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A New BDS-2 Satellite Clock Bias Prediction Algorithm with an Improved Exponential Smoothing Method

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Abstract: High-accuracy and dependable prediction of the bias of space-borne atomic clocks is extremely crucial for the normal operation of the satellites in the case of interrupted communication. Currently, the clock bias prediction for the Chinese BeiDou Navigation Satellite System (BDS) remains still a huge challenge. To develop a high-precision approach for forecasting satellite clock bias (SCB) in allusion to analyze the shortcomings of the exponential smoothing (ES) model, a modified ES model is proposed hereof, especially for BDS-2 satellites. Firstly, the basic ES models and their prediction mechanism are introduced. As the smoothing coefficient is difficult to determine, this leads to increasing fitting errors and poor forecast results. This issue is addressed by introducing a dynamic “thick near thin far (TNTF)” principle based on the sliding windows (SW) to optimize the best smoothing coefficient. Furthermore, to enhance the short-term forecasted accuracy of the ES model, the gray model (GM) is adopted to learn the fitting residuals of the ES model and combine the forecasted results of the ES model with the predicted results of the GM model from error learning (ES + GM). Compared with the single ES models, the experimental results show that the short-term forecast based on the ES + GM models is improved remarkably, especially for the combination of the three ES model and GM model (ES3 + GM). To further improve the medium-term prediction accuracy of the ES model, the new algorithms in ES with GM error learning based on the SW (ES + GM + SW) are presented. Through examples analysis, compared with the single ES2 (ES3) model, results indicate that (1) the average forecast precision of the new algorithms ES2 + GM + SW (ES3 + GM + SW) can be dramatically enhanced by 49.10% (56.40%) from 5.56 ns (6.77 ns) to 2.83 ns (2.95 ns); (2) the average forecast stability of the new algorithms ES2 + GM + SW (ES3 + GM + SW) is also observably boosted by 53.40% (49.60%) from 8.99 ns (16.13 ns) to 4.19 ns (8.13 ns). These new coupling forecast models proposed in this contribution are more effective in clock bias prediction both forecast accuracy and forecast stability.

Keywords: BeiDou satellite navigation system (BDS); satellite clock bias (SCB) prediction; exponential smoothing (ES) method; smoothing coefficient; thick near thin far (TNTF) principle; sliding windows (SW)

1. Introduction

The Chinese Beidou Navigation Satellite System (BDS), to date, has completed the construction of its regional system BDS-2, which consists of 15 satellites [1]. The system can provide positioning, navigation, timing, and short message communication services for users in the Asia and Pacific.

Over the past few years, it has made great strides in building global navigation systems such as BDS-3. By the end of 2018, a total of 19 BDS-3 satellites had been launched to enable the primary system for global services. With a total of 33 operational satellites, namely, 5 geostationary earth orbits (GEO), 7 inclined geosynchronous orbits (IGSO), and 21 medium earth orbits (MEO), the standard positioning accuracy of the orbits is achieved 5 m in both horizontal and vertical directions for the Asia-Pacific users and 10 m for global users approximately. By 23 June 2020, with the successful launch of the 55th networking satellite, it marks the complete networking of the BDS [2].

As one of the vital payloads of the Global Navigation Satellite System (GNSS), the performance of the satellite clocks and the accuracy, stability, and reliability of satellite clock bias (SCB) prediction are one of the critical factors affecting the positioning, navigation, and timing (PNT) service capabilities of the whole system [3–5]. Due to the rapid development of navigation satellite technology and its widespread use in practical applications, users' requirements for accuracy are getting higher and higher. Many factors affect the capability of the PNT services, among which the impact of the SCBs is one of the most important factors [6,7]. Even a clock bias of 1 ns will produce a range error of 0.3 m approximately, which seriously influences the positioning and timing accuracy of navigation systems [8,9]. In order to meet the requirements of users for centimeter-level positioning, it is essential to establish a high-precision time system. The clock bias products released by the international IGS server are a post-precision clock bias file with high accuracy, but the acquisition time is longer and cannot fulfill the necessities of real-time positioning [10–12]. The improvement of the precision of clock bias forecast will contribute to the acquisition of a priori information required for satellite autonomous navigation and the ability to improve precise point positioning (PPP) accuracy [13–15]. Therefore, studying how to further the stability and accuracy of clock bias prediction is of paramount importance for boosting the quality of clock bias products and the ability of navigation and positioning.

At present, in order to enhance the precision level of SCBs prediction, many scholars had conducted multi-angle, multi-directional, and in-depth studies on SCBs prediction from various aspects, and a variety of clock bias prediction models had been established, e.g., polynomial prediction model, gray model (GM), auto-regressive moving average model (ARMA), spectrum analysis model (SAM), Least Squares Support Vector Machines (LSSVM), functional network, etc. [16–21]. Since the interruption of satellite clock information will affect the accuracy of real-time PPP, some methods, such as the optimal arc length identifying, diverse polynomial power forecasting and empirical model made of a sixth-power harmonic function, etc., were proposed as a way to build up the performance of real-time PPP [22–24]. Furthermore, a predicted method of clock bias with a wavelet neural network (WNN) model was put forth, and some outcomes were obtained via the prediction tests. Compared with the IGS-P clock bias products, this method could better improve the prediction accuracy, but this approach did not consider the periodic errors and the frequency shift errors [25]. Then, considering the characteristics of the rate of change of the clock bias, an improved WNN model (its activation function was reconstructed) was put forward to predict SCBs of the GPS satellites, and the prediction results showed the model had higher prediction accuracy and robustness [26]. A mathematical approach based on stochastic differential equations was presented to forecast the behavior of in-orbit atomic clocks. Concurrently, they considered respective diverse situations with conclusive and stochastic signatures and optimized prediction results of clock bias. The most considerable issue was that the structure of this model was complex [27].

