Article

Frequency Characteristics of Multiscale Hybrid Nanocomposite Annular Plate Based on a Halpin–Tsai Homogenization Model with the Aid of GDQM

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Abstract: In this article, we study the vibration performance of multiscale hybrid nanocomposite (MHC) annular plates (MHCAP) resting on Winkler–Pasternak substrates exposed to nonlinear temperature gradients. The matrix material is reinforced with carbon nanotubes (CNTs) or carbon fibers (CF) at the nano- or macroscale, respectively. The annular plate is modeled based on higher-order shear deformation theory (HSDT). We present a modified Halpin–Tsai model to predict the effective properties of the MHCAP. Hamilton’s principle was employed to establish the governing equations of motion, which is finally solved by the generalized differential quadrature method (GDQM). In order to validate the approach, numerical results were compared with available results from the literature. Subsequently, a comprehensive parameter study was carried out to quantify the influence of different parameters such as stiffness of the substrate, patterns of temperature increase, outer temperature, volume fraction and orientation angle of the CFs, weight fraction and distribution patterns of CNTs, outer radius to inner radius ratio, and inner radius to thickness ratio on the response of the plate. The results show that applying a sinusoidal temperature rise and locating more CNTs in the vicinity of the bottom surface yielded the highest natural frequency.

Keywords: higher-order shear deformation theory; nonlinear temperature gradient; vibration; annular plate; multiscale hybrid nanocomposite; Halpin–Tsai homogenization model

1. Introduction

Annular plates are among the most important structural components widely employed in different branches of engineering like civil applications, aerospace technologies, manufacturing industry, and so on. Exploring the mechanical performance of annular plates with respect to different types of loading and boundary conditions is of great importance for engineers. In order to achieve desired thermo-mechanical properties, carbon-based materials are among the best choices to reinforce engineering structures at different length scales. Composites consists of a matrix and a macroscale reinforcement such as carbon fibers (CF) oriented in specific directions to enrich mechanical performance of the structure. Chakrapani et al. [1] used multiscale analysis to investigate the role of the fiber orientation and sequence of the laminate on the forced oscillation response of CF-reinforced composite beams. Moreover, they designed experiments confirming the accuracy of their numerical results. Emam and Eltaher [2] studied the post-buckling and buckling performance of fiber-reinforced beams exposed to a hygro-thermal environment.

Reinforcing composites with nano-fibers instead of macroscale fibers might significantly enhance the mechanical properties of the composite [3–5]. Maghamikia and Jam [6] employed the finite element method (FEM) to study the buckling behavior of carbon nanotube-reinforced (CNTR) annular and circular plates on the basis of third-order shear deformation theory (TSDT). They reported that the critical buckling loads obtained by their approach were lower than those obtained according to classical theory as a result of considering the shear strain terms. Tahouneh and Yas [7] applied the differential quadrature method (DQM) to study the vibration characteristics of two-dimensional continuously graded CNTR thick annular plates resting on an elastic substrate within the framework of elasticity theory. Tahouneh and Jam [8] investigated the free vibration behavior through the thickness-graded CNTR annular plates resting on an elastic foundation on the basis of elasticity theory and employing the DQM. In both studies, the Eshelby–Mori–Tanaka micromechanics approach was used to approximate the effective elastic properties of the composite. Functionally graded materials (FGM) are a very popular group of advanced materials [9–13]. Applying the variational differential quadrature method (VDQM) and assuming first-order shear deformation theory (FSDT), Ansari et al. [14] carried out vibration and buckling analyses of functionally graded carbon nanotube reinforced composites (FG-CNTRC) surrounded by an elastic substrate and subjected to thermal loads. Keleshteri et al. [15,16] focused on the post-buckling and large amplitude vibration characteristics of FG-CNTRC annular sector plates with integrated piezoelectric layers on the basis of FSDT considering von Karman nonlinearity and utilizing generalized differential quadrature method (GDQM). Following the same procedure, Keleshteri et al. [17] explored the large amplitude performance of FG-CNTRC annular plates coupled with piezoelectric layers resting on elastic substrate. Torabi and Ansari [18] claimed that it is necessary to develop governing equations for large amplitude vibration of FG-CNTRC annular plates in the form of general asymmetric relations accounting for initial thermal stresses to obtain accurate results. Within the framework of FSDT, Pang et al. [19] studied the vibration response of thick FG-CNTRC annular sector plates utilizing the artificial spring boundary method. The buckling performance of moderately thick FG-CNTRC annular sector plates resting on Pasternak elastic substrates and subjected to shear and in-plane loading has been analyzed by Ansari et al. [20]. Gholami and Ansari [21] carried out a nonlinear resonance analysis of FG-CNTRC annular sector plates excited by transverse harmonic loads uniformly distributed through the plate area employing VDQM and Reddy's TSDT integrated with von Karman nonlinear strain relations. Mercan et al. [22] obtained natural frequencies of laminated thick shear deformable FG-CNTRC annular plates employing discrete singular convolution (DSC) regularized Shannon delta (RSD) and Lagrange delta sequence (LDS) kernels to the motion equations extracted from conical shell relations. Ansari et al. [23] presented a general approach to explore free vibration response of FG-CNTRC cylindrical, conical shells and annular plates by applying VDQM on the governing equations established based on FSDT and Hamilton's principle. Considering von Karman nonlinearities in the HSDT-based axisymmetric governing equations, Ansari et al. [24] studied the vibration response of sandwich annular plates with isotropic core and FG-CNTRC interfaces embedded in elastic substrates. On the basis of FSDT, Zhong et al. [25] employed Ritz method to investigate vibration characteristics of FG-CNTRC annular, circular, and sector plates. Civalek and Baltacıoğlu [26] employed DSC to analyze free vibration behavior of CNTRC annular and annular sector plate within the framework of FSDT.

Graphene platelets (GPLs) are an alternative to CNTs for enriching the mechanical characteristics of composite structures [27–35]. Within the framework of theory of elasticity, Yang et al. [36] employed the Mian–Spencer approach and carried out a three-dimensional axisymmetric thermo-mechanical bending analysis of FG-CNTRC annular and circular plates. Malekzadeh et al. [37] focused on the vibration response of functionally graded graphene-platelets reinforced composites (FG-GPLRC). Gholami and Ansari [38] utilized VDQM and Reddy's TSDT to carry out a nonlinear asymmetric bending investigation of FG-GPLRC annular plates subjected to bi-harmonic and uniform transverse loadings.

Multiscale hybrid composites (MHC) exploit both a nanoscale as well as a macroscale reinforcement. Related articles can be classified into three main categories: Those investigating the manufacturing

process, those exploring the characteristics of MHCs, and those related to the mechanical behavior of MHC structures. The article published by Thostenson et al. [39] is among the first attempts to obtain a multiscale composite using chemical vapor deposition. Song [40] introduced a novel manufacturing process called vacuum-assisted resin transfer molding (VARTM) to synthesize carbon nanotube (CNT)/aramid fiber/epoxy MHCs and reported an improvement in bending characteristics compared to aramid fiber/epoxy composite. Bekyarova et al. [41,42] designed mechanical tests to prove that CNT/CF/epoxy multiscale composites manufactured by VARTM exhibit higher shear strength and electrical conductivity compared with CF/epoxy composite. Jiang et al. [43] used electrophoretic deposition (EPD) to obtain graphene oxide/CF/polyurethane composite in order to achieve higher tensile strength. Cougo et al. [44] reviewed some manufacturing methods of MHCs with carbon-based nano-fillers such as high energy sonication [45], resin infusion technique [46], VARTM [47], and EPD technique [48,49].

