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Letter

Rearrangeable Nonblocking Conditions for Four Elastic Optical Data Center Networks

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Abstract: Four variants of elastic optical data center network (DCN) architectures based on optical circuit switching were proposed in an earlier study. The necessary and sufficient values of frequency slot units (FSUs) per fiber required for these four DCNs in the sense of there being strictly nonblocking (SNB) were derived, but no results in the sense of being rearrangeable nonblocking (RNB) were presented. In reality, only limited bandwidths are available, and reducing the value of FSUs per fiber has become a critical task to realize nonblocking optical DCN architectures in practice. In this paper, we derive the sufficient value of FSUs per fiber required for the four DCNs to be RNB by two multigraph approaches. Our results show that the proposed RNB conditions in terms of FSUs per fiber for a certain two of the four DCNs reduce their SNB results down to at least half for most cases, and even down to one-third.

Keywords: data center networks; elastic optical networks; rearrangeable nonblocking (RNB); edge-coloring

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1. Introduction

Recently, high transmission speed between the servers in data centers [1] has become an increasing requirement to meet the needs of current applications such as cloud computing and data mining. To support such high transmission speed, various data center network (DCN) architectures have been proposed [1–4]. One among them integrates an electronic packet switching (EPS) network and an optical circuit switching (OCS) network [1,2,4]. In such a hybrid electrical/optical architecture, both the EPS and OCS networks connect to each top-of-rack (ToR) switch simultaneously, where EPS serves small flows and OCS serves big flows. It has been shown that such a hybrid electrical/optical architecture reduces the power consumption and the operating expense [5,6].

An elastic optical network (EON) [7–11] is a candidate for being the OCS part of a DCN [2]. In EONs, flexible frequency grids proposed by ITU-T [12] are used, and a different number, say m, of *adjacent* frequency slot units (FSUs) are assigned to an optical connection, where m is usually upper bounded by a value, say $m_{\max}$. The bandwidth of an FSU is 12.5 GHz [12], and a connection is called an m-slot connection if it is assigned m adjacent FSUs. Four variants of optical DCN architectures based on elastic optical switches, called DCN1, DCN2, DCN3 and DCN4, were proposed in [13]. The four DCN architectures are similar to wavelength-space-wavelength (W-S-W) networks [8–10], which are Clos-like architectures, but they do not adopt costly tunable wavelength converters as W-S-W networks do. In addition, the maximum number of connections generated from each input fiber in the four DCNs and W-S-W networks is limited due to the different components used. This leads to the nonblocking conditions derived for these four DCNs being different from those derived for W-S-W networks.

When a network is called nonblocking, it is in reference to the nonblocking traffic assigned to the network [14]. To prevent excessive blocking of connections, the network should be nonblocking. A network is called strictly nonblocking (SNB) if a connection will never be blocked by existing connections, and a network is called rearrangeable nonblocking (RNB) if a new connection can be accommodated by rearranging some existing connections [14]. An RNB network is also defined as one where any set (or frame) of connections can be routed simultaneously. The necessary and sufficient number of FSUs per fiber required for these four DCNs in the sense of their being SNB were given in [13], but no results in the sense of being RNB were proposed.

The four DCNs usually require a great number of FSUs per fiber to be SNB, especially when $m_{\max}$ is growing. However, the resource of FSUs per fiber in practical systems is limited since for the EON switches, and the C-band has only around 350 available FSUs (1530–1565 nm). Reducing the value of FSUs per fiber is a challenging task, and this issue has been studied in various research on EONs [8–11]. In order to reduce the value of FSUs per fiber to realize nonblocking optical DCN architectures in practice, we studied the four DCNs in the sense of being RNB in this paper, and derived the sufficient number of FSUs per fiber by adopting two multigraph approaches. Our results show that two of the proposed RNB conditions reduced the SNB results significantly.

The rest of the paper is organized as follows: In Section 2, we give a brief review of the four DCN architectures and introduce the notations used in the paper. In Section 3, we prove the RNB conditions for the DCN1 and DCN3 networks. In Section 4, we prove the RNB conditions for the DCN2 and DCN4 networks. Section 5 concludes the paper.