Nevertheless, the characteristics of the clock bias of the BDS are different from those of Galileo, GLONASS, and GPS because of its special constellation, which involves MEO satellites, GEO satellites, and IGSO satellites. Hence, the forecasting models of the Galileo, GLONASS, and GPS clock bias cannot be simply applied mechanically in the BDS-2 clock bias forecast. In 2020, the fractal behavior of the BDS clock bias was demonstrated by using the moving-average rescaled range analysis approach, and the results of this study suggested that the BDS clock bias had a memory [28]. Further, because of the advantages of simple modeling, easy calculation, less data to be stored, and the ability to quickly calculate new predicted values from recent observed values, the exponential smoothing (ES)

model is widely concerned in time series forecasting, such as transportation, power load forecasting, industry, and finance [29–32]. Up to now, very few studies have focused on the ES model in satellite clock forecasting. The ES model was first introduced into the short-term, medium-term, and long-term prediction of clock bias in 2017, but the selection of the smoothing coefficient was blind [33]. The 0.618 method was used to search for the best smoothing coefficient, and then, the model was established to predict the power load by using the found optimal smoothing coefficient. However, the optimization method was complex in operation and the forecast result was poor [34]. In the ES model, the determination of the smoothing coefficient is very complicated. In order to address this challenge, the “thick near thin far (TNTF)” principle is introduced to search for the best smoothing coefficient for better prediction. Then, in order to weaken the cumulative effect of errors due to the increase in forecast time, the idea of a sliding window (SW) is used to boost the forecast performance of the ES model employing the approach of a dynamic smoothing coefficient (ES based on SW). Besides, to further improve the forecast performance of the ES model, the GM model is used to learn the fitting error sequence of the ES model in the modeling process, and an SW is also adopted in the process (ES + GM based on SW), and then, the prediction results of the ES model and the GM model are combined as the final prediction clock bias. All these coupling algorithms are demonstrated by examples.

The remaining paper is organized as follows. The prediction theory of the ES + GM based on the SW idea is introduced—which includes the prediction principle of the ES method, the process of searching for the best smoothing coefficients utilizing the TNTF principle, the prediction principle of the ES model based on the SW idea, and the prediction principle of the ES + GM based on the SW idea—in Section 2. The sources of experimental data and the evaluation criteria used to evaluate the prediction performance of these coupling algorithms are introduced in Section 3. In Section 4, the forecasting performance of these coupling algorithms is demonstrated through examples and the forecasting advantages of these coupling algorithms are also analyzed in this section. Finally, some outcomes are drawn in Section 5.

2. Theory of Improved Exponential Smoothing Method Prediction

The modeling principles of the basic one exponential smoothing (ES1) model, two exponential smoothing (ES2) model, and three exponential smoothing (ES3) model are introduced first. Then, the principle and process of searching for the optimal smoothing coefficient using the TNTF principle is described in detail. Meanwhile, the prediction principles of the ES model based on the SW idea are introduced. Finally, the forecasting principles of the combined prediction method of the ES model and the GM model based on the SW idea are presented.

2.1. Exponential Smoothing Prediction Model

The ES model is a predicting method that is a weighted average of the clock bias time series observed in the past. The basic idea is that each time series has its inherent characteristics, that is, there is a certain basic mathematical model. However, the time series of actual observed clock bias not only embodies the above basic ideas but also reflects the characteristics of its random variation. In general, the impact of historical clock bias time series on future clock bias decreases with time gradually. Therefore, a more practical approach should be that the ES value of clock bias at the time t is equal to the weighted average of the actual observed value of clock bias at the time t and the ES value of clock bias at the time $t - 1$. In other words, the influence of the maximum and minimum is eliminated in the historical clock bias data to obtain the smoothed value of the clock bias time series. These smoothed clock biases are used as the forecast value of the clock bias at the future time.

In the whole forecasting process, the ES model can continuously correct new forecast values using forecast errors, i.e., the principle of “error feedback” can be utilized to correct the sequences of the forecasted clock bias, to boost the accuracy of the forecasted values of clock bias. The detailed process of the ES model is calculated as follows.

Suppose the current time is t , and the observed value of the corresponding clock error series is $x_1, x_2, \dots, x_t$, then the ES1 value of the clock bias time series at this time is [35,36]

$$S'_t = \alpha \cdot x_t + (1 - \alpha) \cdot S'_{t-1} \quad (1)$$

where S'_t and S'_{t-1} refer to the ES1 value of the observed clock bias time series at the time t and $t - 1$, respectively; α denotes the smoothing coefficient; and x_t is the time series of the observed clock bias at the time t .

Equation (1) can be described as follows:

$$\begin{cases} S'_{t-1} = \alpha \cdot x_{t-1} + (1 - \alpha) \cdot S'_{t-2} \\ S'_{t-2} = \alpha \cdot x_{t-2} + (1 - \alpha) \cdot S'_{t-3} \\ \vdots \\ S'_3 = \alpha \cdot x_3 + (1 - \alpha) \cdot S'_2 \\ S'_2 = \alpha \cdot x_2 + (1 - \alpha) \cdot S'_1 \end{cases} \quad (2)$$

Combined Equations (1) and (2) can be obtained as follows:

$$S'_t = \alpha \cdot \left\{ \sum_{i=0}^{t-1} [(1 - \alpha)^i \cdot x_{t-i}] \right\} + (1 - \alpha)^{t-1} \cdot S'_1 \quad (3)$$

At the time $t + 1$, the prediction model for the ES1 method can be written as

$$F_{t+1} = S'_t \quad (4)$$

where F_{t+1} indicates the predicted value of the observed clock bias time series at the time $t + 1$. It can be seen from Equation (4) that the predicted value of clock bias at time $t + 1$ is the exponential smoothing value at time t .

From Equations (1) and (4), the following equation can be derived.

$$F_{t+1} = \alpha \cdot x_t + (1 - \alpha) \cdot F_t \quad (5)$$

The ES2 is performed based on ES1, and then, the value of ES2 at time t is

$$S''_t = \alpha \cdot S'_t + (1 - \alpha) \cdot S''_{t-1} \quad (6)$$

where S'_t refer to the ES1 value of the observed clock bias sequences at time t ; S''_t and S''_{t-1} represent the ES2 values of the observed clock bias sequences at the time t and $t - 1$, respectively; and α is the smoothing coefficient.

In the same way, the ES3 is performed based on ES2, then the value of ES3 at time t is

$$S'''_t = \alpha \cdot S''_t + (1 - \alpha) \cdot S'''_{t-1} \quad (7)$$

where S''_t refers to the ES2 value of the observed clock bias sequences at time t ; S'''_t and S'''_{t-1} represent the ES3 values of the observed clock bias sequences at the time t and $t - 1$, respectively; and α is the smoothing coefficient.

