



# Performance Analysis in Heterogeneous Networks with Spatiotemporal Traffic and Scheduling

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**Abstract:** In this work, we consider interference performance under direct data transmission in a heterogeneous network. The heterogeneous network consists of *K*-tier base stations and users, whose locations follow independent Poisson point processes (PPPs). Packet arrivals of users follow independent Bernoulli processes. Two different scheduling policies, round-robin (RR) and random scheduling (RS), are employed to all the Base Stations (BS). The universal frequency reuse mode is adopted to reveal actual spectrum reuse. By leveraging stochastic geometry and queueing theory, the interference interactions of the proposed network are accurately modelled. Accurate expressions for the mean packet throughputs of the network under universal frequency reuse mode are derived. The simulation results explore the optical bias factors in heterogeneous networks to maximize the mean packet throughput. Under a given user density, by changing BS densities, we achieved a certain mean packet throughput level.

**Keywords:** stochastic geometry; queueing theory; universal frequency reusing; scheduling schemes; bias association; mean packet throughput

# 1. Introduction

With the development of the fifth generation (5G) communication network, the increase in data traffic has increasing raised service quality requirements. The delayed performance of 5G networks is highly influenced by dynamic traffic variations. Therefore, to design systems related to packet delay or packet throughput, randomly deployed networks with queueing interactions must be accurately modelled and analyzed. To account for the actual dynamic traffic in cellular networks, stochastic geometry was combined with queueing theory to establish spatiotemporal models to capture both spatial distribution randomness and packet arrival randomness [1–3].

Stamatiou et al. [4] bridged the gap between the work considering Poisson point process (PPP) modelling with backlogged nodes and the work on random access protocol did not consider the random spatial distributions of nodes. Zhong et al. [5] employed tools from queueing theory and point process theory to study the sufficient and necessary conditions for queue stability of a static Poisson network. The results were based on constant link distances and failed to capture random network topology. Yang et al. [6] deduced mean packet throughput using the random scheduling (RS) strategy in small cell networks. However, the method limits the maximum number of users served per cell and ignores the user association randomness. Zhong et al. [7] analyzed the packet delay in



a cellular heterogeneous network, considering randomly generated data traffic. They evaluated the delay performance of three scheduling schemes: RS, first-in first-out scheduling, and round-robin (RR) protocol. However, only bounds for the delay distribution were obtained and active probabilities of base stations (BSs) were fixed. The signal-to-interference-ratio (SIR) analysis and the status of the queues were not decoupled. Yang et al. [8] derived the delay outage of a Poisson point processe (PPP) distributed small cell network with RS or RR being adopted. The user number per cell was fixed. Assumptions in these analyses were usually simplified. Accurate modelling is still necessary to help gain useful insights and design suitable methods.

In the letter, a spatiotemporal model is developed in a *K*-tier heterogeneous network (HetNet), where the data packets' arrival of each user are initiated randomly and the BS locations in each tier are modelled as independent PPPs. The effects of bias association, scheduling schemes, and universal frequency reuse (UFR) mode are all considered to more realistically reveal the HetNet scenario. Interference interactions in the spatiotemporal HetNet are accurately captured. By adopting stochastic geometry and queueing theory, accurate expressions of mean packet throughputs of the network are derived, which can help provide insights into system designs associating with packet throughput.

#### 2. System Model

In this paper, we consider interference performance under direct data transmission in a downlink *K*-tier heterogeneous network. The research on heterogeneous networks dates back to the BARWAN (Bay Area Research Wireless Access Network) project initiated by the University of California, Berkeley in 1995. The project leader, RH Katz, first mentioned the integration of different types of networks that overlap each other to form heterogeneous networks. Network to meet the business diversity needs of future terminals. Each converged network is called a layer of heterogeneous network, and a layer of heterogeneous network usually contains a base station and several mobile terminals. In the following decades, heterogeneous networks have attracted widespread attention in the field of wireless communications, and have also become the development direction of next-generation wireless networks. Locations of BSs from the *i*th (i = 1, 2, ..., K) tier formed an independent homogeneous PPP  $\Phi_i$  with density  $\lambda_i$  and the users formed another PPP  $\Phi_u$  with density  $\lambda_u$ . The BS transmission power of the *i*th tier is  $P_{ti}$ .

