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The Influence of Main Design Parameters on the Overall Cost of a Gearbox

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Abstract: This study is aimed at determining optimum partial gear ratios to minimize the cost of a three-stage helical gearbox. In this work, eleven input parameters were investigated to find their influence on the optimum gear ratios of the second and the third stages (u_2 and u_3). To reach the goal, a simulation experiment was designed and implemented by a cost optimization program. The results revealed that in addition to the input parameters, their interactions also have important effects in which the total ratio gearbox ratio (u_t) and the cost of shaft (C_s) have the most impact on u_2 and u_3 responses, respectively. Moreover, the proposed models of the two responses are highly consistent to the experimental results. The proposed regression equations can be applied to solve optimization cost problems.

Keywords: gearbox; gear ratio; optimum gearbox design; three-stage helical gearbox

1. Introduction

In gearbox optimization design, determining optimum gear ratios has been a greatly important task. It can be explained by the fact that the size, the mass, and therefore, the cost of a gearbox is significantly affected by the gear ratios. To illustrate, Figure 1 shows the relation between the gear mass and the gear ratio of the second stage u_2 [1]. It can be seen from the figure that with the optimum value of u_2 ($u_2 = 2$), the mass of gears is merely about 178 (kg) whereas it reaches about 275 (kg) when $u_2 = 6$. Therefore, there has been various research work dealing with optimizing gear ratios so far [1–11]. The methodology of gear ratio optimization can basically be divided into three groups, e.g., graph method, practical method, and model method. The oldest method is the graph method [2,12], whereby the gear ratios are found based on the graph of the relationship between the component ratios and the total gearbox ratio. Figure 2 is an example of this method in which the gear ratios of the first and the second stages u_1 and u_2 of a three-stage helical gearbox are determined graphically. The practical method is introduced in [13], in which the optimum gears are determined based on the actual data from gearbox companies. For example, the mass of a two-stage helical gearbox is minimum when the ratio of the center distances of the second to the first stage is 1.4–1.6 [13]. From that comment, the optimum gear ratios are given. The most common method is the model method [3,4,14–16]. In this method, models for calculating optimum gear ratios are determined by solving optimum problems with different target functions such as minimal gearbox length [3,4,15], minimal mass of gears [4] or minimal gearbox cross section [4,14,16].

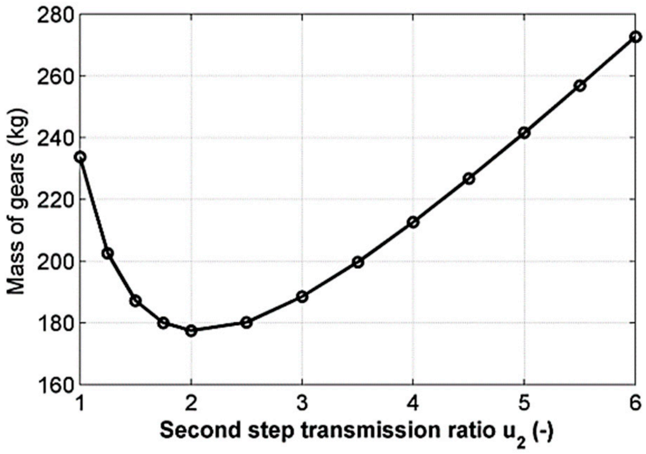


Figure 1. Gear mass versus second stage gear ratio [1].

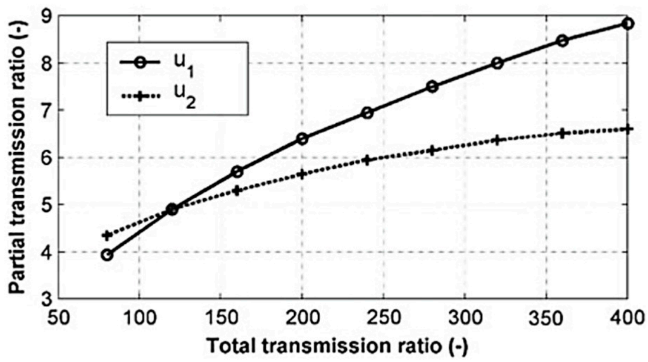


Figure 2. Partial gear ratios versus total gearbox ratio.

In literature these studies have investigated various levels of gear stages such as two-stage gearboxes [17,18], three-stage gearbox [4,14], and four-stage gearboxes [19]. Also, the determination of optimum gear ratios of bevel gearbox was carried in [20–23]. Recently, the optimum partial gear ratios have been found for mechanical driven systems using a gearbox and a chain drive [5,9,23] or a V-belt drive [22,24,25].

As previously mentioned, the optimal gear ratios directly impact the cost of the gearbox. However, up to now, there has been no research on calculating the optimal gear ratios with cost objective function. For this reason, this article presents a study on cost optimization in terms of finding the optimum gear ratio of three-stage Helical Gearboxes. The objective functions selected were the optimum gear ratios for second and third stage gears. Eleven input parameters were taken to investigate each parameter's influence and their interaction on the objective functions. A simulation experiment was planned using computer program to carry out the above issue.

2. Optimization Problem

2.1. Cost Analysis of Three-Stage Helical Gearbox

In practice, the cost of a gearbox depends on many cost elements, including the cost of the casing, shafts and gears, and bearings. However, due to the complicated cost calculation, the cost of bearings has not been considered in this study. As a result, the cost of a three-stage helical gearbox, namely C_{gb}, can be calculated as the Equation (1):

$$C_{gb} = C_g + C_{gh} + C_s \tag{1}$$

where C_g, C_{gh}, and C_s indicate the cost of gears, the cost of gearbox housing and the cost of shafts respectively.

Theoretically, the cost of a gear (the price of a gear) includes material costs, machining costs, heat treatment costs, labor costs including management and overhead costs, etc. The gear cost also depends on the gear shape and the gear size. The above component costs help to calculate the cost of a gear. In addition, in practice, the gear cost is usually calculated by unit price per kilogram and it varies by company policy and periodically. Therefore, in this study, the gear cost is investigated as a variable and calculated by the Equation (2):

$$C_g = c_{g.m} \cdot m_g \tag{2}$$

in which, $c_{g.m}$ is the cost per a kilogram of gears (USD/kg), and m_g is representative for the mass of all gears in the gearbox (kg).

The cost of gearbox housing can be determined by Equation (3):

$$C_{gh} = c_{gh.m} \cdot m_{gh} \tag{3}$$

in this situation, $c_{gh.m}$ is the cost per a kilogram of gearbox housing (USD/kg), and m_{gh} is the mass of the gearbox housing (kg).

Finally, the cost of shafts is determined by Equation (4):

$$C_s = c_{s.m} \cdot m_s \tag{4}$$

where $c_{s.m}$ is the cost per a kilogram of shaft (USD/kg), and m_s is the mass of all shafts in the gearbox (kg).

Based on previously mentioned equations, it can be drawn that in order to get the cost of the gearbox (C_{gb}) two factors should be identified. The first is the cost per a kilogram of gears, gearbox housing, and shafts which are varied according to the market. The second is the mass of gears, the gearbox housing, and shafts corresponding to m_g, m_{gh}, and m_s. However, it is noticed that the first factor is beyond the scope of this study, because it depends on the price of commercial markets. Then the later will be obtained by the detailed calculations in the next part of this study.

2.2. The Determination of Gearbox Housing Mass

The mass of gearbox housing (m_{gh}) can be simply calculated by using Equation (5):

$$m_{gh} = \rho_{gh} \cdot V_{gh} \tag{5}$$

where, ρ_{gh} is the weight density of gearbox housing materials referred in Table 1; V_{gh} is the volume of the gearbox housing (m^3).

Table 1. Weight density of used materials.

