

Editorial

Special Issue on Mathematical Modeling Using Differential Equations and Network Theory

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Received: 1 March 2020; Accepted: 3 March 2020; Published: 10 March 2020



1. Introduction

This special issue collects the latest results on differential/difference equations, the mathematics of networks, and their applications to engineering, and physical phenomena. The Special Issue has 42 submissions and eight high-quality papers which got published with original research results. The Special Issue brought together mathematicians with physicists, engineers, as well as other scientists. Topics covered in this issue:

- Differential/difference equations
- Mathematics of networks
- Fractional calculus
- Partial differential equations
- Discrete calculus
- Mathematical models using dynamical systems

2. Acoustic Wave Equations Using Fractional-Order Differential Equations

In [1], the authors present a newly developed technique, defined as a variational homotopy perturbation transform method in order to solve fractional-order acoustic wave equations. The basic idea behind this article is to extend the variational homotopy perturbation method to the variational homotopy perturbation transform method.

The proposed method is an accurate and straightforward technique to solve fractional-order partial differential equations, and can be considered as a practical analytical technique to solve non-linear fractional partial differential equations compared to other analytical techniques existing in the literature. Several illustrative examples verify the method.

3. Analytical Solutions of Dimensional Physical Models Using Modified Decomposition Method

In [2], the authors present a new analytical technique based on an innovative transformation in order to solve (2+time fractional-order) dimensional physical models. The proposed method is based on the hybrid methodology of Shehu transformation along with the Adomian decomposition method.

The solutions of the targeted problems are represented by graphs and are obtained in a series form that has the desired rate of convergence. The method is, in general, a practical analytical technique to solve linear and non-linear fractional partial differential equations. Numerical examples are given using the proposed method.

4. Multi-Switching Combination Synchronization of Fractional-Order Delayed Systems

In [3] the authors investigate multi-switching combination synchronization of three fractional-order delayed systems. This is actually a generalization of previous multi-switching combination synchronization of fractional-order systems by introducing time-delays.

Based on the stability theory of linear fractional-order systems with multiple time-delays, the article provides appropriate controllers to obtain multi-switching combination synchronization of three non-identical fractional-order delayed systems. In addition, numerical simulations show that they are in accordance with the theoretical analysis given.

5. An Overview of Early Developments of the Hardy–Cross-Type Methods

In [4], the authors provide an overview of early developments of the Hardy–Cross-type methods for computation of flow distribution in pipe networks.

Cross originally proposed a method for analysis of flow in networks of conduits or conductors in 1936. His method was the first really useful engineering method in the field of pipe network calculation. Only electrical analogs of hydraulic networks were used before the Hardy–Cross method. A problem with flow resistance versus electrical resistance makes these electrical analog methods obsolete. The method by Hardy–Cross is taught extensively at faculties, and it remains an important tool for the analysis of looped pipe systems. Engineers today mostly use a modified Hardy–Cross method that considers the whole looped network of pipes simultaneously (use of these methods without computers is practically impossible).

In addition, in [4] a method from a Russian practice published during the 1930s, which is similar to the Hardy–Cross method, is also described. Some notes from the work of Hardy–Cross are also presented. Furthermore, an improved version of the Hardy–Cross method, which significantly reduces the number of iterations, is presented and discussed.

Finally, the authors present results on tested multi-point iterative methods, which can be used as a substitution for the Newton–Raphson approach used by Hardy–Cross.

6. Parametrical Non-Complex Tests to Evaluate Partial Decentralized Linear-Output Feedback Control Stabilization Conditions

In [5], the authors formulate sufficiency-type linear-output feedback decentralized closed-loop stabilization conditions if the continuous-time linear dynamic system can be stabilized under linear output-feedback centralized stabilization.

The provided tests are simple to evaluate, while they are based on the quantification of the sufficient smallness of the parametrical error norms between the control, output, interconnection and open-loop system dynamics matrices and the corresponding control gains in the decentralized case related to the centralized counterpart.

The tolerance amounts of the various parametrical matrix errors are described by the maximum allowed tolerance upper-bound of a small positive real parameter that upper-bounds the various parametrical error norms. Such a tolerance is quantified by considering the first or second powers of such a small parameter.

The results are seen to be directly extendable to quantify the allowed parametrical errors that guarantee the closed-loop linear output-feedback stabilization of a current system related to its nominal counterpart. Several numerical examples are included and discussed in the article.

7. Transient-Flow Modeling of Vertical Fractured Wells with Multiple Hydraulic Fractures

Massive hydraulic fracturing of vertical wells has been extensively employed in the development of low-permeability gas reservoirs. The existence of multiple hydraulic fractures along a vertical well makes the pressure profile around the vertical well complex.

