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Component Combination Rules of Wind Load Effects of Building Structures

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Received: 10 November 2020; Accepted: 4 December 2020; Published: 8 December 2020



Abstract: Under the action of the same wind azimuth, the extreme values of the wind load effect components of building structures are generated in the along-wind, cross-wind, vertical, and torsional directions. In designing the wind-resistant structure, the extreme values of effect components need to be combined to determine the internal force envelope values of members. Complete quadratic combination (CQC) and Turkstra combination rules are often used to determine the combination value of extreme values of wind effect components. The extreme probability distribution expressions of the CQC, and the Turkstra and approximate rules, are derived. The simplified combination Equations and combination coefficients of the CQC and Turkstra approximate rules are proposed in this paper. We use the combination Equations and Monte Carlo simulation method to analyze the accuracy of Turkstra and its approximate rules. The results show that the combination extreme is associated with the correlation coefficients, mean values, ratios of standard deviations, and fluctuating extremes of effect components. The errors between Turkstra and its approximate rules are small when load effect components show a positive correlation. The errors are largest when the standard deviations of components are equal. Our research results provide a theoretical basis for the combination method of wind load effect components of building structures.

Keywords: building structure; combination rule; extreme value probability distribution; wind effect component; combination coefficient

1. Introduction

The stationary random wind loads acting on each surface of a building structure have spatial correlation. The structural effect of the building structure under the action of wind load includes not only the downwind effect component, but also the cross-wind, vertical, and torsional effect components, and each maximum effect component does not occur concurrently [1–3]. It is unreasonable to consider only one or two of the effect components to evaluate the structural combination effect in the structural design, and it is also unreasonable to combine the maximum of each effect component to evaluate the structural combination effect. It is, therefore, critical to accurately calculate the combined effect of the extreme value of the effect component of the structural member under the action of wind load, which is of great significance for the study of the extreme value combination of the effect component of the building structure under the action of wind load. The load effect combination rules widely used today are mainly based on engineering experience and reliability theory. In the field of wind engineering, complete quadratic combination (CQC) and Turkstra (TR) rules are commonly used.

For the scalar sum of the effect components of the stationary Gaussian wind load, when the components are uncorrelated and their peak factors vary only slightly, the combination extreme is equal to the square root of the sum of squares of the extremum of the effect components, which is also

known as the square root of the sum of the squares (SRSS) rule; when the components are correlated, and the coupling of the cross terms between the components needs to be considered, the combination extreme is equal to the square root of the complete quadratic term of the extremum of each effect component, which is also known as the CQC rule. The CQC rule is based on the reliability theory, which is the statistical average of considerable samples. The accuracy is high when there are many samples, but the error may be large for a specific single sample [4]. The CQC rule is applied in the AS/NZS 1170.2:2011 wind load standard [5].

To facilitate application in engineering design, based on engineering experience, it is assumed that the wind load effect components obey a stable Gaussian distribution and are independent of each other. The maximum of one effect component and the accompanying values of the other effect components are combined sequentially, and the maximum is selected as the scalar sum of the effect components, which is also known as the TR rule. Based on the TR rule, Tamura et al. [6–11] undertook intensive research on the combination extreme of wind load effects of low-rise and medium-rise building structures, and the results showed that the extreme value of wind load effect combination derived from the TR rule is related to the correlation coefficient of the effect component, the aspect ratio of the building plane, the building height, the wind direction angle, and the wind pressure distribution when the wind load combination effect reaches the maximum. CHEN [12] studied the combination of wind load effect components according to the TR rule, and the results showed that the combination extreme value is affected by the correlation coefficient of the effect component and its amplitude ratio, whereas the correlation coefficient of the wind load effect component differs from the correlation coefficient of the wind load component. The load component correlation coefficient, therefore, should not be used for effect component combination. TR rule is applied in GB50009-2012 [13] and AIJ-RLB-2004 [14,15] codes.

The theoretical solution of the probability distribution of the TR rule cannot be expressed, but the upper and lower bounds of its probability distribution can. Naess et al. [14] obtained the minimum of the combination extreme according to the upper bound of the TR rule probability distribution. This rule is called the TR1 rule. The upper bound of the probability distribution of the TR1 rule can be expressed; the maximum of combination extreme is obtained according to the lower bound of the TR rule probability distribution. This rule is called the TR2 rule, and the lower bound of the probability distribution of the TR2 rule can also be expressed. Naess et al. [16–18] extended the TR1 and TR2 rules to the situation where the load effect components are related, and noted that the accuracy of the combination rule is related to the correlation coefficient of the effect component, the probability distribution type, and the quantile level. When one of the load effect component amplitudes is significantly larger than other components, the TR1 and TR2 rules are more accurate. CHEN and GONG [4,19,20] studied the combination rules of wind load effects, and the results showed that when the effect components are strongly correlated or have large amplitude differences, and the effect components have non-zero mean values, the TR1 and TR2 rules are more accurate, but when the effect components are partially correlated or uncorrelated, the TR1 and TR2 rules will underestimate the combination extreme.

Some approximate combination rules have also been adopted in practice for the convenience of calculation, i.e., 40% rule and 75% rule. Solari, G. and Pagnini [21] used the external dodecagon of an ellipse as the envelope of the most unfavorable condition for the combination of downwind and crosswind combination responses. On the basis of Solari's method, Asami [22] made use of the external octagon of an elliptic trajectory to estimate the most unfavorable response.

