

Article



A New Transmission Theory of "Global Dynamic Wrap Angle" for Friction Hoist Combining Suspended and Wrapped Wire Rope

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Received: 13 January 2020; Accepted: 12 February 2020; Published: 14 February 2020



Abstract: A new transmission theory of "global dynamic wrap angle" for friction hoist is proposed. The theory is based on a mine hoist simulation model which combines the suspended rope with the wrapped rope. Rope dynamics in a suspended section are verified by the field experiment results. The theory holds that the mechanical state of wire rope is dynamic through the whole wrap angle, including deformation, contact and friction. When the rope enters the wrap angle, it provides positive friction and changes direction at a certain boundary point. The demarcation of the boundary depends on the rope load on both sides of the friction pulley. The theory is suitable for accurately analyzing the kinetics of high-speed and heavy-load friction hoisting.

Keywords: Hoisting Rope; Rope Dynamics; dynamic wrap angle; Friction Transmission

1. Introduction

Multi-rope friction hoists are widely used in deep and ultra-deep coal mine hoisting for their remarkable advantages, such as their large capacity, high lifting height, high safety factor, low power consumption, small machine size and cost-effectiveness. The friction hoist (Figure 1) relies on the friction force between the rope and the pulley to transport goods or staff. With the development of modern large-scale friction hoisting, the problems of dynamic characteristics, which are not obvious in low-speed and light-load conditions, are gradually reflected, for example, in rope vibration and slippage [1,2]. During dynamic operations, especially in severe conditions such as acceleration and emergency braking, the long-distance suspended rope will produce large elastic vibrations and tension impacts under the influence of the terminal heavy container [3–5]. The vibration impact produces not only longitudinal tension, but also horizontal and torsional vibration due to the transmission of the shock wave [6,7]. Especially when the depth and speed of the mine are continuously increasing, the time-varying characteristics of the rope length and the swirl of the wellbore gas flow [8] at the bottom of the container caused by the rapid speed change will intensify the vibration [3,9]. When the vibration is transmitted upward to the friction pulley along the rope, it leads to abnormal contact, dynamic slip and even disengagement [10] between the rope and pulley, which will seriously reduce the friction stability. Therefore, it is of great importance to combine the suspended ropes on both sides of the pulley with the wrapped rope on the pulley in order to accurately obtain their dynamic behaviors. Especially for the rope on the pulley, its effective friction contact with the pulley is an important guarantee for the working efficiency and anti-skid safety of the hoist. Many scholars have conducted abundant research on the dynamics of suspension rope hoisting systems. Kaczmarczyk [11,12] established the classical distributed parameter dynamic model of the kilometre hoisting rope based

on Hamilton's principle and described the transverse-longitudinal coupling dynamic response for the first time. On this basis, Zhu [13] calculated the dynamic responses of a one-dimensional system with a spatial discretization method. The method was applied to study the longitudinal, transverse, and coupled vibrations in moving elevator cable-car systems [14]. Meanwhile, a rope vibration parameter analysis based on Hamilton's principle has also been widely conducted [15–17].



Figure 1. Schematic diagram of a large cage hoisting system in an auxiliary shaft of a coal mine.

The above studies are mainly focused on the dynamic characteristics of the rope itself. However, in the present numerical analysis method based on Hamilton's principle, the initial mechanical boundary conditions at the point where the rope meets or separates from the pulley are either set to zero or applied with a constant-amplitude sinusoidal excitation [3,18], which causes the "mechanical link" between the suspended rope and the wrapped rope to be truncated. Therefore, these numerical methods do not establish the relationship between the suspended rope dynamics and the friction contact in the wrap angle, and are thus unable of determining the rope vibration entering the wrap angle during operation. Currently, either the classical Euler formula or the deformation formula are used in the design or calculation of friction transmission. The theoretical calculation theory of friction transmission of modern large-scale equipment seems inaccurate to some extent. At the beginning, the formula was used to calculate the belt transmission, while the calculation method of wire rope friction transmission is a direct reference to the Euler formula. It is considered that the contact process between the belt and pulley can be divided into two stages. First, the viscous stage is when the belt and pulley are relatively static, which is also called the static angle. Next, the creep stage is when the belt and pulley slip relative to one another, which is also called the dynamic angle. Lubarda [19] calculated the changes in the friction and pressure in the contact area between the belt and pulley in the dynamic angle, and noted the non-orthogonal relationship between the friction and positive pressure. However, he did not consider the mechanical input outside the wrap angle. Wang [20,21] explored the effects of the payload, speed, and acceleration on the dynamic friction transmission and creeping properties. However, it is limited to the longitudinal tension characteristics without considering the transverse and longitudinal vibration.