2. Preliminaries and Notations

In this section, we will review the four elastic optical DCN architectures proposed in [13] and introduce the notations used in this paper. The four elastic optical DCN architectures require bandwidth-variable, waveband-selective switches (BV-WSSs) [15,16], bandwidth-variable space switches (BV-SSs), passive combiners (PCs) and ToR switches. Both BV-WSSs and BV-SSs, the latter of which consist of BV-WSSs and PCs, can switch wavebands with flexible bandwidths without spectrum conversion capabilities. Each ToR switch consists of q bandwidth-variable transponders (BVTs), which are divided into two parts: the transmission part, denoted by BVT-T, and the receiving part, denoted by BVT-R. The part of each ToR switch consisting of q BVT-Ts (or BVT-Rs) and a PC (or BV-WSS) is denoted by ToR-T (or ToR-R) (see Figure 1). A BVT-T can use any m consecutive FSUs of its output; i.e., the frequency of its output is arbitrarily tunable. In addition, a BVT-T is connected to a BVT-R in a strict one-to-one manner, and thus a BVT-T does not simultaneously send connections to two or more BVT-Rs. All connections generated from the same ToR switch occupy different FSUs, so that all of them can be sent through one fiber connecting the ToR-T (or ToR-R) to the OCS network.

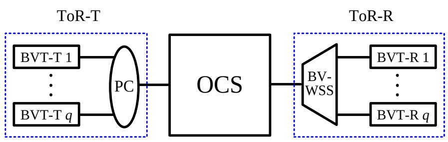

Figure 1. A ToR switch consisting of q BVTs.

The DCN1 architecture, denoted by DCN1(r, q, k), is given in Figure 2a. A DCN1(r, q, k) network contains one $r \times r$ BV-SS and r ToR switches, each of which consists of q BVT-Ts (or BVT-Rs) and is attached to an input (or output) fiber with k FSUs of the BV-SS. The DCN2 architecture, denoted by DCN2(s, r, q, k), is a variant of DCN1(r, q, k) and is given in Figure 2b. A DCN2(s, r, q, k) network contains one $r \times r$ BV-SS and r groups of s ToR switches, which are combined by one PC into (or directed from one BV-WSS to) one input (or output) fiber connecting to the BV-SS. We use ToR-T (or ToR-R) u-i to denote the ith ToR-T (or ToR-R) in group u, where $1 \le u \le r$ and $1 \le i \le s$. The DCN3 architecture is denoted by DCN3(r, q, k, p), and it contains p $r \times r$ BV-SSs and r ToR switches (Figure 3a). The output (or input) fiber of ToR-T u (or ToR-R v) is connected to one BV-WSS (or PC) which connects to the uth input (or vth output) of each BV-SS. Finally, the DCN4 architecture is denoted by DCN4(s, r,

q, k, p), and it is obtained from a DCN2(s, r, q, k) network by adopting p BV-SSs to connect ToR-Ts and ToR-Rs (Figure 3b).

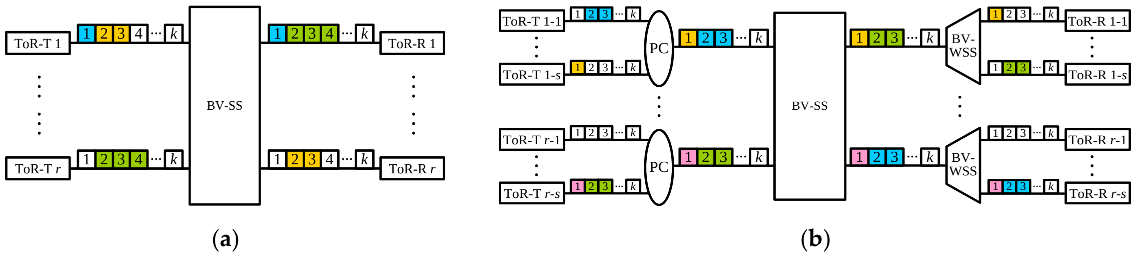

Figure 2. (**a**) A DCN1(r, q, k) network and (**b**) a DCN2(s, r, q, k) network, where each ToR-T (or ToR-R) consists of q BVT-Ts (or BVT-Rs), as given in Figure 1.