The prediction model for the ES2 method can be written as [37]

$$F_{t+m} = A_t + B_t \cdot m \quad (8)$$

where $A_t = 2 \cdot S'_t - S''_t$, $B_t = \frac{\alpha(S'_t - S''_t)}{1 - \alpha}$, $m = 1, 2, \dots$; F_{t+m} represents the prediction value of the observed clock bias sequences at the time $t + m$.

The prediction model for the ES3 method can be written as [37]

$$F_{t+m} = A_t + B_t \cdot m + \frac{1}{2} \cdot C_t \cdot m^2 \quad (9)$$

where,

$$\begin{cases} A_t = 3 \cdot S'_t - 3 \cdot S''_t + S'''_t \\ B_t = \frac{\alpha \cdot [(6-5\alpha) \cdot S'_t - (10-8\alpha) \cdot S''_t - (4-3\alpha) \cdot S'''_t]}{2 \cdot (1-\alpha)^2} \\ C_t = \frac{\alpha^2 \cdot (S'_t - 2 \cdot S''_t + S'''_t)}{(1-\alpha)^2} \\ m = 1, 2, \dots \end{cases}$$

and F_{t+m} represents the forecast value of the observed clock bias sequences at the time $t + m$.

Since the ES method is a recursive calculation formula, for the convenience of calculation, it is usually expressed as follows:

$$S'_t = S''_t = S'''_t = x_1 \quad (10)$$

The most critical problem of the ES prediction model is how to determine an appropriate smoothing coefficient α accurately. Since different α will have a larger influence on the forecast result, the value of α should be chosen between the interval $(0, 1)$ according to the specific situation of the clock bias sequences. The value of α reflects the degree of correction. The larger the value of α , the larger the degree of correction; the smaller the value of α , the smaller the degree of correction. Generally, when there is a big random fluctuation in the time series data of the clock bias, a big α should be selected to keep up with the recent changes in the clock bias quickly. When the time series of the clock bias is relatively stable, a smaller α should be selected. To address this conundrum that the smoothing coefficient α is hard to determine in the ES forecast model, the principle of the TNTF is adopted to optimize the smoothing coefficient α to achieve a better forecasting results in this contribution.

The ES1 prediction model is applicable to time series with less pronounced change trends. The ES2 prediction model is suitable for time series like linear variation trends. The ES3 prediction model is mainly applied to time series with quasi-nonlinear variation trends. Considering the characteristics of clock bias time series changes, we analyze the forecasting performance of the ES2 model, the ES3 model, and the forecasting models using different modeling strategies for SCBs.

2.2. Optimize the Smoothing Coefficient Using the "TNTF" Principle

The TNTF principle means that the trend of clock bias over the future period depends more on the recent development law in the historical clock bias, while the correlation of the development trend between long-term historical and the future clock bias is weak. Principle TNTF has a variety of specific implementation methods, the method adopted in this contribution is to introduce a fitting error weight coefficient β (varies from 0 to 1), and the fitting error weight coefficient β is divided into 9 equal parts (0.1, 0.2, ..., 0.9). The fitting errors of the model are determined by employing the weighted mean absolute percentage error (WMAPE) as follows [38].

$$WMAPE = \frac{1}{n} \cdot \sum_{t=1}^n \frac{|\hat{X}_t - X_t| \cdot \beta^{n-t}}{X_t} \quad (11)$$

where the value of β represents the degree of TNTF principle; $\hat{X}_t$ is the analog value of the clock bias; X_t is the observed value of the clock bias; n is the number of the clock bias participating in modeling.

The most commonly used methods to search for the optimal smoothing coefficient α include the theoretical method and the empirical approach. In this contribution, the equipartition method is adopted. Compared with other methods, the equipartition method has the advantages of simplicity and intuition. The search process of α is as follows.

- (1) Values assigned to $\beta = 0.1, 0.2, \dots, 0.9$, respectively.

- (2) Divide α ($0 < \alpha < 1$) into 999 parts evenly, namely $\alpha_i = 0.001 \cdot i, i = 1, 2, \dots, 999$ ($\alpha_1 = 0.001, \alpha_2 = 0.002, \dots, \alpha_{999} = 0.999$).
- (3) We substitute α_i into Equations (3), (6), and (7) in turn, and then the corresponding S'_t, S''_t , and S'''_t can be calculated by combining Equation (10).
- (4) The analog values of the ES method can be obtained by substituting the calculated S'_t, S''_t , and S'''_t into Equations (8) and (9), respectively.
- (5) By substituting the analog values of the calculated clock bias and the actual observed values of the clock bias and the given β into Equation (11), the values WMAPE corresponding to a set of α_i can be obtained.
- (6) Repeat operation steps from (2) to (5) for each given β to obtain 9×999 corresponding WMAPE, and then, compare these values. The α_i corresponding to the smallest WMAPE is the optimal smoothing coefficient searched out. The flow of the specific optimization process is depicted in Figure 1.

Figure 1. Flow chart of the optimization process of the thick near thin far (TNTF) principle.

2.3. Prediction Principle of ES Based on Sliding Window

Since the smoothing coefficients of the ES method are difficult to determine and the error accumulation is relatively large as the forecast time increases, the algorithm is improved using the idea of sliding windows (SW) in forecasting SCBs. The flow of the SW method for forecasting SCBs is shown in Figure 2. The flow of the improved algorithm using the SW is shown in Figure 3. The process of forecasting the clock bias is divided into n steps. In each step, the best smoothing coefficient is chosen to reduce the prediction errors. The range of the smoothing coefficient is (0,1). In the first step, to obtain the optimal smoothing coefficient, the interval of (0,1) is divided into 999 parts on average, and the first value is 0.001 and the last value is 0.999. The above TNTF principle is employed in searching for the best smoothing coefficient. The found best smoothing coefficient is then used to develop an ES model to forecast the clock bias with the following length of L1. In the second step, the clock bias forecasted in the first step is used as the fitting sample for the ES model, and then, the best smoothing coefficient can be found employing the same method as in the first step. Based on the found optimal smoothing coefficient, an ES model is established to forecast the clock bias of the following length of L2. Repeat this until the end of the n th step. The final forecast clock bias is the sum of the forecast clock bias for each step. Taken together, the detailed steps for the ES model using the SW (ES + SW) are as follows. The flow of the ES + SW models are depicted in Figure 4.