Consequently, the traditional theoretical model of friction transmission based on static and dynamic angle theory is not applicable when the load, depth or speed of the hoist increase continuously. The objective of the present study is to establish a new transmission theory for friction hoisting system combining suspended and wrapped wire rope. The research results lay a solid foundation for studying the dynamic characteristics of rope transmitting behaviors with high-speed and heavy-load working conditions in a modern friction hoisting system.

2. Hoisting Model

2.1. Hoist Equipment Parameters

Figure 1 shows a large cage hoisting system in the auxiliary shaft. Table 1 lists the main parameters of the large cage hoisting system in an auxiliary shaft of coal mine. Table 2 lists the speeds and times of each stage of the lifting process.

	Item		Unit	Friction Pulley	Guide Pulley
	Туре			JKM-5 × 6	
	Lifting height		m		425
	Pulleys Spacing m	Vertical:1	Vertical:13/horizontal:3.35		
Hoist	Diameter		m	5	5
	Number			6	6
	Friction coefficient			0.25	0.25
	wrap angle		0	187.42	
	Item		Unit	Hoisting rope	Tail rope
	Туре			$54 \times 36WS + FC$	$216 \times 34 \times 4 \times 19 + FC$
	Type Diameter/Width × Thickness mm	54	216×34		
Rope	Number			6	3
1	Density		kg/mm ³	$4.72 imes 10^{-6}$	23
	Total length		m	523.08	481.5
	Item		Unit		
	Cage mass			0.65×10^5	
	Balance hammer ma	SS		0.05×10^{5} 0.95×10^{5}	
Load	MAX Test load(total) MID MIN		kg	1.2×10^{5}	
				0.95×10^5 (mainly discussed)
				0.7×10^{5}	

Table 1. Parameters of large cage hoisting system.

Table 2. Operating parameters (Acceleration and Constant Speed stage).

Stage		Acceleration (m/s ²)	Speed (m/s)	Time (s)
	Varying	0~0.6	0.51	1.71
	Constant	0.6	9.43	14.87
Main acceleration	Varying	0.6~0	9.95	1.71
		Total		18.29
Constant speed	Constant	0	9.95	24.45

2.2. Modelling Process

According to the relevant parameters of the large cage hoisting system above, the global dynamic simulation model of the actual hoisting system was established using multi-body dynamics simulation software "Adams". The characteristics of the model state that the dynamics of the wire rope on the lifting side and the lowering side, as well as the coupling effect of the rope with the friction pulley and the guide pulley, are unified as a whole. The mine system is a multi-rope friction hoisting system of the shaft tower. The global simulation model is then simplified as follows:

The pulleys are simplified as uniform materials, and the groove shape is ideal and regular;

The weight of the tail rope is simplified as a time-varying force corresponding to the lifting height, which is applied to both the cage and the balance hammer;

The actual lifting system is supported by six steel ropes. In the model, only a single steel rope is established, and the load is calculated according to 1/6 of the original load;

On the basis of the above simplification, the friction pulley, guide pulley, wire rope, cage and balance hammer are established. The model is shown in Figure 2. The parameters of the model are consistent with those of the hoist equipment mentioned above. According to the relevant provisions of the China coal industry standard MT 234-1991 "3T Tramcar, Multi-rope tank cage for vertical shaft" and MT 235-2011 "Vertical multi-rope cage balance hammer", the 3D models of the cage and balance hammer are established.



