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Observability Analysis and Navigation Filter Optimization of High-Orbit Satellite Navigation System Based on GNSS

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Abstract: Global Navigation Satellite System (GNSS) can be applied for the navigation of the high-orbit satellites. The system observability will change due to the changes in the visible satellite numbers and the spatial geometry between the navigation satellites and the users in the navigation system. The influence of the observability changing is not considered in the traditional navigation filter algorithm. In this paper, an optimized navigation filter method based on observability analysis is proposed. Firstly, a novel criterion for the relative observable degree is proposed for each observation component by making use of observation data from previous and posterior time simultaneously. Secondly, according to the relationship between observability and navigation filter accuracy, a novel optimized navigation filter method is constructed by introducing an adjusting factor based on the relative observable degree. Through the comparative simulations with the traditional Extended Kalman Filter (EKF), the optimized navigation filter method can reduce the estimation error of position and velocity by about 36% and 44% respectively. Therefore, the superiority of the proposed filter optimization algorithm is verified.

Keywords: GNSS; high-orbit satellites; observability; observable degree; adjusting factor; filter optimization

1. Introduction

High-orbit satellites [1] have better performance for ground coverage, safety, and stability compared with low-orbit and medium-orbit ones. For this reason, they play an important role in practice, e.g., satellite navigation and positioning systems. GNSS is the general term for all satellite navigation and positioning systems in orbit [2,3]. The position and velocity estimation of the users can be obtained based on GNSS by measuring the pseudo-range and Doppler frequency shift between the users and the navigation satellites. It has the advantages of comprehensive navigation information, high navigation accuracy, and simple measurement equipment [4], and has been widely applied in reality [5–10]. The navigation of the high-orbit satellites based on GNSS is one of the important research areas in the high-orbit satellite navigation technology.

The observability is the premise and the foundation of the navigation system with good navigation performance. The concept of observability was firstly proposed by Kalman [11], which refers to the ability of the navigation system to estimate the state variables through the outputs or the observations of the navigation system. In the engineering practice, we consider that the observability can be extended to the following three levels: the geometric observability, the physical observability, and the engineering

observability. For the navigation system of high-orbit satellites based on GNSS, the geometrical observability refers to that the line of sight of the high-orbit satellite to the navigation constellation is visible, the physical observability refers to that the effective observation data of the high-orbit satellite measurement equipment on the GNSS navigation satellite can be obtained and the engineering observability refers to that after obtaining effective observation data, the value of the user's state variables that meet the engineering accuracy requirements can be estimated. These three levels of observability are named as 'visibility', 'testability', and 'availability'. Among them, the 'availability' in this article refers to the observability in the control field. It is the basis for judging the performance of the navigation system and the reflection of the ability of the control system to determine the state variables at the initial moment [12]. In addition, good observability is the guarantee for the state filter convergence of the navigation system of high-orbit satellite [13,14]. Therefore, research on the observability of the navigation system is an important link in the design of the navigation system and the optimization of the navigation filter algorithm and has important theoretical and practical significance.

Since the navigation system of high-orbit satellites based on GNSS is a nonlinear system, there are currently two main methods to analyze the observability for the nonlinear system: one is to transfer the nonlinear system into a linear system by linearizing the nonlinear system and then distinguish the observability by using the Gram matrix of the linear system [15,16]. The second method analyzes the observability of the navigation system through the observability discriminant matrix which can be computed through the Lie derivative in the differential manifold [17,18]. However, the calculation complexity of the second method is high and its operating efficiency may also be reduced when the system equation is complex. The observability can only qualitatively judge whether the navigation system is observable. To quantify the observability of the system and its state components, the observable degree needs to be applied [19]. The analysis method of the observable degree has not been clearly defined. Ham [20] firstly proposed the method in 1983 which uses the eigenvalues and eigenvectors of the estimated error covariance matrix to describe the observability of the system state. However, the method can only work after the filter, and the system observability cannot be given in real-time. Reference [21] proposed a method to analyze the observable degree of the system through the singular value of the observable matrix. The method can judge the observability of the system before filter and is a pre-analysis method. However, this method can only give the overall observability of the system and cannot obtain the observability for each state component of the system. Reference [22] defined a method which analyzes the observable degree according to the error attenuation degree of the initial state variables, but the method paid much attention to the initial error covariance and ignored the real-time performance of filter; besides, some studies proposed to use the Fisher information matrix are also used to measure the observability of the navigation system in some studies [23–25].

Currently, there are many navigation filter algorithms applied to the navigation systems of the high-orbit satellites, including traditional Kalman filter and various improved forms. In addition, many scholars have proposed a large number of corresponding improved filter algorithms for the nonlinearity and uncertainty of the actual navigation system as well as the non-gaussian and correlation of noise, etc. [26–30]. Reference [31] proposed an adaptive filter method based on the state observability. The method constructed some adaptive factors according to the proposed quantized observability of each state component. The filter gain attenuation method is simultaneously applied to adjust the components whose observability is weak. Reference [32] proposed an information fusion algorithm based on the observable degree. The algorithm determines the information distribution coefficient according to the observable degree of the observation model and takes the unscented filter as the federated filter algorithm. Reference [33] proposed a smart Kalman filter method based on the adaptive observability to solve the problem of inaccurate parameters of the observation equation. The method improves the filter robustness by iteratively solving the adaptive adjustment factor.

However, the observable degree of the different state variables of the navigation system is inconsistent and the information transmitted to the subsequent filter estimation is also inconsistent. The previous research mainly used the observability directly as an adjustment factor and fed it back to the filter to improve the importance of the state variables with strong observability, while to reduce the influence of state variables with weak observability. Since the observability results obtained by different observability measurement methods are not consistent, it may cause the navigation filter divergence and reduce the overall navigation performance. In addition, the high-orbit satellite navigation is different from the general navigation system because the high-orbit satellite is higher than most orbits of the four major satellite navigation systems at present. Superadd the transmitting antenna of the GNSS satellite is facing the earth [34], and the angle between the main lobe signal transmission is limited. When the high-orbit satellite moves beyond the GNSS constellation, it can only receive the navigation satellite signal from the other side of the earth [35]. Therefore, the number of navigation satellites (i.e. the number of the visible satellites) that can be observed by the satellite at the receiver antenna position is limited. In a cycle of satellite operation, the number of visible navigation satellites changes constantly, which leads to the change of the observability. Therefore, it is of great significance to optimize the existing navigation filter algorithm and improve the overall performance of the navigation system of high-orbit satellites based on GNSS with the observability and the observable degree of the navigation system.