In the wind-resistant design of building structures, it is necessary to directly multiply the extreme value of the wind load effect component by the combination coefficient to obtain a simple Equation for the combination extreme of the effect, and the accuracy of the combination extreme should meet the engineering requirements. Therefore, it is very important to determine the simple and reasonable combination rules for extreme value combinations of wind load effect components. The TR rule is based on engineering experience and does not have a rigorous theoretical foundation. In contrast, the CQC

rule is based on reliability theory and has rigorous theoretical foundations, but it is not convenient in engineering applications, especially when multi-effect components are combined. This paper derives the probability distribution expressions of combination extremes of wind load effect components according to CQC combination rules and Turkstra and its approximate combination rules, and proposes simplified combination Equations and combination coefficients of CQC combination rules and Turkstra approximate combination rules. The approximate combination Equations and combination coefficients proposed in this paper enable engineers to calculate the extreme values of the wind load combination effect directly and simply by applying the extreme values of effect components in engineering design. The combination Equation and Monte Carlo simulation method are used to analyze the accuracy of Turkstra and its approximate combination rules, and the application range of relative error within 10% is proposed. The simplified combination Equations and combination coefficients of the CQC rule and Turkstra approximation rules proposed in this paper provide a theoretical basis for the extreme value combination method of wind load effect components of a building structure, which is convenient for engineering application.

2. CQC Combination Rules and Simplified Combination Coefficients

The CQC rule takes into account cross-term coupling between components. The calculation of the square root of the complete quadratic term of the extreme value of each effect component is complex, especially for the case of multiple effect components. To facilitate engineering applications, in this section, the extreme value probability distribution functions of the wind load effect components scalar sum are used to obtain the expression of the extreme value of the combined effect, by which the simplified combination coefficient of the extreme value of the effect component is given. The extreme value of the effect component and its combination coefficient are used to express the extreme value of the combined effect, and a simplified Equation for the combination of wind load effect components' extreme value is given according to the CQC rule. It is straightforward to expand the two-effect component extreme value combination to a multi-effect component extreme value combination, and the simplified Equation can then be derived according to CQC rule.

Assuming that the wind load effect components $R_1(t)$ and $R_2(t)$ obey the Gaussian distribution, the scalar sum is:

$$R(t) = R_1(t) + R_2(t), \quad (1)$$

This also obeys the Gaussian distribution, and the extreme probability distribution function of the scalar sum is as follows:

$$F(R) = \exp \left[-v_{0,R}^+ T \cdot \exp \left(-\frac{(R - \mu_1 - \mu_2)^2}{2\sigma_R^2} \right) \right], \quad (2)$$

In the Equation, μ_1 and μ_2 respectively represent the mean value of $R_1(t)$ and $R_2(t)$; T represents the given duration; $v_{0,R}^+$ is the mean upcrossing rate of the extreme value of $R(t)$, and the expression is as follows. The expression is different from the mean upcrossing rate in EN 1991-1-4 [23] which corresponds to the natural frequency of the structure, the background factor, and the resonance response factor. Furthermore, when calculating the total wind load of the structure according to EN 1991-1-4, the reduction effect on the wind action due to the non-simultaneity of occurrence of the peak wind pressures on the surface is taken into account by the size factor C_s . The resulting force is multiplied by 0.85–1 according to the ratio of the height to the depth of the structure.

$$v_{0,R}^+ = \frac{1}{2\pi} \cdot \frac{\dot{\sigma}_R}{\sigma_R}, \quad (3)$$

In the Equation, σ_R represents the standard deviation of $R(t)$, and $\dot{\sigma}_R$ represents the standard deviation of the derivative of $R(t)$; the expressions of σ_R and $\dot{\sigma}_R$ are, respectively, Equations (4) and (5).

$$\sigma_R = \sqrt{\sigma_1^2 + 2\rho_{12}\sigma_1\sigma_2 + \sigma_2^2}, \quad (4)$$

$$\dot{\sigma}_R = \sqrt{\dot{\sigma}_1^2 + 2\rho_{12}\dot{\sigma}_1\dot{\sigma}_2 + \dot{\sigma}_2^2}, \quad (5)$$

In the Equation, σ_1 and σ_2 represent the standard deviations of $R_1(t)$ and $R_2(t)$ respectively; $\dot{\sigma}_1$ and $\dot{\sigma}_2$ represent the standard deviations of the derivatives of $R_1(t)$ and $R_2(t)$ respectively; ρ_{12} represents the correlation coefficient of $R_1(t)$ and $R_2(t)$. Substituting Equations (4) and (5) into Equation (3), we get:

$$v_{0,R}^+ = \sqrt{\frac{1 + 2\rho \cdot \frac{v_{0,1}^+}{v_{0,2}^+} \cdot c_{21} + \left(\frac{v_{0,2}^+}{v_{0,1}^+}\right)^2 \cdot c_{21}^2}{1 + 2\rho_{12} \cdot c_{21} + c_{21}^2}} \cdot v_{0,1}^+, \quad (6)$$

In the Equation, c_{21} is the ratio of σ_2 to σ_1 ; $v_{0,1}^+$ and $v_{0,2}^+$ are the mean upcrossing rate of the extreme values of $R_1(t)$ and $R_2(t)$, respectively. If $v_{0,1}^+$ and $v_{0,2}^+$ are equal, $v_{0,R}^+$ and $v_{0,1}^+$ are equal.

The expressions of the maximum and minimum of the combined effect can be obtained from Equation (2):

$$R_{\max} = (\mu_1 + \mu_2) + g_R\sigma_R, \quad (7)$$

$$R_{\min} = (\mu_1 + \mu_2) - g_R\sigma_R, \quad (8)$$

In the Equation, g_R represents the peak factor of the combined effect, and its expression [24] is as follows. The peak coefficient K_p in the EN 1991-1-4 is changed from 0.5772 in Equation (9) to 0.6:

$$g_R = \sqrt{2 \ln(v_{0,R}^+ T)} + \frac{0.5772}{\sqrt{2 \ln(v_{0,R}^+ T)}}, \quad (9)$$

Assuming that the peak factors of the effect component and the combined effect are approximately equal:

$$g_R \cong g_1 \cong g_2 \cong g, \quad (10)$$

Then, we can obtain:

$$g_R\sigma_R \cong \sqrt{g_1^2\sigma_1^2 + 2\rho_{12}g_1\sigma_1g_2\sigma_2 + g_2^2\sigma_2^2}, \quad (11)$$

Substituting Equation (11) into Equation (7) yields the maximum of the combined effect:

$$\begin{aligned} R_{\max} &= (R_{1\max} - g_1\sigma_1) + (R_{2\max} - g_2\sigma_2) + \sqrt{g_1^2\sigma_1^2 + 2\rho_{12}g_1\sigma_1g_2\sigma_2 + g_2^2\sigma_2^2} \\ &= R_{1\max} + R_{2\max} \cdot \lambda_{2,\max} \end{aligned} \quad (12)$$

In the Equation, $\lambda_{2,\max}$ is called the combination coefficient of extreme value $R_{2\max}$, and its expression is:

$$\lambda_{2,\max} = 1 - \frac{1 + c_{12} - \sqrt{1 + 2\rho_{12}c_{12} + c_{12}^2}}{\frac{\mu_2}{g\sigma_2} + 1}, \quad (13)$$

In the Equation, c_{12} is the ratio of σ_1 to σ_2 .