ρ_{gh} (kg/m^3)	ρ_g (kg/m^3)	ρ_s (kg/m^3)
7.2	7.82	7.85

Figure 3 presents the schematic relations of the gearbox housing dimensions. It is realized that the shape of gearbox housing is constructed by various component rectangulars. Hence, the volume of the gearbox housing can be determined by Equation (6).

$$V_{gh} = 2 \cdot V_b + 2 \cdot V_{A1} + 2 \cdot V_{A2} \tag{6}$$

where V_b, V_{A1}, and V_{A2} are the volumes of bottom housing, side A1, and side A2 (kg), respectively.

$$V_b = L \cdot B_1 \cdot 1.5 \cdot S_G \tag{7}$$

$$V_{A1} = L \cdot H \cdot S_G \tag{8}$$

$$V_b = B_2 \cdot H \cdot S_G = (B_1 - 2 \cdot S_G) \cdot H \cdot S_G \tag{9}$$

Substituting (7), (8), and (9) into (6) gets:

$$V_{gh} = 3 \cdot L \cdot B_1 \cdot S_G + 2 \cdot L \cdot H \cdot S_G + 2 \cdot (B_1 - 2 \cdot S_G) \cdot H \cdot S_G \tag{10}$$

In which, L, H, B_1, and S_G can be determined by [26]:

$$L = (d_{w11} + d_{w21}/2 + d_{w12}/2 + d_{w22}/2 + d_{w13}/2 + d_{w22}/2 + 22.5)/0.975 \tag{11}$$

$$H = d_{w23} + 6.5 \cdot S_G \tag{12}$$

$$B_1 = b_{w2} + b_{w3} + 6 \cdot S_G \tag{13}$$

$$S_G = 0.005 \cdot L + 4.5 \tag{14}$$

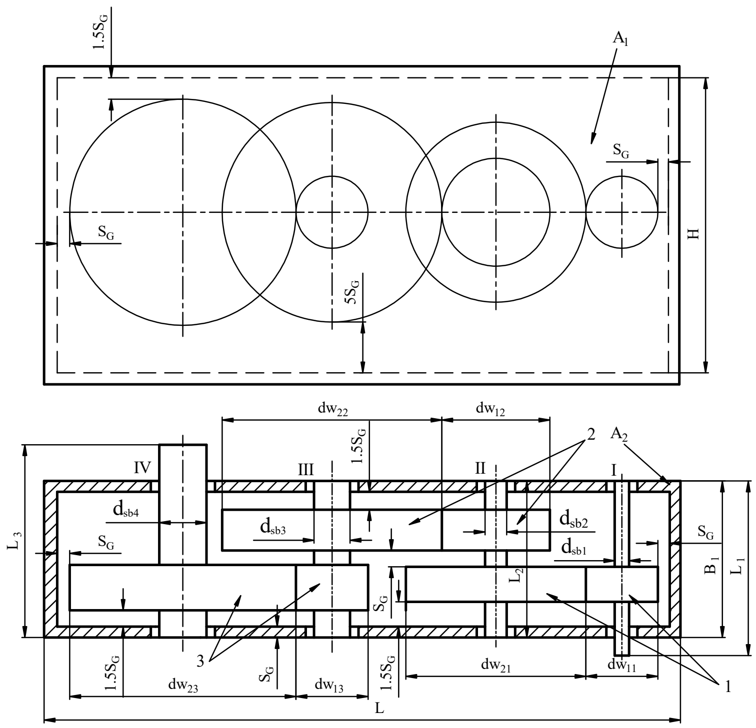


Figure 3. Schema for determination of gearbox mass.

2.3. Gear Mass Calculations

The studied gearbox includes three stages, consequently the total mass of gears can be summed up as follow:

$$m_g = m_{g1} + m_{g2} + m_{g3} \tag{15}$$

where, m_{g1}, m_{g2}, and m_{g3} represent the gear mass of the first, the second, and the third stages (kg) in which the first one can be determined by the following equations:

$$m_{g1} = \rho_g \cdot \left(\frac{\pi \cdot e_1 \cdot d_{w11}^2 \cdot b_{w1}}{4} + \frac{\pi \cdot e_2 \cdot d_{w21}^2 \cdot b_{w1}}{4} \right) \tag{16}$$

where ρ_g is the weight density of gear material (kg/m^3), cf. Table 1; e_1 and e_2 are the volume coefficients of the drive gear and the driven gear of the first stage, respectively. In practice, e_1 and e_2 can be orderly selected by the values of 1 and 0.6; b_{w1} is the width of the gears calculated by: $b_{w1} = X_{ba1} \cdot a_{w1}$ (mm).

Similarly, we have:

$$m_{g2} = \rho_g \cdot \left(\frac{\pi \cdot e_1 \cdot d_{w12}^2 \cdot b_{w2}}{4} + \frac{\pi \cdot e_2 \cdot d_{w22}^2 \cdot b_{w2}}{4} \right) \tag{17}$$

and

$$m_{g3} = \rho_g \cdot \left(\frac{\pi \cdot e_1 \cdot d_{w13}^2 \cdot b_{w3}}{4} + \frac{\pi \cdot e_2 \cdot d_{w23}^2 \cdot b_{w3}}{4} \right) \tag{18}$$

where b_{w2} and b_{w3} are the gear widths which can be determined in order by (mm); $b_{w2} = X_{ba2} \cdot a_{w2}$ and $b_{w3} = X_{ba3} \cdot a_{w3}$.

2.4. Shaft Mass Calculation

It is known that a three stage gearbox contains four shafts constructing three stages. For this reason, the mass of the gearbox shafts can be determined by:

$$m_s = m_{s1} + m_{s2} + m_{s3} + m_{s4} \tag{19}$$

In which,

$$m_{s1} = \rho_s \cdot \pi \cdot d_{s1}^2 \cdot l_{s1} / 4 \tag{20}$$

$$m_{s2} = \rho_s \cdot \pi \cdot d_{s2}^2 \cdot l_{s2} / 4 \tag{21}$$

$$m_{s3} = \rho_s \cdot \pi \cdot d_{s3}^2 \cdot l_{s3} / 4 \tag{22}$$

$$m_{s4} = \rho_s \cdot \pi \cdot d_{s4}^2 \cdot l_{s4} / 4 \tag{23}$$

m_{s1}, m_{s2}, m_{s3}, and m_{s4} are the mass of shafts 1, 2, 3, and 4 of the gearbox (kg) respectively; ρ_s is the weight density of shaft material (cf. Table 1); l_{s1}, l_{s2}, l_{s3}, and l_{s4} are orderly the length of shaft 1, 2 , 3, and 4 of the gearbox established by (cf. Figure 1):

$$l_{s1} = B_1 + 1.2 \cdot d_{s1} \tag{24}$$

$$l_{s1} = l_{s2} = B_1 \tag{25}$$

$$l_{s4} = B_1 + 1.2 \cdot d_{s4} \tag{26}$$

In the above equations [12]:

$$d_{s1} = [T_{11} / (0.2 \cdot [\tau])]^{1/3} \tag{27}$$

$$d_{s2} = [T_{12} / (0.2 \cdot [\tau])]^{1/3} \tag{28}$$

$$d_{s3} = [T_{13} / (0.2 \cdot [\tau])]^{1/3} \tag{29}$$

$$d_{s4} = [T_{14} / (0.2 \cdot [\tau])]^{1/3} \tag{30}$$

where $[\tau]$ is the allowable shear stress. In this study, its value is chosen as $[\tau]$ = 17 MPa.

2.5. Determination of the Centre Distances of the Gear Stages

In addition to the module of gears, center distance is also an important factor for designing as well as optimizing gearbox. According to [12], the center distance of the i stage of the gearbox can be calculated by equation (31):

$$a_{wi} = k_a \cdot (u_i + 1) \cdot \sqrt[3]{T_{1i} \cdot k_{H\beta} / \left([\sigma_{Hi}]^2 \cdot u_i \cdot X_{ba1} \right)} \quad (31)$$

where:

- $k_{H\beta}$ is the contacting load ratio for pitting resistance selected by 1.1 [12];
- $[\sigma_{Hi}]$ is the allowable contact stress of the i stage (MPa);
- k_a is the material coefficient; As the gear material is steel, k_a = 43;
- X_{ba1} is the coefficient of wheel face width of the i stage;
- T_{1i} is the torque on the drive shaft of the i stage (Nmm) determined by:

$$T_{1i} = \frac{T_r}{\prod_{j=i}^{3} \left(u_i \cdot \eta_{hg}^{4-i} \cdot \eta_{be}^{5-i} \right)} \quad (32)$$

According to [12], the pinion and the gear pitch diameters of the i stage can be calculated by the value of the center distance as the following equations:

$$d_{w1i} = 2 \cdot a_{wi} / (u_1 + 1) \quad (33)$$

$$d_{w2i} = 2 \cdot a_{wi} \cdot u_i / (u_i + 1) \quad (34)$$

2.6. Optimization Problem

Based on previously mentioned analyses, it can be emphasized that in order to reduce the cost of gearbox, minimizing the objective function (C_{gb}) or C_{gb} should satisfy the following constraints:

$$1 \le u_1 \le 91 \le u_2 \le 91 \le u_3 \le 9 \quad (35)$$

It can be clarified that to solve the optimizing solution, it is essential to optimize the values of partial gear ratios of u_1, u_2, and u_3. On the other hand, we have the relation between transmission ratios and partial ratios, $u_t = u_1 \cdot u_2 \cdot u_3$. Hence, in this study, instead of optimizing all three mentioned partial ratios, the optimization of only two partial ratios (u_2 and u_3) are considered. The partial ratio can be determined by equation: $u_1 = u_t / (u_2 \cdot u_3)$.

