Supplementary Materials: On the Use of a Simplified Slip Limit Equation to Predict Screw Self-Loosening of Dental Implants Subjected to External Cycling Loading

Mikel Armentia [1,2,*], Mikel Abasolo 1, Ibai Coria [1] and Abdel-Hakim Bouzid [3]

Supplementary Material: Development of the expression (8) of the manuscript.

The study of the effect of transverse loading on the self-loosening of bolted joints can be found in [31,32]. As mentioned in the manuscript, the model developed in this work for dental implants is based on those references. A brief overview of the formulation will be provided in this section as a background for model developed in the manuscript.

The external transverse load helps overcome the friction resistance in the screw head and thread contacts. Consequently, resistance torques under the screw head and in the thread contact surface T_h and T_t become smaller and therefore a lower loosening torque T_L is needed to untighten the screw, as it can be deducted from Equation (2). According to [31,32], and considering the contact stress to be uniform, the simplified underhead bearing torque T_h required to turn the screw head when the latter is subjected to a transverse frictional force F_e can be expressed as follows:

$$F_e = \frac{\mu_h F_a}{\pi \cdot ({r_{hmax}}^2 - {r_{hmin}}^2)} \int_0^{2\pi} \int_{r_{hmin}}^{r_{hmax}} \frac{r_{ch} + r sin\theta}{\sqrt{{r_{ch}}^2 + r^2 + 2 r_{ch} r sin\theta}} r dr d\theta \tag{i}$$

$$T_h = \frac{\mu_h F_a}{\pi \cdot ({r_{hmax}}^2 - {r_{hmin}}^2)} \int_0^{2\pi} \int_{r_{hmin}}^{r_{hmax}} \frac{r + r_{ch} sin\theta}{\sqrt{{r_{ch}}^2 + r^2 + 2 r_{ch} r sin\theta}} r^2 dr d\theta \tag{ii}$$

Where F_a is the axial load in the screw (equal to the preload F_p if no external loads are applied) and F_e is the transverse load acting on the screw. For a given F_e value, Equation (i) is solved to obtain the value of the radius of rotation r_{ch} the center of which acts the bolt head frictional force, the details of which are given in [33]. Then, substituting the value of r_{ch} in Equation (ii)(i), T_h is calculated, i.e. the torque needed to cause slippage under the screw head surface when an external transverse load F_e is applied. Similarly, the frictional torque T_t required to cause slippage in the screw thread when an external thread force F_e is applied is:

$$F_e = \frac{\mu_t \cdot F_a \cdot \sqrt{sec^2\alpha + tan^2\beta}}{n \cdot \pi \cdot ({r_{tmax}}^2 - {r_{tmin}}^2) \cdot \left(1 - \mu_t \cdot sin\beta \cdot \sqrt{sec^2\alpha + tan^2\beta}\right)} \int_0^{2\pi} \int_{r_{tmin}}^{r_{tmax}} \frac{(r_{ct} +}{\sqrt{{r_{ct}}^2 \cdot (1 + tan^2\alpha cos^2\theta)}} \tag{iii}$$

$$T_t = \frac{\mu_t \cdot F_a \cdot \sqrt{sec^2\alpha + tan^2\beta}}{n \cdot \pi \cdot ({r_{tmax}}^2 - {r_{tmin}}^2) \cdot \left(1 - \mu_t \cdot sin\beta \cdot \sqrt{sec^2\alpha + tan^2\beta}\right)} \cdot \int_0^{2\pi} \int_{r_{tmin}}^{r_{tmax}} \frac{(r + r_{ct} sin\theta) r^2 dr d\theta}{\sqrt{{r_{ct}}^2 \cdot (1 + tan^2\alpha cos^2\theta) + r^2 \cdot (1 + tan^2\beta) + 2 r_{ct} r \cdot sin\theta}} \tag{iv}$$