Based on the above analysis, we propose an adaptive navigation filter optimization method based on the observability and the observable degree of local components. According to the attenuation degree between the error of the optimal solution obtained from the observation in a period and the error of the state variable estimated by the filter at the current time, a new observability calculation matrix is defined, and the relative observable degree of each state component of the system is obtained. The advantages of this optimization method are that it can improve the traditional navigation filter method which only uses the observation information of the previous time, and can adjust the original filter process with the feedback of the observation information during the later time and the degree of error attenuation, which effectively improves the navigation filter accuracy and reduces the estimation error. The main contributions of this paper are as follows:

- (1) We propose a new method for calculating the observable degree of a high-orbit satellite navigation system based on GNSS, which can simultaneously give the relative observability of each state component at each moment and the overall observability of the system;
- (2) We design an adaptive optimization method of navigation filter based on this observable degree, which maps the observable degree of the state component to a feedback weighting factor to improve the performance of the navigation filter;
- (3) Based on the GNSS navigation system, we combine the proposed observability calculation method as an adjustment factor of the adaptive filter for filter optimization. Numerical simulation shows that the method effectively improves the navigation filter accuracy of the navigation systems of the high-orbit satellite based on GNSS.

The rest of this article is organized as follows. In Section 2, we briefly introduce the state equation and measurement equation of the high-orbit satellite navigation system based on GNSS. In Section 3, we analyze the observability of the system and propose the observability matrix of the system. Furthermore, the observable degree calculation matrix of the system is defined. In Section 4, according to the proposed framework for the observable degree, we propose an adaptive filter method. In Section 5, we verify the effectiveness of the proposed method by simulation of the high-orbit satellite navigation system based on GNSS. Section 6 is the summary and prospect of the full text.

2. Navigation Model Based on GNSS

The models of the navigation system include the state model and the measurement model, which is the basis for analyzing the observability of the navigation system and designing the navigation filter algorithm. Under the high-orbit environment, the force on the satellite is very complex. In addition to the gravity, the satellite is also affected by other perturbation forces. For the measurement equation, we only give the corresponding measurement model by measuring the inter-satellite pseudo-range based on the GNSS.

2.1. State Equation of the Navigation System

In the Earth’s inertial coordinate system, in order to analyze the navigation model, we define the state variable as

$$\mathbf{X} = \begin{bmatrix} \mathbf{r}^T & \mathbf{v}^T \end{bmatrix}^T$$

where the $\mathbf{r} = \begin{bmatrix} x & y & z \end{bmatrix}^T$ is the position vector of the high-orbit satellite in the Earth’s inertial coordinate system and the $\mathbf{v} = \begin{bmatrix} v_x & v_y & v_z \end{bmatrix}^T$ is the corresponding speed vector. Therefore the state equation of the orbital motion for the high-orbit satellite can be expressed as

$$\dot{\mathbf{X}} = f(\mathbf{X}(t), t) + \mathbf{w}(t). \tag{1}$$

where $f(\mathbf{X}(t), t)$ is the perturbation acceleration of the high-orbit satellite, the specific form can be found in the reference [36], and $\mathbf{w}(t)$ is the error vector of the perturbation model.

2.2. Measure Equation of the Navigation System

In the navigation system of high-orbit satellite based on GNSS, the observation is the pseudo-range measurement between the navigation satellites and the user satellite, and the measurement equation is as follows:

$$\begin{aligned} \rho_j &= d_j + v_j, j = 1, 2, \dots \\ d_j &= \sqrt{(x - x_j)^2 + (y - y_j)^2 + (z - z_j)^2}. \end{aligned} \tag{2}$$

The subscript j represents the j -th visible navigation satellite, and v_j is the measurement error, and ρ_j is the pseudo-range from j -th navigation satellite to the user satellite, and d_j is the geometrical distance from j -th navigation satellite to the user satellite. Assuming that there are m navigation satellites, then m measurement equations can be obtained, and the measurement model can be obtained by combining these m pseudo-range measurement equations. Let $\mathbf{Z}(t) = [\rho_1, \dots, \rho_j, \dots, \rho_m]^T$, $\mathbf{h} = [d_1, \dots, d_j, \dots, d_m]^T$, $\mathbf{v} = [v_1, \dots, v_j, \dots, v_m]^T$, the measurement model of the high-orbit satellite navigation system based on GNSS can be expressed as:

$$\mathbf{Z}(t) = \mathbf{h}(\mathbf{X}(t)) + \mathbf{v}(t). \tag{3}$$

2.3. Model Linearization

In the navigation system composed of Equations (1) and (3), since the navigation models are all nonlinear models, hence, we discretize the state equation and measure equation, and then linearizes it as follows:

$$\begin{cases} \mathbf{X}_{k+1} = \mathbf{F}_{k+1,k} \mathbf{X}_k + \mathbf{w}_k \\ \mathbf{Z}_k = \mathbf{H}_k \mathbf{X}_k + \mathbf{v}_k \end{cases} \tag{4}$$

where $\mathbf{X}_k \in \mathbf{R}^{6 \times 1}$ is the state vector. $\mathbf{Z}_k \in \mathbf{R}^{m \times 1}$ is the measurement vector. m is the number of visible navigation satellites. $\mathbf{F}_{k+1,k} \in \mathbf{R}^{6 \times 6}$ is the state one-step transition matrix, the expression is as shown

in Formula (5). $\mathbf{H}_{k+1,k} \in \mathbf{R}^{m \times 6}$ is the measurement matrix, the expression is as shown in Formula (5). $\mathbf{w}_k \in \mathbf{R}^{6 \times 1}$ is the system noise vector, and $\mathbf{v}_k \in \mathbf{R}^{m \times 1}$ is the measurement noise vector. Both of them are zero-mean Gaussian white noise vector sequences, and they are not correlated with each other, that

$$\text{is, satisfying: } \begin{cases} E[\mathbf{w}_k] = 0, & E[\mathbf{w}_k \mathbf{w}_j^T] = \mathbf{Q}_k \delta_{kj} \\ E[\mathbf{v}_k] = 0, & E[\mathbf{v}_k \mathbf{v}_j^T] = \mathbf{R}_k \delta_{kj} \\ E[\mathbf{w}_k \mathbf{v}_j^T] = 0 \end{cases} \quad \mathbf{Q}_k, \mathbf{R}_k \text{ is the system noise covariance matrix and the}$$

measurement noise covariance matrix respectively.

$$\begin{aligned} \mathbf{F}_{k+1,k} &= \nabla_{\mathbf{X}_k} \tilde{\mathbf{f}}(\mathbf{X}_k)^T = \left. \frac{\partial (\mathbf{X} + \mathbf{f}(\mathbf{X}, k))^T}{\partial \mathbf{X}} \right|_{\mathbf{X}=\hat{\mathbf{x}}_k} = \mathbf{I}_{6 \times 6} + \left. \frac{\partial \mathbf{f}(\mathbf{X})}{\partial \mathbf{X}} \right|_{\mathbf{X}=\hat{\mathbf{x}}_k} T \\ \mathbf{H}_k &= \nabla_x h(\mathbf{X}_k)^T = \left. \frac{\partial h(\mathbf{X})}{\partial \mathbf{X}} \right|_{\mathbf{X}=\hat{\mathbf{x}}_k} \end{aligned} \tag{5}$$

where T is the sample interval.