Figure 2. Dynamic model of the friction hoist. (a) Cage; (b) Balance hammer; (c) Pulleys.

2.2.1. Contact Definition

As the transmission bridge of the entire hoisting system, the accuracy of the rope modelling is the key of the entire simulation model, especially for unifying the construction of the rope in the suspension section and the rope in the wrap angle section. Therefore, it is necessary to dynamically define the friction contact parameters between the rope and the pulley surface after entering the friction pulley. The rope is then discretized into spheres. Compared with the conventional cylindrical model, the spherical model has a higher accuracy, which can accurately reflect the geometric shape of the rope in contact with the pulleys, and can realize the lateral displacement limitation of the rope groove. The mass and inertia effects are also included in the contact area. The contact force between the discrete rope sphere and the surface of the pulleys is expressed by a function F_{im} . More details about the function can be found in [1].

The rope is located in the groove of the pulley. The sizes of the rope grooves are shown in Figure 3. The contact geometry of the groove and the discrete rope sphere is spherical to cylindrical, i.e., the inner cylindrical surface of the groove contacts the outer spherical surface of the rope. Hertzian contact theory is then applied to calculate both the contact force and friction force, and then the resultant force is obtained. The definition of friction is given in [1], and the dimensional parameters include the Depth, Radius, Angle and contact parameters k, e, and c_{max} . Their specific values are shown in Table 3.



Figure 3. Parameters of the pulley and rope grooves.

2.2.2. Discrete Element Mechanics Beam

Each adjacent pair of discrete spheres are connected by a mechanical beam element. The non-linear Euler-Bernoulli beam [22], which is more suitable for mine hoisting with a large axial load, is applied

in the Adams simulation to obtain the exact solution in this paper. The mechanical beam includes six axial forces and six torsional forces. A detailed description of the model is given in [2]. It should be noted that the stiffness matrix of the model is modified by three correction factors. The stiffness matrix K_{ij} is expressed as:

$$K_{11} = EA/L \times Rkx$$

$$K_{22} = 12E \cdot Izz / [L^{3}(1 + P_{y})]$$

$$K_{26} = -6E \cdot Izz / [L^{2}(1 + P_{y})]$$

$$K_{33} = 12E \cdot Iyy / [L^{3}(1 + P_{z})] \times Rkb$$

$$K_{35} = 6E \cdot Izz / [L^{2}(1 + P_{z})] \times Rkb$$

$$K_{44} = G \cdot Ixx / L \times Rkt$$

$$K_{55} = (4 + P_{z})E \cdot Iyy / [L(1 + P_{z})] \times Rkb$$

$$K_{66} = (4 + P_{y})E \cdot Izz / [L(1 + P_{y})] \times Rkb$$
(1)

The purpose of the *Rkx*, *Rkb*, and *Rkt* inputs is to represent an orthotropic material property. These are multipliers on the stiffness in the axial (*Rkx*), bending (*Rkb*) and twisting (*Rkt*) directions. Cables commonly bend much more easily than they stretch and twist. Therefore, the following is set: Rkx = 1.0, Rkb << 1.0 and Rkt < 1.0. The specific values of each parameter are shown in Table 3. The notes and definitions of the other parameters in the above formula are given in [2]. The final model of the friction hoisting system and the contact section of the friction pulley are shown in Figure 4.



Figure 4. Model of the contact section between the rope and pulley.

2.2.3. Parameters

The parameters of the friction pulley, guide pulley and wire rope in the global hoisting model are shown in Table 3.

	Item	Unit	Value
	Diameter	m	5
Pulleys	Centre distance	m	Vertical 13, horizontal
			3.35
	Width	mm	600
	Depth	mm	50
Rope groove	Radius	mm	27
	Angle	0	20
Comtoot	Friction coefficient		0.25
Contact	Hertz $k/e/c_{max}$		10,000/2/0.1
	Diameter	mm	54
	Density	Kg/mm ³	4.72×10^{-6}
Rope	Elastic modulus	MPa	1.15×10^{5}
	Tensile(<i>Rkx</i>)/Bending(<i>Rkb</i>)/Torsional (<i>Rkt</i>)stiffness coefficient		1.0/0.001/0.01
	Damping		0.01
	Discrete number		1701

Table 3	Parameters of	the global	hoisting	model
Table 5.	1 arameters or	the global	noisting	mouel.