The propagation channel suffers from small-scale Rayleigh fading with unit mean power and large-scale path loss, which can be written as  $l(r) = r^{-\alpha}$ , with  $\alpha > 2$  being the path loss exponent according to the power law model. The Rayleigh fading channel is a statistical model of the radio signal propagation environment. In the wireless communication channel environment, after the electromagnetic waves reach the receiver through multiple paths such as reflection, refraction, and scattering, the intensity of the total signal obeys the Rayleigh distribution. At the same time, due to the movement of the receiver and other reasons, characteristics such as signal strength and phase are fluctuating, so it is called Rayleigh fading. Rayleigh fading is applicable to the case where there is no direct signal from the transmitter to the receiver, and it is especially suitable for complex heterogeneous network scenarios. To balance the load among different tiers, we assume the *i*th tier has the bias factor  $b_i$ .

Each user is associated with the BS, providing the maximum average biased received power. The universal frequency reuse mode is adopted, and users suffer from both intra-and inter-tier interference. The general frequency reuse mode is to reuse frequencies and was first used in GSM networks. Frequency multiplexing means that the same frequency covers different areas. These areas using the same frequency need to be separated from each other by a certain distance to suppress the same frequency interference to within the allowable index. In order to make full use of frequency resources, frequency reuse technology has also become an important research content of next-generation wireless networks.

We introduce discrete time queueing model to capture the sporadic wireless data generation conditions. In particular, time is divided into a series of equal time intervals, i.e., time slots.

Packet arrivals at each user follow an independent Bernoulli process at a rate of  $\xi \in [0, 1]$  (packets/slot). Under normal circumstances, the data in the network is bursty. To facilitate the analysis and analysis of system performance, it is assumed that the arrival of the data packet follows the Bernoulli process. The size of each packet is unified. We assume the BS is equipped with an infinite buffer and all the incoming packets of a cell follow first come first serve (FCFS). In a heterogeneous network, all nodes will try to send data in the same time slot. Whether the data is successfully transmitted depends on the data buffer and channel conditions. At the beginning of the time slot, the base station with data packets waiting to be sent starts a transmission attempt. After the user receives the signal from the BS, if the instantaneous SIR exceeds a predefined threshold, the user sends an acknowledge character(ACK) through the error-free feedback channel, and the BS deletes the data packet from its buffer; otherwise, the BS receives negative acknowledge(NACK). The data packet remains at the head of the queue and waits for the next transmission opportunity. Figure 1 depicts a two-tier heterogeneous network with spatiotemporal model. The disc is used to indicate the coverage of the base station wireless signal. Among various scheduling schemes, RS and RR are the two popular policies from which operators may choose for resource allocation among users as their implementation costs are low [9]. We assume that  $K_1$  tiers use RS and  $K_2$  tiers use RR  $(K_1, K_2 \in \{0, 1, \dots, K\}, K_1 + K_2 = K)$ .

- Random Scheduling (RS): BS randomly selects a user from its cell to serve during each time slot.
- Round-Robin (RR): The BS schedule users in a sequential order in a cell.



Figure 1. A two-tier heterogeneous network with spatiotemporal model.

#### 3. Mean Packet Throughput Analysis

## 3.1. Preliminaries

Due to Slivnyark's theorem [10], at time slot *t*, the Signal-to-Interference Ratio (SIR) of the typical user, which is located at the origin, is:

$$\gamma_{i,t} = \frac{h_{x_i} \|x_i\|^{-\alpha}}{\sum\limits_{y_k \in \Phi \setminus x_i} \hat{P}_{tk} \zeta_{y_k,t} h_{y_k} \|y_k\|^{-\alpha}},\tag{1}$$

where  $h_{x_i}$  and  $h_{y_k}$  represent small-scale fading of the transmission link and interfering link, respectively;  $||x_i||$  and  $||y_k||$  denote the serving distance and interfering distance, respectively;  $\Phi$  is the spatial realization of all the point processes, denoted as  $\Phi$ : =  $\bigcup_{i \in 1, \dots, K} \Phi_i$ ; and  $\zeta_{y_k, t} \in \{0, 1\}$  denotes the transmission status (transmission or not) of user at  $y_k$  at time slot t, i.e.,  $\zeta_{y_k, t} = 1$  shows the user is transmitting.  $\hat{P}_{tk} = \frac{P_{tk}}{P_{ti}}$ . Please note that the interference contains intra-tier and inter-tier interference.