3. Observability of the Navigation System

The observability of the navigation system is the prerequisite and foundation for the system to have good navigation performance, and it is also the key to optimize the navigation filter algorithms. In this section, according to the definition of ‘observability in engineer’ of the navigation system, we propose a new method of observability calculation for high-orbit satellite navigation system based on GNSS, including observability discrimination methods and measurement criterion, aiming at the linearized navigation model (4) and considering the difference of relative observability of each state component of the navigation system.

3.1. Observability Qualitative Analysis

For the navigation model (4), according to the state equation, we can get

$$\mathbf{X}_i = \mathbf{F}_{i,i-1} \mathbf{F}_{i-1,i-2} \cdots \mathbf{F}_{k+1,k} \mathbf{X}_k + \bar{\mathbf{w}}_i = \mathbf{F}_{i,k} \mathbf{X}_k + \bar{\mathbf{w}}_i \tag{6}$$

where $\bar{\mathbf{w}}_i = \sum_{j=k+1}^i \mathbf{F}_{i,j} \mathbf{w}_{j-1}, i = k, k+1, \dots, k+m, \mathbf{F}_{i,j} = \prod_{s=1}^{i-j} \mathbf{F}_{i-s+1,i-s}$. According to the observability definition, given m measurement data, m measurement equations are combined, and Equations (4) and (6) can be obtained. We can get

$$\begin{cases} \mathbf{Z}_k = \mathbf{H}_k \mathbf{X}_k + \mathbf{v}_k \\ \mathbf{Z}_{k+1} = \mathbf{H}_{k+1} \mathbf{X}_{k+1} = \mathbf{H}_{k+1} (\mathbf{F}_{k+1,k} \mathbf{X}_k + \bar{\mathbf{w}}_{k+1}) + \mathbf{v}_{k+1} = \mathbf{H}_{k+1} \mathbf{F}_{k+1,k} \mathbf{X}_k + \mathbf{H}_{k+1} \bar{\mathbf{w}}_{k+1} + \mathbf{v}_{k+1} \\ \mathbf{Z}_{k+2} = \mathbf{H}_{k+2} \mathbf{X}_{k+2} = \mathbf{H}_{k+2} (\mathbf{F}_{k+2,k} \mathbf{X}_k + \bar{\mathbf{w}}_{k+2}) + \mathbf{v}_{k+2} = \mathbf{H}_{k+2} \mathbf{F}_{k+2,k} \mathbf{X}_k + \mathbf{H}_{k+2} \bar{\mathbf{w}}_{k+2} + \mathbf{v}_{k+2} \\ \vdots \\ \mathbf{Z}_{k+m-1} = \mathbf{H}_{k+m-1} \mathbf{X}_{k+m-1} = \mathbf{H}_{k+m-1} (\mathbf{F}_{k+m-1,k} \mathbf{X}_k + \bar{\mathbf{w}}_{k+m-1}) + \mathbf{v}_{k+m-1} \\ = \mathbf{H}_{k+m-1} \mathbf{F}_{k+m-1,k} \mathbf{X}_k + \mathbf{H}_{k+m-1} \bar{\mathbf{w}}_{k+m-1} + \mathbf{v}_{k+m-1} \end{cases} \tag{7}$$

Let $\mathbf{Z}_{k,m} = [\mathbf{Z}_k^T, \mathbf{Z}_{k+1}^T, \dots, \mathbf{Z}_{k+m-1}^T]^T, \mathbf{O}_{k,m} = [\mathbf{H}_k^T, \mathbf{F}_{k+1,k}^T \mathbf{H}_k^T, \dots, \mathbf{F}_{k+m-1,k}^T \mathbf{H}_{k+m-1}^T]^T, \mathbf{V}_{k,m} = [\mathbf{v}_k^T, (\mathbf{H}_{k+1} \bar{\mathbf{w}}_{k+1} + \mathbf{v}_{k+1})^T, \dots, (\mathbf{H}_{k+m-1} \bar{\mathbf{w}}_{k+m-1} + \mathbf{v}_{k+m-1})^T]^T$ Then the Equation (6) can be transferred to

$$\mathbf{Z}_{k,m} = \mathbf{O}_{k,m} \mathbf{X}_k + \mathbf{V}_{k,m} \tag{8}$$

Based on the weighted least squares, we can get

$$O_{k,m}^T G_{k,m}^{-1} Z_{k,m} = O_{k,m}^T G_{k,m}^{-1} O_{k,m} X_k \tag{9}$$

where $G_{k,m}$ is the covariance matrix of $V_{k,m}$. To make the above equation have a unique solution, $O_{k,m}^T G_{k,m}^{-1} O_v$ should be nonsingular.

Let $B_{k,m} = O_{k,m}^T G^{-1} O_{k,m} = \sum_{i=k}^{k+m-1} F_{i,k}^T H_i^T \left(\sum_{j=k+1}^i H_i F_{i,j} Q_{j-1} F_{i,j}^T H_i^T + R_i \right)^{-1} H_i F_{i,k}$, it is defined as the observability discrimination matrix. If the system is observable, since $B_{k,m}$ is a positive semi-definite matrix, $B_{k,m}$ being nonsingular is equivalent to that being a positive definite matrix, that is, $B_{k,m} > 0$. Calculate the condition number of $B_{k,m}$ in different periods, $cond(B_{k,m}) = \sigma_{\max} / \sigma_{\min}$, where σ_{\max} and σ_{\min} are the largest and the smallest singular values of the matrix respectively. If the condition number is large, it indicates that the overall observability of the system at that moment is weak; if the condition number is small, it indicates that the overall observability of the system at that moment is strong.

3.2. Observable Degree Analysis

Due to the inconsistency of the information validity between different state variables, in order to more accurately pass the state variable information to the subsequent filter, the observable degree of each state component of the system needs to be given. The method of the observable degree of each state variable in this article is given below.

In the filter process, the filter estimation of state variables is a sequential process, and there is a relationship between different moments. Regarding the estimated value of the state variable at the previous moment as a random variable, the estimated error covariance of the random variable will affect the prediction accuracy of the filter at the current moment and subsequent moments. The traditional filter only quotes the measurement information at various previous moments to estimate the state value at the current moment, which is not enough for a stable sequence process. The measurement information at subsequent moments can also provide useful information for the current state estimation. Therefore, we set an objective function to minimize the error value of the current state variable estimation and the subsequent measurement information, in order to determine the best estimate at the current moment.