Many researchers conducted experimental and theoretical investigations on the properties of MHCs, i.e., electrical anisotropy [50], microwave properties [51], electrical [52] and thermal conductivity [53], and mechanical properties [54,55] of CNT/CF/matrix composites. Using molecular dynamics, Radue and Odegard [56] studied the effect of different types of epoxy on the elasticity modulus of CNT/CF/epoxy composites. Li et al. [57] improved the thermo-mechanical properties of glass fiber/epoxy composites combined with CNTs. Other studies on mechanical properties of MHCs can be found, for instance in [58–62] and references therein.

Studying the mechanical behavior of MHC structures has become of great interest in recent years. Utilizing FEM within the framework of hyperbolic shear deformation theory and taking von Karman-type nonlinearity into account, Ebrahimi and Habibi [63,64] analyzed the nonlinear dynamic and low-velocity eccentric impact response of CNT/CF/polymer plates resting on elastic substrates in a thermal environment. Ahmadi et al. [65] applied FEM to study the buckling behavior of CNT/CF/polyimide composite rods subjected to axial compression. Ahmadi et al. [66] studied the forced and free vibration characteristics of CNT/CF/polymer matrix. Ebrahimi and Dabbagh [67] studied the thermo-elastic vibration of a multiscale composite beam made of CNT/CF/resin using an analytical method. Karimiasl et al. [68] utilized the homotopy perturbation method to analyze the nonlinear oscillation response of MHC doubly curved shells coupled with piezoelectric layer exposed to a hygro-thermal environment. In further studies, they [69,70] focused on nonlinear forced and free oscillation as well as post-buckling and buckling analyses of MHC doubly curved shells. In both studies, they considered GPLs/CF/polymer and CNT/CF/polymer as the MHC and employed Halpin–Tsai micromechanics to model the elastic characteristics. Based on the assumptions of classical plate theory, Ebrahimi and Dabbagh [71,72] predicted the natural frequencies and critical buckling force of CNT/CF/matrix MHC plate resting on Winkler–Pasternak elastic foundation. Tornabene et al. [73] conducted a study dealing with free vibration performance of three-phase MHC structures made of CNT/polymer/fiber within the framework of FSDT.

To our best knowledge, free vibration performance of MHCAP made of CNT/CF/matrix, exposed to nonlinear temperature gradients and resting on Winkler–Pasternak substrate has not been explored yet, and is the focus of this manuscript. The mechanical properties of MHCAP was obtained based on the Halpin–Tsai model integrated with a micromechanical model. The governing differential equations were established through Hamilton’s principle within the framework of HSDT. The GDQM was employed to solve for the vibration response of the plate.

2. Theory and Formulation

2.1. Problem Description

Let us consider the annular plate with inner radius a , outer radius b and thickness h , resting on a two-parameter elastic substrate, as illustrated in Figure 1. As mentioned before, CNTs and CF are used as nanoscale and macroscale reinforcement, respectively.

Figure 1. Schematic view of the multiscale hybrid annular plate, related parameters, and coordination.

2.2. The Homogenization Process of MHC

An overview of MHC is presented in Figure 2. Accordingly, the homogenization approach was based on Halpin–Tsai model and a micromechanical theory and consisted of two main steps. First, we computed the effective properties of the composite, i.e.,

$$E_{11} = V_F E_{11}^F + V_{NCM} E^{NCM} \quad (1)$$

$$\frac{1}{E_{22}} = \frac{1}{E_{22}^F} + \frac{V_{NCM}}{E^{NCM}} - V_F V_{NCM} - \frac{\frac{(\nu^F)^2 E^{NCM}}{E_{22}^F} + \frac{(\nu^{NCM})^2 E_{22}^F}{E^M} - 2\nu^F \nu^{NCM}}{V_F E_{22}^F + V_{NCM} E^{NCM}} \quad (2)$$

$$\frac{1}{G_{12}} = \frac{V_F}{G_{12}^F} + \frac{V_{NCM}}{G^{NCM}} \quad (3)$$

$$\rho = V_F \rho^F + V_{NCM} \rho^{NCM} \quad (4)$$

$$\nu_{12} = V_F \nu^F + V_{NCM} \nu^{NCM} \quad (5)$$

where ρ , E , G and ν represent the mass density, elasticity modulus, shear modulus, and Poisson's ratio, respectively; the superscripts NCM and F indicate 'nanocomposite matrix' and 'fiber', respectively. The sum of volume fraction of the carbon fiber V_F and the volume fraction of the nanocomposite matrix V_{NCM} is equal to 1.

$$V_F + V_{NCM} = 1 \quad (6)$$

Second, we obtained the effective properties of the nanocomposite using the extended Halpin–Tsai micromechanics theory:

$$E^{NCM} = E^M \left(\frac{5}{8} \left(\frac{1 + 2\beta_{dd} V_{CNT}}{1 - \beta_{dd} V_{CNT}} \right) + \frac{3}{8} \left(\frac{1 + 2(l^{CNT}/d^{CNT})\beta_{dl} V_{CNT}}{1 - \beta_{dl} V_{CNT}} \right) \right) \quad (7)$$

in which β_{dd} and β_{dl} are given by

$$\beta_{dl} = \frac{(E_{11}^{CNT}/E^M) - (d^{CNT}/4t^{CNT})}{(E_{11}^{CNT}/E^M) + (l^{CNT}/2t^{CNT})}, \quad \beta_{dd} = \frac{(E_{11}^{CNT}/E^M) - (d^{CNT}/4t^{CNT})}{(E_{11}^{CNT}/E^M) + (d^{CNT}/2t^{CNT})} \quad (8)$$

l^{CNT} , t^{CNT} , d^{CNT} , E^{CNT} , and V_{CNT} denoting the length, thickness, diameter, elasticity modulus, and volume fraction of CNTs, respectively. In addition, E^M and V_M are the elasticity modulus and volume fraction of the matrix, respectively. The volume fraction of CNTs can be calculated explicitly based on its weight fraction W_{CNT} according to following relation

$$V_{CNT}^* = \frac{W_{CNT}}{W_{CNT} + (\frac{\rho^{CNT}}{\rho^M})(1 - W_{CNT})}. \quad (9)$$

Figure 2. Overview of multiscale hybrid composite.

According to subsequent relations and Figure 3, three functionally graded and one uniform pattern were considered for the distribution of CNTs through the thickness of MHCAP.

$$\begin{aligned}
 V_{CNT} &= V_{CNT}^* \frac{|\xi_j|}{h} & \text{FG - X} \\
 V_{CNT} &= V_{CNT}^* \left(1 + \frac{2\xi_j}{h}\right) & \text{FG - V} \\
 V_{CNT} &= V_{CNT}^* \left(1 - \frac{2\xi_j}{h}\right) & \text{FG - A} \\
 V_{CNT} &= V_{CNT}^* & \text{FG - UD}
 \end{aligned} \tag{10}$$

where $\xi_j = \left(\frac{1}{2} + \frac{1}{2N_t} - \frac{j}{N_t}\right)h$ $j = 1, 2, \dots, N_t$.

