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A Novel Fast Parallel Batch Scheduling Algorithm for Solving the Independent Job Problem

Bin Zhang [1,2], Dawei Wu [3], Yingjie Song [2,*], Kewei Liu [1] and Juxia Xiong [4]

[1] School of Computer Science and Technology, Shandong Technology and Business University, Yantai 264005, China; zhangb@sdtbu.edu.cn (B.Z.); liukewei0110@163.com (K.L.)
[2] Shandong Co-Innovation Center of Future Intelligent Computing, Shandong Technology and Business University, Yantai 264005, China
[3] School of Traffic, Northeast Forestry University, Harbin 150040, China; wdw2017211666@nefu.edu.cn
[4] Guangxi Key Laboratory of Hybrid Computation and IC Design Analysis, Guangxi University for Nationalities, Nanning 530006, China; xiongjuxia1107@163.com
* Correspondence: songjy@sdtbu.edu.cn; Tel.: +86-0535-690-3541

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Abstract: With the rapid economic development, manufacturing enterprises are increasingly using an efficient workshop production scheduling system in an attempt to enhance their competitive position. The classical workshop production scheduling problem is far from the actual production situation, so it is difficult to apply it to production practice. In recent years, the research on machine scheduling has become a hot topic in the fields of manufacturing systems. This paper considers the batch processing machine (BPM) scheduling problem for scheduling independent jobs with arbitrary sizes. A novel fast parallel batch scheduling algorithm is put forward to minimize the makespan in this paper. Each of the machines with different capacities can only handle jobs with sizes less than the capacity of the machine. Multiple jobs can be processed as a batch simultaneously on one machine only if their total size does not exceed the machine capacity. The processing time of a batch is determined by the longest of all the jobs processed in the batch. A novel and fast 4.5-approximation algorithm is developed for the above scheduling problem. For the special case of all the jobs having the same processing times, a simple and fast 2-approximation algorithm is achieved. The experimental results show that fast algorithms further improve the competitive ratio. Compared to the optimal solutions generated by CPLEX, fast algorithms are capable of generating a feasible solution within a very short time. Fast algorithms have less computational costs.

Keywords: independent job sizes; fast scheduling algorithm; machine capacities; makespan; parallel batch machines

1. Introduction

How to reduce the production cycle and improve the utilization rate of resources is an important problem under the constraints of workshop production, such as delivery time, technical requirements and resource status, etc. Most enterprises adopt workshop scheduling technology to solve this problem. An effective scheduling optimization method can take advantage of many production resources in the workshop. The research and application of a workshop scheduling optimization method has become one of the basic contents of advanced manufacturing technology [1–3].

Batch processing machines (BPMs) are widely applied in many enterprises, for example, steel casting, chemical and mineral processing, and so on [4–6]. BPMs scheduling problem is a hot topic in workshop scheduling problem. In the traditional scheduling problem, each machine can only process, at most, one job at a time [7]. However, BPMs can process a number of jobs simultaneously

as a batch and all jobs in a batch have the same processing time [8]. The processing time of one batch is equal to the maximum processing time of all the jobs processed by the batch [9].

In this paper, we analyze the parallel batch processing machine scheduling problem where the jobs have arbitrary size and the machines have different capacities. We give a set of n jobs $\mathcal{J} = \{J_1, J_2, ..., J_n\}$ and a set of m parallel batch machines $\mathcal{M} = \{M_1, M_2, \ldots, M_m\}$, and we let all the jobs be released simultaneously (release time of job is 0). Let p_j and s_j denote the processing time and the size of job $J_j \in \mathcal{J}$ ($j = 1, 2, \ldots, n$), respectively, where $p_j \geq 0$ and $s_j > 0$. Machine M_i ($i = 1, 2, \ldots, m$) has a finite capacity K_i. Without loss of generality, we assume $K_1 \leq K_2 \leq \cdots \leq K_m$ and $s_j \leq K_m$ for each job J_j in order to ensure job J_j can be processed by at least one machine. However, there might be $s_j > K_i$ for some J_j and M_i. Machine M_i ($i = 1, 2, \ldots, m$) can handle multiple jobs at the same time, but the total size of these jobs cannot exceed K_i. The longest processing time of all jobs in a batch determines the processing time of a batch. The purpose of this problem is to allot each job to a batch and to schedule the batch on the machine to minimize the maximum completion time of the schedule, $C_{\max} = \max_j C_j$, where C_j represents the completion time of job j in the schedule [10,11]. Using the notations proposed in [12,13], this problem can be denoted as $P | s_j, p-batch, K_i | C_{\max}$.

The notations used in this paper are summarized in Table 1.

Table 1. Notations.

Notation	Description
$\mathcal{J}$	set of jobs, $\mathcal{J} = \{J_1, J_2, ..., J_n\}$
J_j	jth job
p_j	processing time of job J_j
s_j	size of job J_j
$\mathcal{M}$	set of machines, $\mathcal{M} = \{M_1, M_2, ..., M_m\}$
M_i	ith machine
K_i	capacities of M_i
$B_{i,g}$	g th batch scheduled on ith machine
UB	upper bound of makespan
LB	lower bound of makespan
$C_{\max}$	makespan

Before we move on, let us introduce some useful notations and terminologies. Let $\mathcal{J}_i = \{J_j \in \mathcal{J} \mid K_{i-1} < s_j \leq K_i\}$, and $i = 1, 2, \ldots, m, j = 1, 2, ..., n$; when $i = 1$, K_0 was used, but it was meaningless, so we set $K_0 = 0$. It is possible that $\mathcal{J}_i = \phi$ for some i. We have $\mathcal{J} = \bigcup_{i=1}^{m} \mathcal{J}_i$. Let $a_j = i$ denote the index of a machine with the minimum capacity that can process the job $J_j \in \mathcal{J}_j$, and then J_j can be assigned to each machine in $\mathcal{M}_j = \{M_{a_j}, M_{a_j+1}, \ldots, M_m\}$ where $1 \leq a_j \leq m$. $M_{a_j}, M_{a_j+1}, \ldots, M_m$ are called the golden machines for job J_j, $\mathcal{M}_j$ is the golden machines set, J_j is called the golden job for $M \in \mathcal{M}_j$, and all of the jobs that can be processed by M_i are called the golden jobs set for M_i. In a scheduling process, the running time of the machine is equal to the total processing time of batches scheduled on this machine.