Based on the above optimization ideas, we give an optimal estimation problem as follows. At the k -th moment, we give a set of measured values $Z_k, Z_{k+1}, \dots, Z_{k+m}$ from k to $k + m$ to find the optimal estimation $\hat{X}_{k,k+m}$ of X_k that minimizes the loss function (10) [37].

$$J_k = (\bar{X}_k - X_k)^T P_k^{-1} (\bar{X}_k - X_k) + \sum_{i=k}^{k+m} \tilde{Z}_i^T R_i^{-1} \tilde{Z}_i \tag{10}$$

where \bar{X}_k is the expectation of the state variable at the k -th moment, P_k is the estimation error covariance matrix at the k -th moment. $\tilde{Z}_i = Z_i - H_i X_i$ is the error between the measured value and the measured estimated value at the i ($i = k, \dots, k + m$)-th moment, R_i is the covariance matrix of the measurement error at the i -th moment. Substitute $\tilde{Z}_i = Z_i - H_i X_i$ into the right side of Equation (10), we can get:

$$J_k = (\bar{X}_k - X_k)^T P_k^{-1} (\bar{X}_k - X_k) + \sum_{i=k}^{k+m} (Z_i - H_i X_i)^T R_i^{-1} (Z_i - H_i X_i) \tag{11}$$

Substitute Formula (10) into (11), we get

$$\begin{aligned}
 J_k &= (\bar{X}_k - X_k)^T P_k^{-1} (\bar{X}_k - X_k) + \sum_{i=k}^{k+m} (Z_i - H_i(F_{i,k}X_k + \bar{w}_i))^T R_i^{-1} (Z_i - H_i(F_{i,k}X_k + \bar{w}_i)) \\
 &= (\bar{X}_k - X_k)^T P_k^{-1} (\bar{X}_k - X_k) + \sum_{i=k}^{k+m} (Z_i - H_i\bar{w}_i - H_iF_{i,k}X_k)^T R_i^{-1} (Z_i - H_i\bar{w}_i - H_iF_{i,k}X_k) \\
 &= X_k^T \left(P_k^{-1} + \sum_{i=k}^{k+m} F_{i,k}^T H_i^T R_i^{-1} H_i F_{i,k} \right) X_k - \left[\bar{X}_k^T P_k^{-1} + \sum_{i=k}^{k+m} (Z_i - H_i\bar{w}_i)^T R_i^{-1} H_i F_{i,k} \right] X_k \\
 &\quad - X_k^T \left[P_k^{-1} \bar{X}_k + \sum_{i=k}^{k+m} F_{i,k}^T H_i^T R_i^{-1} (Z_i - H_i\bar{w}_i) \right] + \left(\bar{X}_k^T P_k^{-1} \bar{X}_k + \sum_{i=k}^{k+m} (Z_i - H_i\bar{w}_i)^T R_i^{-1} (Z_i - H_i\bar{w}_i) \right)
 \end{aligned} \tag{12}$$

In order to minimize the objective function, since J_k is a quadratic function, the optimal estimate $\hat{X}_{k,k+m}$ of X_k is expressed as follows:

$$\hat{X}_{k,k+m} = \left(P_k^{-1} + \sum_{i=k}^{k+m} F_{i,k}^T H_i^T R_i^{-1} H_i F_{i,k} \right)^{-1} \left(P_k^{-1} \bar{X}_k + \sum_{i=k}^{k+m} F_{i,k}^T H_i^T R_i^{-1} (Z_i - H_i\bar{w}_i) \right) \tag{13}$$

Let $K_{k,k+m} = \sum_{i=k}^{k+m} F_{i,k}^T H_i^T R_i^{-1} (Z_i - H_i\bar{w}_i)$, $L_{k,k+m} = \sum_{i=k}^{k+m} F_{i,k}^T H_i^T R_i^{-1} H_i F_{i,k}$, then $\hat{X}_{k,k+m}$ can be expressed as:

$$\hat{X}_{k,k+m} = \left(P_k^{-1} + L_{k,k+m} \right)^{-1} \left(K_{k,k+m} + P_k^{-1} \bar{X}_k \right) \tag{14}$$

Since P_k^{-1} is a positive definite matrix, $L_{k,k+m}$ is a positive semi-definite matrix, the above optimal solution $\hat{X}_{k,k+m}$ must exist. Then we can get the expression of the error at the k -th moment of the state variable as [38]:

$$\tilde{X}_{k,k+m} = X_k - \hat{X}_{k,k+m} = X_k - \left(P_k^{-1} + L_{k,k+m} \right)^{-1} \left(K_{k,k+m} + P_k^{-1} \bar{X}_k \right) \tag{15}$$

That is

$$\begin{aligned}
 \tilde{X}_{k,k+m} &= X_k - \left(P_k^{-1} + L_{k,k+m} \right)^{-1} \left[\sum_{i=k}^{k+m} F_{i,k}^T H_i^T R_i^{-1} (H_i X_i + v_i - H_i \bar{w}_i) + P_k^{-1} \bar{X}_k \right] \\
 &= X_k - \left(P_k^{-1} + L_{k,k+m} \right)^{-1} \left[\sum_{i=k}^{k+m} F_{i,k}^T H_i^T R_i^{-1} (H_i F_{i,k} X_k + v_i) + P_k^{-1} \bar{X}_k \right] \\
 &= X_k - \left(P_k^{-1} + L_{k,k+m} \right)^{-1} \left(\sum_{i=k}^{k+m} F_{i,k}^T H_i^T R_i^{-1} H_i F_{i,k} X_k + P_k^{-1} \bar{X}_k \right) - \left(P_k^{-1} + L_{k,k+m} \right)^{-1} \sum_{i=k}^{k+m} F_{i,k}^T H_i^T R_i^{-1} v_i \\
 &= X_k - \left(P_k^{-1} + L_{k,k+m} \right)^{-1} \left(L_{k,k+m} X_k + P_k^{-1} \bar{X}_k \right) - \left(P_k^{-1} + L_{k,k+m} \right)^{-1} \sum_{i=k}^{k+m} F_{i,k}^T H_i^T R_i^{-1} v_i \\
 &= \left[I - \left(P_k^{-1} + L_{k,k+m} \right)^{-1} L_{k,k+m} \right] X_k - \left(I + P_k L_{k,k+m} \right)^{-1} \bar{X}_k - \left(P_k^{-1} + L_{k,k+m} \right)^{-1} \sum_{i=k}^{k+m} F_{i,k}^T H_i^T R_i^{-1} v_i \\
 &= \left[I - \left(I + P_k L_{k,k+m} \right)^{-1} P_k L_{k,k+m} \right] X_k - \left(I + P_k L_{k,k+m} \right)^{-1} \bar{X}_k - \left(P_k^{-1} + L_{k,k+m} \right)^{-1} \sum_{i=k}^{k+m} F_{i,k}^T H_i^T R_i^{-1} v_i \\
 &= \left[\left(I + P_k L_{k,k+m} \right)^{-1} \left(I + P_k L_{k,k+m} - P_k L_{k,k+m} \right) \right] X_k - \left(I + P_k L_{k,k+m} \right)^{-1} \bar{X}_k \\
 &\quad - \left(P_k^{-1} + L_{k,k+m} \right)^{-1} \sum_{i=k}^{k+m} F_{i,k}^T H_i^T R_i^{-1} v_i \\
 &= \left(I + P_k L_{k,k+m} \right)^{-1} \left(X_k - \bar{X}_k \right) - \left(P_k^{-1} + L_{k,k+m} \right)^{-1} \sum_{i=k}^{k+m} F_{i,k}^T H_i^T R_i^{-1} v_i \\
 &= \left(I + P_k L_{k,k+m} \right)^{-1} \tilde{X}_{k,k} - \left(P_k^{-1} + L_{k,k+m} \right)^{-1} \sum_{i=k}^{k+m} F_{i,k}^T H_i^T R_i^{-1} v_i
 \end{aligned} \tag{16}$$