Substituting Equation (11) into Equation (8) to obtain the minimum of the combined effect:

$$\begin{aligned} R_{\min} &= (R_{1\min} + g_1\sigma_1) + (R_{2\min} + g_2\sigma_2) - \sqrt{g_1^2\sigma_1^2 + 2\rho_{12}g_1\sigma_1g_2\sigma_2 + g_2^2\sigma_2^2} \\ &= R_{1\min} + R_{2\min} \cdot \lambda_{2,\min} \end{aligned} \quad (14)$$

In the Equation, $\lambda_{2,\min}$ is called the combination coefficient of extreme value $R_{2\min}$, and its expression is:

$$\lambda_{2,\min} = 1 + \frac{1 + c_{12} - \sqrt{1 + 2\rho_{12}c_{12} + c_{12}^2}}{\frac{\mu_2}{g\sigma_2} - 1}, \quad (15)$$

Derived above is the simplified Equation for the combination of the two effect components according to the CQC rule, and the extreme combination coefficient of the two effect components is proposed. Extending the above principle to the combination of multi-effect components, the maximum of the combination extreme of the scalar sum of n -effect components can be written as:

$$R_{\max} = R_{1\max} + \sum_{i=2}^n R_{i\max} \cdot \lambda_{i,\max}, \quad (16)$$

In the Equation, $\lambda_{i,\max}$ is called the combination coefficient of the i -th effect component extreme value $R_{i\max}$, and its expression is:

$$\lambda_{i,\max} = 1 - \frac{1 + \sqrt{\sum_{j=1}^{n-1} \sum_{k=1}^{n-1} \rho_{jk} c_{ji} c_{ki}} - \sqrt{\sum_{j=1}^n \sum_{k=1}^n \rho_{jk} c_{ji} c_{ki}}}{\frac{\mu_i}{g\sigma_i} + 1}, \quad (17)$$

In the Equation, ρ_{jk} is the correlation coefficient of $R_j(t)$ and $R_k(t)$; c_{ji} is the ratio of σ_j to σ_i ; c_{ki} is the ratio of σ_k to σ_i .

The minimum of the combination extreme of the scalar sum of n -effect component can be written as:

$$R_{\min} = R_{1\min} + \sum_{i=2}^n R_{i\min} \cdot \lambda_{i,\min}, \quad (18)$$

In the Equation, $\lambda_{i,\min}$ is called the combination coefficient of the i -th effect component extreme value $R_{i\min}$, and its expression is:

$$\lambda_{i,\min} = 1 + \frac{1 + \sqrt{\sum_{j=1}^{n-1} \sum_{k=1}^{n-1} \rho_{jk} c_{ji} c_{ki}} - \sqrt{\sum_{j=1}^n \sum_{k=1}^n \rho_{jk} c_{ji} c_{ki}}}{\frac{\mu_i}{g\sigma_i} - 1}, \quad (19)$$

In engineering design, the maximum and minimum combination coefficients of each effect component can be calculated according to Equations (17) and (19), and the maximum and minimum values of the extreme value of combined effect can then be calculated according to Equations (16) and (18). Structural wind-resistant design can then be carried out. It can be seen from Equations (17) and (18) that the combination coefficient is related to the correlation coefficient, variance ratio, mean value, and fluctuating value. The combination coefficients proposed above provide convenience for engineers to directly apply combination coefficients to combine extreme values of wind load effect components in structural wind-resistant design. In addition to the CQC rule, the widely used load effect combination rules also include the TR rule. The following is a study of the TR rule and its approximate combination rules.

3. TR Rule and Its Approximate Combination Rules

The TR rule is based on engineering experience. It is assumed that the maximum values of the effect components and the combined effect occur simultaneously, but there is no rigorous theoretical basis. In order to provide the TR rule with a rigorous theoretical basis, the simplified combination

Equation, combination coefficient of TR1 rule and the numerical integral expression of the TR2 rule are proposed based on the theoretical knowledge of random vibration and mathematical statistics.

3.1. TR combination rule

The TR rule combines the maximum of one effect component with the accompanying values of the other effect components, and the maximum is selected as the combination extreme. Assuming that the extreme values of the sum of the effect components and each component occur simultaneously, the scalar sum of the n -effect components is expressed as the following Equation:

$$R_{\max} = \max\{\hat{X}_1, \hat{X}_2, \dots, \hat{X}_i, \dots, \hat{X}_n\}, \quad (20)$$

In the Equation, $\hat{X}_i$ is the combined effect when the i -th effect component takes the extreme value and the other effect components take the accompanying values, the expression is:

$$\hat{X}_i = R_1^{(i)} + R_2^{(i)} + \dots + R_{i\max} + \dots + R_n^{(i)}, \quad (21)$$

In the Equation, $R_{i\max}$ is the maximum of $R_i(t)$ at point-in-time t ; $R_n^{(i)}$ is the accompanying value of $R_n(t)$ at point-in-time t .

The expression of the probability distribution function of the combination effect extreme by the TR rule is not available in theory, but the upper and lower bounds of the probability distribution function of the combination effect extreme by the TR rule can be obtained, and the approximate combination value is obtained according to the upper and lower bounds, which are called TR1 and TR2 combination rules, respectively. The following is a study of the simplified combination Equation of two approximate rules.

