / σ_3 .
3. The triaxial peak strain ε_3 is larger than the corresponding uniaxial peak strain ε_m , and increases first and then decreases as σ_2 / σ_3 increases. The peak strain ε_2 increases with the increase of the stress ratio σ_2 / σ_3 . The value of ε_1 increases and increases as σ_2 / σ_3 increases but decreases as σ_1 / σ_3 increases. ε_3 , ε_2 and ε_1 increased first and then decreased as the replacement ratio increases.
4. The unified theory established the failure criterion of SSC under triaxial compression on experimental data. It can accurately predict the envelope of concrete under different Lode angles.

Author Contributions: Conceptualization, Z.D.; methodology, B.C.; software, J.Z.; validation, Z.D., B.L.; investigation, B.C.; resources, Z.D.; writing—original draft preparation, J.Z.; writing—review and editing, X.W. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the National Natural Science Foundation of China, Grant Nos. 51478126, 51868005.

Data Availability Statement: Data is contained within the article.

Acknowledgments: The support from the program is gratefully acknowledged.

Conflicts of Interest: The authors declare no conflict of interest.

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