After simplification, we can get:

$$\tilde{\mathbf{X}}_{k,k+m} = (\mathbf{I} + \mathbf{P}_k \mathbf{L}_{k,k+m})^{-1} \left(\tilde{\mathbf{X}}_{k,k} - \mathbf{P}_k \sum_{i=k}^{k+m} \mathbf{F}_{i,k}^T \mathbf{H}_i^T \mathbf{R}_i^{-1} \mathbf{v}_i \right) \tag{17}$$

It can be seen from Equation (17) that the state estimation error obtained is proportional to $(\mathbf{I} + \mathbf{P}_k \mathbf{L}_{k,k+m})^{-1}$, and $(\mathbf{I} + \mathbf{P}_k \mathbf{L}_{k,k+m})^{-1}$ represents the ratio of the state estimation error value obtained at the k -th moment from the previous moment to the state estimation error estimated by the measurement at the later moment, and it can be expressed as the error loss of the state estimation.

Define $\mathbf{D}_{k,k+m} = (\mathbf{I} + \mathbf{P}_k \mathbf{L}_{k,k+m})^{-1}$ as the calculation matrix of the observable degree, and its diagonal element is the observable degree of the corresponding state variable, namely

$$\eta_{k,i} = (\mathbf{D}_{k,k+m})_{i,i} \tag{18}$$

where $(\mathbf{D}_{k,k+m})_{i,i}$ is the i -th main diagonal element of $\mathbf{D}_{k,k+m}$, $\eta_{k,i}$ is the observable degree of the i -th state variable at the k -th moment. The larger $\eta_{k,i}$ is, the weaker the observability of the i -th state variable, and the smaller $\eta_{k,i}$ is, the stronger the observability of the i -th state variable at that time.

4. Optimized Filter Algorithm for Navigation System Based on the Observable Degree

As the observability is related to the filter accuracy tightly, one of the most efficient methods to increase the accuracy is to optimize the filter algorithm based on the observability. In this paper, a novel Adaptive extend Kalman Filter (AKF) base on the novel observability criterion is proposed for the navigation system of high-orbit satellites. The framework is summarized in Figure 1. The details of the method will be reported in the following part.

The traditional process of EKF is reported as follows.

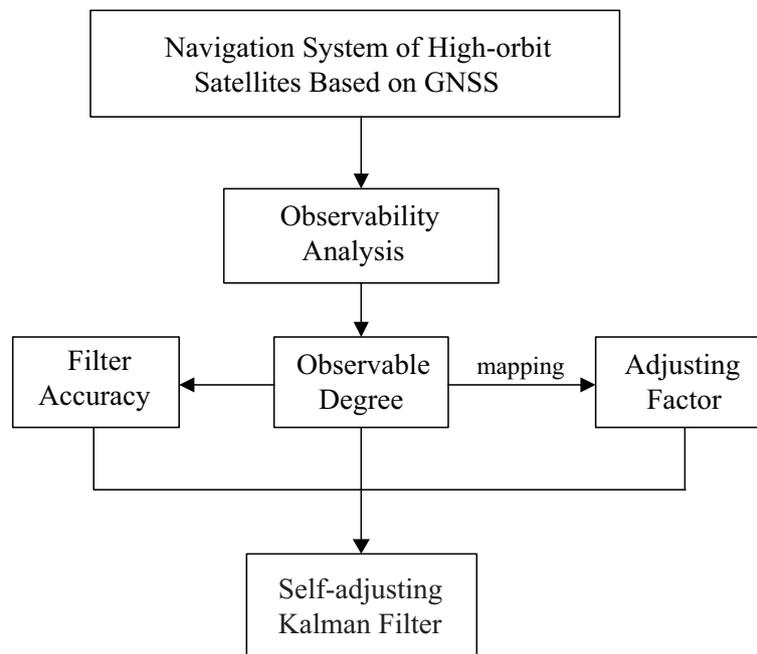


Figure 1. The framework of navigation filter optimization algorithm proposed in the paper.

The one-step prediction of the state vector is

$$\hat{\mathbf{X}}_{k/k-1} = f(\hat{\mathbf{X}}_{k-1}) . \tag{19}$$

The error covariance matrix of $\hat{X}_{k/k-1}$ is

$$P_{k/k-1} = F_{k/k-1} P_{k-1} F_{k/k-1}^T + Q_{k-1}. \tag{20}$$

The filter gain matrix is

$$K_k = P_{k/k-1} H_k^T (H_k P_{k/k-1} H_k^T + R_k)^{-1}. \tag{21}$$

The prediction of state vector \hat{X}_k is

$$\hat{X}_k = \hat{X}_{k/k-1} + K_k (Z_k - h(\hat{X}_{k/k-1})). \tag{22}$$

The error covariance matrix of \hat{X}_k is

$$P_k = (I - K_k H_k) P_{k/k-1}. \tag{23}$$

Recent works contribute a variety of the frameworks of the adaptive Kalman Filter, in the paper, we choose the framework with an adjusting factor ψ_k in front of the filter gain K_k in Equation (21). Then the adaptive filter gain matrix can be written as

$$K_k^* = \psi_k P_{k,k-1} H_k^T (H_k P_{k,k-1} H_k^T + R_k)^{-1}, \tag{24}$$

where ψ_k is a diagonal matrix. The value of the i -th diagonal element of ψ_k is the adjusting factor of the corresponding i -th state variable.

Increasing the value of the adaptive adjustment factor ψ_k will change the value of the original gain matrix which can be considered as a balance parameter between the one-step predicted value of the state variable and the observed value. When the navigation system suffers a weak observable degree, in order to suppress the influence of the weak observable degree, adjusting the weighting method can be used to correct the bias of predicted state variables.

As we discussed above, the observability plays an important role in the navigation system. The adjusting factor should also be designed for each state variable in order to suppress the influence of weak observable degree. This conclusion is obviously correct because if we use an identical factor for all the state variables, the effect of the adjusting factor will modify the state variables at an equal level. The state variable with a high observable degree will be incorrect and the filter accuracy will be decreased.

In the previous section, we have proposed a relative observable degree for each state variable. Meanwhile, the prediction of each state variable is closely related to the observable degree. For this reason, a criterion for defining the adaptive adjusting factor based on the relative observable degree is proposed.

The definition of ψ_k can be expressed as

$$(\psi_k)_i = \phi(\eta_{k,i}), \quad i = 1, \dots, n, \tag{25}$$

where the subscript (i) represents the i -th diagonal element and η_k which donates the observable degree in k -th moment is defined in Equation (18). $\phi(\eta)$ is a mapping function which represents the relationship between the observable degree and the adjusting factor.