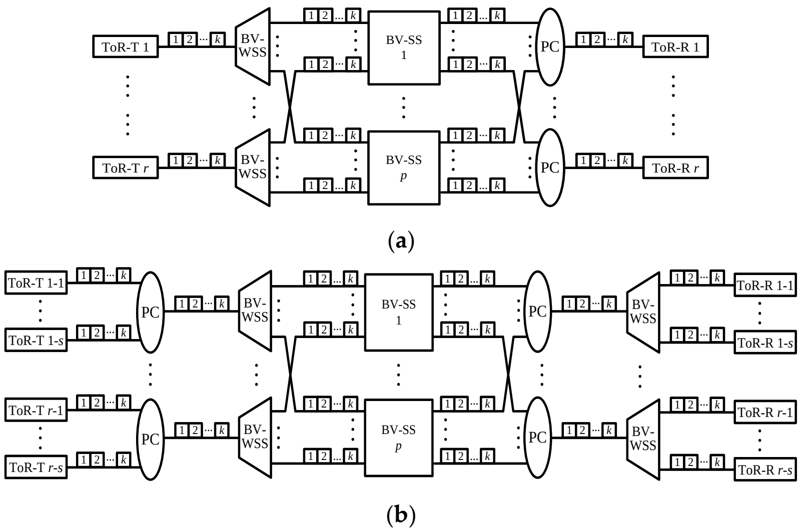

Figure 3. (**a**) A DCN3(r, q, k, p) network, and (**b**) a DCN4(s, r, q, k, p) network, where each ToR-T (or ToR-R) consists of q BVT-Ts (or BVT-Rs), as given in Figure 1.

The four DCN architectures serve m-slot connections with $m \le m_{\max}$. To guarantee that each fiber occupying k FSUs is sufficient to carry connections served by all BVT-Ts, the value of k is assumed to be $k \ge qm_{\max}$ (or $k \ge sqm_{\max}$) for the DCN1 and DCN3 (or DCN2 and DCN4) architectures. An m-slot connection from a BVT-T in ToR-T u (or ToR-T u-i) to a BVT-R in ToR-R v (or ToR-R v-j) in a DCN1 or DCN3 (or a DCN2 or DCN4) is denoted by (u, v, m) (or (u-i, v-j, m)), where $1 \le u, v \le r$ and $1 \le i, j \le s$.

FSUs in each fiber are numbered from 1 to k. To set up a connection (u, v, m) (or (u-i, v-j, m)), the same sets of m adjacent FSUs must be found in both the fiber connecting ToR-T u (or ToR-T u-i) with one BV-SS and the fiber connecting this BV-SS with ToR-R v (or ToR-R v-j). If those sets do not exist, the connection is blocked. The necessary and sufficient values of k for DCN1 to DCN4 in the sense of being SNB were given in [13]. We quote the SNB results for the DCN1 and DCN2 networks in Lemmas 1 and 2 for further comparison in Sections 3 and 4.

Lemma 1. A DCN1(r, q, k) network for m-slot connections with $1 \le m \le m_{\max}$ is SNB if and only if

$$k \ge k_{\mathrm{SNB}} = 2(q-1)\cdot(2m_{\max}-1) + m_{\max} \tag{1}$$

Lemma 2. A DCN2(s, r, q, k) network for m-slot connections with $1 \le m \le m_{\max}$ is SNB if and only if

$$k \geq k'_{SNB} = 2(sq - 1)\cdot(2m_{max} - 1) + m_{max} \quad (2)$$

3. RNB DCN1 and DCN3 Networks

In this section, we first consider the RNB DCN1 network and then the RNB DCN3 network. In order to derive the sufficient value of k for a DCN1(r, q, k) network in the sense of being RNB, we propose a multigraph approach and a routing algorithm in the following.

3.1. Multigraph Approach and Routing Algorithm

Given a DCN1(r, q, k) network and a frame F of connections, we propose Multigraph Approach A, given below, to convert the DCN1(r, q, k) network for frame F into a multigraph G_F.