3.2. TR1 Combination Rule and Simplified Combination Coefficients

In the actual structure design, the probability distribution function of the combined effect is difficult to obtain, but the lower bound of the combination extreme can be obtained from the upper bound of the probability distribution of the combined effect, and the mean value of the lower bound of the combination extreme can be calculated according to the conditional probability. Then, the simplified combination Equation of the TR1 rule of the combination effect is derived.

Take the extreme value combination of two effect components as an example. From the research of Naess et al. [16], the probability distribution function of the extreme value of the effect component scalar sum has the following relationship:

$$F_{\hat{X}_1 \hat{X}_2}(x, x) \leq \min\{F_{\hat{X}_1}, F_{\hat{X}_2}\}, \quad (22)$$

In the Equation, $F_{\hat{X}_1}$ and $F_{\hat{X}_2}$ are the probability distribution functions of $\hat{X}_1$ and $\hat{X}_2$, respectively.

When $R_1(t)$ reaches the maximum, the derivative of the conditional probability distribution of $R_2(t)$ is the conditional probability density $f_{R_2|R_1}(R_2)$. The conditional probability density is equal to the ratio of the joint probability density to the conditional edge probability density. In the case of a one-sided safety limit, $f_{R_2|R_1}(R_2)$ is:

$$f_{R_2|R_1}(R_2) = \frac{1}{\sqrt{2\pi} \cdot (\sigma_2 \cdot \sqrt{1 - \rho_{12}^2})} \cdot \exp \left[\frac{-1}{(2\sigma_2 \cdot \sqrt{1 - \rho_{12}^2})^2} \cdot \left(R_2 - \left(\mu_2 + \rho_{12}^2 \cdot \frac{\sigma_2}{\sigma_1} (R_{1\max} - \mu_1) \right) \right)^2 \right], \quad (23)$$

Calculate the expectation of Equation (23), and obtain the conditional probability expectation value of $R_2(t)$:

$$\begin{aligned} E[R_2^{(1)}] &= \mu_2 + \rho_{12} \cdot \frac{\sigma_2}{\sigma_1} (R_{1\max} - \mu_1) \\ &= \mu_2 + \rho_{12} g \sigma_2 \end{aligned} \quad (24)$$

In Equations (20) and (21), when n is equal to 2, the lower bound $R_{\max}$ of the combined effect extreme value of the two effect components combined is derived according to the TR1 rule:

$$R_{\max} = \max\{\hat{X}_1, \hat{X}_2\}, \quad (25)$$

$$\hat{X}_1 = R_{1\max} + E[R_2^{(1)}], \quad (26)$$

$$\hat{X}_2 = R_{2\max} + E[R_1^{(2)}], \quad (27)$$

Substituting Equation (24) into Equation (26), we get:

$$\begin{aligned} \hat{X}_1 &= R_{1\max} + \mu_2 + \rho_{12} g \sigma_2 \\ &= R_{1\max} + \lambda_2 \cdot R_{2\max} \end{aligned} \quad (28)$$

In the Equation, λ_2 is called the combination coefficient of extreme value $R_{2\max}$, and its expression is as follows:

$$\lambda_2 = 1 - \frac{(1 - \rho_{12})}{\frac{\mu_2}{g\sigma_2} + 1}, \quad (29)$$

Similarly:

$$\hat{X}_2 = \lambda_1 \cdot R_{1\max} + R_{2\max}, \quad (30)$$

In the Equation, λ_1 is called the combination coefficient of extreme value $R_{1\max}$, and its expression is as follows:

$$\lambda_1 = 1 - \frac{(1 - \rho_{21})}{\frac{\mu_1}{g\sigma_1} + 1}, \quad (31)$$

Substitute Equations (28) and (30) into Equation (25) to obtain a simplified combination Equation of the TR1 rule for the extreme value combination of the two effect components

$$R_{\max} = \max\{R_{1\max} + \lambda_2 \cdot R_{2\max}, \lambda_1 \cdot R_{1\max} + R_{2\max}\}, \quad (32)$$

If the mean values of $R_1(t)$ and $R_2(t)$ are both zero, the combination coefficient is equal to ρ_{12} , and the combination coefficient under this specific condition is identical to the combination coefficient proposed by CHEN [4].

The combination coefficients of the extreme values of the two effect components combined according to the TR1 rule are proposed above, and the simplified Equation for combination of the two effect components is derived according to the TR1 rule. It is straightforward to expand the two-effect component extreme value combination to the multi-effect component extreme value combination. The simplified combination Equation of the TR1 rule for the extreme value combination of n -effect component can be written as:

$$\begin{aligned}
R_{\max} &= \max\{\hat{X}_1, \dots, \hat{X}_i, \dots, \hat{X}_n\} \\
&= \max\left\{\sum_{i=1}^n \lambda_{1i} \cdot R_{i,\max}, \dots, \sum_{i=1}^n \lambda_{ii} \cdot R_{i,\max}, \dots, \sum_{i=1}^n \lambda_{ni} \cdot R_{i,\max}\right\} \\
&= \max\left\{\sum_{i=1}^n \lambda_{ji} \cdot R_{i,\max}\right\}, j = 1, 2, \dots, n, j = i, \lambda_{ji} = 1
\end{aligned} \tag{33}$$

In the Equation, λ_{ji} is called the combination coefficient of the i -th effect component extreme value $R_{i,\max}$ of the j -th combination (the j -th effect component takes the extreme value and the remaining components take the accompanying values), and the expression is as follows:

$$\lambda_{ji} = 1 - \frac{(1 - \rho_{ji})}{\frac{\mu_i}{g\sigma_i} + 1}, \tag{34}$$

In the Equation, ρ_{ji} is the correlation coefficient between the j -th effect component and the i -th effect component.