In the adjusting process, since the original state estimations are biased under weak observable degree, in order to reduce the bias, the adjusting factor should be well designed. For example, when the observation component has a high observable degree, the estimation should rely more on the observation information. In opposite, the estimation should rely more on state prediction.

Based on the navigation system in this paper, the observation Equation (3) shows that

$$Z = h(r), \tag{26}$$

where r is the position vector of the high-orbit satellites. This Equation (26) indicates that the state observation is related to the position vector immediately and has little relationship with the velocity vector. Hence, the setting of the adjusting factor should consider the position vector and the velocity vector respectively. For the position vector, as the observable degree is a kind of relative value, the component with the best observable degree should hold the original estimation and its adjusting factor should be 1. When the state is ideal, if we notate the error of \hat{X}_k as \tilde{X}_k , then the optimal filter gain is the solution of

$$K_k^* = \underset{K}{\operatorname{argmin}} E(\tilde{X}_k \tilde{X}_k^T), \tag{27}$$

$$P_k^* = (I - K_k^* H_k) P_{k/k-1}. \tag{28}$$

However, when suffering a weak observable degree, the estimation error has a larger variance. An inequality can be derived that

$$P_k = (I - K_k H_k) P_{k/k-1} \tag{29}$$

$$\geq (I - K_k^* H_k) P_{k/k-1} = P_k^*. \tag{30}$$

Compared with Equations (29) and (30), we can find that when the observable degree is weak, the $K_k H_k$ is larger than $K_k^* H_k$. Hence, in order to reduce the error covariance, the adjusting factor should be larger than 1.

Based on the analysis above, we can optimize the EKF through the adjusting factor. The process of our Algorithm 1 can be expressed as follows

Algorithm 1 Adaptive Extend Kalman Filter

1: **Input:** Optimal estimation of \hat{X}_{k-1}^* and covariance matrix P_{k-1}^* in moment $k - 1$

2: **Output:** Optimal estimation of \hat{X}_k^* and covariance matrix P_k^* in moment k

Step 1: Calculate the state transition F_k , measurement matrix H_k and state covariance matrix P_k by Equations (19)–(23)

Step 2: Repeat **Step 1** and calculate the state transition matrix and measurement matrix at moment $k + 1, \dots, k + m$, written as $H_{k+1}, F_{k+1}, \dots, H_{k+m}, F_{k+m}$.

Step 3: Use the $H_k, F_k, H_{k+1}, F_{k+1}, \dots, H_{k+m}, F_{k+m}$ in **Step 1** and **Step 2** then calculate the $\eta_{k,i}$ by Equation (18).

Step 4: Define a mapping function ϕ based on the criterion in Section 3 then calculate the (ψ_k) by Equation (25).

Step 5: Calculate the new filter gain matrix K_k^* by Equation (24).

Step 6: Calculate $\hat{X}_k^* = \hat{X}_{k/k-1} + K_k^* (Z_k - h(\hat{X}_{k/k-1}))$ and $P_k^* = (I - K_k^* H_k) P_{k/k-1}$ and output them.

5. Simulation Analysis

In this section, we take the navigation system of high-orbit satellites based on GNSS in the high-orbit environment as the simulation background and use the observability theory and navigation filter optimization as the research methods to simulate and analyze the EKF and the navigation filter optimization method proposed in this paper.

5.1. Simulation Conditions

In this paper, we assume that the navigation system is the Beidou satellite navigation system. The initial epoch is UTC 0h0min0s on 28 May 2020. The Beidou navigation satellite orbit simulation data is obtained from the TLE data by the North American Aerospace Defence Command. Figure 2 is an illustration of the orbits of three BEIDOU navigation systems (contains BDS-1, BDS-2, BDS-3) including 49 satellites, and these orbits are represented by different colors. An inclined geo-synchronization orbit (IGSO) satellite is the user satellite. The coordinate system used in the simulation is the J2000.0 geocentric inertial coordinate system, the time system is Coordinated Universal Time (UTC). The simulation duration is 43080 s, and the sampling time interval is 4 s. In the experiment, only the main lobe signal of the navigation satellite has been considered, and the receiver sensitivity has not been considered. Set the half-angle of the main lobe signal beam as 21.3° , and the half-angle of the part occluded by the earth as 13.35° . The satellite orbit dynamics model of user satellite is set as a two-body model, with a mean square error of position error of 10 m and a mean square error of velocity error of 0.1 m/s [39,40], the mean square error of measured white noise is set to be 1 m, and the position and velocity at the initial moment are

$$X_0 = [2.242 \times 10^7 \text{ m } 3.257 \times 10^7 \text{ m } 1.539 \times 10^7 \text{ m } -2.139 \times 10^3 \text{ m/s } 469.418 \text{ m/s } 2.122 \times 10^3 \text{ m/s}],$$

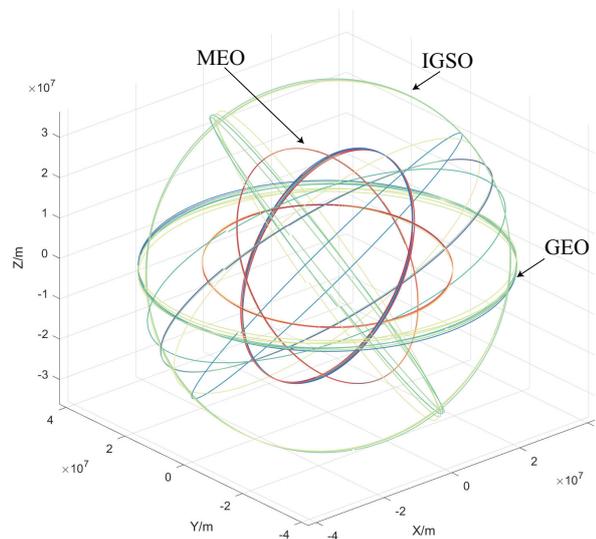


Figure 2. This is the Beidou navigation constellation map under J2000.0 geocentric inertial coordinate system.