Figure 2. Prediction flow chart of the sliding window method.

Figure 3. Flow chart of exponential smoothing (ES) clock bias prediction using a sliding window.

Algorithm 1 The ES model using the SW

Input: $X_t, \beta, i,$

Output: the values of the forecast clock bias

1: Initializes all parameters;

2: **Repeat**

3: **for** $i=1, 2, \dots, 999; \alpha=0.001, 0.002, \dots, 0.999$ **do**

4: Establish ES model;

5: Calculate corresponding WMAPE(i);

6: **until** find min WMAPE(i);

7: **return** the best $\alpha \leftarrow \min \text{WMAPE}(i);$

8: SW processing;

9: forecast length $L_1, L_2, \dots, L_n;$

10: the final clock bias $\leftarrow$ the sum of the clock bias corresponding to $L_1, L_2, \dots, L_n;$

11: **end**

Figure 4. Flow of the ES model using a sliding window.

Take the first part as an example, and the rest of the parts are similar to the first one.

Step 1 (parameter initialization). All parameters of the algorithm are initialized.

Step 2 (searching for the best smoothing coefficient). The interval of (0,1) is divided into 999 parts equally, and the first value is 0.001, and the last value is 0.999. The TNTF principle is adopted to search for the best smoothing coefficient.

Step 3 (modeling and forecasting). An ES model is developed for forecasting based on the searched best smoothing coefficients.

Step 4 (SW processing). Take the previous steps as an example; in the first step, the forecast is stopped when the length of the forecast reaches L_1 . In the second step, the clock bias from the first step forecast is used as the fitting sample for modeling, and the forecast is stopped when the length of the forecast reaches L_2 . Repeat until the end of the n th step, and the final complete forecast clock bias sequences are the sum of the clock errors corresponding to $L_1 + L_2 + \dots + L_n$.

2.4. Prediction Principle of ES + GM Using Sliding Window

To further boost the accuracy of clock bias prediction, the GM model is used to learn the fitting errors generated by the ES model during the modeling process, and forecast the value of the fitting errors of the ES model. The final prediction values of clock bias can be obtained by combining the predicted values of the ES model and the GM model. The detailed process of this fusion algorithm is shown in Figure 5. The specific steps for this fusion algorithm of the ES model and the GM model using the SW (ES + GM + SW) are as follows.

Figure 5. Flow chart of ES + GM clock bias prediction using a sliding window. GM, gray model.

Take the first part as an example, and the rest of the parts are similar to it. Steps 1 to 3 of ES + GM + SW are the same as ES + SW (see Section 2.3).

Step 4 (GM forecast). During step 3, an ES model is built based on the optimal smoothing coefficients, which generates a fitting error series during the modeling process, and then, the GM model is used to learn the fitting error series and forecast.

Step 5 (SW processing). The processing method of this step is similar to step 4 of the ES model based on the SW.

Step 6 (fusion forecasting). Combining the prediction value of the ES model and the prediction value of the GM model in each part, the final prediction value of clock bias can be obtained.

In step 4, the GM model is used to learn the fitting error sequence of the ES model, and the modeling process is as follows.

Suppose that the fitting error sequence of the ES model is $\varepsilon^{(0)} = \{\varepsilon^{(0)}(1), \varepsilon^{(0)}(2), \dots, \varepsilon^{(0)}(n)\}$, a new sequence is generated by one-time accumulation as follows [39].

$$\varepsilon^{(1)} = \left\{ \varepsilon^{(0)}(1), \sum_{i=1}^2 \varepsilon^{(0)}(i), \dots, \sum_{i=1}^n \varepsilon^{(0)}(i) \right\} \quad (12)$$

Establish a first-power differential equation for the sequences $\varepsilon^{(1)}$ as shown below.

$$\frac{d\varepsilon^{(1)}}{dt} + a\varepsilon^{(1)} = u \quad (13)$$

The first-power differential equation (13) could be described by the matrix equation form (14) after discretization.

$$\mathbf{Y} = \mathbf{G} \cdot \mathbf{A} \quad (14)$$

where,

$$\mathbf{Y} = \begin{bmatrix} \varepsilon^{(0)}(2) \\ \varepsilon^{(0)}(3) \\ \vdots \\ \varepsilon^{(0)}(n) \end{bmatrix}, \mathbf{G} = \begin{bmatrix} -\frac{[\varepsilon^{(1)}(2) + \varepsilon^{(1)}(1)]}{2} & 1 \\ -\frac{[\varepsilon^{(1)}(3) + \varepsilon^{(1)}(2)]}{2} & 1 \\ \vdots & \vdots \\ -\frac{[\varepsilon^{(1)}(n) + \varepsilon^{(1)}(n-1)]}{2} & 1 \end{bmatrix}, \mathbf{A} = \begin{bmatrix} a \\ u \end{bmatrix} \quad (15)$$

The least-squares solution to the matrix Equation (14) is obtained employing the least-squares method (LSM) as follows.

$$\hat{\mathbf{A}} = \begin{bmatrix} \hat{a} & \hat{u} \end{bmatrix} = (\mathbf{G}^T \mathbf{G})^{-1} \mathbf{G}^T \mathbf{Y} \quad (16)$$

Substituting Equation (16) into Equation (13), the following Equation (17) can be obtained.

$$\frac{d\varepsilon^{(1)}}{dt} + \hat{a}\varepsilon^{(1)} = \hat{u} \quad (17)$$

The solution of the time response function (17) can be expressed as

$$\varepsilon^{(1)}(k+1) = \left[\varepsilon^{(0)}(1) - \frac{\hat{u}}{\hat{a}} \right] \cdot e^{-\hat{a} \cdot k} + \frac{\hat{u}}{\hat{a}} \quad (18)$$

Since sequence $\varepsilon^{(1)}$ is the cumulative sequence of sequence $\varepsilon^{(0)}$, the prediction model for the fitting error sequence $\varepsilon^{(0)}$ of the ES model can be written as follows:

$$\hat{\varepsilon}^{(0)}(k+1) = (1 - e^{\hat{a}}) \cdot \left[\varepsilon^{(0)}(1) - \frac{\hat{u}}{\hat{a}} \right] \cdot e^{-\hat{a}k} \quad (19)$$

3. Experimental Data and Evaluation Indicators

The data source selected for the experiment and analysis is first introduced in detail. Furthermore, evaluation indicators used to evaluate the prediction performance of these models are then presented.