3. Experimental Work

To investigate the influences of factors on objective functions of u_2 and u_3, simulation experiments, namely screening experiments, are carried out. Eleven factors (or input parameters) listed in Table 2 are selected for the exploration. Low and high values are considered to test each input factor. As the experiment in this work is a simulation experiment, it is not necessary to reduce the number of experiments required to be performed like real experiments. Therefore, it is desired to perform full factorial design of 2^{11} instead of the Taguchi method as the usual practice. Nevertheless, the expecting function is not available in Minitab@19, therefore the model of 2^{11-4} and 1/16 fraction is purposely adopted. Consequently, $2^{11-4} = 128$ tests for the simulation experiment are utilized. This method is also the way for the largest number of experiments. Moreover, the use of a screening design is aimed at eliminating influential parameters. This is the simplest method to determine the effects of parameters as well as their interactions on the target function. On the other hand, it is possible to provide mathematical models that the Taguchi method cannot.

Table 2. Input parameters.

Real Factor	Minitab®19	Name	Unit	Low	High
Total gearbox ratio	A	u_t	-	10	100
Coefficient of wheel face width of stage 1	B	X_{ba1}	-	0.3	0.35
Coefficient of wheel face width of stage 2	C	X_{ba2}	-	0.33	0.38
Coefficient of wheel face width of stage 3	D	X_{ba3}	-	0.35	0.4
Allowable contact stress of stage 1	E	AS_1	MPa	350	420
Allowable contact stress of stage 2	F	AS_2	MPa	350	420
Allowable contact stress of stage 3	G	AS_3	MPa	350	420
Output torque	H	T_{out}	Nm	1000	10000
Cost of gearbox housing	I	C_{gh}	USD/kg	1	5
Cost of gears	J	C_g	USD/kg	2	9
Cost of shafts	K	C_s	USD/kg	1.5	5

The demonstration of the input parameters and the responses can be seen in Table 2, where the factors are orderly assigned as factors A, B, etc. The output responses are presented in Table 3.

Table 3. Experimental plans and output responses.

Run Order	Center Pt	Blocks	u_t	X_{ba1}	X_{ba2}	X_{ba3}	AS_1	AS_2	AS_3	T_{out}	C_{gh}	C_g	C_s	u_2	u_3
1	1	1	30	0.35	0.38	0.4	350	420	350	1000	1	2	1.5	4.02	3.64
2	1	1	100	0.35	0.33	0.36	420	350	420	1000	5	2	1.5	4.08	4.24
3	1	1	100	0.3	0.33	0.4	420	350	350	1000	5	9	5	4.11	3.73
4	1	1	30	0.35	0.33	0.4	350	350	350	1000	5	2	1.5	3.63	3.58
5	1	1	100	0.3	0.33	0.36	350	420	350	1000	5	9	1.5	5.22	2.59
6	1	1	30	0.35	0.38	0.4	420	350	350	10000	5	9	1.5	3.03	3.28
...															
(Appendix A)															
127	1	1	30	0.3	0.38	0.4	420	420	420	10000	1	2	5	4.11	5.83
128	1	1	100	0.3	0.38	0.36	420	350	350	10000	5	9	5	4.02	3.55

4. Results and Discussions

4.1. The Influence of Input Parameters and Their Interactions

The evolution of the optimum gear ratio of the second step (u_2) as functions of each input parameter is presented in Figure 4. It is observed that u_2 increases when Total gearbox ratio (u_t), Allowable contact stress of stage 2 (AS_2), and Cost of shafts (C_s) increase also. Nevertheless, for this tendency it is realized that Total gearbox ratio (u_t) has greater influence than that of other factors. Conversely, u_2 decreases with the growth of Allowable contact stress of stage 1 and 3 (AS_1 and AS_3), and Output torque (T_{out}). Moreover, it is shown that Coefficients of wheel face width of stage 1, 2, and 3 $(X_{ba1}, X_{ba2}$ and $X_{ba3})$ do not have influence on u_2. Regarding to the case of u_3 (c.f. Figure 4b), the experimental results reveal that Cost of gears $\left(C_g\right)$ and Cost of shafts (C_s) have a significant effect on the value of u_3. It means that u_3 develops when Cost of shafts rises or Cost of gears declines. Furthermore, the first five input parameters mentioned in Table 2 do not have impact on the evolution of u_3. It is noticed that the influence investigation of the input factors on response as previously mentioned do not take their interactions into account. This will be considered in the next part of the current study.

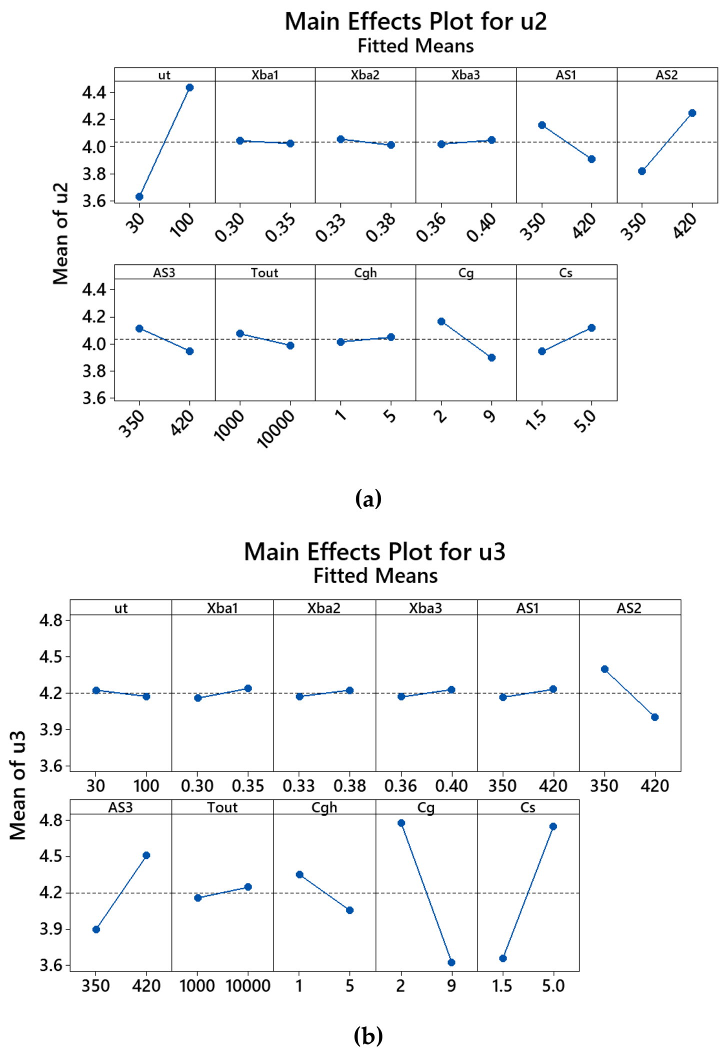
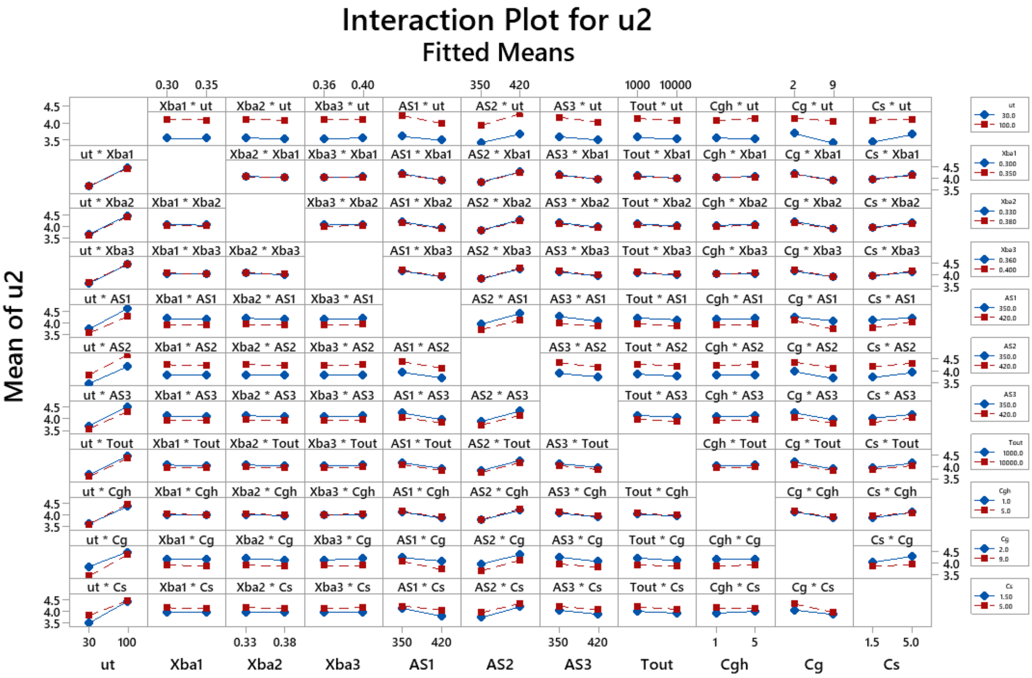