The error covariance matrix at the initial moment is

$$P_0 = \text{diag}([10 \ 10 \ 10 \ 0.1 \ 0.1 \ 0.1]).$$

According to Section 3.2, due to the real-time requirements of the navigation system of high-orbit satellites, in the observability calculation matrix, the parameter m equals to 1. According to the selection criteria of the adjustment factor in Section 4, if the number of visible satellites is larger than 3, the observability mapping function is constructed as

$$(\psi_k)_i = \begin{cases} \frac{\eta_i}{\min_{i=1,2,3} \eta_i}, & i = 1, 2, 3 \\ \frac{\eta_i}{16}, & i = 4, 5, 6 \end{cases}$$

If the number of visible satellites is smaller than 3,

$$(\psi_k)_i = \begin{cases} 1.1, & i = 1, 2, 3 \\ \frac{\eta_i}{2}, & i = 4, 5, 6 \end{cases}$$

5.2. Simulation Results and Analysis

First, we obtain the number of the visible satellites at different times according to the geometric relationship between the navigation satellites and the user satellites, which is shown in Figure 3a. From the observability analysis of the system in Section 3.1, the rank of the observability discriminant matrix during the entire sampling duration is calculated to be 6 all the time, which shows that the system is observable during the entire simulation process. Then the condition number is calculated. The condition number of the observable matrix during the simulation process is shown in Figure 3b. According to the observability calculation formula in Section 3.2, the observable degree of each state component of the system is obtained. Figure 4a shows the observable degree of the position vector at different moments, and Figure 4b shows the observable degree of the velocity vector at different moments. In this paper, EKF and the improved adaptive filter are used to evaluate the overall system and each state variable. Figure 5 shows the average error of each state component in the corresponding sampling period. The filter results obtained are shown in Figure 6. Figure 6 reflects the comparison result between the residual error of EKF and adaptive filter estimation results for each state variable. Figure 7a,b reflect the comparison result between diagrams of EKF and adaptive filter position estimation error and velocity estimation error respectively. Figure 8a,b are the comparison figures of the filter position and velocity error of the period with high observability respectively. Taking 8000–10,000 sampling points as an example, Figure 9 shows the comparison result between the observability of the state variable component y and its residual error result.

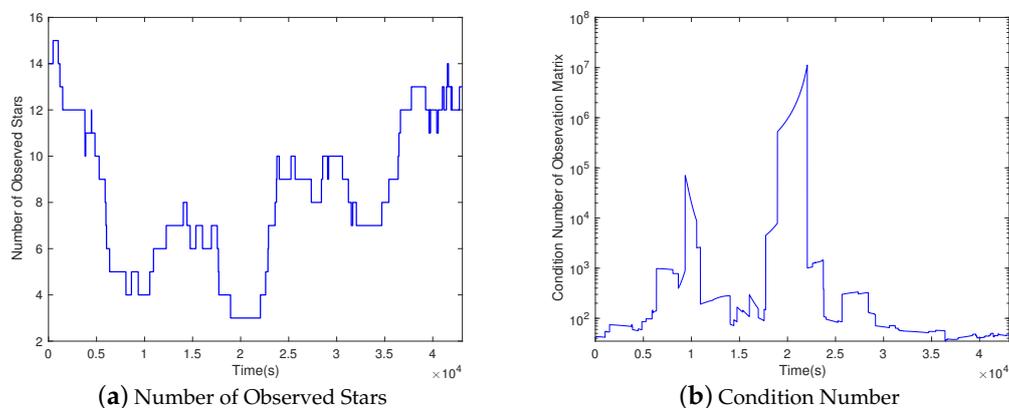


Figure 3. Figure (a) is the figure of number of observed stars. The abscissa is sampling time. Figure (b) is the figure of condition number of observation matrix. The abscissa is sampling time. The abscissas in follow figures are the same.

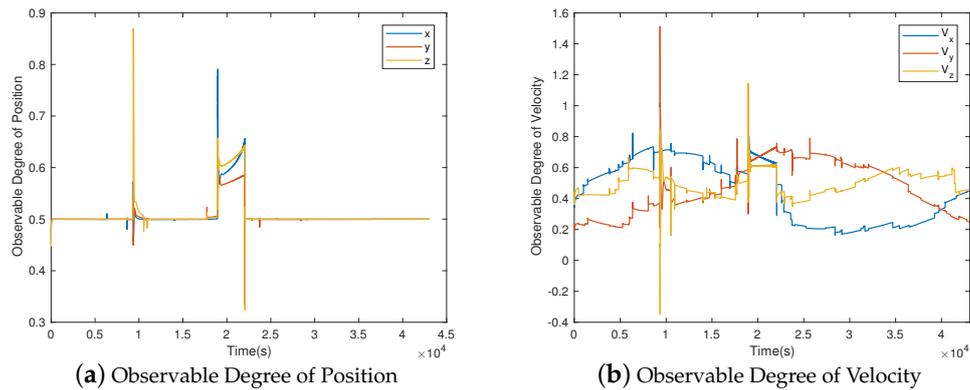


Figure 4. (a,b) show the observable degree of position and velocity respectively. The abscissa is sampling time.

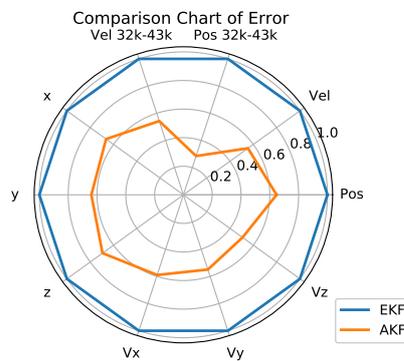


Figure 5. The radar figure shows the rate between estimation errors of EKF and AKF. The evaluation criterion is the mean error. The 'x','y','z','Vx','Vy','Vz' represent the each state variable respectively. The 'Pos','Vel' represent the position and velocity respectively. The 'Pos 32k-43k','Vel 32k-43k' represent the position and velocity during 32,000 s–43,080 s respectively.

According to the simulation experiment results above, this paper mainly analyzes from the following aspects:

(1) The results of the high-orbit satellite navigation are directly related to the number of the visible satellites and changes in observability. From Figures 3a,b and 4, it can be seen that the number of the visible satellites changes constantly in the simulation period. When the number decreases, the observability matrix condition number becomes larger, and the observability of the position of each state component becomes weaker, the filter estimation error also becomes larger. When the number of the visible satellites increases, the condition number of the observability matrix becomes smaller, and the observability of the position state component becomes stronger and the filter estimation has lower error.

(2) The accuracy of the adaptive filter based on observability has been significantly improved compared with conventional methods. It can be seen from Figures 5–9 that in the entire navigation process, the optimized navigation filter method can reduce the estimation error of position and velocity by about 36% and 44% respectively compared with the original EKF, which verifies the effectiveness of this method proposed in this article. It can be seen from Figure 8 that in the period when the number of visible satellites is large, the adaptive filter method will reduce the position error more significantly, and the result is reduced by nearly 70%. This shows that when there exist enough visible satellites, the observability of the overall state variable will be strong, and the proposed navigation filter optimization method will achieve better performance than the other cases.

(3) The observability calculation method for state components proposed in this paper can reflect the changes in the filter accuracy of each component. It can be seen from Figure 9 that the observable

degree of the state component y becomes smaller, its observability becomes relatively better, and the residual error in the y direction also rapidly becomes smaller. This result further verifies the relationship between observability and filter accuracy. When the observability is strong, the filter accuracy of the state variable is high.