The sum of V_M and V_{CNT} as the two constituents of the nanocomposite matrix is equal to 1.

$$V_{CNT} + V_M = 1 \tag{11}$$

The corresponding mass density, Poisson's ratio, and shear modulus of the nanocomposite matrix can be determined as follows

$$\rho^{NCM} = V_{CNT}\rho^{CNT} + V_M\rho^M \tag{12}$$

$$\nu^{NCM} = \nu^M \tag{13}$$

$$G^{NCM} = \frac{E^{NCM}}{2(1 + \nu^{NCM})} \tag{14}$$

Moreover, the expansion coefficients of the MHC is determined as [64]

$$\alpha_{11} = \frac{V_f E_{11}^f \alpha_{11}^f + V_{NCM} E^{NCM} \alpha^{NCM}}{V_f E_{11}^f + V_{NCM} E^{NCM}} \quad (15)$$

$$\alpha_{22} = (1 + V_f) V_f \alpha_{22}^f + (1 + V_{NCM}) V_{NCM} \alpha_{NCM} - \nu_{12} \alpha_{11} \quad (16)$$

where α^{NCM} indicates the corresponding thermal expansion coefficient of the nanocomposite matrix which is equal to [67]

$$\alpha_{NCM} = \frac{1}{2} \left\{ \left(\frac{V_{CNT} E_{11}^{CNT} \alpha_{11}^{CNT} + V_m E_m \alpha_m}{V_{CNT} E_{11}^{CNT} + V_m E_m} \right) (1 - \nu^{NCM}) + (1 + \nu_m) \alpha_m V_m + (1 + \nu^{CNT}) \alpha^{CNT} V_{CNT} \right\} \quad (17)$$

Figure 3. Distribution patterns of carbon nanotube (CNT) through the thickness of the multiscale hybrid annular plate. (a) UD; (b) FG-X; (c) FG-A; (d) FG-V.

Properties of functionally graded (FG) and uniform distribution of NCM along the thickness direction of the annular plate are displayed in Figure 4 and expressed according to the following relations.

Figure 4. Through-the-thickness variation of mechanical properties ($\theta = \pi/4$, $W_{CNT} = 0.02$, $V_F = 0.2$).

2.3. Kinematic Relations

The cylindrical coordinate system (R, θ, z) and axial symmetry in loading and geometry were taken into account. With the aid of the Reddy's shear-deformation plate theory based high-order shear deformation theory (HSDT) and symmetric deformation, the displacement fields of a circular plate are expressed by the following equations:

$$\begin{aligned} U(R, \theta, z, t) &= u(R, t) - z \frac{\partial w(R, t)}{\partial R} + (z - c_1 z^3) \left(\frac{\partial w(R, t)}{\partial R} + \phi(R, t) \right) \\ V(R, \theta, z, t) &= 0 \\ W(R, \theta, z, t) &= w(R, t) \end{aligned} \quad (18)$$

where u , w , and ϕ indicate the displacements of the midsurface in R and Z directions and rotations of the transverse normal around θ direction, respectively. It is well known that in the higher-order shear deformation theory c_1 is equal to $4/3 h^2$. The nonzero components of the strain tensor can be decomposed as

$$\begin{Bmatrix} \varepsilon_{RR} \\ \varepsilon_{\theta\theta} \\ \gamma_{RZ} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{RR}^0 \\ \varepsilon_{\theta\theta}^0 \\ \gamma_{RZ}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_{RR} \\ \kappa_{\theta\theta} \\ \kappa_{RZ} \end{Bmatrix} + z^3 \begin{Bmatrix} \kappa_{RR}^* \\ \kappa_{\theta\theta}^* \\ \kappa_{RZ}^* \end{Bmatrix} \quad (19)$$

where ε_{RR} and $\varepsilon_{\theta\theta}$ indicate the corresponding normal strains in R and θ directions and γ_{RZ} represents the shear strain in the RZ plane. Equation (18) can be rewritten as

$$\begin{Bmatrix} \varepsilon_{RR}^0 \\ \varepsilon_{\theta\theta}^0 \\ \gamma_{RZ}^0 \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial R} \\ \frac{u}{R} \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial R} \end{Bmatrix}, \begin{Bmatrix} \kappa_{RR} \\ \kappa_{\theta\theta} \\ \kappa_{RZ} \end{Bmatrix} = \begin{Bmatrix} -\frac{\partial^2 w}{\partial R^2} + \frac{\partial^2 w}{\partial R^2} + \frac{\partial \phi}{\partial R} \\ -\frac{1}{R} \frac{\partial w}{\partial R} + \frac{1}{R} \left(\frac{\partial w}{\partial R} + \phi \right) \\ -\frac{\partial^2 w}{\partial R \partial z} + \frac{\partial^2 w}{\partial R \partial z} + \frac{\partial \phi}{\partial z} \end{Bmatrix}, \begin{Bmatrix} \kappa_{RR}^* \\ \kappa_{\theta\theta}^* \\ \kappa_{RZ}^* \end{Bmatrix} = \begin{Bmatrix} -c_1 \left(\frac{\partial^2 w}{\partial R^2} + \frac{\partial \phi}{\partial R} \right) \\ -\frac{c_1}{R} \left(\frac{\partial w}{\partial R} + \phi \right) \\ -c_1 \left(\frac{\partial^2 w}{\partial R \partial z} + \frac{\partial \phi}{\partial z} \right) \end{Bmatrix} \quad (20)$$

2.4. Hamilton's Principle

In this study, the Hamilton's principle was employed to derive the governing equations of motion

$$\int_{t_1}^{t_2} (\delta T - \delta U + \delta W_1 - \delta W_2) dt = 0 \quad (21)$$

in which T and U stand for the kinetic energy and strain energy, respectively, and W_1 and W_2 denote the work done by the thermal load and the elastic substrate, respectively. Taking the first variation of the strain energy yields

$$\delta U = \frac{1}{2} \int_V \sigma_{ij} \delta \varepsilon_{ij} dV = \int_{R_1}^{R_2} \left[\begin{aligned} & \left\{ \frac{\partial N_{RR}}{\partial R} + \frac{N_{\theta\theta}}{R} \right\} \delta u \\ & + \left\{ \frac{\partial M_{RR}}{\partial R} + \frac{M_{\theta\theta}}{R} - c_1 \frac{\partial P_{RR}}{\partial R} - \frac{c_1}{R} P_{\theta\theta} + (Q_{RZ} - 3c_1 S_{RZ}) \right\} \delta \phi \\ & + \left\{ -c_1 \frac{\partial^2 P_{RR}}{\partial R^2} - \frac{c_1}{R} \frac{\partial P_{\theta\theta}}{\partial R} + \left(\frac{\partial Q_{RZ}}{\partial R} - 3c_1 \frac{\partial S_{RZ}}{\partial R} \right) \right\} \delta w \end{aligned} \right] dR \quad (22)$$

The resultants of the force and moment can be expressed as

$$\{N_{RR}, M_{RR}, P_{RR}\} = \int_z \sigma_{RR} \{1, z, z^3\} dz \quad (23)$$

$$\{N_{\theta\theta}, M_{\theta\theta}, P_{\theta\theta}\} = \int_z \sigma_{\theta\theta} \{1, z, z^3\} dz \quad (24)$$

$$\{Q_{Rz}, S_{Rz}\} = \int_z \sigma_{Rz} \{1, z^2\} dz \quad (25)$$

The variation of the work induced by thermal gradient is given by

$$\delta W_1 = \int_{R_1}^{R_2} \left[N^T \frac{\partial w}{\partial x} \frac{\partial \delta w}{\partial x} \right] dR \quad (26)$$

with the force resultant of the thermal loading N^T

$$N^T = \int_{-h/2}^{h/2} (Q_{11} \alpha_{11} + Q_{12} \alpha_{22}) (T - T_0) dz \quad (27)$$

Note that three different patterns were considered for the temperature gradient across the thickness: Uniform temperature rise (UTR), power form temperature rise (PTR), and sinusoidal temperature rise (STR) [74]. As linear temperature rises are unrealistic in real applications, we studied two other conditions to finally compare the difference to the first scenario.