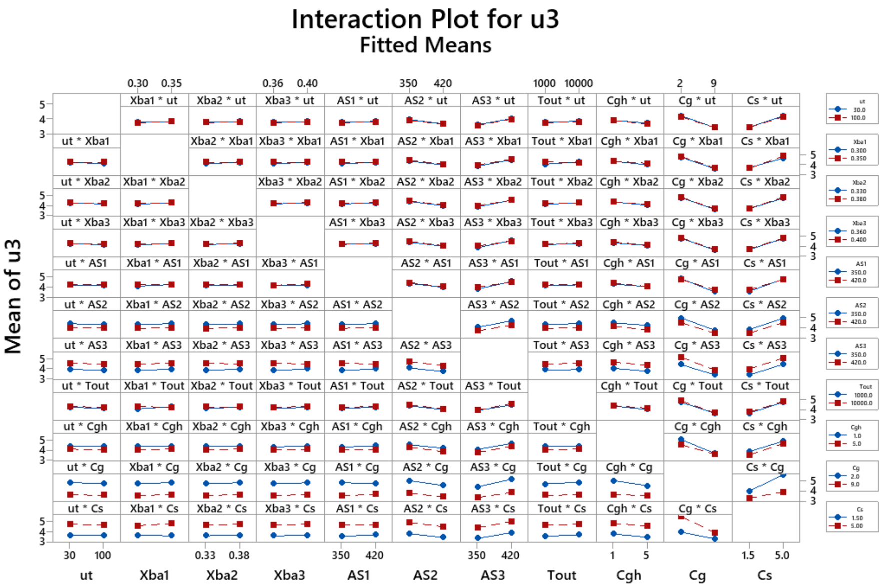
Figure 4. Main effects plot for (**a**) u_2 and (**b**) u_3.

Figure 5a displays the interactions between the input parameters on the response of u_2. It is observed that the interactions between u_t and some input parameters such as AE (u_t*AS_1), AF (u_t*AS_2), AG (u_t*AS_3), AK (u_t*C_g), and AL (u_t*C_s) in both values of 30 and 100 has the most significant influence on the u_2 response, while the stable tendency is observed for the interactions between u_t and remaining input parameters, e.g., AB (u_t*X_{ba1}), AC (u_t*X_{ba2}), AD (u_t*X_{ba3}), AH (u_t*T_{out}), and AJ (u_t*C_{gh}). Similarly, we can realize the interactions which have significant influence on the u_2 response but are lesser than the ones of u_t like BF (X_{ba1}*A_{S2}), BE (X_{ba1}*A_{S1}), CL (X_{ba2}*C_s), CK (X_{ba2}*C_g), CF (X_{ba2}*AS_2), FH (AS_2*T_{out}), FG (AS_2*AS_3), FL (AS_2*C_s), FK (AS_2*C_g), FL (AS_2*C_s), FJ (AS_2*C_{gh}), HK (T_{out}*C_g), GL (AS_3*C_s), GJ (AS_3*C_{gh}), and GH (AS_3*T_{out}). Referring to the case of the response u_3 (cf. Figure 5b), it is visualized that the interactions JK (C_{gh}*C_g), JL (C_{gh}*C_s), GK (AS_3*C_g), GL (AS_3*C_s), FK(AS_2*C_g), FL(AS_2*C_s), KL (C_g*C_s), BH (X_{ba1}*T_{out}), BL (X_{ba1}*C_s), DG (X_{ba3}*AS_3), EG (AS_1*AS_3), and EK (AS_1*C_g) have significant influences on the u_3 response.

Figure 6 presents the Normal Plot of the standardized effects in which the relationship between the responses (u_2 and u_3) and the input parameters as well as their interactions are exposed. Based on the results presented in the figure, it is seen that u_t and AS_2 have the greatest influence on the response u_2 as previously documented. Furthermore, it is realized that, in addition to single input parameters as early presented (u_t, AS_2, C_s, C_g, AS_1, and AS_3) the interactions of some input parameters also have both positive and negative impacts on the response of u_2. For instance, the increase in the interactions of AK, EL, AJ, AF, BH, HK, and GK leads to the augment of the u_2 response. Conversely, the decrease in the interactions of AL, KL, EK, AE, JL, and AG causes the reduction of u_2 response. Considering the case of u_3, the results anew reveal that Cost of gears (C_g) and Cost of shafts (C_s) have dominant impact on the value of u_3 as mentioned above. Besides, the interactions between the input parameters also have influence in both positive and negative trends. For example, the response of u_3 is positively influenced by the interactions of JK, EK, and BL, while being negatively affected by those of KL, GK, EG, and BH. Based on the results shown in the Normal Plot of the Standardized Effects, the parameters or interactions with insignificant influence can be eliminated, while those with strong impact are remained. The testing process can go further and in more detailse with the remained parameters. In these situations, the remained parameters are listed in Tables 4 and 5 in the case of u_2 and u_3, respectively.

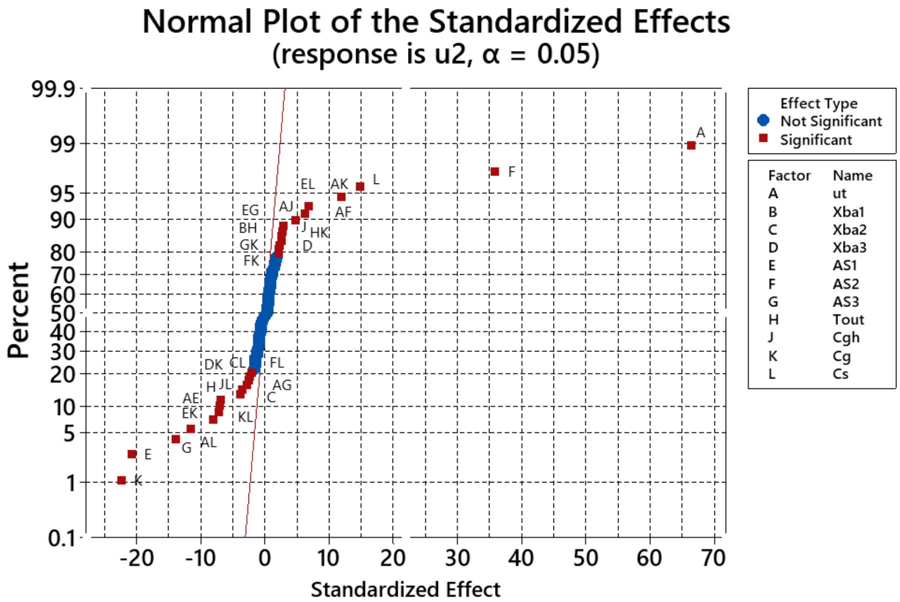


(a)