3. Results

3.1. Verification Test

3.1.1. Test Scheme

The vibration of the steel wire rope on the top of cage is tested by a portable multi-channel vibration measuring instrument system. The test location and the definition of vibration direction in this paper are described in Figure 5, and the field test process is shown in Figure 6.



Figure 5. Vibration testing scheme of the wire rope above the cage.



Figure 6. Field test photos.

3.1.2. Test Result

Figure 7 shows the transverse rope vibration waveform (as shown in Figure 5, cage short side direction, measuring point 1, 9.95 m/s speed lifting). The main characteristic of each measuring point is unstable vibration similar to spindle amplitude during the acceleration process. The unstable vibration of spindle type lasts about 3 seconds, and then the rope operates smoothly. The amplitudes at points 1 and 2 are greater than those at points 3 and 4 because points 3 and 4 are closer to the container. The positions of point 1 and point 2 are relatively far from the binding connector, and the flexible characteristics of the wire rope are more easily stimulated.

The basic vibration level of the rope is analyzed to determine the vibration magnitude of the hoist system. Figure 8 illustrates the effective vibration velocity (maximum / $\sqrt{2}$) at each measuring point of the rope in different directions. The influence of the time-varying parameter of the lifting speed on the vibration stability of the system is reflected. The increase in speed increases the instability of the system, resulting in enhanced rope vibration. The vibration in the X direction (long side direction

of the cage) is the smallest, and that of the Z direction (short side direction of the cage) is the largest. Position difference of vibration intensity is discussed in the 4th section of the paper.



Figure 7. Local nonstationary rope vibration diagram for each measuring point.



Figure 8. Effective vibration velocity of the rope in different directions. (**a**) Long side direction, axis X; (**b**) Vertical direction, axis Y; (**c**) Short side direction, axis Z.

3.2. Suspended Rope Dynamics

3.2.1. Vibration

Figure 9 shows the simulation and test signals of the transverse (cage short side) vibration of the 5th rope element (0.5m above the cage). The simulation results of the first 14 s are intercepted. The simulation results are in good agreement with the field test. Similar dynamic nonstationary phenomena occur in the acceleration stage, i.e., the "spindle" vibration. The characteristics of both can be divided into two stages. In the period of unstable vibration, the amplitude of the first third increases, while that of the second third decreases. They are close in amplitude, extremum and duration. Spectrum analysis shows that the frequencies of each signal are close to 16 Hz. The high similarity indicates that the simulation model achieves high accuracy and reliability.



Figure 9. Test and simulation of transverse rope vibration acceleration at 0.5 m above the cage.

Figure 10 shows the vibration acceleration in three directions of the rope at 0.5 m above the cage and hammer. The vibration intensity in the X direction is low, and only a few sudden peaks appear. The vibration of the Y direction is ladder-shaped under the influence of lifting acceleration. Multiple nonstationary vibrations appear in the Z direction, and the strength is the highest among the three directions. The Y-direction vibration amplitude above the cage increases in the constant speed stage due to the continuous shortening of the rope length (lifting movement). In addition, on the side of the hammer, this phenomenon is exactly the opposite.



Figure 10. Vibration acceleration in three directions of the rope at 0.5 m. (**a**) Above the cage; (**b**) above the hammer.

3.2.2. Tension

The rope dynamics at the special locations are selected to reveal the differences caused by the rope length and position. Figure 11 shows the dynamic rope tension at the anchors and midpoints. The relative tensions at each midpoint are similar to those in [3]. As seen from Figure 11a, the overall rope tensions of the lifting side increase with the lifting process. The increases occur because the tension reflects the load of the component below the point, and the tail rope beneath the tension acquisition point continues to lengthen as the lifting proceeds.