As shown in Figure 1, a heterogeneous network usually includes a macro station and several small base stations. The object of this study is a wireless terminal in a small base station, called typical

user. Let  $\mu_{\Phi}$  be the transmission success probability of the typical user, which depends on the probability that the SIR of the typical user  $\gamma_{\Phi}$  is above a certain threshold  $\theta$  with the condition of  $\Phi$ , i.e.,  $\mu_{\Phi} = \mathbb{P}(\gamma_{\Phi} > \theta \mid \Phi)$ . For deriving mean packet throughput, we provide a formal definition as follows:

**Definition 1.**  $A_x(t)$  is the average number of data packets arriving at a transmitter x in time slot [0, t], and  $D_{i,x}$  is sojourn time from the arrival of the ith packet to its successful sending. The definition of mean packet throughput is as follows:

$$\tau \triangleq \lim_{R \to \infty} \frac{\sum_{x \in \Phi \cap B(0,R)} \lim_{t \to \infty} \frac{A_x(t)}{\sum_{i=1}^{A_x(t)} D_{i,x}}}{\sum_{x \in \Phi} \chi_{\{x \in B(0,R)\}}},$$
(2)

Please note that the spatiotemporal characteristics of the network involve two aspects: (1) the service rate of the queueing depends on spatial distribution of users in the network; and (2) the active probability of each BS relies on the queueing status of the transmitters.

From Lemma 1 and Lemma 2 in [8], we obtain the mean packet throughput and activity probability of a BS with *N* associated users under the RS strategy, subject to the realization of the point processing  $\Phi$ . The expressions are as follows:

$$\tau_{RS} = \begin{cases} \frac{\mu_{\Phi}/N - \xi}{1 - \xi}, & \text{if } \frac{\mu_{\Phi}}{N} > \xi\\ 0, & \text{if } \frac{\mu_{\Phi}}{N} \le \xi \end{cases}$$
(3)

$$p_a^{RS} = \min\{N\xi/\mu_{\Phi}, 1\}.$$
 (4)

With the same method, the mean packet throughput and active probability of a BS with N associated users under the RR strategy condition on  $\Phi$  are given, respectively, as follows:

$$\tau_{RR} = \begin{cases} \frac{\mu_{\Phi}/N - \xi}{1 - \frac{N}{2}\mu_{\Phi}/N - \xi}, & if \quad \frac{\mu_{\Phi}}{N} > \xi\\ 0, & if \quad \frac{\mu_{\Phi}}{N} \le \xi \end{cases}$$
(5)

$$p_a^{RR} = \min\{N\xi/\mu_{\Phi}, 1\}.$$
 (6)

#### 3.2. Mean Packet Throughput of the Heterogeneous Network with UFR

Use the same method as in [11,12], we know that the probability mass function (PMF) of the user number per cell for a single-tier PPP network with density  $\lambda$  is

$$\mathbb{P}(N=n) = \frac{\Gamma(n+q)}{\Gamma(n+1)\Gamma(q)} \frac{\lambda_u^n (q\lambda)^q}{(\lambda_u + q\lambda)^{n+q}},$$
(7)

where  $\Gamma(\cdot)$  is the gamma function, q = 3.575.

For a *K*-layer heterogeneous network, the area distribution of Voronoi cells in a single-layer PPP network with a density of  $\lambda_i/a_i$  [13] can be used to accurately estimate the area distribution of weighted Voronoi cells in the *i*th tier, where  $a_i$  is the probability that a typical user is associated with the *i*th tier. The expression is as follows:

$$a_i = \frac{\lambda_i (P_{ti} b_i)^{2/\alpha}}{\sum\limits_{k=1}^K \lambda_k (P_{tk} b_k)^{2/\alpha}}.$$
(8)