3.1. Experimental Datasets

To fully investigate the forecasting performance of these model aforementioned in this contribution, the clock bias of the BDS-2 satellites with the first day to the second day of the 737 weeks (16–17 February 2020) released by the International GNSS Monitoring and Assessment System (iGMAS) were selected as experimental data.

In order to make the research results provide some references for PPP, these data were interpolated into the clock bias of the 30 s intervals before modeling and predicting. There are three types of BDS-2 satellites: the GEO satellites, the IGSO satellites, and the MEO satellites, with a total of 15 satellites, containing 5 GEO satellites, 7 IGSO satellites, and 3 MEO satellites. The clock bias of two satellites with continuous and complete data with no clock jumps and outliers were selected from each type of satellite for the experiments. The selected satellites are C 01 (GEO), C 05 (GEO), C 10 (IGSO), C 16 (IGSO), C 11 (MEO), and C 12 (MEO), respectively, with a total of six satellites. As of February 2020, information about these satellites is listed in Table 1 [40].

Table 1. Information on selected satellites (as of February 2020).

PRN	Satellite No.	NORAD.ID	SVN	Int. sat. ID	Manuf.	Notes
C 01	GEO-01	36287	C 003	2010-001 A	2G-CAST	140.0° E; launched 17 January 2010
C 05	GEO-05	38091	C 011	2012-008 A	2G-CAST	58.75° E; launched 25 February 2012
C 10	IGSO-05	37948	C 010	2011-073 A	2I-CAST	~95° E; launched 2 December 2011
C 16	IGSO-07	43539	C 019	2018-057 A	2I-CAST	~95° E; launched 10 July 2018
C 11	MEO-03	38250	C 012	2012-018 A	2M-CAST	Slot A-7; launched 30 April 2012
C 12	MEO-04	38251	C 013	2012-018 B	2M-CAST	Slot A-8; launched 30 April 2012

3.2. Evaluation Criteria

To fully assess the predicted performances of these models, the root mean square (RMS) and Range (the absolute value of the error between the maximum errors and the minimum errors) are used as the statistics to evaluate the models' forecast results. Among them, the RMS represents the forecast accuracy of the model, and the Range denotes the stability of the model. The expression of RMS and Range can be written as [41]

$$RMS = \sqrt{\frac{\sum_{i=1}^n (error^{(i)})^2}{n}} \quad (20)$$

$$Range = |error_{\max}^{(i)} - error_{\min}^{(i)}| \quad (21)$$

where, $error^{(i)} = \hat{x}_i - x_i$, ($i = 1, 2, \dots, n$) refer to the forecast error of the model; $\hat{x}_i$ and x_i indicate the predicted and real clock bias in epoch i , respectively; n is the number of observed clock bias. $error_{\max}^{(i)}$ and $error_{\min}^{(i)}$ represent the maximum and minimum value of the model prediction errors, respectively.

As shown in Equations (20) and (21), values of RMS and Range will be obtained by using different models. Also, the smaller the values of RMS and Range represent the better-predicted performances of the model.

4. Results and Discussion

Five modeling strategies are used to predict SCBs for short-term prediction first, and the forecasting performance of these models is analyzed in detail. Then, the medium-term prediction of SCBs is performed using seven modeling strategies, and the prediction performance of these models is analyzed.

4.1. Short-Term Forecast Performance Analysis

The first day clock bias data of the 737 weeks (16 February 2020) are used to build a model. Then, the established model can be applied to forecast the clock bias data for the next 6 h. Since the length of the forecast is not long, SW processing is not adopted. The modeling strategies are as follows.

(1) The ES2 and ES3 models are built using the first day clock bias data of the 737 weeks (16 February 2020), respectively, and the established ES2 and ES3 models can be employed to predict the next 6 h clock bias data.

(2) To boost the forecast accuracy of the ES2 (ES3) model, the GM model is utilized to learn the fitting error of the ES2 (ES3) model during the modeling process. In other words, a combined forecasting model of the ES2 (ES3) model and the GM model (abbreviated as ES2 + GM and ES3 + GM) is developed. The coupling model algorithm is as follows. First, an ES2 (ES3) model is set up employing the clock bias data of the first day of the 737 weeks (16 February 2020), and the established ES2 (ES3)

model can be applied to predict the next 6 h clock bias data. Then, the GM model is applied to learn the fitting error of the ES2 (ES3) model during the modeling process, and the predicted results of the fitting error for the next 6 h will be obtained. The final predicted clock bias is the sum of the clock bias predicted by the ES2 (ES3) model and the predicted value of the fitting error predicted by the GM model.

(3) Besides this, to serve as a comparison algorithm, the single GM model is built using the clock error of the first day of the 737 weeks (16 February 2020) and forecast the clock bias data of the next 6 h.

For convenience, the labels “GM,” “ES2,” “ES2 + GM,” “ES2 + SW,” “ES2 + GM + SW,” “ES3,” “ES3 + GM,” “ES3 + SW,” and “ES3 + GM + SW,” stand for the gray model, two exponential smoothing model, the combined model using the gray model to learn the fitting error of two exponential smoothing model, two exponential smoothing model using sliding windows, the combination of two exponential smoothing model and gray model using sliding windows, three exponential smoothing model, the combined model using the gray model to learn the fitting error of three exponential smoothing model, three exponential smoothing model using sliding windows, and the combination of three exponential smoothing model and gray model using sliding windows, respectively, in named system of Figures 6–13 and Tables 2–5.

Figure 6. Variation diagram of 6 h satellite clock bias forecast errors using different models: (a) is the changes of the C 01 SCBs forecast, (b) is the changes of the C 05 SCBs forecast, (c) is the changes of the C 10 SCBs forecast; (d) is the changes of the C 16 SCBs forecast, (e) is the changes of the C 11 SCBs forecast, and (f) is the changes of the C 12 SCBs forecast.

Figure 7. Variation diagram of 6 h forecast accuracy (RMS) and stability (Range) using different models: (a) is the changes of RMS, and (b) is the changes of Range.