$$UTR : \Delta T = T - T_0 \quad (28)$$

$$PTR : T = T_0 + \Delta T \left(\frac{1}{2} + \frac{z}{h} \right)^\beta \quad (29)$$

$$STR : T = T_0 + \Delta T \left(1 - \cos \frac{\pi}{2} \left(\frac{1}{2} + \frac{z}{h} \right) \right)^\beta \quad (30)$$

while the applied work by the elastic substrate can be expressed as

$$\delta w_2 = \int_{R_1}^{R_2} \left(-K_w w \delta w + K_p \left[\left(\frac{\partial w}{\partial x} \right) \left(\frac{\partial}{\partial x} \delta w \right) \right] \right) dR \quad (31)$$

with K_p and K_w being the Pasternak and Winkler coefficients of the substrate, respectively. The kinetic energy and its first variation can be written as

$$T = \int_A \frac{1}{2} \rho \left[\left(\frac{\partial U}{\partial t} \right)^2 + \left(\frac{\partial V}{\partial t} \right)^2 + \left(\frac{\partial W}{\partial t} \right)^2 \right] dR dZ \quad (32)$$

$$\delta T = \int_{R_1}^{R_2} \rho \left[\frac{\partial U}{\partial t} \frac{\partial \delta U}{\partial t} + \frac{\partial V}{\partial t} \frac{\partial \delta V}{\partial t} + \frac{\partial W}{\partial t} \frac{\partial \delta W}{\partial t} \right] dR \quad (33)$$

$$\delta T = \int_{R_1}^{R_2} \rho \left[\begin{aligned} & \left\{ -I_0 \frac{\partial^2 u}{\partial t^2} - I_1 \frac{\partial^2 \phi}{\partial t^2} + I_3 c_1 \left(\frac{\partial^2 \phi}{\partial t^2} + \frac{\partial^3 w}{\partial R \partial t^2} \right) \right\} \delta u \\ & + \left\{ -I_1 \frac{\partial^2 u}{\partial t^2} - I_2 \frac{\partial^2 \phi}{\partial t^2} + I_4 c_1 \left(\frac{\partial^2 \phi}{\partial t^2} + \frac{\partial^3 w}{\partial R \partial t^2} \right) \right\} \delta \phi \\ & + \left\{ c_1 I_3 \frac{\partial^2 u}{\partial t^2} + c_1 I_4 \frac{\partial^2 \phi}{\partial t^2} - I_6 c_1^2 \left(\frac{\partial^3 \phi}{\partial R \partial t^2} + \frac{\partial^3 w}{\partial R \partial t^2} \right) \right\} \delta \phi \\ & + \left\{ -c_1 I_3 \frac{\partial^3 u}{\partial R \partial t^2} - c_1 I_4 \frac{\partial^3 \phi}{\partial R \partial t^2} + I_6 c_1^2 \left(\frac{\partial^3 \phi}{\partial R \partial t^2} + \frac{\partial^4 w}{\partial R^2 \partial t^2} \right) \right\} \delta w \\ & + \left\{ -I_0 \frac{\partial^2 w}{\partial t^2} \right\} \delta w \end{aligned} \right] dR \quad (34)$$

where $\{I_0, I_1, I_2, I_3, I_4, I_5, I_6\} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho^{NCM} \{1, z, z^2, z^3, z^4, z^5, z^6\} dZ$ represent the mass inertias.

Substituting Equations (22), (26), (31), and (34), into Equation (21) yields the Euler–Lagrange equations of MHCAP

$$\delta u : \frac{\partial N_{RR}}{\partial R} - \frac{N_{\theta\theta}}{R} = I_0 \frac{\partial^2 u}{\partial t^2} + I_1 \frac{\partial^2 \phi}{\partial t^2} - c_1 I_3 \left(\frac{\partial^2 \phi}{\partial t^2} + \frac{\partial^3 w}{\partial R \partial t^2} \right) \quad (35)$$

$$\begin{aligned} \delta w : c_1 \frac{\partial^2 P_{RR}}{\partial R^2} - \frac{c_1}{R} \frac{\partial P_{\theta\theta}}{\partial R} + \frac{\partial Q_{Rz}}{\partial R} - 3c_1 \frac{\partial S_{Rz}}{\partial R} - K_w w \\ + K_p \frac{\partial^2 w}{\partial R^2} - N^T \frac{\partial^2 w}{\partial R^2} = c_1 I_3 \frac{\partial^3 u}{\partial R \partial t^2} + c_1 I_4 \frac{\partial^3 \phi}{\partial R \partial t^2} \\ - c_1^2 I_6 \left(\frac{\partial^3 \phi}{\partial R \partial t^2} + \frac{\partial^4 w}{\partial R^2 \partial t^2} \right) + I_0 \frac{\partial^2 w}{\partial t^2} \end{aligned} \quad (36)$$

$$\begin{aligned} \delta \phi : \frac{\partial M_{RR}}{\partial R} - c_1 \frac{\partial P_{RR}}{\partial R} - \frac{M_{\theta\theta}}{R} + \frac{c_1}{R} P_{\theta\theta} - Q_{Rz} + 3c_1 S_{Rz} = I_1 \frac{\partial^2 u}{\partial t^2} + I_2 \frac{\partial^2 \phi}{\partial t^2} \\ - c_1 I_4 \left(\frac{\partial^2 \phi}{\partial t^2} + \frac{\partial^3 w}{\partial R \partial t^2} \right) - c_1 I_3 \frac{\partial^2 u}{\partial t^2} - c_1 I_4 \frac{\partial^2 \phi}{\partial t^2} + c_1^2 I_6 \left(\frac{\partial^2 \phi}{\partial t^2} + \frac{\partial^3 w}{\partial R \partial t^2} \right) \end{aligned} \quad (37)$$

The boundary conditions are given as

$$\begin{aligned} \delta u = 0 \quad \text{or} \quad N_{RR} n_R = 0 \\ \delta w = 0 \quad \text{or} \quad \left[c_1 \frac{\partial P_{RR}}{\partial R} - c_1 \frac{P_{\theta\theta}}{R} + Q_{Rz} - 3c_1 S_{Rz} - K_p \frac{\partial w}{\partial R} + N^T \frac{\partial w}{\partial R} \right] n_R = 0 \\ \delta \phi = 0 \quad \text{or} \quad [M_{RR} - c_1 P_{RR}] n_R = 0 \end{aligned} \quad (38)$$

2.5. Governing Equations

The corresponding stress–strain relation of orthotropic composites can be written as

$$\begin{Bmatrix} \sigma_{RR} \\ \sigma_{\theta\theta} \\ \tau_{RZ} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & 0 \\ \bar{Q}_{12} & \bar{Q}_{22} & 0 \\ 0 & 0 & \bar{Q}_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{RR} \\ \varepsilon_{\theta\theta} \\ \gamma_{RZ} \end{Bmatrix} \quad (39)$$

with

$$\begin{aligned} \bar{Q}_{11} &= Q_{11} \cos^4 \theta + 2Q_{12} \sin^2 \theta \cos^2 \theta + Q_{22} \sin^4 \theta \\ \bar{Q}_{12} &= (Q_{11} + Q_{22}) \sin^2 \theta \cos^2 \theta + Q_{12} (\sin^4 \theta + \cos^4 \theta) \\ \bar{Q}_{22} &= Q_{11} \sin^4 \theta + 2Q_{12} \sin^2 \theta \cos^2 \theta + Q_{22} \cos^4 \theta \\ \bar{Q}_{55} &= Q_{55} \cos^2 \theta \end{aligned} \quad (40)$$

θ being the orientation angle and with

$$Q_{11} = \frac{E_{11}}{1 - \nu_{12}\nu_{21}}, Q_{12} = \frac{\nu_{12}E_{22}}{1 - \nu_{12}\nu_{21}}, Q_{22} = \frac{E_{22}}{1 - \nu_{12}\nu_{21}}, Q_{55} = G_{12} \quad (41)$$