Then, the probability mass function (PMF) of *n* user per cell in the *i*th tier is as follows:

$$\mathbb{P}(N_{i} = n) = \frac{\Gamma(n+q)}{\Gamma(n+1)\Gamma(q)} \frac{(\lambda_{u})^{n}(q\lambda_{i}/a_{i})^{q}}{(\lambda_{u}+q\lambda_{i}/a_{i})^{n+q}} = \frac{\Gamma(n+q)}{\Gamma(n+1)\Gamma(q)} \frac{(a_{i}\lambda_{u})^{n}(q\lambda_{i})^{q}}{(a_{i}\lambda_{u}+q\lambda_{i})^{n+q}}.$$
(9)

**Lemma 1.** The expression of the average active probability of the *i*th tier is as follows:

$$p_{ai} = \sum_{k=1}^{\lfloor \frac{\mu_{\phi_i}}{\zeta} \rfloor} \frac{k\zeta}{\mu_{\Phi_i}} \cdot \mathbb{P}\left(N_i = k\right) + 1 - \sum_{k=0}^{\lfloor \frac{\mu_{\phi_i}}{\zeta} \rfloor} \mathbb{P}\left(N_i = k\right).$$
(10)

where  $\mu_{\Phi_i}$  represents the probability of success transmission of the *i*th tier.

**Proof.** We assume  $M_i = \lfloor \frac{\mu_{\phi_i}}{\zeta} \rfloor$ . According to Loynes theorem,  $M_i$  is then the maximum average number of users per cell in the *i*th tier to maintain the stability of the cell queue states. For cells of the *i*th tier that have  $k \ (k = 1, 2, \dots, M_i)$  associated users, they are active with probability  $k\zeta/\mu_{\phi_i}$ . For cells that have more than  $M_i$  users, the queueing status is unstable, which means that the BSs are always active. By deconditioning Equations (4) or (6) on N, we obtain get the BS active probability in Equation (10).  $\Box$ 

According to the above derivations, we are now ready to derive the mean packet throughput via queueing theory and stochastic geometry.

**Theorem 1.** Under UFR mode, Equation (11) gives the mean packet throughout of the network in which  $K_1$  tiers adopt RS policy and  $K_2$  tiers adopt RR policy. In Equation (11),  $I_1$  and  $I_2$  are the collections of the tiers that adopt RS and RR, respectively;  $K_1 = |I_1|$ ,  $K_2 = |I_2|$ ,  $|\cdot|$  denotes the cardinality of the set; the success transmission probabilities of all tiers satisfy the following equations:

$$\tau = \sum_{i \in I_1} a_i \sum_{k=1}^{\lfloor \frac{\mu_{\phi_i}}{\xi} \rfloor} \frac{\mathbb{P}(N_i = k)(\mu_{\phi_i}/k - \xi)}{1 - \xi} + \sum_{i \in I_2} a_i \sum_{k=1}^{\lfloor \frac{\mu_{\phi_i}}{\xi} \rfloor} \frac{\mathbb{P}(N_i = k)(\mu_{\phi_i}/k - \xi)}{1 - \frac{k-1}{2} \cdot \mu_{\phi_i}/k - \xi},$$
(11)

$$\begin{cases} \mu_{\Phi_1} = \frac{1}{1+a_1 \prod\limits_{k=1}^{K} \left[ p_{ak} \cdot \hat{\lambda}_k Z(\theta, \alpha, \hat{b}_k) \hat{P}_{ik}^{\frac{2}{\alpha}} \right]}, \\ \mu_{\Phi_2} = \frac{1}{1+a_2 \prod\limits_{k=1}^{K} \left[ p_{ak} \cdot \hat{\lambda}_k Z(\theta, \alpha, \hat{b}_k) \hat{P}_{ik}^{\frac{2}{\alpha}} \right]}, \\ \vdots \\ \mu_{\Phi_K} = \frac{1}{1+a_K \prod\limits_{k=1}^{K} \left[ p_{ak} \cdot \hat{\lambda}_k Z(\theta, \alpha, \hat{b}_k) \hat{P}_{ik}^{\frac{2}{\alpha}} \right]}. \end{cases}$$
(12)

where  $\hat{\lambda}_k$  is the density of the kth tier normalized by the density of the ith tier, i.e.,  $\hat{\lambda}_k = \frac{\lambda_k}{\lambda_i}$ ,  $\hat{b}_k$  is the bias factor of the k-th tier normalized by the bias factor of the i-th tier, i.e.,  $\hat{b}_k = \frac{b_k}{b_i}$ ,

$$Z(\theta, \alpha, \hat{b}_k) = \frac{2\theta \hat{b}_k^{\frac{2}{\alpha}-1}}{\alpha-2} \cdot {}_2F_1\left[1, 1-\frac{2}{\alpha}; 2-\frac{2}{\alpha}; -\frac{\theta}{\hat{b}_k}\right].$$
(13)

where  $_{2}F_{1}[\cdot, \cdot; \cdot; \cdot]$  is the Gauss hypergeometric function.