Figure 8. Boxplot change diagram of 6 h prediction accuracy (a) and stability (b) using different models.

Figure 9. The average accuracy (RMS), stability (Range), and improvement percentage of 6 h forecast using different models: (a) is the changes of RMS and Range, and (b) is the changes of improvement percentage of RMS and Range.

Figure 10. Variation diagram of 24 h satellite clock bias forecast errors using different models: (a) is the changes of the C 01 SCBs forecast, (b) is the changes of the C 05 SCBs forecast, (c) is the changes of the C 10 SCBs forecast; (d) is the changes of the C 16 SCBs forecast, (e) is the changes of the C 11 SCBs forecast, and (f) is the changes of the C 12 SCBs forecast.

Figure 11. Variation diagram of 24 h forecast accuracy (RMS) and stability (Range) using different models: (a) is the changes of RMS, and (b) is the changes of Range.

Figure 12. Boxplot change diagram of 24 h prediction accuracy (a) and stability (b) using different models.

Figure 13. Change of average accuracy (RMS), stability (Range), and improvement rate of 24 h forecast using different models: (a) is the changes of RMS and Range, and (b) is the changes of improvement percentage of RMS and Range.

Table 2. Statistics of the different models for 6 h prediction (ns).

Satellite No.		GM ¹		ES2 ¹		ES2 + GM ¹		ES3 ¹		ES3 + GM ¹	
		RMS	Range	RMS	Range	RMS	Range	RMS	Range	RMS	Range
GEO	C 01	6.49	9.60	6.34	2.12	6.23	2.08	6.47	2.05	5.09	1.86
	C 05	14.05	7.47	4.94	1.57	3.69	1.16	3.29	7.99	1.23	4.21
	C 10	9.90	3.48	6.77	2.15	6.03	1.57	7.18	3.38	6.13	1.49
IGSO	C 16	0.69	1.03	0.34	0.99	0.29	0.83	0.41	1.21	0.41	1.21
	C 11	5.24	2.87	2.44	1.02	1.90	1.01	2.92	2.44	2.08	2.04
MEO	C 12	2.09	0.54	1.23	0.66	1.00	0.69	1.29	0.79	1.16	1.22

¹: GM means the gray model, ES2 means the two exponential smoothing model, ES2 + GM means the combined model using the gray model to learn the fitting error of the two exponential smoothing model, ES3 means the three exponential smoothing model, and ES3 + GM means the combined model using the gray model to learn the fitting error of the three exponential smoothing model.

Table 3. Statistics of the average prediction precision (RMS), stability (Range), and improvement of the different models for 6 h (ns).

GM		ES2		ES2 + GM		IMP ¹ (%)		ES3		ES3 + GM		IMP ² (%)	
RMS	Range	RMS	Range	RMS	Range	RMS	Range	RMS	Range	RMS	Range	RMS	Range
6.41	4.17	3.68	1.42	3.19	1.22	13.30	14.10	3.59	2.98	2.68	2.01	25.30	32.60

¹: IMP presents the improvement of the ES2 + GM compared with the single ES2 model (Unit: %). ²: IMP presents the improvement of the ES3 + GM compared with the single ES3 model (Unit: %).

Table 4. Statistics of the different models for 24 prediction (ns).

Strategies	Criteria	GEO		IGSO		MEO	
		C 01	C 05	C 10	C 16	C 11	C 12
GM	RMS	36.90	43.91	16.38	3.77	12.93	3.51
	Range	73.10	68.98	20.45	7.05	19.64	4.09
ES2	RMS	8.79	7.95	9.08	1.89	3.39	2.24
	Range	21.38	11.00	9.09	5.22	3.96	3.27
ES2 + SW ¹	RMS	7.10	3.72	3.78	1.40	1.95	1.13
	Range	17.08	9.86	7.36	3.22	3.82	2.94
ES2 + GM + SW ¹	RMS	6.14	3.16	3.91	1.27	1.67	0.81
	Range	5.73	6.86	4.61	2.79	3.00	2.48
ES3	RMS	9.21	8.11	9.01	2.96	8.41	2.93
	Range	20.06	29.00	19.58	6.58	14.69	6.85
ES3 + SW ¹	RMS	8.30	7.36	5.67	1.04	4.47	1.18
	Range	15.75	27.69	15.96	4.82	14.03	4.88
ES3 + GM + SW ¹	RMS	5.41	3.75	4.61	0.67	2.21	1.05
	Range	6.09	13.49	13.12	3.35	8.20	4.53

¹: ES2 + SW implies the two exponential smoothing model with the sliding window, ES2 + GM + SW implies the combined model of the ES2 + GM with the sliding window, ES3 + SW implies the three exponential smoothing model with the sliding window, and ES3 + GM + SW implies the combined model of the ES3 + GM using the sliding window.

Figure 6 shows the error change of 6 h SCBs forecast using different models, where Figure 6a is the changes of the C 01 (GEO) SCBs forecast, Figure 6b is the changes of the C 05 (GEO) SCBs forecast, Figure 6c is the changes of the C 10 (IGSO) SCBs forecast, Figure 6d is the changes of the C 16 (IGSO) SCBs forecast, Figure 6e is the changes of the C 11 (MEO) SCBs forecast, and Figure 6f is the changes of the C 12 (MEO) SCBs forecast.

Figure 7 shows the variation of 6 h forecast accuracy (RMS) and stability (Range) using different models, where Figure 7a is the changes of forecast accuracy (RMS) and average forecast accuracy (RMS), and Figure 7b is the changes of forecast stability (Range) and average forecast stability (Range).

Figure 8 shows the boxplot changes of 6 h prediction accuracy (RMS) and stability (Range) using different models, where Figure 8a is the boxplot changes of RMS of prediction errors, and Figure 8b is the boxplot changes of Range of prediction errors.

Figure 9 shows the changes of the average accuracy (RMS), stability (Range), and improvement percentage for 6 h prediction using different models, where Figure 9a is the changes of the average of RMS and Range, Figure 9b is the improvement of the ES2 + GM (ES3 + GM) model compared to the ES2 (ES3) model. The 6 h prediction statistics for the different satellites with different models are listed in Table 2, and the statistics of the average forecast precision (RMS), stability (Range), and improvement of the different models for 6 h prediction are shown in Table 3.