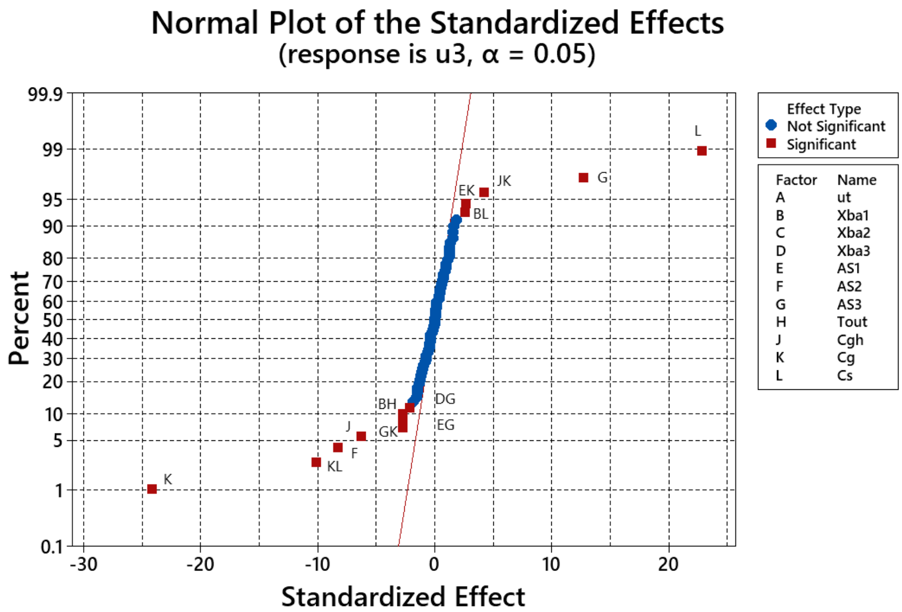


(b)

Figure 5. The interactions between input parameters on the response of u_2 (**a**) and u_3 (**b**).



(a)



(b)

Figure 6. The evolution of response as a function of input parameters and their interactions (**a**) u_2 and (**b**) u_3

4.2. Proposed Regression Model of the Response

In order to achieve equations of the response u_2 and u_3, a regression process with two interaction factors is carried out using Minitab@19. The significance of this regression is $\alpha = 0.05$. The estimated effects and the coefficients for u_2 response are exhibited in Table 4 where the factors with no influence on them are eliminated. It is noticed that if the effect of each input parameter or interaction has p-value higher than the significance of α, it does not strongly impact the response. For example, the factor of X_{ba1} has p-value of 0.111 superior to $\alpha = 0.05$, which means that X_{ba1} is not significant to the response u_2. The regression equation of the u_2 response is described by following model (Regression Equation in Uncoded Units):

$$u_2 = 4.241 + 0.01830\ u_t - 1.106\ X_{ba1} + 0.190\ X_{ba2} + 1.925\ X_{ba3} - 0.00601\ AS_1 + 0.004379\ AS_2 - 0.00686\ AS_3 - 0.000058\ T_{out} + 0.00397\ C_{gh} + 0.1013\ C_g + 0.1261\ C_s - 0.000036\ u_t{*}AS_1 + 0.000030\ u_t{*}AS_2 - 0.000014\ u_t{*}AS_3 + 0.000405\ u_t{*}C_{gh} + 0.000587\ u_t{*}C_g - 0.001144\ u_t{*}C_s + 0.000131\ X_{ba1}{*}T_{out} - 0.327\ X_{ba2}{*}C_s - 0.2243\ X_{ba3}{*}C_g + 0.000012\ AS_1{*}AS_3 - 0.000404\ AS_1{*}C_g + 0.000670\ AS_1{*}C_s + 0.000101\ AS_2{*}C_g - 0.000218\ AS_2{*}C_s + 0.000105\ AS_3{*}C_g + 0.000001\ T_{out}{*}C_g - 0.00676\ C_{gh}{*}C_s - 0.007003\ C_g{*}C_s \quad (36)$$

It can be said that the experimental data are greatly consistent with the proposed model when the minimum value of R-square is approximately 98% (all of them are more than 98%).

Table 4. Estimated Effects and Coefficients for u_2.

Term	Effect	Coef	SE Coef	T-Value	p-Value	VIF
Constant		4.03383	0.00598	674.32	0.000	
ut	0.80109	0.40055	0.00598	66.96	0.000	1.00
Xba1	−0.01922	−0.00961	0.00598	−1.61	0.111	1.00
Xba2	−0.04359	−0.02180	0.00598	−3.64	0.000	1.00
Xba3	0.02766	0.01383	0.00598	2.31	0.023	1.00
AS1	−0.25172	−0.12586	0.00598	−21.04	0.000	1.00
AS2	0.43266	0.21633	0.00598	36.16	0.000	1.00
AS3	−0.16828	−0.08414	0.00598	−14.07	0.000	1.00
Tout	−0.08484	−0.04242	0.00598	−7.09	0.000	1.00
Cgh	0.03328	0.01664	0.00598	2.78	0.006	1.00
Cg	−0.27141	−0.13570	0.00598	−22.68	0.000	1.00
Cs	0.17766	0.08883	0.00598	14.85	0.000	1.00
ut*AS1	−0.08766	−0.04383	0.00598	−7.33	0.000	1.00
ut*AS2	0.07359	0.03680	0.00598	6.15	0.000	1.00
ut*AS3	−0.03422	−0.01711	0.00598	−2.86	0.005	1.00
ut*Cgh	0.05672	0.02836	0.00598	4.74	0.000	1.00
ut*Cg	0.14391	0.07195	0.00598	12.03	0.000	1.00
ut*Cs	−0.14016	−0.07008	0.00598	−11.71	0.000	1.00
Xba1*Tout	0.02953	0.01477	0.00598	2.47	0.015	1.00
Xba2*Cs	−0.02859	−0.01430	0.00598	−2.39	0.019	1.00
Xba3*Cg	−0.03141	−0.01570	0.00598	−2.63	0.010	1.00
AS1*AS3	0.03047	0.01523	0.00598	2.55	0.012	1.00
AS1*Cg	−0.09891	−0.04945	0.00598	−8.27	0.000	1.00
AS1*Cs	0.08203	0.04102	0.00598	6.86	0.000	1.00
AS2*Cg	0.02484	0.01242	0.00598	2.08	0.040	1.00
AS2*Cs	−0.02672	−0.01336	0.00598	−2.23	0.028	1.00
AS3*Cg	0.02578	0.01289	0.00598	2.15	0.034	1.00
Tout*Cg	0.03234	0.01617	0.00598	2.70	0.008	1.00
Cgh*Cs	−0.04734	−0.02367	0.00598	−3.96	0.000	1.00
Cg*Cs	−0.08578	−0.04289	0.00598	−7.17	0.000	1.00

Coded Coefficients.

S	R-sq	R-sq(adj)	R-sq(pred)
0.0676794	98.77%	98.41%	97.90%

Model Summary.

In the case of u_3 response, the results obtained from regression process show the difference from those of u_2 response. Indeed, u_t and X_{ba2} have no influence on the response, moreover, only eight interactions between input parameters have impact on it (cf. Table 5). It is observed that the factors of B, D, E, and H have p-value of 0.078, 0.184, 0.146, and 0.052 respectively, larger than significance α (0.05). Hence, these parameters have little influence on the u_3 response. However, the interactions of BH, BL, DG, EG, and EK have p-value inferior to α. For this reason, they strongly influence the response of u_3.

The regression equation of this response can be presented as following model (Regression Equation in Uncoded Units):

$$\begin{aligned} u_3 = &-16.99 + 0.19\, X_{ba1} + 29.9\, X_{ba3} + 0.01838\, AS_1 - 0.005652\, AS_2 + 0.0599\, AS_3 \\ &+ 0.000197\, T_{out} - 0.1537\, C_{gh} - 0.079\, C_g + 0.072\, C_s - 0.000575\, X_{ba1} {*} T_{out} \\ &+ 1.404\, X_{ba1} {*} C_s - 0.0737\, X_{ba3} {*} AS_3 - 0.000053\, AS_1 {*} AS_3 + 0.000524\, AS_1 {*} C_g \\ &- 0.000528\, AS_3 {*} C_g + 0.01440\, C_{gh} {*} C_g - 0.03934\, C_g {*} C_s \end{aligned} \tag{37}$$

Table 5. Estimated effects and coefficients for u_3.