**Proof.** The transmission success probability of the *i*th tier is derived as:

$$\mu_{\Phi_{i}} = \mathbb{P}[\gamma_{i,t} > \theta]$$

$$= \mathbb{E}\left[\exp\left(-\theta \|x_{i}\|^{\alpha}\left(\sum_{\substack{y_{k} \in \Phi \setminus x_{i} \\ \|y_{k} \in \Phi \setminus x_{i}}} \frac{\hat{P}_{tk}\zeta_{y_{k},t}h_{y_{k}}}{\|y_{k}\|^{\alpha}}\right)\right)\right]$$

$$= \mathbb{E}\left[\prod_{\substack{y_{k} \in \Phi \setminus x_{i} \\ \|y_{k} \in \Phi \setminus x_{i}}} \exp\left(-\frac{\theta \hat{P}_{tk}\zeta_{y_{k},t}h_{y_{k}}\|x_{i}\|^{\alpha}}{\|y_{k}\|^{\alpha}}\right)\right]$$

$$\stackrel{(a)}{=} \int_{z_{k}}^{\infty} \prod_{k=1}^{K} \exp\left[-p_{ak}\lambda_{k}Z(\theta,\alpha,\hat{b}_{k})\hat{P}_{tk}^{\frac{2}{\alpha}}\pi r^{2}\right]f_{R}(r)dr$$

$$= \frac{1}{1+a_{i}\prod_{k=1}^{K}\left[p_{ak}\cdot\hat{\lambda}_{k}Z(\theta,\alpha,\hat{b}_{k})\hat{P}_{tk}^{\frac{2}{\alpha}}\right]},$$
(14)

where (*a*) follows from the probability generating function (PGFL) and conditional PGFL of PPP [14],  $f_R(r) = 2\pi \frac{\lambda_i}{a_i} r \exp(-\pi \frac{\lambda_i}{a_i} r^2)$  is the probability density function (PDF) of the serving distance.  $z_k = (\hat{P}_{tk} \hat{b}_k)^{\frac{1}{\alpha}} r$  is the minimum distance between the typical user and its closest BS from the *k*th tier based on the bias association policy.

Therefore,  $\mu_{\Phi_1}, \mu_{\Phi_2}, \dots, \mu_{\Phi_K}$  satisfy the coupling equations in Equation (12). Please note that the success transmission probability of each tier is associated with that of all other tiers.

By deconditioning Equation (3) on *N*, the mean packet throughput of the *i*th tier with RS policy is obtained as:

$$\tau_{RSi} = \sum_{k=1}^{\lfloor \frac{r \cdot \varphi_i}{\xi} \rfloor} \frac{\mathbb{P}(N_i = k)(\mu_{\phi_i}/k - \xi)}{1 - \xi},$$
(15)

By deconditioning Equation (5) on *N*, the mean packet throughput of the *i*th tier with RR policy is derived as:

$$\tau_{RRi} = \sum_{k=1}^{\lfloor \frac{|\gamma_{ij}|}{\xi} \rfloor} \frac{\mathbb{P}(N_i = k)(\mu_{\phi_i}/k - \xi)}{1 - \frac{k-1}{2} \cdot \mu_{\phi_i}/k - \xi},$$
(16)

Then, the mean packet throughput can be obtained based on the total probability law as:

$$\tau = \sum_{i \in I_1} a_i \tau_{RSi} + \sum_{i \in I_2} a_i \tau_{RRi}.$$
(17)