By integrating Equation (40) over the thickness and substituting the obtained resultants into Equations (35)–(37), the governing equations of the MHCAP are obtained as

$$\begin{aligned} \delta u : & \left\{ A_{11} \frac{\partial^2 u}{\partial R^2} + B_{11} \frac{\partial^2 \phi}{\partial R^2} - D_{11} c_1 \left(\frac{\partial^2 \phi}{\partial R^2} + \frac{\partial^3 w}{\partial R^3} \right) \right\} + \left\{ \frac{A_{12}}{R} \frac{\partial u}{\partial R} + \frac{B_{12}}{R} \frac{\partial \phi}{\partial R} - D_{12} c_1 \left(\frac{1}{R} \frac{\partial \phi}{\partial R} + \frac{1}{R} \frac{\partial^2 w}{\partial R^2} \right) \right\} \\ & - \left\{ \frac{A_{12}}{R} \frac{\partial u}{\partial R} + \frac{B_{12}}{R} \frac{\partial \phi}{\partial R} - \frac{D_{12} c_1}{R} \left(\frac{\partial \phi}{\partial R} + \frac{\partial^2 w}{\partial R^2} \right) \right\} - \left\{ \frac{A_{22}}{R^2} u + \frac{B_{22}}{R^2} \phi - \frac{D_{22} c_1}{R} \left(\frac{\phi}{R} + \frac{1}{R} \frac{\partial w}{\partial R} \right) \right\} \\ & = I_0 \frac{\partial^2 u}{\partial t^2} + I_1 \frac{\partial^2 \phi}{\partial t^2} - I_3 c_1 \left(\frac{\partial^2 \phi}{\partial t^2} + \frac{\partial^3 w}{\partial R \partial t^2} \right) \end{aligned} \quad (42)$$

$$\begin{aligned} \delta w : & c_1 \left\{ D_{11} \frac{\partial^3 u}{\partial R^3} + E_{11} \frac{\partial^3 \phi}{\partial R^3} - G_{11} c_1 \left(\frac{\partial^3 \phi}{\partial R^3} + \frac{\partial^4 w}{\partial R^4} \right) \right\} \\ & + c_1 \left\{ \frac{D_{12}}{R} \frac{\partial^2 u}{\partial R^2} + \frac{E_{12}}{R} \frac{\partial^2 \phi}{\partial R^2} - G_{12} c_1 \left(\frac{1}{R} \frac{\partial^2 \phi}{\partial R^2} + \frac{1}{R} \frac{\partial^3 w}{\partial R^3} \right) \right\} \\ & - \frac{c_1}{R} \left\{ D_{12} \frac{\partial^2 u}{\partial R^2} + E_{12} \frac{\partial^2 \phi}{\partial R^2} - G_{11} c_1 \left(\frac{\partial^2 \phi}{\partial R^2} + \frac{\partial^3 w}{\partial R^3} \right) \right\} \\ & - \frac{c_1}{R} \left\{ \frac{D_{22}}{R} \frac{\partial u}{\partial R} + \frac{E_{22}}{R} \frac{\partial \phi}{\partial R} - G_{22} c_1 \left(\frac{1}{R} \frac{\partial \phi}{\partial R} + \frac{1}{R} \frac{\partial^2 w}{\partial R^2} \right) \right\} \\ & + (A_{55} - 3C_{55}c_1) \left(\frac{\partial \phi}{\partial R} + \frac{\partial^2 w}{\partial R^2} \right) - 3c_1 (C_{55} - 3E_{55}c_1) \left(\frac{\partial \phi}{\partial R} + \frac{\partial^2 w}{\partial R^2} \right) - K_W w + K_P \frac{\partial^2 w}{\partial R^2} - N^T \frac{\partial^2 w}{\partial R^2} \\ & = c_1 I_3 \frac{\partial^3 u}{\partial R \partial t^2} + c_1 I_4 \frac{\partial^3 \phi}{\partial R \partial t^2} - c_1^2 I_6 \left(\frac{\partial^3 \phi}{\partial R \partial t^2} + \frac{\partial^4 w}{\partial R^2 \partial t^2} \right) + I_0 \frac{\partial^2 w}{\partial t^2} \end{aligned} \quad (43)$$

$$\begin{aligned} \delta \phi : & \left\{ B_{11} \frac{\partial^2 u}{\partial R^2} + C_{11} \frac{\partial^2 \phi}{\partial R^2} - E_{11} c_1 \left(\frac{\partial^2 \phi}{\partial R^2} + \frac{\partial^3 w}{\partial R^3} \right) \right\} + \left\{ \frac{B_{12}}{R} \frac{\partial u}{\partial R} + \frac{C_{12}}{R} \frac{\partial \phi}{\partial R} - \frac{E_{12}}{R} c_1 \left(\frac{\partial \phi}{\partial R} + \frac{\partial^2 w}{\partial R^2} \right) \right\} \\ & - c_1 \left\{ D_{11} \frac{\partial^2 u}{\partial R^2} + E_{11} \frac{\partial^2 \phi}{\partial R^2} - G_{11} c_1 \left(\frac{\partial^2 \phi}{\partial R^2} + \frac{\partial^3 w}{\partial R^3} \right) \right\} - c_1 \left\{ \frac{D_{12}}{R} \frac{\partial u}{\partial R} + \frac{E_{12}}{R} \frac{\partial \phi}{\partial R} - \frac{G_{12}}{R} c_1 \left(\frac{\partial \phi}{\partial R} + \frac{\partial^2 w}{\partial R^2} \right) \right\} \\ & - \frac{1}{R} \left\{ B_{12} \frac{\partial u}{\partial R} + C_{12} \frac{\partial \phi}{\partial R} - E_{12} c_1 \left(\frac{\partial \phi}{\partial R} + \frac{\partial^2 w}{\partial R^2} \right) \right\} - \frac{1}{R} \left\{ B_{22} \frac{u}{R} + C_{22} \frac{\phi}{R} - E_{22} c_1 \left(\frac{\phi}{R} + \frac{1}{R} \frac{\partial w}{\partial R} \right) \right\} \\ & + \frac{c_1}{R} \left\{ D_{12} \frac{\partial u}{\partial R} + E_{12} \frac{\partial \phi}{\partial R} - G_{12} c_1 \left(\frac{\partial \phi}{\partial R} + \frac{\partial^2 w}{\partial R^2} \right) \right\} + \frac{c_1}{R} \left\{ D_{22} \frac{u}{R} + E_{22} \frac{\phi}{R} - G_{22} c_1 \left(\frac{\phi}{R} + \frac{1}{R} \frac{\partial w}{\partial R} \right) \right\} \\ & - (A_{55} - 3C_{55}c_1) \left(\phi + \frac{\partial w}{\partial R} \right) + 3c_1 (C_{55} - 3E_{55}c_1) \left(\phi + \frac{\partial w}{\partial R} \right) = I_1 \frac{\partial^2 u}{\partial t^2} + I_2 \frac{\partial^2 \phi}{\partial t^2} - I_4 c_1 \left(\frac{\partial^2 \phi}{\partial t^2} + \frac{\partial^3 w}{\partial R \partial t^2} \right) \\ & - c_1 I_3 \frac{\partial^2 u}{\partial t^2} - c_1 I_4 \frac{\partial^2 \phi}{\partial t^2} + I_6 c_1^2 \left(\frac{\partial^2 \phi}{\partial t^2} + \frac{\partial^3 w}{\partial R \partial t^2} \right) \end{aligned} \quad (44)$$

with $\{A_{ij}, B_{ij}, C_{ij}, D_{ij}, E_{ij}, F_{ij}, G_{ij}\} = \int_{-\frac{h}{2}}^{\frac{h}{2}} \bar{Q}_{ij} \{1, z, z^2, z^3, z^4, z^5, z^6\} dz$.