Multigraph Approach A:

Let each left vertex u (or right vertex v) in multigraph G_F be ToR-T u (or ToR-R v) of the DCN1(r, q, k) network. In multigraph G_F, there is an edge connecting vertexes u and v if there is an m-slot connection from a BVT-T in ToR-T u and it is destined to a BVT-R in ToR-R v, i.e., (u, v, m) (see Figure 4a). Note that we call G_F a multigraph [17] because multiple connections between ToR-T u and ToR-R v are allowed, and thus there could be more than one edge connecting vertexes u and v in G_F.

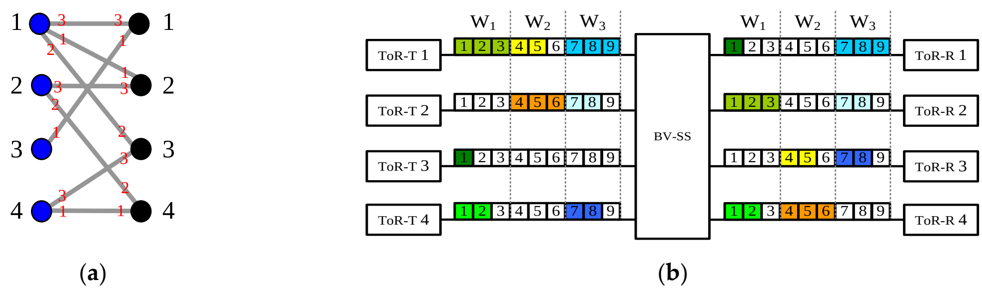

Figure 4. Given a DCN1(4, 3, 9) network and a frame F of connections (1, 1, 3), (1, 2, 3), (1, 3, 2), (2, 2, 2), (2, 4, 3), (3, 1, 1), (4, 3, 2) and (4, 4, 2): (**a**) The corresponding multigraph G_F constructed by Multigraph Approach A. Note that G_F is edge-colored by colors 1, 2, and 3 (marked in red), and left and right vertices are marked in blue and black, respectively. (**b**) A routing of connections in frame F according to Routing Algorithm A in association with the edge-coloring of G_F.

In Property 1, we show that G_F is q-edge-colorable.

Property 1. Given a DCN1(r, q, k) network and a frame F of connections, let G_F be the corresponding multigraph constructed by Multigraph Approach A. Multigraph G_F is q-edge-colorable.

Proof. Let $\Delta(G_F)$ be the maximum degree of G_F. Since each ToR switch consists of q BVT-Ts and q BVT-Rs, at most q m-slot connections can be generated from a ToR-T (or destined to a ToR-R). Thus, we have $\Delta(G_F) \leq q$. From the construction of G_F, we can see that G_F is a bipartite multigraph. In addition, G_F is q-edge-colorable according to graph theory [17] if G_F is a bipartite multigraph with $\Delta(G_F) \leq q$. □

In a DCN1(r, q, k) network, we use I_u (or O_v) to denote the fiber connecting ToR-T u (or ToR-R v) and the BV-SS, where $k \geq qm_{max}$ and $1 \leq u, v \leq r$. In addition, we partition each fiber with k FSUs into q parts, each of which consists of m_{max} consecutive FSUs. Each part is called a window, and these q windows, denoted by W_l for $1 \leq l \leq q$, are numbered from 1 from left to right. We use $|W_l|$ to represent the size of window W_l, and also use $I_{u,l}$ (or $O_{v,l}$) to represent the lth window in fiber I_u (or O_v) for $1 \leq u, v \leq r$ and $1 \leq l \leq q$.

Recall that G_F is q-edge-colorable (Property 1). Let colors 1, 2, … , q be adopted to edge color G_F. We route each (u, v, m) for the RNB condition [14] using Routing Algorithm A given below.

Routing Algorithm A:

Connection (u, v, m) is routed in windows $\mathrm{I}_{u,c}$ and $\mathrm{O}_{v,c}$ if color c is assigned to the corresponding edge of (u, v, m) in G_F (see Figure 4b).