In the acceleration stage of Figure 11b,c, there is a significant abrupt amplitude that corresponds to the "spindle" rope vibration in the Z direction and the amplitude intensification in the Y direction in Figure 10, i.e., dynamic nonstationary vibration of the rope occurs on both sides of the friction pulley at the same time. This shows that the "mechanical transfer" effect in the wrap angle cannot be neglected. Compared with the traditional independent modelling method of single-side wire rope dynamics [10,13,14], the global dynamic model considering dynamic friction transmission within the wrap angle has higher accuracy and reliability. In the single-side rope model, the starting point of modeling is the boundary condition, and the displacement and velocity are both 0. However, under

actual working conditions, this point is not only affected by the friction contact of the pulley, but also by the lifting effect of the rope in the wrap angle. All of these are considered in the model of this paper.



Figure 11. Dynamic rope tension at the different positions. (**a**) Tension at anchor and midpoint of lifting side rope; (**b**) Tension at anchor and midpoint of lowering side rope; (**c**) Tension at midpoint of the string rope.

3.3. Wrapped Rope Dynamics

3.3.1. Rope Dynamics in the Wrapping Process

It is necessary to extract some discrete elements of the model that can bypass the pulley to obtain the wrapping dynamics of the rope. The discrete starting point of the rope model is located at the anchor of the hammer, with the serial number of the 1st element, and then the number increases until the anchor of the cage, i.e., the final 1701th element. We extract some rope elements at the whole meter to facilitate the intuitive understanding of its position. Figure 12 shows the vibrations of the 702th and 369th rope elements with initial positions 300 m and 400 m above the cage, respectively. The main vibrations in the X direction are still the sudden peaks at individual positions. The distinct demarcation characteristics in the Z and Y directions indicate that the rope is bypassing the friction pulley. The rope vibration in the Z direction near the friction pulley is the most severe. Even unstable "spindle" vibration appeared at 300 m above the cage. The rope at both positions has significant nonstationary vibration between 5 and 15 s. However, the rope of 400 m is close enough to the friction pulley, which causes the nonstationary vibration to be cross-linked with the amplitude intensification caused by approaching the pulley. The phenomenon reveals the reason that vibration intensification and abnormal noise appear in many friction hoisting system containers [13,14] at the end of the lifting stage.



Figure 12. Rope vibration in three directions in the process of wrapping the friction pulley. (a) X-axis—short side direction of the cage; (b) Y axis—vertical direction of the cage; (c) Z-axis—long side direction of the cage.

The vibration in the Y direction is similar to that in the Z direction. Macroscopic trapezoidal acceleration is not obvious, but instead is the reverse peak with increasing amplitude. When the rope runs out of the friction pulley, i.e., the string rope, the Z and Y direction rope vibrations present typical "beat-frequencies" [4,5]. This is due to the motion of the string being stable and its tension fluctuating steadily in the constant speed stage (Figure 11). The vibration frequencies of the two sides are similar, but there are small differences due to the length and the load of the rope. As a result, the string as the transition section of the two forms a mixing of the two approximate frequencies, which induces the formation of a "beat frequency".

3.3.2. Local Friction and Contact

Figure 13 shows the force lines between the rope and the pulleys. Each force line represents the interaction force between a rope element and the groove on the pulley. This force can be decomposed into normal contact and friction.

Figure 14 shows the process of the force change in the transmission operation. Nine consecutive calculation points are selected in the constant speed stage. The blue arrow indicates the entire process of a rope element from the beginning to the wrap angle until it fully contacts the friction pulley. Numbers a–f indicate an increasing force. For numbers e–f, the force line direction has a significant downward migration compared with a–c. By number g, the force line shifts upward in the reverse

direction and even crosses the force line of the previous element. It can be seen from g, i that as the rope gradually enters the wrap angle, the force line starts to be stable and evenly distributed.



Figure 13. Distribution of the contact force line between the rope and pulleys.



Figure 14. Dynamic force line on the friction pulley during the transmission process.