The structure of the paper is as follows. Section 2 reviews the previous research in related areas. Section 3 gives the definition of the research problem. In Section 4, a novel fast 4.5-approximation algorithm is proposed for problem $P | s_j, p-batch, K_i | C_{\max}$. In Section 5, a fast 2-approximation algorithm is proposed for problem $P | s_j, p_j = p, p-batch, K_i | C_{\max}$. Section 6 designs several

computational experiments to show the effectiveness of fast algorithms. Finally, conclusions are given in Section 7.

2. Literature Review

Since the 1980s, scholars have studied the job scheduling problem of parallel batch machines extensively [1]. In this section, we review the results of research dealing with different job sizes and minimization of the maximum completion time [14–20].

In the one-machine case of problem $P|s_j, p-batch, K_i|C_{\max}$, we denote $1|s_j, p-batch, B|C_{\max}$. Uzsoy [21] proved that $1|s_j, p-batch, B|C_{\max}$ is a strong NP-hard (non-deterministic polynomial) problem, and presented four heuristics. Zhang et al. [22] also proposed a 1.75-approximation algorithm for $1|s_j, p-batch, B|C_{\max}$. Dupont and Flipo presented a branch and bound method for $1|s_j, p-batch, B|C_{\max}$. Dosa et al. [23] presented a 1.7-approximation algorithm for $1|s_j, p-batch, B|C_{\max}$. Li et al. [24] presented a $(2+\varepsilon)$ -approximation algorithm for $1|r_j, s_j, p-batch, B|C_{\max}$ (the more general case where jobs have different release times), where ε is a number greater than 0 and is arbitrarily small.

The special case of $P|s_j, p-batch, K_i|C_{\max}$ is where all $K_i = B$ ($B < n$) is represented as $P|s_j, p-batch, B|C_{\max}$. Chang et al. [25] studied $P|s_j, p-batch, B|C_{\max}$ and provided an algorithm that is based on the simulated annealing approach.

Dosa et al. [23] demonstrated that although the processing time for all jobs is the same (unless P = NP), $P|s_j, p-batch, B|C_{\max}$ cannot be approximated to a ratio less than 2. Dosa et al. presented a $(2+\varepsilon)$ -approximation algorithm. Cheng et al. [26] presented a 8/3-approximation algorithm for $P|s_j, p-batch, B|C_{\max}$ with running time $O(n\log n)$. Chung et al. [27] developed a mixed integer programming model and some heuristic algorithms for $P|r_j, s_j, p-batch, B|C_{\max}$ (the problem where jobs have different release times). A 2-approximation algorithm for $P|r_j, s_j, p_j = p, p-batch, B|C_{\max}$ (the special case of $P|r_j, s_j, p-batch, B|C_{\max}$ where all jobs have the same processing times) was given by Ozturk et al. [28]. Li [29] obtained a $(2+\varepsilon)$ -approximation algorithm for $P|r_j, s_j, p-batch, B|C_{\max}$.

More recently, several research groups have focused on the scheduling problems on parallel batch machines with different capacities and applications in many fields [30–41]. The special case of $P|s_j, p-batch, K_i|C_{\max}$, where all $s_j \le K_1$ (i.e., all jobs can be assigned to any machine), is denoted as $P|s_j \le K_1, p-batch, K_i|C_{\max}$. Costa et al. [30] studied $P|s_j \le K_1, p-batch, K_i|C_{\max}$ and developed a genetic algorithm for it. Wang and Chou [31] proposed a metaheuristic for $P|r_j, s_j \le K_1, p-batch, K_i|C_{\max}$ (the problem where jobs have different release times). Damodaran et al. [32] proposed a PSO method for $P|s_j, p-batch, K_i|C_{\max}$. Jia et al. [33] presented a heuristic and a metaheuristic for $P|s_j, p-batch, K_i|C_{\max}$. Wang and Leung [34] analyzed the problem $P|s_j, p_j = 1, p-batch, K_i|C_{\max}$ where each job has its own unit processing time. They designed a 2-approximation algorithm for the problem. They also obtained an algorithm with asymptotic approximation ratio 3/2. Li [35] proposed a fast 5-approximation algorithm and a $(2+\varepsilon)$ -approximation algorithm for $P|s_j, p-batch, K_i|C_{\max}$, but the presented $(2+\varepsilon)$ -approximation algorithm has high time complexity when ε is small. Jia et al. [36] presented several heuristics for $P|r_j, s_j, p-batch, K_i|C_{\max}$ (the problem where jobs have different release times) and evaluated the validity of the heuristics by computational experiments. Other methods have also been proposed in the literature [42–53].

In this paper, a novel fast 4.5-approximation algorithm was developed for problem $P|s_j, p-batch, K_i|C_{\max}$, and we evaluate the algorithm performance via computational experiments. We also provide a simple and fast 2-approximation algorithm for the case that all jobs have the same

processing time, ($P|s_j, p_j = p, p-batch, K_i|C_{\max}$), improving upon and generalizing the results in [54–57]. The approximation ratio of the 2-approximation algorithm in this paper is equal to the presented algorithm in [26], but is now simpler to understand and easier to implement.

3. Mathematic Formulation of the Problem

In this section, we present the problem under consideration as a mixed integer linear programming (MILP) model. First, the problem parameters and decision variables are given, and then the model is provided. Table 2 shows the problem indices.

Table 2. Indices.

Indices	Description
i	index of machine, $i = \{1, 2, ..., m\}$
j	index of job, $j = \{1, 2, ..., n\}$
l	index of batch, $l = \{1, 2, ..., n\}$

Table 3 shows the problem decision variables.

Table 3. Decision variables.

Decision Variables	Description
x_{jil}	1, if job J_j is assigned to the lth batch processed on machine M_i; 0, otherwise.
y_{il}	the processing time of lth batch processed on machine M_i.
$C_{\max}$	makespan.