The combination extreme based on the upper bound of the TR rule probability distribution is known as the TR1 rule, while obtaining the combination extreme based on the lower bound of the TR rule probability distribution is known as the TR2 rule. The TR2 combination rule expression is derived below.

3.3. TR2 Combination Rule

The bounds of the probability density function of the extreme value of the combined effect can be obtained by taking the derivative of probability distribution function. According to the probability distribution function and the density function of the extreme value of the combined effect, the expectations of the bounds of the extreme value of the combined effect are obtained. The combined effect extreme of two effect components is derived according to the TR2 rule, and finally proposed is the TR2 rule combination expression of the multi-effect component.

The bounds of the extreme probability distribution of the combined effect of two effect components is expressed as follows:

$$F_{R_{\max}}(x) = F_{\hat{X}_1}(x) \cdot F_{\hat{X}_2}(x), \tag{35}$$

When the two effect components are positively correlated, the lower bound of $F_{\hat{X}_1 \hat{X}_2}(x, x)$ is $F_{\hat{X}_1}(x) \cdot F_{\hat{X}_2}(x)$, and the following inequality is obtained:

$$F_{\hat{X}_1 \hat{X}_2}(x, x) \geq F_{\hat{X}_1}(x) \cdot F_{\hat{X}_2}(x), \tag{36}$$

The expected value on the right side of Equation (36) is the upper bound of the extreme value of the combined effect. When the two effect components are negatively correlated, the upper bound of $F_{\hat{X}_1 \hat{X}_2}(x, x)$ is $F_{\hat{X}_1}(x) \cdot F_{\hat{X}_2}(x)$, and the following inequality is obtained:

$$F_{\hat{X}_1 \hat{X}_2}(x, x) \leq F_{\hat{X}_1}(x) \cdot F_{\hat{X}_2}(x) \leq \min\{F_{\hat{X}_1}, F_{\hat{X}_2}\}, \tag{37}$$

In Equation (37), the mathematical expectation of combination extremum obtained by $F_{\hat{X}_1}(x) \cdot F_{\hat{X}_2}(x)$ is the TR2 rule-based combination effect extremum. The combination extremum expected value obtained by $\min\{F_{\hat{X}_1}, F_{\hat{X}_2}\}$ is the TR1 rule-based combination effect extremum. From Equation (37), when the two effect components are negatively correlated, the TR2 rule-based combination extreme is closer to the lower bound of the TR rule-based combination extreme. The TR2 rule is, therefore, more accurate under this condition.

Taking the derivative of Equation (35), then the bounds of the probability density function of the extreme value of the combined effect of the two effect components are obtained:

$$f_{R_{\max}}(x) = f_{\hat{X}_{11}}(x) \cdot F_{\hat{X}_2}(x) + F_{\hat{X}_1}(x) f_{\hat{X}_2}(x), \quad (38)$$

Take the derivative of $F_{\hat{X}_1}$, and get the probability density function of $\hat{X}_1$:

$$f_{\hat{X}_1}(x) = \int_{-\infty}^{+\infty} f_{R_2|R_1}(x - R_1) \cdot f_{R_{1\max}}(R_1) dR_1, \quad (39)$$

In the Equation, $f_{R_{1\max}}(R_1)$ is the extreme probability density function of the effect component $R_1(t)$, and its expression [25] is as follows:

$$f_{R_{1\max}}(R_1) = v_{0,1}^+ T \cdot \frac{(R_1 - \mu_1)}{\sigma_1^2} \cdot \exp\left[-\frac{(R_1 - \mu_1)^2}{2\sigma_1^2} - v_{0,1}^+ T \cdot \exp\left(-\frac{(R_1 - \mu_1)^2}{2\sigma_1^2}\right)\right], \quad (40)$$

Similarly, the probability distribution function and probability density function of $\hat{X}_2$ can be obtained. They are omitted here for brevity.

According to Equations (23), (38)–(40), the expected value of the lower bound of the scalar sum of two effect components is obtained, and the expected value of the extreme value of the combined effect of two effect components according to the TR2 rule is derived as follows:

$$\bar{R}_{\max} = E[R_{\max}] \approx \int_{-\infty}^{+\infty} x \cdot f_{R_{\max}}(x) dx, \quad (41)$$

The expected value of combination effect extremum of the n -effect component combined according to TR2 rule is expressed as follows:

$$\begin{aligned} \bar{R}_{\max} &= E[R_{\max}] \approx \int_{-\infty}^{+\infty} x \cdot f_{R_{\max}}(x) dx \\ &= \int_{-\infty}^{+\infty} x \cdot \left[f_{\hat{X}_1} \cdot F_{\hat{X}_2} \cdot \dots \cdot F_{\hat{X}_i} \cdot \dots \cdot F_{\hat{X}_n} \right. \\ &\quad \left. + \dots + F_{\hat{X}_1} \cdot F_{\hat{X}_2} \cdot \dots \cdot f_{\hat{X}_i} \cdot \dots \cdot F_{\hat{X}_n} \right. \\ &\quad \left. + \dots + F_{\hat{X}_1} \cdot F_{\hat{X}_2} \cdot \dots \cdot F_{\hat{X}_i} \cdot \dots \cdot f_{\hat{X}_n} \right] dx \\ &= \int_{-\infty}^{+\infty} x \cdot \left[\sum_{i=1}^n (f_{\hat{X}_i} \cdot \prod_{j=1}^n F_{\hat{X}_j}) \right] dx, i \neq j \end{aligned} \quad (42)$$

In the Equation, $f_{\hat{X}_i}(x)$ is the probability density function of the extreme value of the combined effect of the i -th effect component taking the extreme value and the remaining components take the accompanying values; the expression is as follows:

$$f_{\hat{X}_i}(x) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} \dots \int_{-\infty}^{+\infty} \frac{f_{(R_1, R_2, \dots, R_n)}(R_1, R_2, \dots, R_n)}{f_{R_i}(R_i)} \cdot f_{R_{i\max}}(R_i) dR_1 dR_2 \dots dR_n, \quad (43)$$