3. Solution Procedure

Numerous computational methods have been introduced to solve differential equations, such as FEM [75], Ritz method [76], deep collocation method [77], etc. In this paper, we employed the

differential quadrature method (DQM), which was proposed by Bellman et al. [78,79] in the 1970s. In DQM, the results depend on the numbers of grid points. As in any computational method, choosing the optimal numbers of grid points is of high importance. Shu [80] proposed an efficient approach to determine the weighting coefficients for an infinite number of grid points which is known as generalized differential quadrature method (GDQ). Shu and Richards [81] applied a domain decomposition method which can be implemented in multidomain problems. In GDQ, the approximate r th derivative of $f(x)$ is determined as

$$\left. \frac{\partial^r f(x)}{\partial R^r} \right|_{x=x_p} = \sum_{j=1}^n g_{ij}^{(r)} f(R_j) \quad (45)$$

with n indicating the number of distributed points along the x -axis. The weighting coefficients C_{ij} are acquired from the first-order derivative as

$$\begin{aligned} g_{ij}^{(1)} &= \frac{M(R_i)}{(R_i - R_j)M(R_j)} \quad i, j = 1, 2, \dots, n \quad \text{and} \quad i \neq j \\ g_{ij}^{(1)} &= - \sum_{j=1, j \neq i}^n C_{ij}^{(1)} \quad i = j \end{aligned} \quad (46)$$

with

$$M(R_i) = \prod_{j=1, j \neq i}^n (R_i - R_j) \quad (47)$$

For higher-order derivatives, the weighting coefficients are determined through the following relations

$$\begin{aligned} g_{ij}^{(r)} &= r \left[g_{ij}^{(r-1)} g_{ij}^{(1)} - \frac{g_{ij}^{(r-1)}}{(R_i - R_j)} \right] \quad i, j = 1, 2, \dots, n, i \neq j \quad \text{and} \quad 2 \leq r \leq n-1 \\ g_{ii}^{(r)} &= - \sum_{j=1, j \neq i}^n g_{ij}^{(r)} \quad i, j = 1, 2, \dots, n \quad \text{and} \quad 1 \leq r \leq n-1 \end{aligned} \quad (48)$$

In this study, a non-uniform set of grid points was selected [82]

$$R_j = a + \frac{b-a}{2} \left(1 - \cos \left(\frac{(j-1)}{(N_j-1)} \pi \right) \right) \quad j = 1, 2, 3, \dots, N_j \quad (49)$$

It can easily be shown that, after some algebra, we obtained the discrete system of equations as

$$\left\{ \begin{bmatrix} [M_{dd}] & [M_{db}] \\ [M_{bd}] & [M_{bb}] \end{bmatrix} \omega^2 + \begin{bmatrix} [K_{dd}] & [K_{db}] \\ [K_{bd}] & [K_{bb}] \end{bmatrix} \right\} \begin{Bmatrix} \delta_d \\ \delta_b \end{Bmatrix} = 0 \quad (50)$$

where the subscripts d and b represent the domain and boundary grid points, respectively, while δ refers to the displacement vector. The natural frequency of the MHCAP was normalized

$$\bar{\omega}_n = \omega_n b^2 \sqrt{\frac{\rho_m}{E_m}} \quad (51)$$

4. Numerical Results and Discussion

Subsequently, we present a comprehensive parameter study to quantify the effects of various parameters on the free vibration response of MHCAP. The geometrical and material characteristics of the constituent materials can be found in Table 1 [64].

Table 1. Material properties of the multiscale hybrid nanocomposite annular plate [64].

Carbon fiber	E_{11}^F [Gpa] 233.05	E_{22}^F [Gpa] 23.1	G_{12}^F [Gpa] 8.96	ρ^F [kg/m ³] 1750	ν^F 0.2	α_{11}^F [×10 ⁻⁶ /k] −0.54	α_{22}^F [×10 ⁻⁶ /k] 10.08		
Epoxy Matrix	E^m [Gpa] 3.51	ν^m 0.34		ρ^m [kg/m ³] 1200	α^m [×10 ⁻⁶ /k] 45				
Carbon nanotube	E_{11}^F [Tpa] 5.6466	$E_{22}^{CNT} = E_{33}^{CNT}$ [Tpa] 7.0800	$G_{12}^{CNT} = G_{13}^{CNT}$ [Tpa] 1.9445	ν_{12}^{CNT} 0.175	ρ^{CNT} [kg/m ³] 1350	α^{CNT} [×10 ⁻⁶ /k] 3.4584	l^{CNT} [μm] 25	d^{CNT} [nm] 1.4	t^{CNT} [nm] 0.34

4.1. Verification

In order to verify the present approach, the nondimensional natural frequency of the annular plate obtained in this study was compared with those reported in [83] for different axisymmetric vibration modes, thickness to outer radius ratios h/b , inner radius to outer radius ratio a/b , and three boundary conditions (see Tables 2–4). The discrepancy between our results and the results of [83] came from the difference in the shear deformation theory [83] and HSDT here. Nonetheless, the discrepancy between the results decreased with increasing modes, and h/b and a/b ratios.

Table 2. Comparison of nondimensional natural frequency of the annular plate for different axisymmetric vibration mode number, inner radius to outer radius ratio, and thickness to outer radius ratio for simply-simply boundary condition.

a/b		h/b	Axisymmetric Vibration Mode Number			
			1	2	3	4
0.1	Ref. [83]	0.001	14.485	51.781	112.99	198.44
	Present	0.001	13.624	50.302	111.86	198.61
	Ref. [83]	0.050	14.324	50.409	107.25	182.55
	Present	0.050	13.528	49.109	106.09	182.08
	Ref. [83]	0.100	13.874	46.947	94.670	151.91
	Present	0.100	13.254	46.061	93.794	151.36
	Ref. [83]	0.150	13.218	42.630	81.519	124.92
	Present	0.150	12.838	42.182	81.036	124.24
	Ref. [83]	0.200	12.450	38.337	70.224	104.20
	Present	0.200	12.326	38.243	70.075	104.04
0.2	Ref. [83]	0.001	16.780	63.370	140.60	248.62
	Present	0.001	16.652	63.003	140.90	250.68
	Ref. [83]	0.050	16.618	61.512	132.21	224.50
	Present	0.050	16.512	61.164	131.99	225.32
	Ref. [83]	0.100	16.164	56.912	114.70	182.50
	Present	0.100	16.118	56.621	114.14	181.93
	Ref. [83]	0.150	15.491	51.317	97.337	147.50
	Present	0.150	15.529	51.108	96.780	146.33
	Ref. [83]	0.200	14.688	45.866	82.963	121.66
	Present	0.200	14.819	45.751	82.567	121.21

Table 3. Comparison of nondimensional natural frequency of the annular plate for different axisymmetric vibration mode number, inner radius to outer radius ratio, and thickness to outer radius ratio for clamp-simply boundary condition.