The research problem can be denoted as $P|s_j, p-batch, K_i|C_{\max}$. The mathematical formulation of the research problem $P|s_j, p-batch, K_i|C_{\max}$ is shown as follows:

$$Minimize \ C_{\max}, \tag{1}$$

which is subject to

$$\sum_{i=1}^{m}\sum_{l=1}^{n} x_{jil} = 1, \quad j = 1, 2, \ldots, n; \tag{2}$$

$$\sum_{j=1}^{n} s_j x_{jil} \le K_i, \quad i = 1, 2, \ldots, m; \ \ l = 1, 2, \ldots, n; \tag{3}$$

$$y_{il} \ge p_j x_{jil}, \ \ j = 1, 2, \ldots, n; \ i = 1, 2, \ldots, m; \ \ l = 1, 2, \ldots, n; \tag{4}$$

$$C_{\max} \ge \sum_{l=1}^{n} y_{il}, \quad i = 1, 2, \ldots, m; \tag{5}$$

$$x_{jil} \in \{0, 1\}, \quad j = 1, 2, \ldots, n; \ i = 1, 2, \ldots, m; \ \ l = 1, 2, \ldots, n. \tag{6}$$

The Objective Function (1) shows that our aim is to find a schedule to minimize the makespan $C_{\max}$. Constraint (2) is to make sure that each job is assigned exactly to one machine. Constraint (3) guarantees that all batches are feasible; in other words, the total size of all jobs assigned to the batch does not exceed the capacity of machine where the batch is scheduled. Constraint (4) indicates that the processing time of a batch is not less than the processing time of the jobs in the batch. Constraint (5) guarantees that the makespan of the schedule is not less than maximum load of all the machines. In Constraint (6), the 0–1 variable x_{jil} indicates whether the j th job is assigned into the l th batch on machine M_i ($x_{jil} = 1$) or not ($x_{jil} = 0$).

4. 5-Approximation Algorithm for $P \mid s_j,\ p-batch, K_i \mid C_{\max}$

We denote the optimal makespan of the problem $P \mid s_j,\ p-batch, K_i \mid C_{\max}$ as OPT. The main focus of the research is to develop a fast scheduling model to get a minimized makespan as close to OPT as possible.

To solve the problem $P \mid s_j,\ p-batch, K_i \mid C_{\max}$, we used the MBLPT (modified longest processing time batch) rule [35], a modification of the BLPT (longest processing time batch) rule. For a given jobs set $\mathcal{J}_i$ that can be assigned to machine M_i, we apply the MBLPT rule, which sorts jobs to get $\mathcal{J}_i^{'}$. We build a batch $B_{i,1}$ on machine M_i, and then the rule repeatedly pops the first job from $\mathcal{J}_i^{'}$ and assigns it to $B_{i,1}$ until the sum of all the jobs assigned to $B_{i,1}$ just exceeds the capacity of M_i. Batch $B_{i,1}$ is called the one-job-overfull batch. Once the one-job-overfull batch exists, a new batch should be built on the same machine, unless the machine runs out of maximum completion time (maximum completion time is in the initialization parameters of the algorithm). We repeat the above job assignment procedure until the job list $\mathcal{J}_i^{'}$ is empty.

Let $\mathcal{B}_i = \{B_{i,g} : g = 1,2,\ldots,h_i\}$ denote the set of batches generated using the MBLPT rule to $\mathcal{J}_i'$ and machine M_i, and h_i is the total number of batches scheduled on machine M_i. Let $p(B_{i,g})$ and $p_s(B_{i,g})$ denote the longest processing time (the processing time of batch $B_{i,g}$ is equal to the longest processing time of jobs on it) and the shortest processing time of the jobs in batch $B_{i,g}$, respectively, such that $p(B_{i,1}) \geq p(B_{i,2}) \geq \cdots \geq p(B_{i,h_i})$. The batches $B_{i,1}, B_{i,2}, \ldots, B_{i,h_i-1}$ are one-job-overfull batches, while B_{i,h_i} can be one-job-overfull or not. We have $p_s(B_{i,g}) \geq p(B_{i,g+1})$ ($g = 1,2,\ldots,h_i - 1$). The Inequality (7) below (refer to [27]) is easy to prove.

$$\sum_{g=1}^{h_i-1}(p(B_{i,g}) - p_s(B_{i,g})) + p(B_{i,h_i}) \leq p(B_{i,1}). \tag{7}$$

By the Inequality (7), we have

Lemma 1. $\sum_{g=1}^{h_i} p(B_{i,g}) \leq \sum_{g=1}^{h_i-1} p_s(B_{i,g}) + p(B_{i,1}).$

We now propose the 4.5-approximation algorithm for $P \mid s_j, p-batch, K_i \mid C_{\max}$. Similar frameworks have been used in [58–62]. In [58], Ou et al. developed a 4/3 approximation algorithm to solve classical scheduling problems with minimized maximum completion time on parallel machines with processing set constraints. In [59], Li proposed a 9/4-approximation algorithm for $P \mid s_j = 1, p-batch, K_i \mid C_{\max}$ (the special case of $P \mid s_j, p-batch, K_i \mid C_{\max}$ where all $s_j = 1$). The algorithm to be described extends the previous research by involving non-identical job sizes.

We first run the 5-approximation algorithm for $P \mid s_j, p-batch, K_i \mid C_{\max}$. The algorithm generates a feasible schedule with a maximum completion time of $UB \leq 5OPT$ in $O(n \log m + n^2)$ time. Let the minimum completion time $LB = UB/5$. We have $LB \leq OPT \leq UB$. We use the binary search method to find the makespan of a feasible solution in the range of the $[LB, UB]$ interval. Firstly, set $T = \left\lceil \frac{LB+UB}{2} \right\rceil$, and then classify both the jobs and batches into long, short, and median. A job J_j is long if $p_j > T/2$, median if $T/4 < p_j \leq T/2$, or short if $p_j \leq T/4$. Similarly, a batch $B_{i,g}$ is long if $p(B_{i,g}) > T/2$, median if $T/4 < p(B_{i,g}) \leq T/2$, or short if $p(B_{i,g}) \leq T/4$. Certainly, long batches may contain median and short jobs, and median batches may contain short jobs. After classification, we use the following SCMF-LPTJF (smallest capacity machine first processed and longest processing time job first processed) procedure to search for a schedule with a makespan at most $9T/4$, which permits one-job-overfull batches. If our above operation fails, we will continue

searching for the upper half of the interval and set $LB = T$; otherwise, we will continue searching for the lower half of the interval, record $OPT = T$, and set $UB = T$. The binary search method is then repeated in the new range of the $[LB, UB]$ interval until $LB \geq UB$.