In Equation (43), the expressions of $f_{(R_1, R_2, \dots, R_n)}(R_1, R_2, \dots, R_n)$, $f_{R_i}(R_i)$, and $f_{R_{i\max}}(R_i)$ are Equations (44)–(46) respectively.

$$f_{(R_1, R_2, \dots, R_n)}(R_1, R_2, \dots, R_n) = \frac{1}{(2\pi)^{n/2} \cdot \sqrt{|C|}} \cdot \exp\left[-\frac{1}{2} \cdot (R_1, R_2, \dots, R_n) C^{-1} \begin{pmatrix} R_1 \\ R_2 \\ \vdots \\ R_n \end{pmatrix}\right], \quad (44)$$

$$f_{R_i}(R_i) = \frac{1}{\sqrt{2\pi} \cdot \sigma_i} \cdot \exp\left[-\frac{(R_i - \mu_i)^2}{2\sigma_i^2}\right], \quad (45)$$

$$f_{R_{\max}}(R_i) = v_{0,i}^+ T \cdot \frac{(R_i - \mu_i)}{\sigma_i^2} \cdot \exp \left[-\frac{(R_i - \mu_i)^2}{2\sigma_i^2} - v_{0,i}^+ T \cdot \exp \left(-\frac{(R_i - \mu_i)^2}{2\sigma_i^2} \right) \right], \quad (46)$$

In Equation (44), C is the covariance matrix of $R_1, R_2, R_3, \dots, R_n$; C^{-1} is the inverse matrix; and $|C|$ is the absolute value of the determinant of C . The expression of C is as follows:

$$C = \begin{bmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \cdots & \rho_{1n}\sigma_1\sigma_n \\ \rho_{21}\sigma_2\sigma_1 & \sigma_2^2 & \cdots & \rho_{2n}\sigma_2\sigma_n \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n1}\sigma_n\sigma_1 & \rho_{n2}\sigma_n\sigma_2 & \cdots & \sigma_n^2 \end{bmatrix}, \quad (47)$$

$E[R_{\max}]$ obtained by numerical integration of Equation (42) is the extreme value of the combination effect of TR2 rule. Because Equation (42) is a multiple integral, the TR2 rule-based combination extreme can only be obtained by numerical integration.

4. Applicability of TR and Its Approximate Combination Rules

The TR1 and TR2 combination rules are approximations of the TR combination rules. To verify the applicability of the TR and its approximate combination rules, the Monte Carlo simulation method was used to analyze the applicability of the TR combination rules first, and then the accuracy of TR1 and TR2 combination rules was analyzed under the applicable conditions of the TR combination rule. The distribution of random numbers generated by the Monte Carlo simulation method is a Gaussian process, but the effect components of building structures in practice may have strong or weak non-Gaussian characteristics. Therefore, the Monte Carlo simulation method used in this paper did not verify the combination of non-Gaussian effect components. However, the combined methods proposed in the paper are based on the condition that the effect components' distributions are Gaussian process. Due to the high precision of CQC or SRSS rules for the scalar sum of the effect components, this section analyzes the relative errors of the TR, TR1, and TR2 combination extremes, and determines the application range of TR and its approximate combination rules. Considering that the simplified combination coefficient of the combination rule proposed above is related to the correlation coefficient and the standard deviation ratio, the applicability of TR, TR1, and TR2 rules is approached from the perspective of the correlation coefficient and standard deviation ratio. When determining the combination coefficients, this paper and EN 1990 (2002) [26] are both based on the probability and statistics theory. However, the effect of wind action is not considered in EN 1990 (2002) when seismic action is involved in the combination of load effect. The probability distribution of extreme wind speed in EN 1991-1-4 is the extreme value Type I distribution. In accordance with EN 1990 the basic wind speed are characteristic values having annual probabilities of exceedance of 0.02. They are both the same as the assumption in GB 50009-2012, which is adopted in this paper. Furthermore, the probability of the calculated wind actions does not exceed the probability of the basic wind speed.

4.1. Applicability of TR Rule

The Monte Carlo simulation method was used to analyze the relative error of the combination extreme of the TR rule and the CQC rule to determine the application range of the TR rule. In Monte Carlo simulation, variance of the first wind load effect component was set as 1, and variance of the second wind load effect component was set as 0.1 to 3, so the variance ratio ranged from 0.1 to 3. The correlation coefficient of effect components varied from -1 to 1 with an increment interval of 0.1 . The number of samples was 1000. The Monte Carlo simulation method was used to generate the random component values of specific conditions, and the combined extremums were calculated according to each combination rule. Figure 1 demonstrates how the relative error between the TR rule- and CQC rule-based combination extreme varies in terms of correlation coefficient and standard deviation ratio.

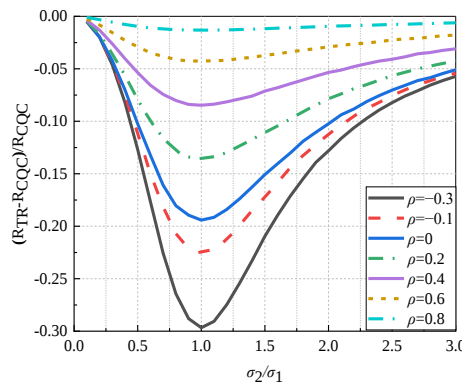


Figure 1. The relative error between the Turkstra (TR) rule- and complete quadratic combination (CQC) rule-based combination extreme.

It can be seen from Figure 1 that when the effect components are positively correlated, uncorrelated, and negatively correlated, the relative error between the TR rule- and CQC rule-based combination extreme first increases and then decreases as σ_2/σ_1 increases, and reaches the maximum when $\sigma_2/\sigma_1 = 1$. The relative error between the TR combination extreme value and the CQC combination extreme value decreases with the increase in ρ . When $\rho \geq 0.4$, regardless of the value of σ_2/σ_1 , the relative error is less than 10%; when $\sigma_2/\sigma_1 \leq 0.4$ or $\sigma_2/\sigma_1 \geq 2.2$, and the effect components are negatively correlated, uncorrelated, and positively correlated, the relative error is less than 10%. In summary, the range where the relative error of the combination extreme between the TR rule and the CQC rule is less than 10% is: $\rho \geq 0.4$; $\sigma_2/\sigma_1 \leq 0.4$; $\sigma_2/\sigma_1 \geq 2.2$.