3.2. RNB Sufficient Conditions

A sufficient value of k for a DCN1(r, q, k) network in the sense of being RNB is derived in Property 2.

Property 2. A DCN1(r, q, k) network for m-slot connections with $1 \le m \le m_{\max}$ is RNB if

$$k \ge k_{\mathrm{RNB}} = q \cdot m_{\max} \tag{3}$$

Proof. This property holds if Routing Algorithm A is feasible, and Routing Algorithm A is feasible if each m-slot connection can be carried by the corresponding windows. Since $m \le m_{\max}$, each m-slot connection can be carried by any window W_l if $|\mathrm{W}_l| = m_{\max}$ for $1 \le l \le q$, which implies that each fiber has $k = q \cdot m_{\max}$ FSUs. Therefore, when Routing Algorithm A is applied, a DCN1(r, q, k) network with $k \ge q \cdot m_{\max}$ is RNB. □

Comparing Equation (3) with Equation (1), we have $k_{\mathrm{RNB}}/k_{\mathrm{SNB}} \le 1/2$ for $m_{\max} \ge 2$ and $q \ge 3$. Property 2 implies that k_{RNB} reduces the SNB DCN1 result given in [13], namely, k_{SNB}, down to at least half for most cases. In addition, numerical results are given in Table 1 which show that k_{RNB} can reduce k_{SNB} down to as low as one third, for example, the cases with $m_{\max} \ge 6$ and $q = 4$, and the cases with $m_{\max} \ge 4$ and $q \ge 8$.

Table 1. Numerical results of k required for being an SNB or RNB DCN1(r, q, k) network for m-slot connections with q = 4, 8, 10 and $1 \le m \le m_{\max}$, where k_{SNB} and k_{RNB} are given in Equations (1) and (3), respectively.

$m_{\max}$	$q = 4$		$q = 8$		$q = 10$	
	k_{SNB}	k_{RNB}	k_{SNB}	k_{RNB}	k_{SNB}	k_{RNB}
2	20	8	44	16	56	20
4	46	16	102	32	130	40
6	72	24	160	48	204	60
8	98	32	218	64	278	80
10	124	40	276	80	352	100

The sufficient condition for being an RNB DCN1(r, q, k) network (Property 2) is also the necessary condition if only one connection rate $m_{\max}$ is considered (Property 3).

Property 3. Suppose only one connection rate, $m_{\max}$, is considered. Then, the DCN1(r, q, k) network is RNB if and only if $k \ge k_{\mathrm{RNB}} = q \cdot m_{\max}$.

Proof. The sufficient condition of this property is true since it is a special case with one connection rate of Property 2. In addition, the necessary condition holds when q connections (u, u, $m_{\max}$) for $1 \le u \le r$ are generated from each ToR-T u. □

From the architectures of the DCN1 and DCN3 networks (see Figures 2a and 3a), we can see that a DCN1(r, q, k) network for $k \geq q{\cdot}m_{\max}$ functions the same as a DCN3(r, q, k, p) network with $p = 1$. Thus, we derive Property 4 immediately.

Property 4. A DCN3(r, q, k, p) network for m-slot connections with $1 \leq m \leq m_{\max}$ and $k \geq q{\cdot}m_{\max}$ is RNB if $p \geq 1$.

Proof. This property is true for two reasons: i) a DCN3(r, q, k, p) network with $p = 1$ functions as well as a DCN1(r, q, k) network, and ii) a DCN1(r, q, k) network for $k \geq q{\cdot}m_{\max}$ is RNB (Property 2). □

For a DCN1(r, q, k) (or DCN3(r, q, k, p)) network, recall that the resource of FSUs per fiber in practical systems is limited, namely, $k \leq 350$. This implies that to have an RNB DCN1(r, q, k) (or DCN3(r, q, k, p)) network for m-slot connections with $1 \leq m \leq m_{\max}$ in the real word, we also need $q{\cdot}m_{\max} \leq 350$ due to Property 2 (or Property 4).