From Figures 6–9 and Tables 2 and 3, we can see that in terms of a single model, the average forecast accuracy of the ES3 model is roughly the same as that of the ES2 model, while the average forecast accuracy of the ES3 model is slightly better than that of the ES2 model. The average forecast accuracy of the GM model is worse than that of the ES2 model. As for the combined model, the GM model is used to learn the fitting errors of the ES model and forecast the values of the fitting errors. The comparative analysis between the combined models (ES2 + GM and ES3 + GM) and the single ES models (ES2 and ES3) show that the prediction results of the combined ES model can boost the forecast performance of the ES model. Especially, the performance enhancement of the ES3 model is remarkable and their prediction accuracy has been further heightened. Compared with the single ES (ES2 and ES3) model, the average forecast accuracy of the ES2 + GM (ES3 + GM) model has been increased by 13.30% (25.30%) from 3.68 ns (3.59 ns) to 3.19 ns (2.68 ns), and the average forecast stability of the ES2 + GM (ES3 + GM) model has been increased by 14.10% (32.60%) from 1.42 ns (2.98 ns) to 1.22 ns (2.01 ns).

Table 5. Statistics of the average prediction precision (RMS), stability (Range), and improvement of the different models for 24 h (ns).

GM		ES2		ES2 + SW		IMP ¹ (%)		ES2 + GM + SW		IMP ² (%)		ES3		ES3 + SW		IMP ¹ (%)		ES3 + GM + SW		IMP ² (%)	
RMS	Range	RMS	Range	RMS	Range	RMS	Range	RMS	Range	RMS	Range	RMS	Range	RMS	Range	RMS	Range	RMS	Range	RMS	Range
19.57	32.22	5.56	8.99	3.18	7.38	42.80	17.90	2.83	4.19	11.00	43.20	6.77	16.13	4.67	13.86	31.00	14.10	2.95	8.13	36.80	41.30

¹: IMP represents the improvement of the ES2 + SW (ES3 + SW) compared with the single ES2 (ES3) model (Unit: %). ²: IMP represents the improvement of the ES2 + GM + SW (ES3 + GM + SW) compared with that of the ES2 + SW (ES3 + SW) model (Unit: %).

In general, apart from the prediction algorithm, the MEO satellites accomplish a fairly superior forecast accuracy, followed by IGSO satellites. The worst is GEO satellites, which are probably related to the poor observed structures and observations.

4.2. Medium-Term Forecast Performance Analysis

The clock bias data of the first day of the 737 weeks (16 February 2020) are employed to build a model. Then, the established model can be applied to predict the next 24 h clock bias. The modeling strategies are as follows.

(1) The ES2 and ES3 models are established employing the clock bias data of the first day of the 737 weeks (16 February 2020), respectively, and the established ES2 and ES3 models can be employed to predict the next 24 h clock bias data.

(2) Because of the longer forecast length, to boost the forecast accuracy of the ES2 (ES3) model, SW processing is adopted, that is, a combined forecasting model of the ES2 (ES3) model using the SW processing (abbreviated as ES2 + SW and ES3 + SW), is established. The coupling model algorithm is as follows. The length of the forecast is divided into two parts equally. First, an ES2 (ES3) model is developed by using the clock bias of the first day of 737 weeks (16 February 2020) to forecast the next 12 h clock bias data. Then, the 12 h clock bias predicted in the previous step and the last 12 h clock bias of the first day of the 737 weeks (16 February 2020), the 24 h clock bias data are used to establish an ES2 (ES3) model to forecast the clock bias for the next 12 h. Finally, the final forecasted clock bias is the combination of the clock bias of the first part of the forecast and the clock bias of the second part of the forecast.

(3) Since the forecast time is relatively long, to further improve the forecast accuracy of the ES2 (ES3) model, the GM model is applied to learn the fitting error of the ES2 (ES3) model during the modeling process, and then, the SW processing method is employed to establish a combination forecast model: the ES2 (ES3) model and GM model using SW processing (abbreviated as ES2 + GM + SW and ES3 + GM + SW). The coupling model algorithm is as follows. The length of the forecast is divided into two parts equally. First, an ES2 (ES3) model is developed by using the clock bias of the first day of 737 weeks (16 February 2020) to predict the next 12 h clock bias data. Then, the GM model is applied to learn the fitting error of the ES2 (ES3) model during the modeling process, and the predicted values of the fitting error for the next 12 h will be obtained. The clock bias of the first part is equal to the sum of the clock bias predicted by the ES2 (ES3) model and the forecast value of the fitting error predicted by the GM model. Then, the SW processing is adopted, and the 12 h clock bias predicted in the previous step and the last 12 h clock bias of the first day of the 737 weeks (16 February 2020), the 24 h clock bias data, are used to establish an ES2 (ES3) model to forecast the clock bias for the next 12 h. Then, using the same processing method as the first part, the clock bias of the second part is equal to the sum of the clock bias predicted by the ES2 (ES3) model and the forecast value of the fitting error predicted by the GM model. Finally, the final forecasted clock bias is the combination of the clock bias of the first part and the clock bias of the second part.

(4) Besides this, the single GM model as a comparison algorithm is established, employing the clock bias of the first day of the 737 weeks (16 February 2020) and forecast the clock bias data of the next 24 h.

Figure 10 shows the error changes of 24 h SCBs forecast using different prediction models, where Figure 10a is the changes of the C 01 (GEO) SCBs forecast, Figure 10b is the changes of the C 05 (GEO) SCBs forecast, Figure 10c is the changes of the C 10 (IGSO) SCBs forecast, Figure 10d is the changes of the C 16 (IGSO) SCBs forecast, Figure 10e is the changes of the C 11 (MEO) SCBs forecast, and Figure 10f is the changes of the C 12 (MEO) SCBs forecast.

The changes in the bar chart for 24 h forecast accuracy (RMS) and stability (Range) using different models are shown in Figure 11, where Figure 11a is the changes of the forecast accuracy (RMS) for the different satellites, and Figure 11b is the changes of the forecast stability (Range) for the different satellites.

Figure 12 shows the boxplot changes of 24 h prediction accuracy and stability using different models, where Figure 12a is the boxplot changes of RMS of prediction errors, and Figure 12b is the boxplot changes of Range of prediction errors.