Algorithm 1. SCMF-LPTJF (smallest capacity machine first processed and longest processing time job first processed)

Input: $\mathcal{J} = \bigcup_{i=1}^{m} \mathcal{J}_i$, T

Output: $C_{\max}$ —best found solution, RT —running time

$Q_0 = \phi$, $AssignedJS = \phi$ // denote the jobs have been assigned to batch as $AssignedJS$
for $i = 1$ *to* m do
  Sort $\mathcal{J}_i$ according to the rule that processing time of jobs is not increased, denote $\mathcal{J}_i'$
end for
for $i = 1$ *to* m do
  $Q_i = Q_{i-1} \cup \mathcal{J}_i' - AssignedJS$
  Apply the MBLPT rule to Q_i and M_i, get $\mathcal{B}_i = \{B_{i,g} : g = 1, 2, \ldots, h_i\}$
  Sort $\mathcal{B}_i$ according to the rule that processing time of batches is not increased, denote $\mathcal{B}_i'$
  Denote long batches set, median batches set, and short batched set as $LongBS$, $MedianBS$, and $ShortBS$
  $LongBS = \phi$, $MedianBS = \phi$, $ShortBS = \phi$
  for batch b in $\mathcal{B}_i'$ do // classify batches into long, median and short.
    if $p(b) > T/2$ then
      append b to $LongBS$
    else if $T/4 < p(b) \leq T/2$ then
      append b to $MedianBS$
    else
      append b to $ShortBS$
    end if
  end for
  if $LongBS \neq \phi$ then
    $LongBS_{\max} = max(LongBS)$ // the longest processing time batch in $LongBS$
    schedule $LongBS_{\max}$ on M_i; remove $LongBS_{\max}$ from $\mathcal{B}_i'$; remove jobs assigned to $LongBS_{\max}$ from Q_i and add these jobs to $AssignedJS$
    $RT = LongBS_{\max}.p$ // $LongBS_{\max}.p$ is the processing time of batch $LongBS_{\max}$
  end if
  for batch b in $MedianBS$ do
    if $RT + b.p \leq 9T/4$ then
      schedule b on M_i; remove b from $\mathcal{B}_i'$; remove jobs assigned to b from Q_i and add these jobs to $AssignedJS$
      $RT = RT + b.p$ // the processing time of batch b
    end if
  end for
  for batch b in $ShortBS$ do

32: if $RT + b.p \le 9T/4$ then

33: schedule b on M_i; remove b from $\mathcal{B}_i^{'}$; remove jobs assigned to b from Q_i and add these jobs to $AssignedJS$

34: $RT=RT+b.p$ // the processing time of batch b

35: end if

36: end for

37: Update $C_{\max}$ // append the batches scheduled on M_i to $C_{\max}$

38: end for

39: return $C_{\max}$, RT

Lemma 2. *If $OPT \le T$, then the SCMF-LPTJF algorithm will generates an optimal schedule for $P|s_j, p-batch, K_i|C_{\max}$ with one-job-overfull batches whose makespan is at most $9T/4$.*

Proof. Let Σ be an optimal schedule whose makespan is OPT. Let H be the set of long jobs and median jobs. □

In Σ, each machine can process up to three median batches or one long batch and one median batch. On the other hand, the SCMF-LPTJF program will allocate a long batch on the machine as much as possible. After it assigns a long batch on a machine, this machine still has enough time (at least $5T/4$ time) to handle at least two median batches. Note that the SCMF-LPTJF procedure forms batches greedily. (It overfills each batch with the longest currently unassigned jobs.) Therefore, the SCMF-LPTJF procedure allocates more processing times for long jobs and median jobs on the machines with smaller capacities than Σ does. Equivalently, we claim that $\sum_{j\in H\cap\bigcup_{i=i}^{m}\mathcal{B}_i} p_j$ is the lower limit of the overall processing time in a long working state, and the median job arranged on machines $M_i, M_{i+1}, \ldots, M_m$ in Σ, $i = 1, 2, \ldots, m$. Hence, if $OPT \le T$, then all long and median jobs will be allocated by SCMF-LPTJF.

Therefore, if there is $OPT \le T$, but there is still a job j when executing to the end of the SCMF-LPTJF process, then job j must be a short job. When job j is assigned, all of machines $M_{a_j}, M_{a_j+1}, \ldots, M_m$ have a load greater than $2T$. Let $i_{\max} < a_j$ be the largest index such that machine $M_{i_{\max}}$ has a load less than or equal to $2T$. If all of the machines have load greater than $2T$, then set $i_{\max} = 0$. Therefore, all of machines $M_{i_{\max}+1}, M_{i_{\max}+2}, \ldots, M_m$ have load greater than $2T$. There is room on the machine $M_{i_{\max}}$ for scheduling any short job. Hence, by the rule of the SCMF-LPTJF procedure, no short job from $\bigcup_{i=1}^{i_{\max}}\mathcal{J}_i$ can be assigned to machines $M_{i_{\max}+1}, M_{i_{\max}+2}, \ldots, M_m$.

By Lemma 1, for $i = i_{\max}+1, i_{\max}+2, \ldots, m$, we have

$$\sum_{g=1}^{h_i-1} p_s(B_{i,g}) \ge \sum_{g=1}^{h_i} p(B_{i,g}) - p(B_{i,1}) > T. \tag{8}$$

It follows that

$$\sum_{j\in \bigcup_{i=i_{\max}+1}^{m}\bigcup_{g=1}^{h_i} B_{i,g}} p_j \cdot s_j > \sum_{i=i_{\max}+1}^{m} K_i \cdot T. \tag{9}$$

In Σ, all the short jobs in $\bigcup_{i=i_{\max}+1}^{m}\bigcup_{g=1}^{h_i} B_{i,g}$ have to be processed on machines $M_{i_{\max}+1}, M_{i_{\max}+2}, \ldots, M_m$. In addition, we have also proved that the overall processing time $\sum_{j\in H\cap\bigcup_{i=i_{\max}+1}^{m}\mathcal{B}_i} p_j$ of the planned long and median jobs on machines $M_{i_{\max}+1}, M_{i_{\max}+2}, \ldots, M_m$ in Σ as a lower bound. Therefore, the above inequality shows that Σ cannot make all of the jobs done in the $T \ge OPT$ time, which is a contradiction.