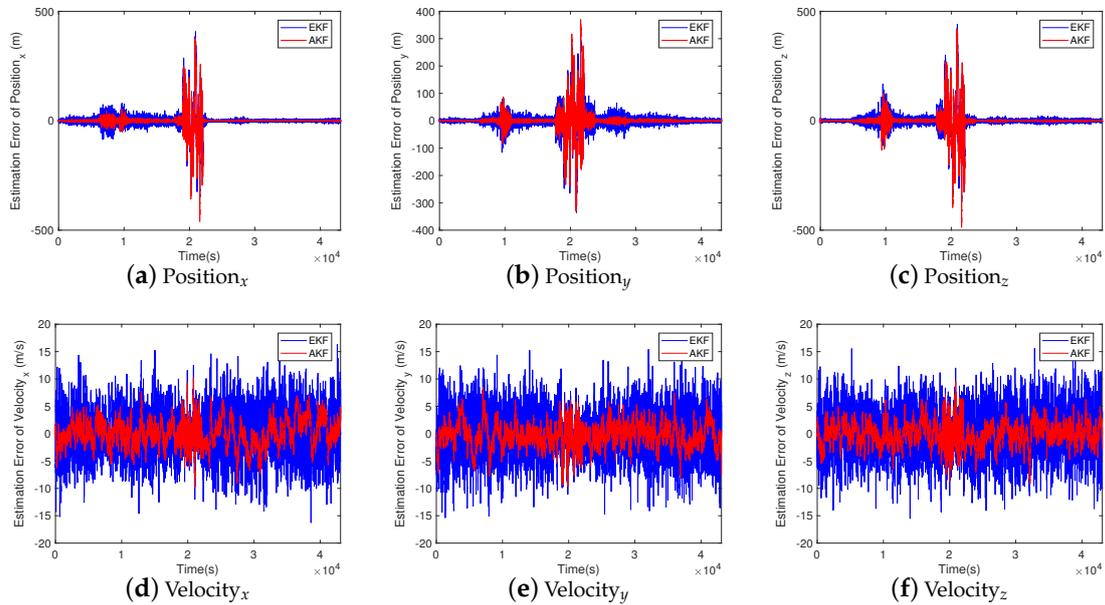


Figure 6. The figure reflects estimation error of each state variable between EKF and AKF.

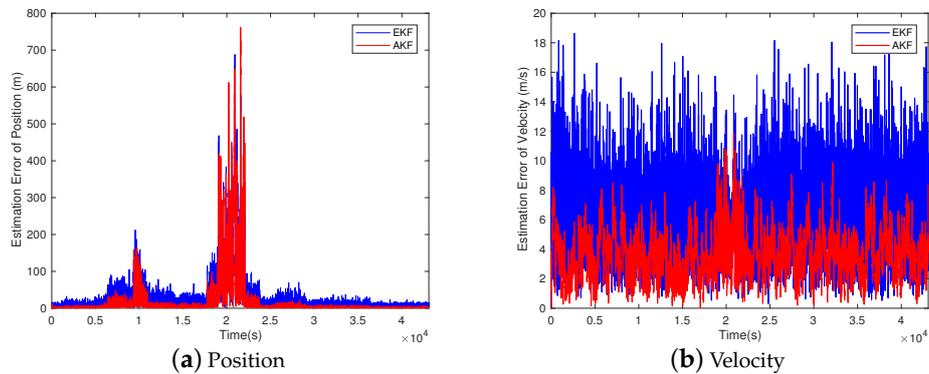


Figure 7. The figure reflects estimation error of position and velocity by EKF and AKF.

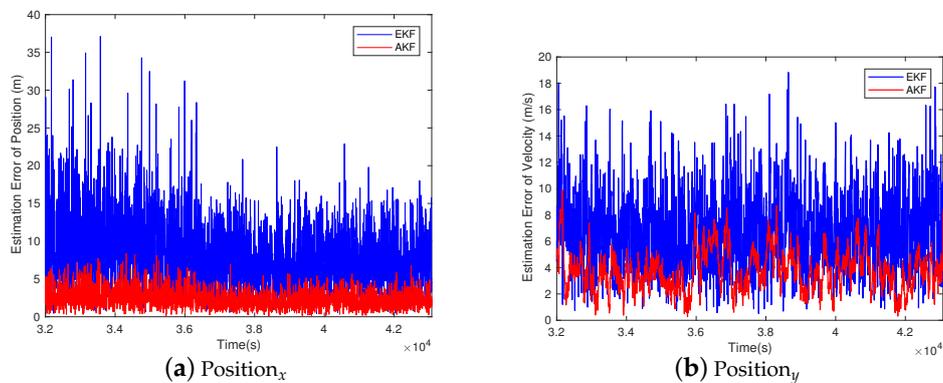


Figure 8. The figure reflects estimation error of position and velocity by EKF and AKF in period between 32,000 s and 43,080 s.

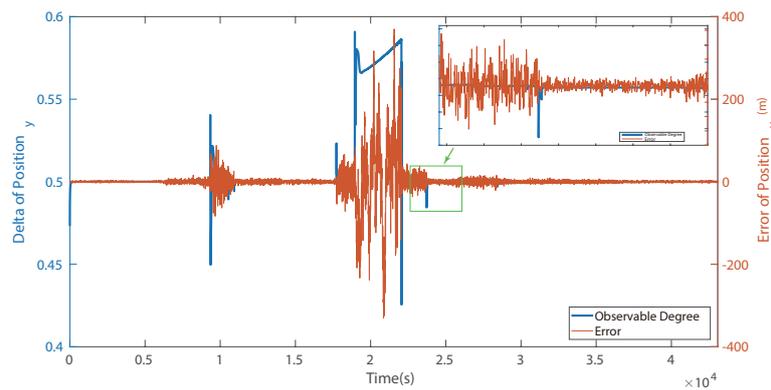


Figure 9. This is a comparison chart between error and observable degree at portion y.

6. Conclusions

To address the problem of the variable number of the visible satellites of the satellite navigation system in a high-orbit environment, this paper discusses the observability of the navigation system and proposes an adaptive navigation filtering optimization method based on the observability analysis. Firstly, the observability of the navigation system is analyzed, and a novel observability calculation method is proposed based on minimizing the objective function of error loss. Then the observable degree of different state variables can be derived. Secondly, according to the relationship between the observability and the filtering accuracy, a mapping value of the observable is applied as the feedback adjustment coefficient to construct an adaptive filter method. This method can combine the information of the previous period and the next period simultaneously to obtain feedback on the filter process. Through this approach, the estimation error can be effectively reduced. Simulation experiments on the high-orbit satellites of the Beidou navigation system show that the proposed adaptive EKF achieves better performance compared with traditional EKF, and verify the effectiveness and practicability of the novel method.

In order to further improve the navigation accuracy, the next main work will focus on the effective method of combining observability with other filter methods, and combining the optimization methods to improve the filter accuracy.

Author Contributions: Q.X. proposed the main idea, finished the draft manuscript; X.Z. and J.W. conceived of the experiments and drew the figures and tables; Methodology, Y.X.; Z.H. analyzed the data; H.Z. conducted the simulations. All authors have read and agreed to the published version of the manuscript.

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Abbreviations

The following abbreviations are used in this manuscript:

GNSS	Global Navigation Satellite System
EKF	Extend Kalman filter
AKF	adaptive Kalman filter
IGSO	inclined geo-synchronization orbit
UTC	Universal Time

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