In [6], the authors study the pressure dependence of permeability in order to develop a seepage model of vertically fractured wells with multiple hydraulic fractures. Both transformed pseudo-pressure and perturbation techniques have been employed to linearize the proposed model.

The proposed work further enriches the understanding of the influence of the stress sensitivity on the performance of a vertical fractured well with multiple hydraulic fractures and can be used to more accurately interpret and forecast the transient pressure.

Some key points in the article are the superposition principle and a hybrid analytical-numerical method that are used to obtain the bottom-hole pseudo-pressure solution, the type curves for pseudo-pressure that are presented and identified, and finally, the discussion that is included on the effects of the relevant parameters on the type curve and the error caused by neglecting the stress sensitivity.

8. Policy-Compliant Maximum Network Flows

Computer network administrators are often interested in the maximal bandwidth that can be achieved between two nodes in the network, or how many edges can fail before the network gets disconnected. Classic maximum flow algorithms that solve these problems are well-known. However, in practice, network policies are in effect, severely restricting the flow that can actually be set up. These policies are put into place to conform to service level agreements and optimize network throughput, and can have a large impact on the actual routing of the flows.

In [7], the authors model the problem and define a series of progressively more complex conditions and algorithms that calculate increasingly tighter bounds on the policy-compliant maximum flow using regular expressions and finite-state automata. This is the first time that specific conditions are deduced, which characterize how to calculate policy-compliant maximum flows using classic algorithms on an unmodified network.

9. The Fractional Form of the Tinkerbell Map Is Chaotic

In [8], the authors are concerned with a fractional Caputo-difference form of the well-known Tinkerbell chaotic map. The dynamics of the proposed map are investigated numerically through phase plots, bifurcation diagrams, and Lyapunov exponents considered from different perspectives.

In addition, a stabilization controller is proposed, and the asymptotic convergence of the states is established by means of the stability theory of linear fractional discrete systems. Numerical results are employed to confirm the analytical findings.

Funding: This Editorial was supported by: Science Foundation Ireland, by funding Ioannis Dassios under Grant No. SFI/15/IA/3074.

Acknowledgments: This issue would not have been possible without the help of a variety of talented authors, professional reviewers, and the dedicated editorial team of Applied Sciences. Thank you to all the authors and reviewers for this opportunity. Finally, thanks to the Applied Sciences editorial team.

Conflicts of Interest: The author declares no conflict of interest.

References

1. Ali, I.; Khan, H.; Shah, R.; Baleanu, D.; Kumam, P.; Arif, M. Fractional View Analysis of Acoustic Wave Equations, Using Fractional-Order Differential Equations. *Appl. Sci.* **2020**, *10*, 610. [[CrossRef](#)]
2. Khan, H.; Farooq, U.; Shah, R.; Baleanu, D.; Kumam, P.; Arif, M. Analytical Solutions of (2+Time Fractional Order) Dimensional Physical Models, Using Modified Decomposition Method. *Appl. Sci.* **2020**, *10*, 122. [[CrossRef](#)]
3. Li, B.; Wang, Y.; Zhou, X. Multi-Switching Combination Synchronization of Three Fractional-Order Delayed Systems. *Appl. Sci.* **2019**, *9*, 4348. [[CrossRef](#)]
4. Brkić, D.; Praks, P. Short Overview of Early Developments of the Hardy Cross Type Methods for Computation of Flow Distribution in Pipe Networks. *Appl. Sci.* **2019**, *9*, 2019. [[CrossRef](#)]
5. De la Sen, M.; Ibeas, A. Parametrical Non-Complex Tests to Evaluate Partial Decentralized Linear-Output Feedback Control Stabilization Conditions from Their Centralized Stabilization Counterparts. *Appl. Sci.* **2019**, *9*, 1739. [[CrossRef](#)]
6. Guo, P.; Sun, Z.; Peng, C.; Chen, H.; Ren, J. Transient-Flow Modeling of Vertical Fractured Wells with Multiple Hydraulic Fractures in Stress-Sensitive Gas Reservoirs. *Appl. Sci.* **2019**, *9*, 1359.

- [CrossRef]
7. Audenaert, P.; Colle, D.; Pickavet, M. Policy-Compliant Maximum Network Flows. *Appl. Sci.* **2019**, *9*, 863. [CrossRef]
 8. Ouannas, A.; Khennaoui, A.; Bendoukha, S.; Vo, T.; Pham, V.; Huynh, V. The Fractional Form of the Tinkerbell Map Is Chaotic. *Appl. Sci.* **2018**, *8*, 2640. [CrossRef]



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