Term	Effect	Coef	SE Coef	T-Value	p-Value	VIF
Constant		4.2002	0.0224	187.27	0.000	
Xba1	0.0797	0.0398	0.0224	1.78	0.078	1.00
Xba3	0.0600	0.0300	0.0224	1.34	0.184	1.00
AS1	0.0656	0.0328	0.0224	1.46	0.146	1.00
AS2	−0.3956	−0.1978	0.0224	−8.82	0.000	1.00
AS3	0.6103	0.3052	0.0224	13.61	0.000	1.00
Tout	0.0881	0.0441	0.0224	1.96	0.052	1.00
Cgh	−0.2981	−0.1491	0.0224	−6.65	0.000	1.00
Cg	−1.1550	−0.5775	0.0224	−25.75	0.000	1.00
Cs	1.0922	0.5461	0.0224	24.35	0.000	1.00
Xba1*Tout	−0.1294	−0.0647	0.0224	−2.88	0.005	1.00
Xba1*Cs	0.1228	0.0614	0.0224	2.74	0.007	1.00
Xba3*AS3	−0.1031	−0.0516	0.0224	−2.30	0.023	1.00
AS1*AS3	−0.1294	−0.0647	0.0224	−2.88	0.005	1.00
AS1*Cg	0.1284	0.0642	0.0224	2.86	0.005	1.00
AS3*Cg	−0.1294	−0.0647	0.0224	−2.88	0.005	1.00
Cgh*Cg	0.2016	0.1008	0.0224	4.49	0.000	1.00
Cg*Cs	−0.4819	−0.2409	0.0224	−10.74	0.000	1.00

Coded Coefficients.

S	R-sq	R-sq(adj)	R-sq(pred)
0.253752	94.10%	93.19%	92.02%

Model Summary.

The results in Table 5 also report that the experimental data are highly consistent with the proposed model when the minimum value of R-square approximately 92.02% (all of them are more than 92.02%). However, this is less reliable when compared to that of u_2 response.

Based on previous analysis, it can be said that the proposed models of u_2 and u_3 can be utilized to get the optimum gear ratio of the second and third stages. As a consequence, the optimum gear ratio of the first stage can be obtained by $u_1 = u_t/(u_2 {\cdot} u_3)$.

4.3. Analysis of Variance—ANOVA

In order to quantitatively conclude the impact of each parameters and their interactions on the responses, Analysis of Variance is necessary. Table 5 reveals the Analysis of Variance in case of u_2 response. It is observed that F-values of some parameters of A, F, K, E, L, G, AK, AL, EK, AE, KL, H, EL, AF, AJ, and JL exhibit the F-value higher than 50, and it can be concluded that these parameters have static significance. The R-square value in this case is high when the lowest R-square approaches 92%. In a similar way, we can also identify the high F-value of parameters in case of the u_3 response, such as K, L, G, KL, F, and J. the lowest value of R-square reaches 92%.

4.4. Validation of Proposed Model

The estimation of errors resulting from the difference between experiments and model of u_2 is qualitatively described in Figure 7. From the Normal Probability Plot, it is observed that the contribution of errors is similar to normal distribution. The Versus fits graph discloses that the relation between residual and fitted value of model is random. Moreover, the Versus Order also exhibits the random relationship between residual and order of data point. The identical tendency is also noted when comparing experiments and proposed model in case of u_3 response. The observed phenomena given by the graphs one more time show the reliability of the proposed model which is highly fitted for the experiments.

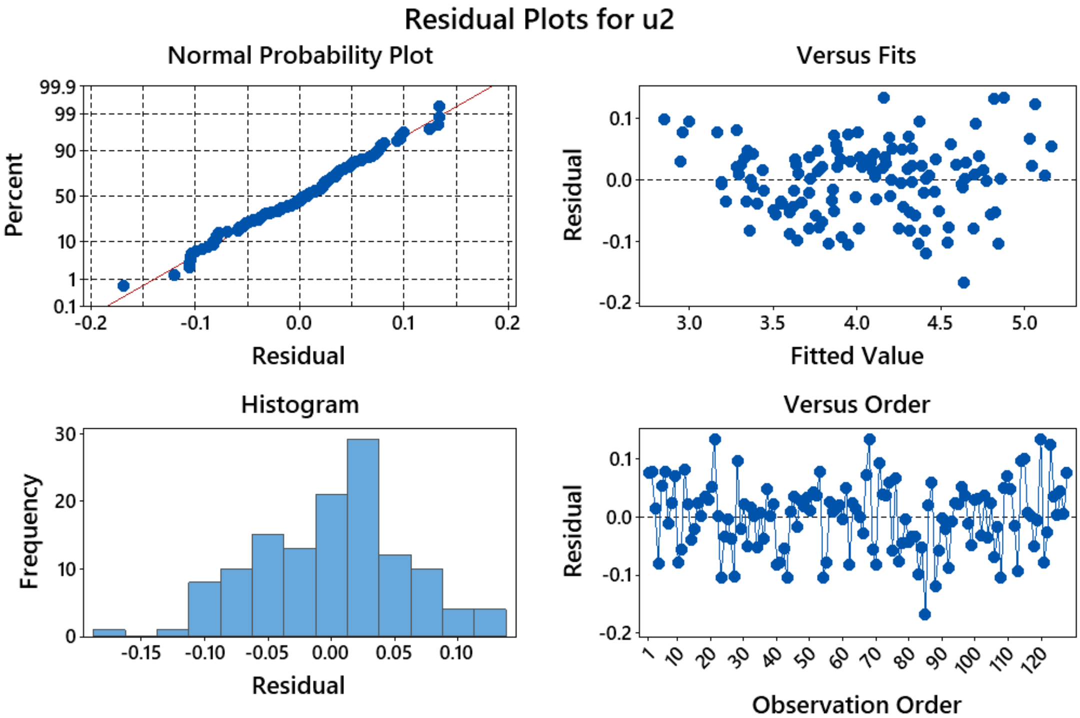


Figure 7. Graphs estimating errors between experiments and model of u_2.

Another way to validate the approximation of data is probability plot exhibited in Figure 8. The Anderson–Darling test in Minitab@19 which is a statistical test to validate the data set come from a specific distribution, e.g., the normal distribution or not. In this way, the data set is representative by blue points. There are three straight lines in the plot where the middle line presents the probability of normal distribution, while two lines in the left and the right refer to limiting boundary with significance of 95%. It is observed that all data set for both case of u_2 and u_3 are sited inside two limiting line when the p-value of 0.289 and 0.097 are greater than α value of 0.05. This indicates that the data set follows the distribution.

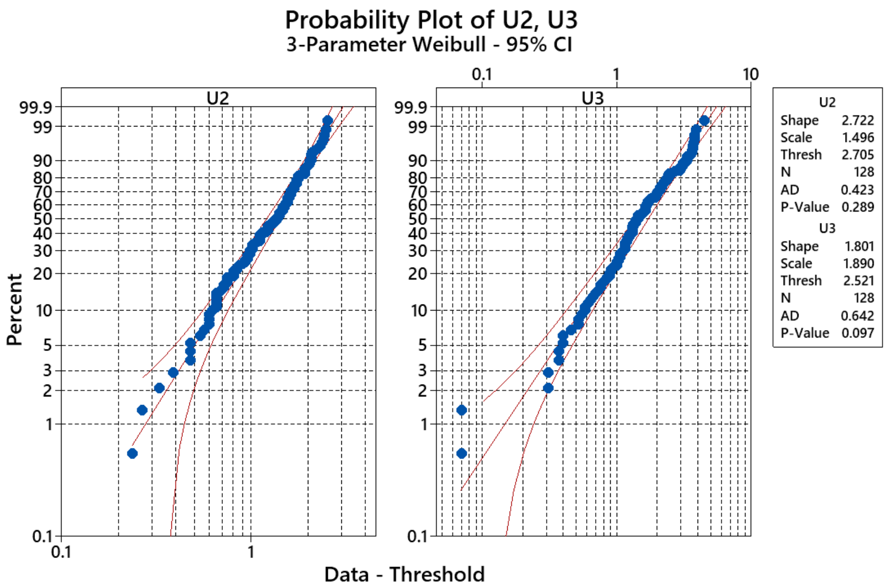


Figure 8. Probability plot of the validity of proposed model for the response of u_2 and u_3.