4.2. Applicability of TR1 and TR2 Rule

In the range of $\rho \geq 0.4$, $\sigma_2/\sigma_1 \leq 0.4$, and $\sigma_2/\sigma_1 \geq 2.2$, the relative errors of the combination extremes between TR1, TR2, and CQC rules were compared and analyzed to determine the application range of TR1 and TR2 rules. Figure 2 demonstrates how the relative error between the TR1 rule- and CQC rule-based combination extremes vary in terms of the correlation coefficient and standard deviation ratio. Figure 3 demonstrates how the relative error between the TR2 rule- and CQC rule-based combination extremes vary in terms of correlation coefficient and standard deviation ratio.

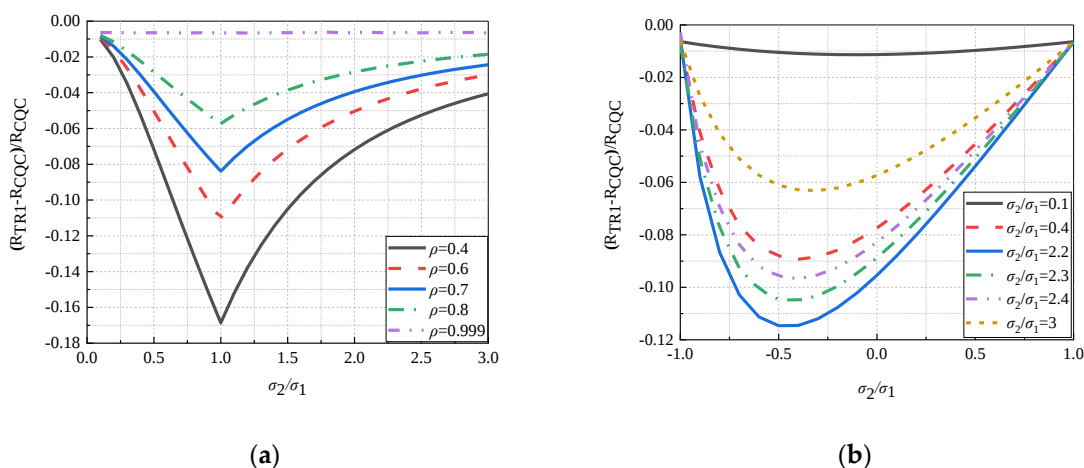


Figure 2. The relative error of the TR1 rule- and CQC rule-based combination extreme: (a) $\rho \geq 0.4$; (b) $\sigma_2/\sigma_1 \leq 0.4$, $\sigma_2/\sigma_1 \geq 2.2$.

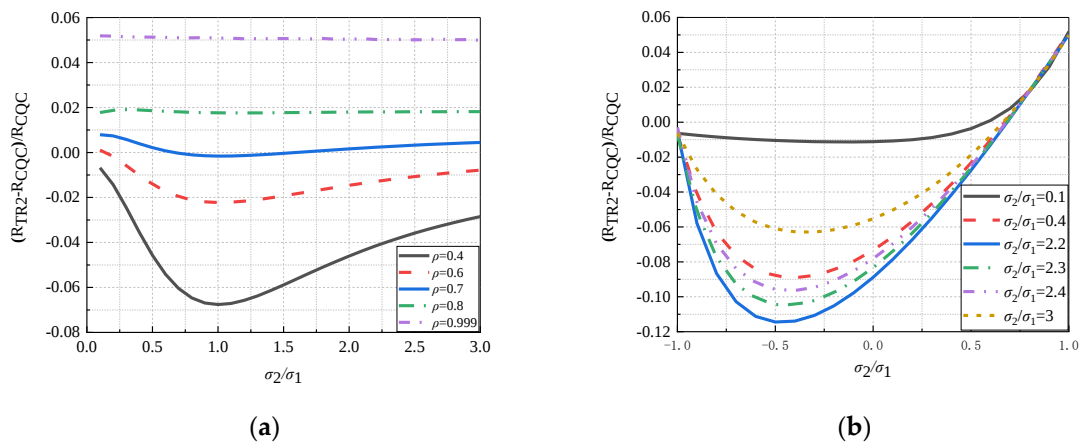


Figure 3. The relative error of the TR2 rule- and CQC rule-based combination extreme: (a) $\rho \geq 0.4$; (b) $\sigma_2/\sigma_1 \leq 0.4$, $\sigma_2/\sigma_1 \geq 2.2$.

As shown in Figure 2a, when the effect components are partially positively correlated, the relative error of the TR1 rule- and CQC rule-based combination extreme first increases and then decreases as σ_2/σ_1 increases, and reaches the maximum when $\sigma_2/\sigma_1 = 1$, and the largest relative error is -8.2% ; when the effect components are completely positively correlated, the relative error of the TR1 rule- and CQC rule-based combination extreme remains unaffected by σ_2/σ_1 , and the relative error is -0.7% . As shown in Figure 2b, the relative error of the TR1 rule- and CQC rule-based combination extreme first increases and then decreases with the increase of ρ when σ_2/σ_1 remains constant, and reaches the maximum when $\rho = -0.4$ and the maximum relative error is -11.5% . The relative error of the TR1 rule- and CQC rule-based combination extreme is always negative, that is, the TR1 rule-based combination extreme always underestimates the combined effect. It can be seen from Figure 2 that the relative error of the TR1 rule- and CQC rule-based combination extreme is less than 10% in the range of $\rho \geq 0.7$, $\sigma_2/\sigma_1 \leq 0.4$, $\sigma_2/\sigma_1 \geq 2.4$.