$\frac{a}{b}$		$\frac{h}{b}$	Axisymmetric Vibration Mode Number			
			1	2	3	4
0.1	Ref. [83]	0.001	22.701	65.538	132.89	224.41
	Present	0.001	20.852	63.734	131.50	223.86
	Ref. [83]	0.050	22.294	63.052	123.85	201.54
	Present	0.050	20.490	61.122	121.73	198.97
	Ref. [83]	0.100	21.200	56.912	105.46	161.99
	Present	0.100	19.512	54.993	102.79	158.00
	Ref. [83]	0.150	19.703	49.879	87.884	129.69
	Present	0.150	18.166	48.119	85.479	126.15
	Ref. [83]	0.200	18.073	43.431	73.899	106.37
	Present	0.200	16.695	41.972	72.128	104.17
0.2	Ref. [83]	0.001	26.737	80.722	165.63	281.34
	Present	0.001	25.772	79.931	165.66	282.52
	Ref. [83]	0.050	26.242	77.053	152.26	247.24
	Present	0.050	25.227	75.901	150.67	244.70
	Ref. [83]	0.100	24.916	68.635	126.72	193.10
	Present	0.100	23.785	66.912	123.76	188.24
	Ref. [83]	0.150	23.113	59.407	103.85	151.97
	Present	0.150	21.869	57.463	100.99	147.52
	Ref. [83]	0.200	21.162	51.247	86.427	123.40
	Present	0.200	19.853	49.456	84.276	120.89

Table 4. Comparison of nondimensional natural frequency of the annular plate for different axisymmetric vibration mode number, inner radius to outer radius ratio, and thickness to outer radius ratio for clamp-clamp boundary condition.

$\frac{a}{b}$		$\frac{h}{b}$	Axisymmetric Vibration Mode Number			
			1	2	3	4
0.1	Ref. [83]	0.001	27.280	75.364	148.21	245.47
	Present	0.001	28.514	77.363	151.05	249.05
	Ref. [83]	0.050	26.534	71.228	135.24	215.08
	Present	0.050	27.679	72.766	136.36	214.46
	Ref. [83]	0.100	24.629	62.140	111.12	167.16
	Present	0.100	25.558	62.834	110.39	163.41
	Ref. [83]	0.150	22.230	52.762	90.286	131.35
	Present	0.150	22.909	52.835	88.962	127.51
	Ref. [83]	0.200	19.843	44.913	74.860	106.81
	Present	0.200	20.294	44.683	73.628	103.75
0.2	Ref. [83]	0.001	34.609	95.738	188.14	311.40
	Present	0.001	35.712	97.395	190.61	314.62
	Ref. [83]	0.050	33.533	89.550	168.60	265.78
	Present	0.050	34.426	90.285	168.15	262.39
	Ref. [83]	0.100	30.841	76.560	134.71	200.02
	Present	0.100	31.274	75.895	131.87	193.37
	Ref. [83]	0.150	27.545	63.827	107.32	154.20
	Present	0.150	27.534	62.466	104.32	152.53
	Ref. [83]	0.200	24.348	53.574	87.870	123.93
	Present	0.200	24.022	52.122	85.531	121.89

4.2. Vibration Study

The vibration studies can be divided into two parts. First, we studied the influence of different parameters (such as the geometric ratios of the plate, distribution patterns of CNT, volume fraction of carbon fibers, weight fraction of CNT, temperature gradient, etc.) on the natural frequencies of the MHCAP. Their effect on the vibration performance of the system was important for the design of such structures. Then, the sensitivity of the vibration response to nonlinear temperature gradients are presented and discussed.

4.2.1. Parametric Vibration Study

Table 5 shows the effects of the distribution patterns and CNT weight fraction on the nondimensional natural frequency of MHCAP. The plate based on FG-A (shown in Figure 3c) exhibited the best vibration response among the CNT patterns. Increasing the volume fraction of carbon fibers also yielded higher frequencies.

Table 5. Effect of CNT pattern on the nondimensional natural frequency of the simply-simply multiscale hybrid composite annular plate (MHCAP) with outer radius to inner radius ratio ($b/a = 4$), thickness ratio ($h/b = 0.2$), temperature at bottom surface ($T_i = 273$ [K]), temperature at top surface ($T_o = 300$ [K]), orientation angle of fibers ($\theta = \pi/4$), Winkler coefficient ($K_w = 0$ [N/m³]), Pasternak coefficient ($K_p = 0$ [N/m]), axial load ($P = 0$ [N]), uniform temperature rise (UTR), and $\beta = 1$ for different weight fractions of CNT and volume fractions of carbon fiber.

V_F	CNT Distribution	W_{CNT}				
		0.02	0.04	0.06	0.08	0.1
0.2	FG-X	2.7059	2.9976	3.2255	3.4195	3.5921
	FG-V	2.7869	3.0596	3.2498	3.4018	3.5325
	FG-A	2.8217	3.1038	3.3006	3.4589	3.5958
	FG-UD	2.8089	3.0889	3.2854	3.4439	3.5812
0.3	FG-X	2.9599	3.2604	3.4808	3.6596	3.8132
	FG-V	3.0806	3.3817	3.5783	3.7265	3.8479
	FG-A	3.1203	3.4291	3.6294	3.7809	3.9058
	FG-UD	3.1070	3.4156	3.6170	3.7695	3.8950
0.4	FG-X	3.1711	3.4722	3.6836	3.8486	3.9859
	FG-V	3.3139	3.6278	3.8245	3.9670	4.0796
	FG-A	3.3563	3.6766	3.8753	4.0191	4.1332
	FG-UD	3.3437	3.6662	3.8675	4.0132	4.1283

The vibration results obtained by GDQM are diagramed in Figure 5 for various numbers of grid points and different boundary conditions. According to the figures, choosing $N = 10$ as the number of grid points provided adequate accuracy to the results.

Figure 6 depicts the effects of the outer radius to inner radius ratio on the vibration performance of the annular plate for different values of Winkler–Pasternak elastic substrate coefficients. Apparently, the Winkler coefficient played a considerable role in the vibration response of the plate with increasing outer radius to inner radius ratio (b/a). Moreover, the minimum value of the natural frequency of the plates resting on Winkler substrate with higher stiffness occurred at lower b/a . For example, the minimum frequency of the plate with Winkler coefficient ($K_w = 300$ (MN/m³)) was associated with $b/a = 2.1$ while the corresponding minimum frequency of $K_w = 100$ (MN/m³) occurred at $b/a = 2.3$. Furthermore, K_w barely affected the vibration behavior of the plate when $b/a < 1.6$. Figure 6b shows the minimum frequency of the system occurred at $b/a = 2.3$ and then increased uniformly for all the values of the Pasternak coefficient. The frequencies for various Pasternak coefficient (K_p) barely changed for different ratios b/a .

Figure 5. Convergence of nondimensional natural frequency of MHCAP with $b/a = 4$, $h/b = 0.1$, $T_i = 273$ [K], $T_o = 300$ [K], weight fraction of CNT ($W_{CNT} = 0.02$), volume fraction of carbon fiber ($V_F = 0.2$), $\theta = \pi/4$, UTR, $\beta = 1$, FG-A, $K_w = 0$ [M N/m³] and $K_p = 0$ [M N/m] for various boundary conditions. (a) Simply-Simply; (b) Clamp-Clamp; (c) Clamp-Simply.

Figure 6. The effect of Winkler–Pasternak coefficient on the nondimensional natural frequency of MHCAP with clamp-clamp boundary conditions for different outer radius to inner radius ratios and $h/b = 0.1$, UTR, $T_i = 273$ [K], $T_o = 300$ [K], $\theta = \pi/4$, $W_{CNT} = 0.02$, $V_F = 0.2$, FG-A, $\beta = 1$, (a) $K_p = 0$, and (b) $K_w = 0$. (a) Winkler coefficient; (b) Pasternak coefficient.

The influence of the temperature rise on the vibration response of the MHCAP is depicted in Figure 7. The sinusoidal temperature rise (STR) exhibited the highest natural frequency of the plate, while the uniform temperature rise (UTR) led to the weakest vibration response. The corresponding frequencies of UTR was close to those of STR. The temperature rise became more influential on the vibration behavior of plates for higher b/a .

Figure 7. The effect of temperature rising pattern on the nondimensional natural frequency of MHCAP with clamp-clamp boundary conditions for different outer radius to inner radius ratios and $h/b = 0.1$, $T_i = 273$ [K], $T_o = 300$ [K], $\theta = \pi/4$, $W_{CNT} = 0.02$, $V_F = 0.2$, $\beta = 1$, FG-A, $K_p = 10$ [MN/m], and $K_w = 100$ [MN/m³].