5. Conclusions

The influence of main design parameters on the optimum partial gear ratios for three-stage helical gearboxes was conducted. The optimum partial gear ratios are derived from the results of optimization problem for getting minimum gearbox cost. This is the first result appearing in scientific publications. To solve the optimum problem, a computer program was built, while a plan of simulation experiments was designed and carried out. The influences of eleven input parameters and their interactions on the output response of u_2 and u_3 were investigated. The input parameters include total gearbox ratio, coefficient of wheel face width of stage 1, coefficient of wheel face width of stage 2, coefficient of wheel face width of stage 3, allowable contact stress of stage 1, allowable contact stress of stage 2, allowable contact stress of stage 3, output torque, cost of gearbox housing, cost of gears, and cost of shafts. The following conclusion can be made:

- ✓ The influence of input parameters and their interactions on u_2 response is different from those of u_3 response. The ANOVA results showed that the parameters of A, F, K, E, L, G, AK, AL, EK, AE, KL, H, EL, AF, AJ, and JL have significant influence on u_2 response (R-square value approaching 98%), while the corresponding to be the parameters of K, L, G, KL, F, and J in case of u_3 (R-square of 92%).
- ✓ The parameters having insignificant influence were eliminated, inversely, the others that had strong influence would be considered for deeper experiments.
- ✓ The proposed models of both u_2 and u_3 responses are highly consistent to experimental data. The reliability of the models is validated. It can be said that the proposed models can be applied to optimize the costs of gearbox.

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Conflicts of Interest: The authors state no conflict of interest.

Appendix A

Run Order	CenterPt	Blocks	u_t	X_{ba1}	X_{ba2}	X_{ba3}	AS_1	AS_2	AS_3	T_{out}	C_{gh}	C_g	C_s	u_2	u_3
1	1	1	30	0.35	0.38	0.4	350	420	350	1000	1	2	1.5	4.02	3.64
2	1	1	100	0.35	0.33	0.36	420	350	420	1000	5	2	1.5	4.08	4.24
3	1	1	100	0.3	0.33	0.4	420	350	350	1000	5	9	5	4.11	3.73
4	1	1	30	0.35	0.33	0.4	350	350	350	1000	5	2	1.5	3.63	3.58
5	1	1	100	0.3	0.33	0.36	350	420	350	1000	5	9	1.5	5.22	2.59
6	1	1	30	0.35	0.38	0.4	420	350	350	10000	5	9	1.5	3.03	3.28
7	1	1	100	0.3	0.33	0.4	420	420	420	10000	5	2	5	4.62	5.14
8	1	1	100	0.35	0.38	0.36	350	420	350	1000	5	2	1.5	5.07	3.25
9	1	1	30	0.3	0.33	0.36	350	420	350	10000	5	2	5	4.26	4.63
10	1	1	100	0.3	0.38	0.4	420	350	350	10000	1	2	1.5	3.93	4.75
11	1	1	30	0.3	0.38	0.36	420	350	350	1000	5	2	1.5	3.45	3.58
12	1	1	30	0.3	0.38	0.4	350	350	420	10000	5	9	5	3.36	4.3
13	1	1	30	0.3	0.38	0.36	420	420	420	10000	5	9	1.5	3.3	3.37
14	1	1	30	0.3	0.38	0.4	350	350	350	1000	5	9	1.5	3.36	3.07
15	1	1	100	0.35	0.38	0.4	350	350	350	1000	1	2	1.5	4.44	3.67
16	1	1	100	0.35	0.33	0.36	420	350	350	10000	5	2	5	4.35	4.69
17	1	1	30	0.35	0.38	0.36	350	350	420	1000	5	2	5	3.72	6.16
18	1	1	100	0.3	0.33	0.36	420	350	420	10000	1	2	5	4.11	6.28
19	1	1	30	0.3	0.33	0.36	420	350	420	1000	1	9	1.5	2.97	3.79
20	1	1	100	0.3	0.38	0.4	420	350	420	1000	1	2	5	4.26	6.04
21	1	1	100	0.3	0.33	0.4	420	420	350	1000	5	2	1.5	4.95	2.89
22	1	1	100	0.35	0.33	0.36	420	420	420	1000	5	9	5	4.41	3.91
23	1	1	100	0.3	0.33	0.4	420	350	420	10000	5	9	1.5	3.72	4.15
24	1	1	30	0.3	0.33	0.4	350	350	420	1000	1	2	1.5	3.51	4.63
25	1	1	30	0.35	0.33	0.36	350	350	420	10000	1	9	1.5	3.18	3.67
26	1	1	30	0.3	0.38	0.36	420	350	420	10000	5	2	5	3.63	6.43
27	1	1	100	0.35	0.33	0.36	420	420	350	10000	5	9	1.5	4.44	3.13
28	1	1	30	0.35	0.33	0.36	420	350	350	1000	5	9	1.5	3.09	3.16
29	1	1	100	0.35	0.38	0.4	420	350	350	1000	5	2	5	4.38	5.65
30	1	1	100	0.3	0.38	0.4	350	350	420	1000	5	2	1.5	4.41	4.15
31	1	1	30	0.3	0.33	0.4	350	420	420	1000	1	9	5	3.81	4.06
32	1	1	100	0.35	0.38	0.36	420	420	350	1000	1	2	5	4.77	4.99
33	1	1	30	0.3	0.33	0.4	350	420	350	10000	1	9	1.5	3.72	2.83
34	1	1	100	0.35	0.38	0.4	350	420	350	1000	1	9	5	4.77	3.97
35	1	1	100	0.35	0.33	0.4	420	420	420	1000	1	2	1.5	4.44	4.3
36	1	1	30	0.35	0.38	0.36	350	350	350	10000	5	2	1.5	3.51	3.82
37	1	1	30	0.35	0.33	0.4	350	350	420	10000	5	2	5	3.81	6.22
38	1	1	100	0.35	0.33	0.4	350	420	420	1000	5	2	5	4.86	5.53
39	1	1	30	0.35	0.38	0.36	420	350	350	10000	1	2	5	3.9	5.5
40	1	1	100	0.3	0.38	0.4	420	420	420	1000	1	9	1.5	4.23	3.76
41	1	1	100	0.3	0.38	0.36	350	420	420	1000	1	2	1.5	4.62	4.51
42	1	1	100	0.35	0.38	0.36	350	350	420	10000	5	9	1.5	4.26	3.52
43	1	1	30	0.3	0.33	0.4	420	350	420	1000	5	2	5	3.84	5.65
44	1	1	100	0.3	0.33	0.36	350	420	420	10000	5	9	5	4.71	3.82
45	1	1	100	0.3	0.38	0.36	420	350	420	1000	5	9	1.5	3.93	3.46
46	1	1	30	0.3	0.33	0.36	350	350	420	1000	5	9	5	3.42	4.12
47	1	1	100	0.3	0.33	0.36	350	350	350	1000	5	2	5	4.68	4.51
48	1	1	100	0.35	0.33	0.36	350	350	420	1000	1	2	5	4.32	7.03
49	1	1	100	0.3	0.33	0.4	350	350	350	1000	1	9	1.5	4.5	2.92
50	1	1	30	0.3	0.38	0.36	420	420	350	1000	5	9	5	3.66	3.76
51	1	1	30	0.35	0.33	0.4	420	350	350	1000	1	2	5	4.14	5.86
52	1	1	30	0.3	0.33	0.4	350	350	350	10000	1	2	5	4.05	6.16
53	1	1	30	0.35	0.38	0.4	420	350	420	1000	5	9	5	3.24	4.63
54	1	1	100	0.35	0.38	0.36	350	350	350	1000	5	9	5	4.26	4.15
55	1	1	30	0.3	0.38	0.4	350	420	420	10000	5	2	1.5	3.69	4.45
56	1	1	30	0.35	0.38	0.4	420	420	350	10000	5	2	5	4.2	4.81
57	1	1	30	0.35	0.33	0.4	420	350	420	10000	1	2	1.5	3.3	4.87
58	1	1	30	0.3	0.33	0.4	420	350	350	10000	5	2	1.5	3.45	3.58