## 4. Numerical and Simulation Results

In this section, we present the MATLAB simulations that were used to verify the validity of the above analytical results. The extensive simulation results are given to illustrate the behavior of the system with respect to the change in its loading parameters. For simplicity, we analyze the throughput performance of a two-tier heterogeneous network. The first and second tiers are assumed to use RR and RS policy, respectively. We considered a simulation area of  $9 \times 10^6$  km<sup>2</sup> with base stations and users scattered across the square plane, whose position remains static once they are settled. The total time slots for simulation was set to  $10^5$ . The major parameters in our simulation were as follows, unless otherwise stated [7,8]: the packet arrival rate was set to  $\xi = 0.05$  packets/slot, the SIR is set as  $\theta = 0$  dB, the path loss exponent  $\alpha$  is 3.8. This paper studies the performance of internal users of small base stations in heterogeneous networks. Because the data is not always available, the data arrival rate is set to 0.05 packets/slot which is the same setting in [7,8]. The BS transmit powers in the first tiers is assumed to be 39 dBm, and the second tiers is respectively to be 24 dBm. The density of the BSs from the two tiers was respectively set to  $\lambda_1 = 10^{-5}$  km<sup>-2</sup>,  $\lambda_2 = \lambda_1$ . We first simulated how bias factors influence the throughput performance of the two-tier network model. We set the bias factor of the first-tier  $b_1$  to 1 and analyzed the effect of the second-tier bias factor  $b_2$ . To validate our derived

results, the simulation results were carried out and repeated for over 10,000 iterations. The simulation parameters that were based on the Monte Carlo simulation method are shown in Figures 2 and 3.



**Figure 2.** The mean packet throughputs of the network versus  $b_2$ .  $b_1 = 1$ .



**Figure 3.** The mean packet throughputs of the network versus  $\lambda_u / \lambda_1$ .  $b_1 = 1$ ,  $b_2 = 4$ .

Figure 2 depicts the mean packet throughput of the two-tier network versus  $b_2$  which validates our analysis. The analytical results match the simulation results very well, which verifies the correctness of mathematical derivation. The figure shows that the average packet throughput increases first and then decreases with the increase in  $b_2$ , i.e., an optimal bias factor  $b_2$  maximizes average packet throughput. This occurs because as  $b_2$  increases, more traffic will be offloaded to the second tier, and the first tier

becomes less congested. Therefore, the first tier can provide higher packet service rate and higher packet throughput, while the throughput of the second tier is lowered due to increased burden. Given  $b_2$  is small, the throughput of first tier dominates the mean packet throughput. The significant increase in the first-tier throughput led to an increase in network throughput. When  $b_2$  increases, more traffic is transferred to the second tier, and the curve approaches the throughput of the second tier. Therefore, our analytical results can help with the design of an optimal bias factor for a network to maximize the mean packet throughput.

Figure 3 depicts the variation of the mean packet throughput of the network with the density ratio  $\lambda_u/\lambda_1$ . The mean packet throughput shows a decreasing tendency. This can be explained by a higher  $\lambda_u/\lambda_1$  leading to more congestion, which leads to a higher probability of BS activity and greater interference with typical users. When  $\lambda_u/\lambda_1$  increases, the mean packet delay is prolonged, which leads to the decline in the mean packet throughput. For achieving a certain packet throughput value, BS density can be designed for a given user density according to our analysis.

## 5. Conclusions

In this letter, the downlink packet throughput of a *K*-tier PPP distributed heterogeneous network with spatiotemporal traffic was analyzed. The dynamic traffic involves both spatial and temporal randomness, which respectively reside in PPP distributed user locations and Bernoulli packet arrivals. To balance traffic among tiers, bias association was employed. For traffic management, we used RR and RS protocols. For spectrum reusing, we assumed all tiers use the same spectrum for more realistic modelling. Interference interactions of the proposed spatiotemporal heterogeneous network were accurately captured. As a result of linking the transmission success probability to the active probability, we obtained accurate expressions of mean throughput of the network under universal frequency. The parameters of our analysis provide guidance for designing spatiotemporal models in heterogeneous networks.

For example, based on the analysis, we can see that the proper design optimal deviation factor can maximize the average packet throughput of the network under various packet arrival rates. Conversely, according to our analysis results, the BS density can also be designed to ensure the network reaches a certain average packet throughput level under a given user density.

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