Figure 15 shows the normal contact force between the 702th, 535th and 369th rope elements and pulleys. The load applied at the lifting side is 0.95×10^5 kg, and the load on a single rope is one sixth of it. The initial positions of each element above the cage are 300, 350 and 400 m, respectively. The contact angle between the rope and the guide pulley is small, so the rope passes through the guide pulley quickly and the contact force presents a more prominent peak. The contact force fluctuates and then rises gradually, and the fluctuation amplitude decreases continuously, which is caused by the rope tension of the lifting side. The variation in the contact force is significantly different from that in [23]. This is due to the existence of the guide pulley, which causes a higher tension level of the separation point (Figure 11).

Compared with the rope at 300 m and 350 m, the contact force at 400 m shows a long significant downward trend. The dividing point of the change trend is the one of acceleration to a constant speed. Therefore, the decreasing trend is due to the lower string rope tension in the acceleration stage (Figure 11), which makes the tension at the meeting point larger than that at the separation point. When the rope is about to leave the friction pulley, the contact force rises abruptly and reaches its maximum value. It can also be determined from Figure 14 that the force line increases significantly when the rope turns 180 degrees, which is due to the dominant effect of the string rope tension.



Figure 15. Contact force produced by the rope wrapping the pulleys. (**a**) Rope 300 m above cage; (**b**) rope 350 m above cage; (**c**) rope 400 m above cage.

Figure 16 shows the friction forces between the rope elements and the pulley's surface. It can be found that there are reverse values of the friction force in both the friction pulley and guide pulley. The friction force increases immediately after the rope is wrapped onto the friction pulley. The friction forces all have reverse peaks in the course of rising, which is similar to the results in [20]. The reason for this is that the rope tension decreases after entering the wrap angle, which results in rope deformation at the tangent point. The deformation direction is the same as the pulley rotation and the deformation speed is not less than the pulley edge linear speed, thus causing a reverse friction force. The maximum friction of the selected rope elements can reach 11,671 N before falling back rapidly to a reverse value. This shows that the rope is again deformed in the same direction as the friction pulley after the rope enters the second half of the wrap angle.



Figure 16. Friction force produced by rope wrapping the pulleys. (**a**) Rope 300 m above cage; (**b**) Rope 350 m above cage; (**c**) Rope 400 m above cage.

4. Discussion

4.1. Position Difference of Vibration Intensity

Figure 17 shows the rope vibrations above the cage and balance hammer. The law of vibration is still strongest in the short side direction (*Z*) and weakest in the long side direction (*X*), which indicates that the accuracy of the model is comprehensive and universal. The overall vibration value of the wire rope on the lowering side (above the hammer) is stronger than that on the lifting side (above the cage).



Figure 17. Rope vibration in three directions at different positions above the cage and the hammer. (**a1**) Z-axis—above the cage, (**a2**) Z-axis—above the hammer; (**b1**) Y axis—above of the cage, (**b2**) Y axis—above the hammer; (**c1**) X-axis—above the cage, (**c2**) X-axis—above the hammer.

With the increase of the distance from the cage, the vibration in all directions is significantly strengthened. In particular, the nonstationary vibration in the Z direction increases both in amplitude and quantity, and the amplitude in Y direction opposite to the acceleration direction increases. Similarly, the amplitude of the X-direction vibration increases with distance. This phenomenon can be explained by the fact that the higher the rope is, the higher the weight it bears. Therefore, as shown in Figure 18, the loading state of the rope goes from loosening to tightening, that is, from elasticity to rigidity.

Greater stiffness will inevitably lead to greater vibration. This theory can also explain that the vibration amplitude in the Y direction increases at the lifting side and decreases at the lowering side. In addition, the nonstationary rope vibrations with increasing amplitude or quantity as distance increases in Figure 17(a1,b2) (purple circles in the figure) also belong to the "loose–tight" mechanism.



Figure 18. A sketch of load variation along rope length.