Run Order	CenterPt	Blocks	u_t	X_{ba1}	X_{ba2}	X_{ba3}	AS_1	AS_2	AS_3	T_{out}	C_{gh}	C_g	C_s	u_2	u_3
59	1	1	30	0.35	0.38	0.36	350	420	350	10000	5	9	5	3.81	3.52
60	1	1	30	0.3	0.38	0.4	350	420	350	1000	5	2	5	4.32	4.54
61	1	1	30	0.35	0.33	0.4	350	420	350	1000	5	9	5	3.93	3.43
62	1	1	100	0.3	0.38	0.4	420	420	350	10000	1	9	5	4.32	4.15
63	1	1	100	0.3	0.38	0.36	420	420	350	10000	5	2	1.5	4.62	3.28
64	1	1	100	0.35	0.38	0.36	420	350	420	10000	1	9	5	3.78	4.48
65	1	1	100	0.35	0.38	0.4	350	350	420	10000	1	2	5	4.2	6.34
66	1	1	30	0.35	0.33	0.36	350	420	350	1000	1	2	1.5	3.96	3.85
67	1	1	100	0.35	0.33	0.4	420	350	420	1000	1	9	5	3.93	4.18
68	1	1	30	0.35	0.38	0.4	350	420	420	1000	1	2	5	4.29	5.86
69	1	1	100	0.35	0.33	0.4	420	420	350	10000	1	2	5	4.74	5.59
70	1	1	30	0.3	0.33	0.36	420	350	350	10000	1	9	5	3.27	3.91
71	1	1	100	0.3	0.38	0.4	350	420	420	1000	5	9	5	4.8	3.82
72	1	1	100	0.35	0.38	0.4	350	420	420	10000	1	9	1.5	4.77	3.4
73	1	1	100	0.3	0.38	0.36	350	350	420	1000	1	9	5	4.2	4.15
74	1	1	30	0.3	0.33	0.36	350	420	420	1000	5	2	1.5	3.93	3.34
75	1	1	30	0.35	0.38	0.4	420	420	420	1000	5	2	1.5	3.69	3.61
76	1	1	100	0.35	0.33	0.4	350	420	350	10000	5	2	1.5	5.1	3.1
77	1	1	100	0.35	0.38	0.4	420	420	350	1000	5	9	1.5	4.47	3.19
78	1	1	30	0.35	0.33	0.36	420	420	350	1000	5	2	5	4.23	4.87
79	1	1	30	0.3	0.33	0.36	420	420	420	1000	1	2	5	4.26	5.86
80	1	1	30	0.35	0.33	0.4	350	420	420	10000	5	9	1.5	3.57	3.4
81	1	1	100	0.35	0.33	0.4	420	350	350	10000	1	9	1.5	3.81	3.7
82	1	1	30	0.35	0.33	0.36	420	350	420	10000	5	9	5	3.18	4.57
83	1	1	30	0.35	0.33	0.4	420	420	420	10000	1	9	5	3.54	4.03
84	1	1	30	0.35	0.38	0.36	420	420	420	1000	1	9	5	3.54	4.36
85	1	1	100	0.35	0.38	0.36	350	420	420	10000	5	2	5	4.47	6.31
86	1	1	100	0.3	0.33	0.4	350	350	420	10000	1	9	5	4.17	4.24
87	1	1	100	0.3	0.33	0.36	420	420	350	1000	1	9	5	4.62	3.22
88	1	1	100	0.35	0.33	0.4	350	350	350	10000	5	9	5	4.29	3.97
89	1	1	100	0.35	0.33	0.4	350	350	420	1000	5	9	1.5	4.29	3.55
90	1	1	100	0.35	0.33	0.36	350	420	420	1000	1	9	1.5	4.77	3.43
91	1	1	30	0.3	0.38	0.36	350	420	420	10000	1	9	5	3.69	3.97
92	1	1	30	0.35	0.33	0.36	350	350	350	1000	1	9	5	3.51	3.73
93	1	1	30	0.35	0.38	0.4	350	350	420	1000	1	9	1.5	3.18	3.82
94	1	1	30	0.3	0.33	0.4	420	420	350	10000	5	9	5	3.66	3.52
95	1	1	30	0.3	0.38	0.36	350	350	350	1000	1	2	5	4.05	5.47
96	1	1	100	0.35	0.33	0.36	350	350	350	10000	1	2	1.5	4.38	3.67
97	1	1	30	0.3	0.33	0.36	420	420	350	10000	1	2	1.5	3.75	3.7
98	1	1	30	0.3	0.33	0.4	420	420	420	1000	5	9	1.5	3.36	3.31
99	1	1	30	0.35	0.38	0.4	350	350	350	10000	1	9	5	3.45	3.85
100	1	1	30	0.35	0.33	0.36	350	420	420	10000	1	2	5	4.11	6.22
101	1	1	100	0.35	0.38	0.4	420	350	420	10000	5	2	1.5	3.99	4.24
1000	1	1	100	0.3	0.33	0.36	420	350	350	1000	1	2	1.5	4.08	4.09
103	1	1	30	0.3	0.38	0.36	350	350	420	10000	1	2	1.5	3.36	4.93
10000	1	1	30	0.3	0.33	0.36	350	350	350	10000	5	9	1.5	3.3	3.1
105	1	1	30	0.35	0.38	0.36	420	350	420	1000	1	2	1.5	3.33	4.69
106	1	1	30	0.3	0.38	0.36	350	420	350	1000	1	9	1.5	3.72	2.83
107	1	1	30	0.35	0.33	0.36	420	420	420	10000	5	2	1.5	3.6	3.91
108	1	1	100	0.3	0.38	0.36	350	420	350	10000	1	2	5	4.74	4.96
109	1	1	100	0.35	0.38	0.36	420	420	420	10000	1	2	1.5	4.32	3.97
110	1	1	100	0.35	0.38	0.4	420	420	420	10000	5	9	5	4.38	3.67
111	1	1	30	0.35	0.38	0.36	420	420	350	10000	1	9	1.5	3.39	2.89
112	1	1	100	0.35	0.38	0.36	420	350	350	1000	1	9	1.5	3.84	3.67
113	1	1	30	0.3	0.38	0.4	420	420	350	1000	1	2	1.5	3.81	4.21
114	1	1	100	0.3	0.38	0.36	350	350	350	10000	1	9	1.5	4.47	2.92
115	1	1	30	0.3	0.38	0.4	420	350	420	10000	1	9	1.5	2.94	3.82
116	1	1	100	0.3	0.33	0.4	350	420	350	1000	1	2	5	5.13	4.93
117	1	1	30	0.3	0.38	0.4	420	350	350	1000	1	9	5	3.36	3.88
118	1	1	100	0.3	0.38	0.4	350	350	350	10000	5	2	5	4.44	5.23

Run Order	CenterPt	Blocks	u_t	X_{ba1}	X_{ba2}	X_{ba3}	AS_1	AS_2	AS_3	T_{out}	C_{gh}	C_g	C_s	u_2	u_3
119	1	1	100	0.3	0.33	0.4	350	420	420	10000	1	2	1.5	4.62	4.48
120	1	1	100	0.35	0.33	0.36	350	420	350	10000	1	9	5	5.01	3.04
121	1	1	100	0.3	0.33	0.36	420	420	420	10000	1	9	1.5	4.17	3.82
122	1	1	100	0.3	0.33	0.36	350	350	420	10000	5	2	1.5	4.17	4.72
123	1	1	100	0.3	0.38	0.4	350	420	350	10000	5	9	1.5	5.19	2.59
124	1	1	30	0.35	0.38	0.36	350	420	420	1000	5	9	1.5	3.66	3.04
125	1	1	100	0.3	0.38	0.36	420	420	420	1000	5	2	5	4.65	4.78
126	1	1	30	0.35	0.33	0.4	420	420	350	1000	1	9	1.5	3.42	2.98
127	1	1	30	0.3	0.38	0.4	420	420	420	10000	1	2	5	4.11	5.83
128	1	1	100	0.3	0.38	0.36	420	350	350	10000	5	9	5	4.02	3.55

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