The algorithm performs a binary search within the range $[LB, UB]$. Finally, we will get a schedule with one-job-overfull batches (Figure 1a) whose makespan is at most $9OPT/4$. We can turn it into a viable scheduling solution (Figure 1b), where the maximum completion time is $9OPT/2$, as follows: for each one-job-overfull batch, move the last packed job into a new batch and calculate the new batch on the same machine. Since the iterative number could be $O(\log(\sum_{j=1}^{n} p_j))$, then the following theorems are obtained.

Theorem 1. *There is a 4.5-approximation algorithm for* $P|s_j, p-batch, K_i|C_{\max}$ *that runs in* $O(n^2 + mn\log p_{sum})$ *time, where* $p_{sum} = \sum_{j=1}^{n} p_j$.

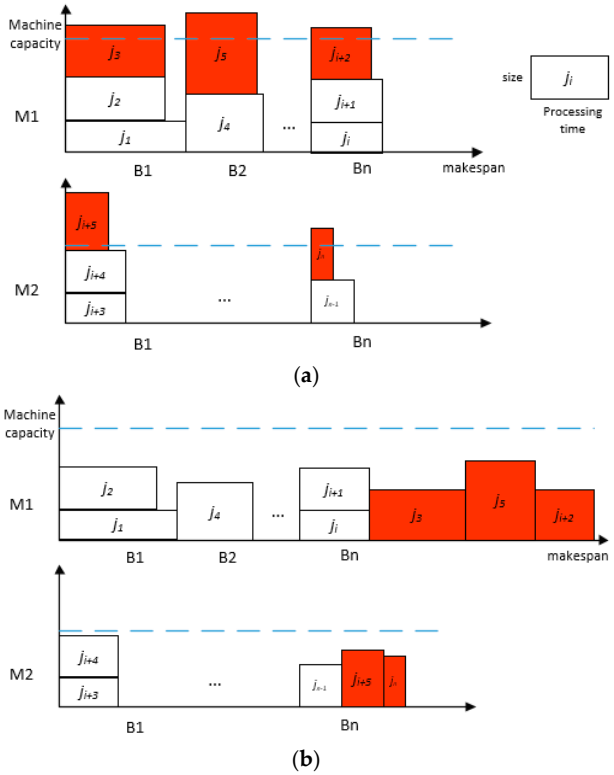


Figure 1. Illustration of the 4.5-approximation algorithm. (**a**) The 4.5-approximation algorithm with one-job-overfull batches. (**b**) The 4.5-approximation algorithm with a feasible schedule.

In order to achieve a strongly polynomial time algorithm, we use a technique described to modify the above algorithm slightly. Therefore, the following theorems are obtained.

Theorem 2. *There is a* $(4.5+\varepsilon)$ *-approximation algorithm for* $P|s_j, p-batch, K_i|C_{\max}$ *that runs in* $O(n^2 + mn\log(1/\varepsilon))$ *time, where* $\varepsilon > 0$ *can be made arbitrarily small.*

5. A 2-Approximation Algorithm for $P|s_j, p_j = p, p-batch, K_i|C_{\max}$

In this section, we study $P|s_j, p_j = p, p-batch, K_i|C_{\max}$, i.e., the problem of minimizing the makespan with equal processing times ($p_j = p$), arbitrary job sizes (may exceed the processing power of certain batches), and non-identical machine capacities.

The 2-approximation algorithm is called LIM (largest index machine first consider). It groups the jobs in $\mathcal{J}_m, \mathcal{J}_{m-1}, \ldots, \mathcal{J}_1$ (this ordering is crucial), respectively, into batches greedily. During the run of the algorithm, $Load_i$ represents the load on machine M_i, i.e., the overall processing time of

the batches on M_i, $i = 1, 2, \ldots, m$. The algorithm dynamically maintains a variable x, which represents the currently largest index such that $Load_x < Load_m$. If there is no such index, then we can set $x = m$. We can assign the next generated batch to machine M_x.

Algorithm 2. LIM (largest index machine first consider)

Input: $\mathcal{J} = \bigcup_{i=1}^{m} \mathcal{J}_i$

Output: $C_{\max}$ —best found solution, RT —running time

1: $AssignedJS = \phi$ //the jobs have been assigned to any batch
2: for $i = 1$ *to* m do
3: $Load_i = 0$ // the load on machine M_i, equals to the overall processing time of the batches on M_i,
4: end for
5: x=m
6: for $i = m$ *to* 1 do
7: if $x == m$ and $load_m > 0$ then
8: $x = i$
9: end if
10: create new batch b, $\mathcal{J}_i = \mathcal{J}_i - AssignedJS$
11: for job j in $\mathcal{J}_i$ do
12: if $b.size \le K_x$ then
13: assign job j to batch b
14: $b.size = b.size + j.size$
15: remove job from $\mathcal{J}_i$ and add it to $AssignedJS$
16: else
17: schedule b to M_x
18: $Load_x = Load_x + b.p$
19: initiate b
20: end if
21: end for
22: if b is not empty and $b.size \le K_x$ then
23: while $b.size \le K_x$ and $\mathcal{J}_{i-1} \| \mathcal{J}_{i-2} \| \cdots \| \mathcal{J}_1 - AssignedJS$ is not empty do
24: get first job j from $\mathcal{J}_{i-1} \| \mathcal{J}_{i-2} \| \cdots \| \mathcal{J}_1 - AssignedJS$
25: assign job j to b
26: $b.size = b.size + j.size$
27: remove job j from $\mathcal{J}_{i-1} \| \mathcal{J}_{i-2} \| \cdots \| \mathcal{J}_1$ and add it to $AssignedJS$
28: end while
29: schedule b to M_x
30: $Load_x = Load_x + b.p$
31: end if
32: if $Load_x \ge Load_m$ then
33: if $x > i$ then
34: $x = x - i$
35: else if $x == i$ then
36: $x = m$
37: end if
38: end if
39: end for
40: for $i = 1$ *to* m do
41: for batch b in M_i do

42: if $b.size \geq K_i$ then
43: create new batch b'
44: pop the last job from b and assign it to b'
45: schedule b' to M_i
46: $Load_i = Load_i + b'.p$
47: end if
48: end for
49: Update $C_{\max}$ // append the batches scheduled on M_i to $C_{\max}$
50: end for
51: return $C_{\max}$, $RT = max(Load_i)$, $i = 1,2,3 \ldots m$.

Theorem 3. *Algorithm LIM is a 2-approximation algorithm for* $P|s_j, p_j = p, p-batch, K_i|C_{\max}$.