4. RNB DCN2 and DCN4 Networks

Similar to Section 3, we first consider the RNB DCN2 network and then the RNB DCN4 network. For the DCN2 network (Figure 2b), $\mathrm{I}_{u\text{-}i}$ (or $\mathrm{O}_{v\text{-}j}$) is used to represent the fiber connecting ToR-T u-i (or ToR-R v-j) and the uth PC (or vth BV-WSS), and I'_u (or O'_v) is used to represent the fiber connecting the uth PC (or vth BV-WSS) and the BV-SS for $1 \leq u, v \leq r$ and $1 \leq i, j \leq s$. Next, we will propose Multigraph Approach B and Routing Algorithm B for the DCN2 network in the sense of being RNB by modifying Multigraph Approach A and Routing Algorithm A, respectively.

Multigraph Approach B:

Given a DCN2(s, r, q, k) network and a frame F of connections, multigraph G'_F is constructed in the following way. Let each left vertex u (or right vertex v) in G'_F be the uth (or vth) group of s ToR-Ts u-i (or ToR-Rs v-j) for $1 \leq i, j \leq s$. An edge is added between two vertexes u and v in G'_F if there is an m-slot connection from the uth ToR-T group destined to the vth ToR-R group (see Figure 5a).

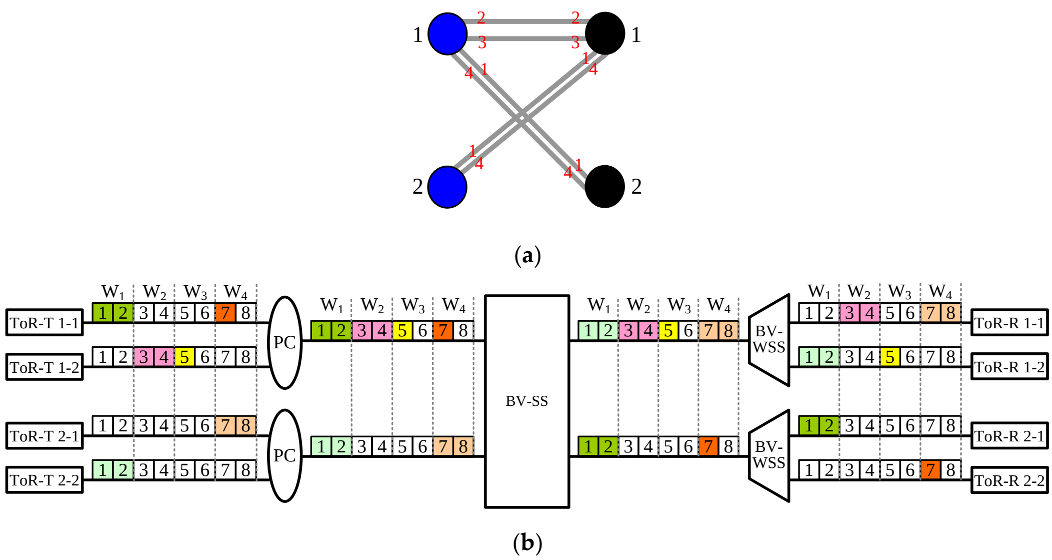

Figure 5. Given a DCN2(2, 2, 2, 8) network and a frame F of connections (1-1, 2-1, 2), (1-1, 2-2, 1), (1-2, 1-1, 2), (1-2, 1-2, 1), (2-1, 1-1, 2) and (2-2, 1-2, 2): (**a**) The corresponding multigraph G'_F generated by Multigraph Approach B. Note that G'_F is edge-colored by colors 1, 2, 3 and 4 (marked in red font), and left and right vertices are marked in blue and black, respectively. (**b**) A routing of connections in frame F according to Routing Algorithm B in association with the edge-coloring of G'_F.

Since each group of ToR switches can generate at most sq m-slot connections, we derive that $\Delta(G'_F) \leq sq$, and thus G'_F is sq-edge-colorable [17]. Let colors 1, 2, . . . , sq be used to edge-color G'_F. We adopt Routing Algorithm B, shown below, to route each (u-i, v-j, m) in association with the edge-coloring of G'_F for the RNB condition.