4.2. New Theory of Friction Transmission

Figure 19 shows the contact force and friction force between the 369th rope element (the initial position is 400 m above the cage) and the friction pulley under different lifting side loads. Among them, 1.2×10^5 kg of lifting side load corresponds to heavy lifting, 0.95×10^5 kg is equal-weight lifting, and 0.7×10^5 kg is light load lifting. It can be found that the initial value of the contact force just entering the pulley depends on the load ratio on both sides. Because there is a constant load at the lowering side, the contact force at different lifting sides changes to the same value after bypassing the wrap angle. However, under any load, the wire rope will produce positive effect friction as soon as it enters the wrap angle, and will provide reverse friction when it is about to be wound out. The greater the lifting side load is, the smaller the reverse friction.



Figure 19. Contact and friction force between the 369th rope element (400 m) and the friction pulley. (a) Contact; (b) friction.

Integrating the friction force of the three rope elements in Figure 16 with time, as shown in Figure 20, the friction impulse of each rope element appears to have a downward inflection point when wrapping the friction pulley. That means that the friction drive of the pulley to the rope has changed from positive to the reverse. Until this point, friction has been beneficial to friction transmission; after this point, it has adverse effects. Combined with Figure 16, the friction force in the wrap angle can be divided into two regions by the judgement of the demarcation point (JDP). This is essentially different from the traditional static and dynamic angle theory—the static angle does not provide friction, and the dynamic angle provides a positive friction effect [19,21,24,25]. The theory holds that the tension does not change when the rope of the lifting side just enters the wrap angle (static angle), i.e., the rope does not provide any friction due to deformation and slippage. When the rope reaches the dynamic angle, the friction is caused by the creep between the rope and pulley [26,27].



Figure 20. Friction impulse of wire rope.

Based on the above dynamic behavior, a new transmission theory of "global dynamic wrap angle" for wire rope is proposed (Figure 21). There is no static angle, and the entire wrap angle is dynamic. The rope entering the wrap angle is immediately subjected to the positive friction provided by the pulley, which is beneficial for the transmission. After that, when the rope reaches a certain position, it starts to provide reverse friction until it is wound out of the wrap angle. The division of positive and reverse friction area depends on the load of wire ropes on both sides. When m1<m2, the reverse effect area increases, and when m1>m2, the positive friction effect area occupies absolute predominance. The traditional static and dynamic angle theories may be more suitable for design calculation in shallow mine hoists. The new transmission theory of "global dynamic wrap angle" is more suitable for the dynamic analysis of deep mine friction hoisting with a high speed and a heavy load.



Figure 21. The new friction transmission theory of "global dynamic wrap angle".

5. Conclusions

A dynamic model of the hoisting system that unifies the dynamic interaction between the suspended and wrapped ropes as a whole is established. The suspension dynamics of wire rope mainly presents nonstationary vibration in the Z direction and tension amplitude that increases with lifting speed. The wrapping dynamics includes the vibration cross-linking near the friction pulley and the dynamic change of the contact force line between the rope and the pulley in the wrap angle. Local dynamic force lines between rope and pulleys show that non-linear positive pressure and friction force exist at any position in the wrap angle.

A new transmission theory of "global dynamic wrap angle" for a rope friction hoist is proposed, i.e., the rope produces a positive friction effect by entering the wrap angle, and the friction will change direction at a certain boundary point until the rope winds out of the wrap angle. The mechanical state

of the wire rope is dynamic throughout the whole wrap angle, including deformation, contact and friction. The division of the positive and reverse effect regions depends on lifting and lowering the side rope end loads. The friction transmission theory proposed in this paper has more accurate dynamic analysis characteristics for high-speed and heavy-load friction hoisting.

Author Contributions: Conceptualization, D.Z.; Formal analysis, Y.G.; Investigation, Y.G.; Methodology, D.W.; Validation, S.W.; Writing—original draft, Y.G.; Writing—review & editing, X.Z. All authors have read and agreed to the published version of the manuscript.

Funding: This project is supported by National Natural Science Foundation of China (Grant No. 51905237, No. 51875565, No. 51705223) and Natural Science Foundation of Jiangsu Province (Grant No. BK20191000).

Conflicts of Interest: The authors declare no conflict of interest.

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