Proof. Let Σ_1 be the schedule with makespan SOL_1 generated by LIM after Step 2. In Σ_1, all batches can be processed at the same time as they are assigned to a machine. During the running of the algorithm, the load on any machine is always less than or equal to the load on M_m. Therefore, M_m finishes last in Σ_1. Let B_{last} be the last batch assigned to M_m. Let $\{M_l, M_{l+1}, \ldots, M_m\}$ be the processing set of B_{last}, which can be defined as the largest size processing set of the job in B_{last}. In Σ_1, let $S(B_{last})$ denote the start time of B_{last}. We have: $SOL_1 = S(B_{last}) + p$. □

Since we assigned B_{last} to M_m, at that moment $x = m$ must hold. Hence, machines $M_l, M_{l+1}, \ldots, M_m$ are busy in the time interval $(0, S(B_{last}))$. All the batches allocated to machines $M_l, M_{l+1}, \ldots, M_m$ before $S(B_{last})$ are one-job-overfull batches. All jobs in these batches, together with the largest size job in B_{last}, must be processed on machines $M_l, M_{l+1}, \ldots, M_m$ in any feasible schedule. Hence, we get $OPT \geq S(B_{last}) + p$. So we can draw the conclusion that $SOL_1 \leq OPT$.

For a feasible schedule with makespan SOL generated by LIM, we have $SOL \leq 2SOL_1 \leq 2OPT$.

6. Computational Experiments

6.1. Experimental Environment

For the performance evaluation of the 4.5-approximation and 2-approximation algorithms, all the instances are generated by a random algorithm, as in the papers [63–68]. In the process of the instances generation, five factors affecting the solution of the problem are determined: the number of jobs, the number of machines, the variation in job sizes, the variation in job processing time, and the variation in machine capacities [69–75].

The experiment is divided into two parts: (1) the 4.5-approximation algorithm is compared with the CPLEX result. (2) The 2-approximation algorithm is compared to CPLEX. The 4.5-approximation algorithm and 2-approximation algorithm were coded in C# and the CPLEX was programed by OPL (Optimization Programming Language), compiled, and run with the IBM ILOG CPLEX Optimization Studio 12.5.1.0 (Education Version). All the algorithms were run on the same machine (Win10, Intel (R) i7-4790, 16 GB).

First, we set the number of machines to two or four, and the capacity of each machine is represented by a uniform integer [10,40]. Then, random problem instances with number of jobs equals to 10, 20, 50, 100, 200, and 300 are generated, and each job processing time P_j is generated by random sampling from a uniform distribution [1,10]. The factor settings of the experiment are summarized in Table 4.

Table 4. Factors setting of the experiment.

Factors	Levels
Number of jobs (n)	10, 20, 50, 100, 200, 300
Number of machines (m)	2, 4
Size of jobs (s)	[1,10], $[11, \max(K_i)]$
Processing time of jobs (P)	[1,10]
Capacity of machines (K)	[10,40]

We combine the parameters and randomly generate 50 instances for each combination (a test suite). Each test suite is denoted by a code. For instance, a test suite with 50 jobs and two machines is denoted by J3M1S1P1K1.

6.2. Comparison of 4.5-Approximation Algorithm and CPLEX

Here, a CPLEX algorithm is used to solve the MILP model given in Section 3, and we compare the CPLEX algorithm with the results of the 4.5-approximation algorithm. CPLEX always gives the optimal solution, but it cannot give the optimal solution for all instances even after operating several hours. Therefore, we set an upper execution time 1800s for CPLEX, and the best-known solution was compared. The job size and machine capacity distribution as shown in Figure 2.

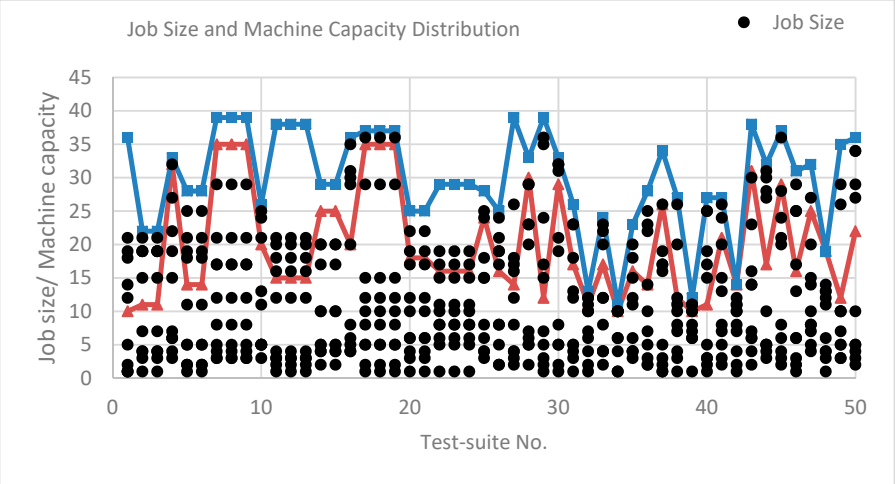
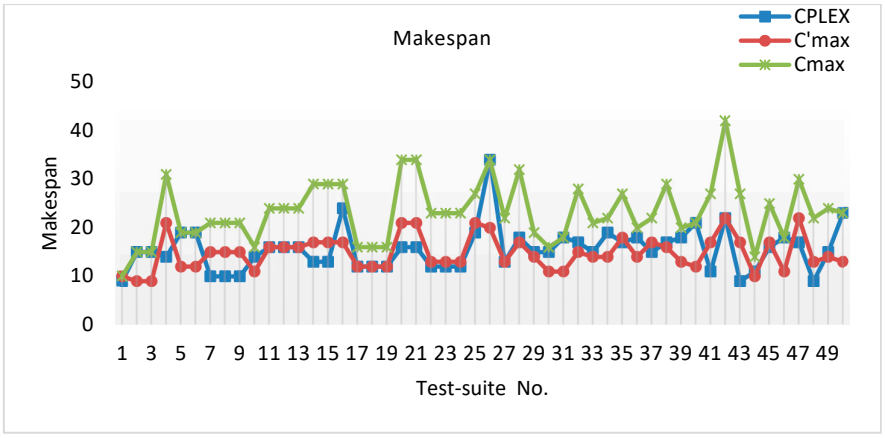


Figure 2. Job size and machine capacity distribution.