Routing Algorithm B:

Connection (u-i, v-j, m) is routed in windows $\mathrm{I}_{u\text{-}i,c}$, $\mathrm{I}'_{u,c}$, $\mathrm{O}'_{v,c}$ and $\mathrm{O}_{v\text{-}j,c}$ if color c is assigned to the corresponding edge of (u-i, v-j, m) in G'_F (see Figure 5b).

Similar to Properties 2–4, we have Properties 5–7, as follows.

Property 5. A DCN2(s, r, q, k) network for m-slot connections with $1 \leq m \leq m_{\max}$ is RNB if

$$k \geq k'_{\mathrm{RNB}} = sq \cdot m_{\max} \tag{4}$$

Proof. The proof is similar to that of Property 2. □

Property 6. Suppose only one connection rate, $m_{\max}$, is considered. Then a DCN2(s, r, q, k) network is RNB if and only if $k \geq k'_{\mathrm{RNB}} = sq \cdot m_{\max}$.

Proof. The proof is similar to that of Property 3. □

Property 7. A DCN4(s, r, q, k, p) network for m-slot connections with $1 \leq m \leq m_{\max}$ and $k \geq sq \cdot m_{\max}$ is RNB if $p \geq 1$.

Proof. From the topology of the DCN4 architecture (see Figure 3b), we can see that a DCN4(s, r, q, k, p) network with $p = 1$ and $k \geq sq \cdot m_{\max}$ functions as well as a DCN2(s, r, q, k) network. According to Property 5, the property holds immediately. □

Comparing Equation (2) with Equation (4), we have $k'_{\mathrm{RNB}}/k'_{\mathrm{SNB}} \leq 1/2$ for $m_{\max} \geq 2$ and $sq \geq 3$. Property 5 implies that k'_{RNB} reduces the SNB DCN2 result given in [13], namely, k'_{SNB}, down to at least half for most cases, and even down to one third. In addition, numerical results are given in Table 2, which shows that k'_{RNB} can reduce k'_{SNB} down to as low as one third, for example, all the cases with $m_{\max} \geq 4$, $s = 3$ and $q \geq 4$. Again, due to the limited resource of FSUs per fiber in practical systems, to have an RNB DCN2(s, r, q, k) (or DCN4(s, r, q, k, p)) network for m-slot connections with $1 \leq m \leq m_{\max}$ in the real word, we need $sq \cdot m_{\max} \leq 350$ due to Property 5 (or Property 7).

Table 2. Numerical results of k required for being an SNB or RNB DCN2(s, r, q, k) network for m-slot connections with $s = 3$, $q = 4$, 8, 10 and $1 \leq m \leq m_{\max}$, where k'_{SNB} and k'_{RNB} are given in Equations (2) and (4), respectively.

$m_{\max}$	$s = 3, q = 4$		$s = 3, q = 8$		$s = 3, q = 10$	
	k'_{SNB}	k'_{RNB}	k'_{SNB}	k'_{RNB}	k'_{SNB}	k'_{RNB}
2	68	24	140	48	176	60
4	158	48	326	96	410	120
6	248	72	512	144	644	180
8	338	96	698	192	878	240
10	428	120	884	240	1112	300

5. Conclusions

Four variants of elastic optical DCN architectures, called DCN1, DCN2, DCN3 and DCN4, were proposed in [13]. The four DCNs in the sense of being SNB usually require a large number of FSUs per fiber. To reduce the value of FSUs, we considered the four DCNs in the sense of their being RNB in this paper. We proposed two multigraph approaches to firstly prove the sufficient number of FSUs per fiber for these four DCNs in the sense of there being RNB. Our results show that the proposed RNB conditions in term of FSUs per fiber for the DCN1 and DCN2 networks reduce their SNB results down to at least half in most scenarios, and even down to one third. In addition, we show that the sufficient condition for an RNB DCN3 (or DCN 4) network is exactly the same as that derived for an RNB DCN1 (or DCN 2) network. The proposed multigraph approaches can be applied to all Clos-like architectures for studying RNB conditions.

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Conflicts of Interest: The authors declare no conflict of interest.

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