As can be seen from Figure 3a, when $0.4 \leq \rho \leq 0.7$, the relative error of the TR2 rule- and CQC rule-based combination extreme increases first and then decreases as σ_2/σ_1 increases, and reaches the maximum when $\sigma_2/\sigma_1 = 1$. The maximum relative error is -6.6% ; when $\rho \geq 0.8$, the relative error of the TR2 rule- and CQC rule-based combination extreme changes very slightly as σ_2/σ_1 goes up, which is almost a constant value. As shown in Figure 3b, the relative error of the TR2 rule- and CQC rule-based combination extreme, first increases and then decreases as the correlation coefficient increases when σ_2/σ_1 remains constant, and reaches the maximum when $\rho = -0.4$ and the largest relative error is -11.4% ; the relative error of the TR2 rule- and CQC rule-based combination extreme is positive or negative. When $\rho \leq 0.7$, the TR2 rule-based combination extreme is smaller than the CQC rule-based combination extreme, and when $\rho > 0.7$, the TR2 rule-based combination extreme is greater than the CQC rule-based combination extreme. It can be seen from Figure 3 that the relative error of the TR2 rule- and CQC rule-based combination extreme is less than 10% in the range of $\rho \geq 0.4$, $\sigma_2/\sigma_1 \leq 0.4$, and $\sigma_2/\sigma_1 \geq 2.4$.

5. Example—Combination of Wind Load Effect of a Portal Frame Structure

In this section, the effect of portal frames under wind load is used to verify the accuracy of the combined rules of CQC, TR, TR1, and TR2. The dimension of the rigid frame is 38.1×24.38 m, the longitudinal spacing is 7.62 m, the eave height is 4.88 m, and the slope of the double-slope roof is 8.33° . Portal frame columns are consolidated with the foundation, columns are rigidly connected with beams, and purlins are hinged with columns and beams. Roof slabs are hinged on purlins. In structural analysis, roof slabs and wall slabs only transfer wind load without considering their influence on structural stiffness. Structural columns, beams, and purlins have the I-type section, whose section sizes are shown in Table 1. The live and permanent loads on the portal frame are excluded, so the effect

of the portal frame under wind load is only included. The corresponding wind pressure data [27] from wind tunnel Test 1 from the University of Western Ontario, Canada, were used to analyze the portal frame, and the structural effects of the portal frame under wind loads on the windward side, leeward side, and windward side with leeward side were obtained separately to verify the accuracy and applicability of the combination rules. The base bending moment of the middle span is selected as the object of wind load effect combination. The analysis results of each combination rule are shown in Table 2.

Table 1. Section profiles for structural members.

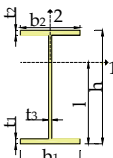
Section Sketch	Structural Member	L (m)	H (m)	b ₁ (m)	b ₂ (m)	t ₁ (m)	t ₂ (m)	t ₃ (m)
	Main frame	0.4	0.8	0.28	0.28	0.012	0.012	0.006
	Purlin	0.1	0.2	0.15	0.15	0.0045	0.0045	0.0032

Table 2. The error comparison table of each combination rule.

Wind Load Effect	σ_2/σ_1	ρ	(Combined Value–the Actual Value)/the Actual Value (%)			
			CQC	TR	TR1	TR2
Base Bending moment	2.13	0.06	0.97	−2.33	−7.34	7.25

As shown in Table 2, the error between the result of CQC rule and the actual value is the smallest, implying that the combined extreme value is closest to the actual value. This verifies that the CQC rule is more accurately applied for the scalar sum of the effect components. Meanwhile, it is reasonable to analyze the relative errors of combined extreme values of TR, TR1, TR2, and CQC rules in Section 4. For a given bit level, the TR2 combination value is always higher than the TR1 value.

6. Conclusions

1. The CQC rule combination extreme is related to the correlation coefficient, the standard deviation ratio, and the ratio of the mean to the fluctuating extreme of the effect component. The TR1 rule combination extreme is correlated with the correlation coefficient of the effect component and the ratio of the mean to the fluctuating extreme. The TR2 rule combination extreme is associated with the correlation coefficient, mean, standard deviation of the effect component, and the mean upcrossing rate.
2. When the effect components are partially correlated, the relative errors of the combination extreme between TR, TR1, TR2, and CQC rules first increase and then decrease with the increase in the standard deviation ratio, and reaches the maximum when $\sigma_2/\sigma_1 = 1$. The relative error of the combination extreme between TR1, TR2, and CQC rules first increases and then decreases with the increase in the correlation coefficient when σ_2/σ_1 remains constant, and reaches the maximum when $\rho = -0.4$.
3. When the wind effect components are positively correlated, the errors of the combination extreme by TR and its approximate combination rules are small, but the errors are large when the wind effect components are negatively correlated. When the effect components are partially positively correlated, the accuracy of TR and its approximation rules improves with the increase in the correlation coefficient. The TR1 rule-based combination extreme is always lower than CQC rule-based combination extreme, but the TR2 rule-based combination extreme is higher than CQC rule-based combination extreme when the correlation coefficient is greater than or equal to 0.7.

4. The application range of the TR rule is $\rho \geq 0.4$, $\sigma_2/\sigma_1 \leq 0.4$, and $\sigma_2/\sigma_1 \geq 2.2$; the application range of the TR1 rule is $\rho \geq 0.7$, $\sigma_2/\sigma_1 \leq 0.4$, $\sigma_2/\sigma_1 \geq 2.4$; the application range of the TR2 rule is $\rho \geq 0.4$, $\sigma_2/\sigma_1 \leq 0.4$, and $\sigma_2/\sigma_1 \geq 2.4$.

Author Contributions: H.G. conceived and designed the analysis, collected the data, contributed data, performed the analysis and wrote the paper; Y.T. conceived and designed the analysis, provided the funding and revised the paper. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the National Natural Science Foundation of China, grant numbers 51878040. The authors are grateful to all the study participants.

Acknowledgments: We acknowledge editors and reviewers for their suggestions.

Conflicts of Interest: The authors declare no conflict of interest.

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