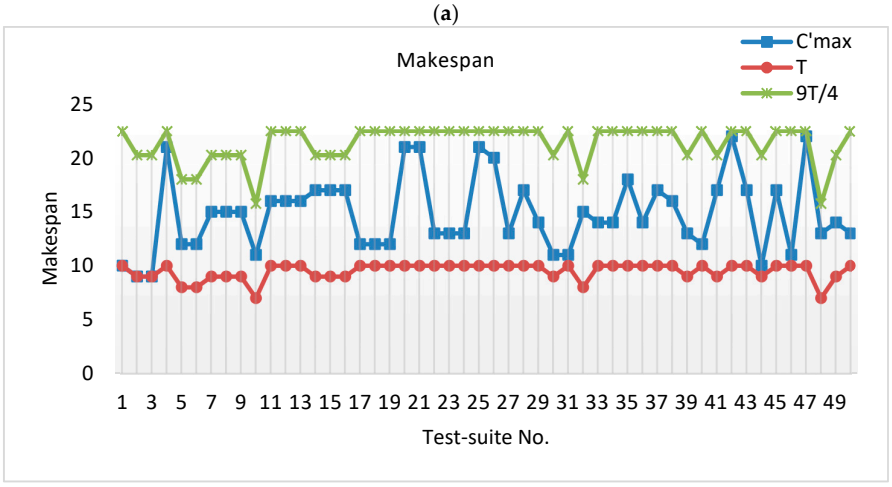
Regarding the 4.5-approximation algorithm, LB and UB are initialized as follows:

$$LB = \max(P_j)$$
$$UB = \sum_{j=1}^{n} P_j \,. \tag{10}$$

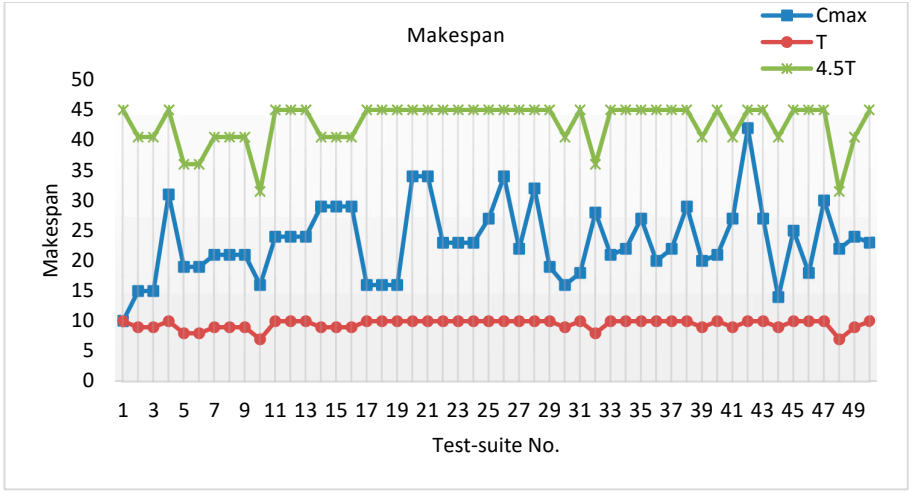
Figure 3 shows the result of test suite J1M1S1P1K1.


Makespan
CPLEX
C'max
Cmax
Makespan
Test-suite No.

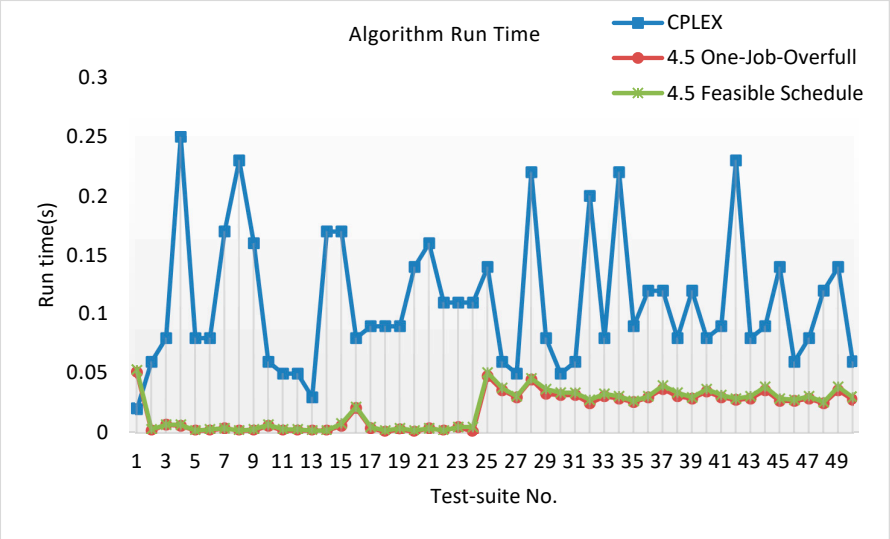
(**a**)


Makespan
C'max
T
9T/4
Makespan
Test-suite No.

(**b**)


Makespan
Cmax
T
4.5T
Makespan
Test-suite No.

(**c**)



(**d**)

Figure 3. Results of J1M1S1P1K1. (**a**) Comparison makespan of CPLEX and the 4.5-approximation algorithm with one-job-overfull batches and the 4.5-approximation algorithm with a feasible schedule. (**b**) Comparison makespan of the 4.5-approximation algorithm with one-job-overfull batches and $9T/4$. (**c**) Comparison makespan of the 4.5-approximation algorithm with a feasible schedule and $4.5T$. (**d**) Comparison running time of CPLEX and the 4.5-approximation algorithm with one-job-overfull batches and the 4.5-approximation algorithm with a feasible schedule.

$C'_{\max}$ is the makespan of the 4.5-approximation algorithm with one-job-overfull batches and $C_{\max}$ is the makespan of 4.5-approximation algorithm with a feasible schedule. Figure 3a shows the makespan of the 4.5-approximation algorithm with one-job-overfull batches and the algorithm with a feasible schedule. Figure 3b,c shows that the 4.5-approximation algorithm substantiates the feasibility of this research method:

$$C'_{\max} \le 9T/4$$

$$C_{\max} \le 4.5T .$$

Figure 3d shows that the run time of the 4.5-approximation algorithm is clearly better than CPLEX. Table 5 shows the results of all test suites. Though CPLEX is the best solver for linear programming problems, it cannot give an optimal solution for a long time, so we terminated CPLEX after running for 1800 s and used the best integer for comparison.