Figure 13 reveals the changes of the average accuracy (RMS), stability (Range), and improvement percentage for 24 h prediction, where Figure 13a is the changes of the average of RMS and Range, Figure 13b is the improvement of the ES2 + SW, ES2 + GM + SW, ES3 + SW, and ES3 + GM + SW model compared to the ES2, ES2 + SW, ES3, and ES3 + SW model, respectively. The 24 h prediction statistics for the different satellites with different models are exhibited in Table 4, and the Statistics of the average forecast precision (RMS), stability (Range), and improvement of the different models for 24 h prediction are shown in Table 5.

From Figures 10–13 and Tables 4 and 5, it can be seen that the medium-term forecast performance of the ES2 model is slightly better than that of the ES3 model with respect to single-model forecasting. These mean that the ES3 model is more suitable for short term forecast, while the ES2 model is more suitable for medium-term forecast by comparison with the abovementioned parts. Compared with the prediction performances of the ES2 model and the ES3 model, the GM model has a slightly worse performance in medium-term prediction. The forecast performance of both the ES2 model and ES3 model has been boosted to a great degree after using the SW processing.

Compared with the single ES2 (ES3) model, the average forecast accuracy of the combined model of the ES2 (ES3) model and the SW processing has been increased by 42.80% (31.00%) from 5.56 ns (6.77 ns) to 3.18 ns (4.67 ns), and the average forecast stability has been increased by 17.90% (14.10%) from 8.99 ns (16.13 ns) to 7.38 ns (13.86 ns). The aforementioned comparative analysis shows that the use of the SW processing strategy can significantly boost the forecast performance of the ES model, especially for the ES2 model, both in terms of forecast accuracy and forecast stability.

The forecast accuracy and stability of these fusion prediction algorithms are further improved by using the GM model to learn the fitting errors of the ES model in the modeling process followed based upon the SW processing. Compared with the single ES2 (ES3) model that does not use the GM model to learn the fitting error, the average forecast accuracy of these fusion algorithms is increased by 11.00% (36.80%) from 3.18 ns (4.67 ns) to 2.83 ns (2.95 ns), and the average forecast stability is increased by 43.20% (41.30%) from 7.38 ns (13.86 ns) to 4.19 ns (8.13 ns). Overall, the GM model is used to learn the fitting error of the ES model, and then the fusion prediction algorithm with the SW processing can significantly improve the performance of the ES model. In particular, the performance of the medium-term prediction of the ES3 model has been improved greatly. In addition, both single algorithm and fusion algorithm have better prediction performance for recently launched satellites than for early launched satellites.

5. Conclusions

In the applications of the GNSS, the clock bias forecast is a critical issue. The accuracy and stability of SCBs prediction determine the accuracy of satellite navigation and positioning directly. For purpose of improving the accuracy and stability of clock bias prediction, a new fusion prediction algorithm is proposed in this contribution to perform short-term and medium-term prediction of clock bias. The clock bias data of the BDS-2 satellites with the first day to the second day of the 737 weeks of the DOY released by the iGMAS are selected as experimental data to assess the prediction performance of these new models. For the comparison and analysis, the single GM, ES2, and ES3 models are regarded as reference, then the prediction performance of the ES2 + SW, ES3 + SW, ES2 + GM + SW, and ES3 + GM + SW model for clock bias are then analyzed.

As for the short-term clock bias prediction, the method of predicting after learning the fitting error for the ES model during the modeling process using the GM model can enhance the forecast performance of the ES model dramatically: both the forecast accuracy and the stability of the forecast. In particular, the average forecast accuracy of the ES3 model has the greatest improvement, up to 25.30% going from 3.59 ns to 2.68 ns, and the average forecast stability of the ES3 model has the

greatest improvement, up to 32.60% going from 2.98 ns to 2.01 ns. With respect to the medium-term prediction of clock bias, the fusion prediction algorithm using two processing strategies can improve the forecasting performance of the ES model, whether in terms of forecast accuracy or forecast stability. Above all, the most significant improvement in the prediction performance of the ES model is achieved by using a GM model to learn the fitting errors during the modeling process, followed by an SW processing strategy. Its average forecast accuracy is improved by 49.10% (56.40%) from 5.56 ns (6.77 ns) to 2.83 ns (2.95 ns), and the average forecast stability is improved by 53.40% (49.60%) from 8.99 ns (16.13 ns) to 4.19 ns (8.13 ns), compared with the single ES2 (ES3) model.

Overall, this new fusion prediction algorithm is advantageous and can be used to forecast the BDS-2 clock bias with high-precision both in the short-term and medium-term. Therefore, it is recommended that this new algorithm can be utilized to predict clock bias for BDS-2 satellites. Furthermore, this study provides a theoretical reference for SCBs in terms of high-precision prediction and practical applications, and also provides a new guideline for continued research in SCBs forecast technology in the future.

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Abbreviations

Abbreviation	The full name
BDS	BeiDou Navigation Satellite System
SCB	Satellite Clock Bias
ES	Exponential Smoothing
TNTF	Thick Near Thin Far
SW	Sliding Windows
GM	Gray Model
ES1	One Exponential Smoothing
ES2	Two Exponential Smoothing
ES3	Three Exponential Smoothing
ES + SW	Exponential Smoothing using Sliding Windows
ES + GM	the combination model of the ES model and the GM
ES2 + GM	the combination model of the ES2 model and the GM
ES3 + GM	the combination model of the ES3 model and the GM
ES2 + GM + SW	the combination of the ES2 model and the GM using sliding windows
ES3 + GM + SW	the combination of the ES3 model and the GM using sliding windows
GNSS	Global Navigation Satellite System
GPS	Global Positioning System
GLONASS	GLObal NAvigation Satellite System
PNT	Positioning, navigation, and timing
IGS	International GNSS Service
PPP	Precise point positioning
ARMA	Auto-Regressive Moving Average model
SAM	Spectrum Analysis Model
LSSVM	Least Squares Support Vector Machines
WNN	Wavelet Neural Network
MEO	Medium Earth Orbit
GEO	Geosynchronous Earth Orbit
IGSO	Inclined Geosynchronous Satellite Orbit

WMAPE	Weighted Mean Absolute Percentage Error
iGMAS	international GNSS Monitoring & Assessment System
PRN	Pseudo-Random Noise code
LSM	Least Squares Method
RMS	Root Mean Square

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