Figure 8 illustrates the effects of the applied temperature on the upper surface on the vibration performance of the annular plate for different outer radius to inner radius ratios. Increasing the temperature gradient resulted in a weaker vibration performance as a consequence of a decreasing structural stiffness. Choosing a higher outer to inner radius caused a minimum natural frequency at higher temperature b/a . Accordingly, we concluded that plates with higher b/a ratios resulted in a better vibration response.

Figure 8. The effect of applied temperature of the top surface on the nondimensional natural frequency of MHCAP with clamp-clamp boundary conditions for different outer radius ratios of inner radius and $h/b = 0.1$, $T_i = 273$ [K], UTR, $\theta = \pi/4$, $W_{CNT} = 0.02$, $V_F = 0.2$, FG-A, $\beta = 1$, $K_p = 10$ [MN/m], and $K_w = 100$ [MN/m³].

The influence of the volume fraction V_F and orientation angle θ of carbon fibers on the free vibration is plotted in Figure 9. The vibration response of the plate was symmetric about $\theta = \frac{\pi}{2}$. Therefore, we analyzed the vibration performance of the plate for $0 \leq \theta \leq \frac{\pi}{2}$. Increasing V_F improved the vibration characteristics of the plate when $0 \leq \theta \leq 0.425\pi$ and increasing θ decreased the natural frequency. The effect of the volume fraction of carbon fibers on the vibration behavior of the plate became less pronounced with increasing orientation angle. The natural frequency of the plate tended to constant values regardless of the volume fraction of carbon fibers at $\theta = 0.425\pi$. For plates with $\theta > 0.425\pi$, increasing V_F had a negative effect on the vibration response of the plate.

Figure 9. The effect of various volume fractions of carbon fiber on the nondimensional natural frequency of MHCAP with clamp-clamp boundary conditions for different orientation angle of carbon fibers and $b/a = 4$, $h/b = 0.1$, sinusoidal temperature rise (STR), $T_i = 273$ [K], $T_o = 300$ [K], $W_{CNT} = 0.02$, FG-A, $\beta = 1$, $K_p = 10$ [MN/m], and $K_w = 100$ [MN/m³].

The effects of the CNTs' weight fraction W_{CNT} on the vibration behavior of the annular plate with respect to different orientation angles can be found in Figure 10. An increasing W_{CNT} improved the vibration response of the plate and was more pronounced for lower values of θ . The CNT weight fraction barely influenced the vibration performance of MHCAP when carbon fibers were arranged in an orientation of $\frac{\pi}{2}$.

Figure 10. The effect of various weight fraction of CNTs on the nondimensional natural frequency of MHCAP with clamp-clamp boundary conditions for different orientation angles of carbon fibers and $b/a = 4$, $h/b = 0.1$, STR, $T_i = 273$ [K], $T_o = 300$ [K], $V_F = 0.2$, FG-A, $\beta = 1$, $K_p = 10$ [MN/m], and $K_w = 100$ [MN/m³].

Figure 11 shows the effect of the outer radius to inner radius ratio b/a on the natural frequency of the plate with and without elastic foundation and different orientation angles of carbon fibers. The effect of β involved in Equations (29) and (30) on the vibration performance of the MHCAP is analyzed in Figure 12 for different values of corresponding temperature of the top surface. Choosing $\beta > 1$ in Equation (29) yielded a nonlinear temperature gradient, while $\beta = 1$ in Equation (29) yielded a linear temperature gradient. Rising β increased the natural frequency. However, the gradient of the ascending trend of the diagrams decreased gradually until it tended near to zero at $\beta > 9.5$. The temperature played a more important role on the vibration performance of the plate for lower β .

Figure 11. The effect of different outer radius to inner radius ratios on the nondimensional natural frequency of MHCAP with clamp-clamp boundary conditions for different orientation angles of carbon fibers and $h/b = 0.1$, STR, $T_i = 273$ [K], $T_o = 273$ [K], $W_{CNT} = 0.02$, $V_F = 0.2$, FG-A, $\beta = 1$, $K_p = 10$ [MN/m], and $K_w = 100$ [MN/m³]. (a) without elastic medium; (b) rested on Winkler medium; (c) rested on Winkler-Pasternak medium.

Figure 12. The effect of applied temperature of the top surface on the nondimensional natural frequency of MHCAP with clamp-clamp boundary conditions for different temperature power β and $h/b = 0.1$, $b = 4 \times a$, $T_i = 273$ [K], $W_{CNT} = 0.02$, $V_F = 0.2$, FG-A, $K_p = 10$ [MN/m], and $K_w = 100$ [MN/m³]. (a) STR; (b) PTR.

4.2.2. Sensitivity Analysis of Vibration

In this section, we focus on the sensitivity of the natural frequency with respect to, the nonlinear temperature gradient. The sensitivity of the i th natural frequency of the MHCAP is defined as

$$S_i = 100 \left(\frac{f_i - f_i^0}{f_i^0} \right) \quad i = 1, 2, \dots \quad (52)$$

with f_i and f_i^0 indicating the corresponding natural frequency of the i th mode number with and without including the effect of temperature gradients, respectively. The sensitivity of the fundamental natural

frequency as a function of the temperature gradient and the β parameter (involved in the STR and PTR) is presented in Figure 13. The absolute values of the frequency sensitivity increased with increasing temperature gradient and β parameter. Also, the absolute value of the sensitivity for PTR was higher than STR for a fixed temperature gradient and β parameter.

Figure 13. Sensitivity of the natural frequency of MHCAP with clamp-clamp boundary conditions to nonlinear temperature gradient for different values of β parameter in STR and power form temperature rise (PTR) and $h/b = 0.1$, $\theta = \pi/4$, $W_{CNT} = 0.02$, $V_F = 0.2$, FG-A, $K_p = 10$ [MN/m], and $K_w = 100$ [MN/m³].

5. Conclusions

We presented a free vibration analysis of MHCAP consisting of CNT/CF/matrix resting on a two-parameter substrate subjected to temperature gradients. The model was based on HSDT, and the Halpin–Tsai model was combined with a micromechanical scheme to extract the effective mechanical properties of MHCAP. The governing equations were obtained through Hamilton’s principle and solved with the GDQM. After verification, we carried out an intensive parameter study which revealed the following conclusions:

- Annular plates with lower thickness to outer radius ratio presented better vibration performance.
- Reinforcing MHCAP with FG-A as the distribution pattern of CNTs exhibited the best vibration response in comparison with any other patterns tested.
- In the case of Pasternak-type elastic substrate, the lowest natural frequency was associated with $b/a = 2.3$ as the outer radius to inner radius ratio of the MHCAP.
- The effect of the Winkler coefficient on the vibration performance of the plate was more significant when the outer radius to inner radius ratio was higher.
- Applying a sinusoidal temperature rise results to a highest natural compared with associated results of other patterns of temperature rise.
- When the outer radius to inner radius ratio of the MHCAP was larger, the minimum frequency occurred at higher, corresponding temperature of the top surface.
- Unlike compression load, applying higher load in extensional direction led to higher natural frequencies.

Increasing the volume fraction of carbon fiber V_F improved the vibration characteristics of the plate when $0 \leq \theta \leq 0.425\pi$, whereas increasing V_F had a negative effect on the vibration response of the MHCAP in the case of $0.425\pi \leq \theta \leq 0.5\pi$.

Higher power index in the relations of STR and PTR led to a better vibration study.

The sensitivity of vibration response was higher in the case of higher nonlinear temperature gradients.

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