The results illustrate that the 4.5 approximation algorithm is more effective than CPLEX in any scale test-suite. For the small-scale test-suite (10 jobs and two machines), the best solution obtained by the 4.5-approximation algorithm is closest to the CPLEX best solution. For the medium-scale and large-scale test-suites, the average result of the 4.5-approximation algorithm is no bigger than $4.5T$.

6.3. Comparison of 2-Approximation Algorithm (LIM) and CPLEX

For the problem $P|s_j, p=p, p-batch, K_i|C_{\max}$, minimizing the makespan with equal running times, arbitrary job sizes (which may exceed the processing power of certain batches), and different machine capacities should be the solution. The running time of jobs was set to a default value of 8. Then, the LB and UB are denoted as

$$LB = 8$$

$$UB = n*8 .$$

Table 6 show the experimental results given by the CPLEX and the LIM algorithm for all the test suites. Column SQL-AVG (the average value of SQL) reports the average makespan obtained using the LIM algorithm. Compared with the CPLEX makespan, the LIM algorithm can obtain the efficient solution in only little running time (Column Run Times).

Table 5. Simulation results of CPLEX and the 4.5-approximation algorithm with one-job-overfull batches and a feasible schedule.

Test Suite	CPLEX			C'_{max}				C_{max}				T	$9T/4$	$4.5T$
	Makespan	GAP (%)	Run Time (s)	Best	AVG	Worst	Run Time(s)	Best	AVG	Worst	Run Time (s)			
J1M1S1P1K1	9	0	0.02	9	14.92	21	0.02	10	23.23	34	0.02	9.49	21.35	42.71
J1M2S1P1K1	8	0	0.09	9	13.02	22	0.02	9	19.06	36	0.02	9.63	21.66	43.32
J2M1S1P1K1	19	42.11	2.10	17	21.62	27	0.01	30	35.76	51	0.01	10.26	23.10	46.19
J2M2S1P1K1	10	40.00	2.26	15	19.44	22	0.01	24	32.04	48	0.01	10.09	22.7	45.4
J3M1S1P1K1	52	79.87	466.47	35	47.39	68	0.13	61	86.61	136	0.13	21.86	49.19	98.37
J3M2S1P1K1	47	80.85	25.39	21	28.84	40	0.02	37	53.53	106	0.02	13.57	30.53	61.07
J4M1S1P1K1	59	90.68	1800	76	94.57	129	0.04	150	175.8	255	0.04	42.73	96.14	192.29
J4M2S1P1K1	29	86.21	1800	40	52.75	108	0.04	76	99.20	180	0.04	24.22	54.50	108.99
J5M1S1P1K1	197	97.46	1800	151	185.53	255	0.08	250	352.33	496	0.08	83.29	187.40	374.81
J5M2S1P1K1	-	-	1800	78	105.49	186	0.08	146	203.96	365	0.08	47.55	106.99	213.98
J6M1S1P1K1	161	97.2	1800	213	283.43	387	0.12	377	535.71	739	0.12	126.65	284.96	569.93
J6M2S1P1K1	-	-	1800	104	143.73	283	0.12	190	275.35	560	0.13	64.65	145.46	290.93

[1] Note: (1) Column 2 is the minimum makespan of 50 instances for each test suite. '-' represents that CPLEX could not find a feasible solution in 1800 s. (2) Each test suite contains 50 instances. Columns 5, 6, and 7 report the best, average, and worst C'_{max}, respectively. Columns 9, 10, and 11 report the best, average, and worst C_{max}, respectively. (3) Columns 13, 14, and 15 report the average T, $9T/4$, and $4.5T$ of 50 instances, respectively.

Table 6. Simulation results of CPLEX and the 2-approximation algorithm with one-job-overfull batches and a feasible schedule.

Test Suite	CPLEX			SOL [1]				SOL			
	Makespan	GAP (%)	Run Time (s)	Best	AVG	Worst	Run Time (s)	Best	AVG	Worst	Run Time (s)
J1M1S1P1K1	24	0	0.25	16	18.82	24	0	24	31.84	48	0
J1M2S1P1K1	24	66.67	0.20	8	9.10	16	0	16	17.10	34	0
J2M1S1P1K1	40	13.93	3.11	24	38.27	56	0	44	66.20	88	0
J2M2S1P1K1	24	33.33	2.13	16	23.22	40	0	32	42.04	72	0
J3M1S1P1K1	80	87.06	1800	64	89.41	120	0	112	159.53	224	0
J3M2S1P1K1	64	87.5	38.34	40	61.96	96	0	72	109.96	160	0
J4M1S1P1K1	200	96	306.73	136	186.67	232	0	240	337.73	432	0
J4M2S1P1K1	144	95.83	1800	80	121.25	192	0	136	219.61	344	0
J5M1S1P1K1	412	98.59	1800	264	374.12	456	0	480	679.06	848	0
J5M2S1P1K1	-	-	1800	160	245.49	336	0	296	453.02	656	0
J6M1S1P1K1	-	-	1800	424	569.10	744	0	744	1031.37	1360	0
J6M2S1P1K1	-	-	1800	232	343.22	512	0	416	633.57	944	0

[1] Note: When run time is labeled as 0, it was less than 10^{-2}.

7. Conclusions and Future Works

The paper analyzed the parallel batch scheduling problem of minimizing the makespan, where arbitrary sizes of scheduling jobs are allowed and machines have different capacities. Each machine can only deal with jobs whose sizes do not exceed that machine's capacity. We developed an efficient 4.5-approximation algorithm for this problem. The experimental results show that the algorithms can obtain a reasonable solution in a finite time. A 2-approximation algorithm is achieved under the particular circumstances of equivalent processing times. Computational experiments show that the fast algorithm can help to improve the efficiency of resource consumption and give researchers more choices to balance the quality of the solution and the running time in the parallel batch scheduling problem.

Several important related directions for this problem are worth researching in the future. First of all, how do we improve the fast algorithm to get closer to the optimal solution in shortest time? In addition, jobs with release times are more common BPM problems in the manufacturing industry. How to develop a fast scheduling algorithm for this problem is an import direction. Finally, BPM problems with different service levels can be considered as well.

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