Where n is the number of threads in contact, and r_{ct} is the radius of rotation the center of which acts the threads frictional force detailed in [33]. Finally, the transverse load F_e is considered not to affect the pitch torque T_p, so its value remains the same as in Equations (3) and (4):

$$T_p = F_a \cdot \frac{tan\beta}{\gamma} \cdot r_t \tag{v}$$

Thus, the loosening torque T_L can be finally calculated with Equation (2). The relationship can be plotted in the normalized graph shown in Figure A, with T_h/F_a-F_e/F_a and T_t/F_a-F_e/F_a curves obtained from Equations (i)-(iv), and the resulting T_L/F_a-F_e/F_a curve calculated with Equation (2). Note that, in the absence of transverse load ($F_e = 0$), the torque values are the ones in Equations (3) and (4). Screw self-loosening occurs when the transverse load F_e reaches a critical value for which the external loosening torque T_L needed to untighten the screw is null, which corresponds to the point marked with a black triangle in the graph.

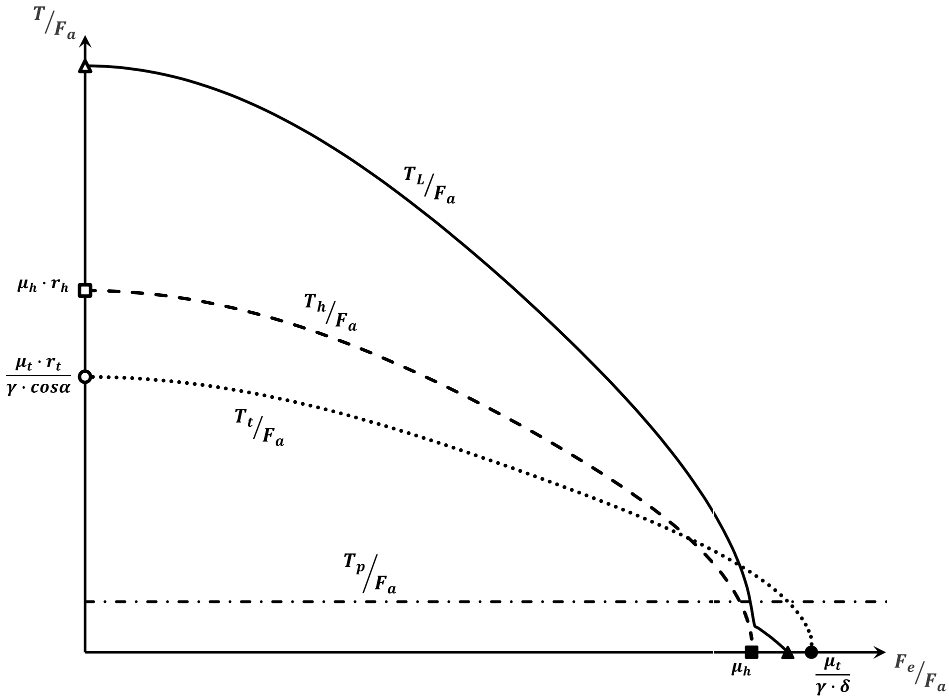

Figure A. Torque vs transverse load: Slip limit curves.

Thus, the sophisticated above model to predict self-loosening in screwed joints under transverse cyclic loading was validated experimentally for the case of two sliding plates, similar to the classical Junker test machine [28,29,33]. This model forms the basis of the mechanical foundations of the self-loosening phenomenon and quantifies the effect of the different design, manufacturing and operational variables involved, thus enabling the designer to select appropriate screw head and thread geometry, friction coefficient, tightening torque, and other relevant parameters. As a drawback, its applicability is not straightforward because the Equations must be solved using numerical integration.

The new analytical model proposed in this work simplifies Equations (i)-(iv), in such a way that the loosening torque T_L and the self-loosening phenomenon can be predicted straightforward. Figure B shows that T_h/F_a-F_e/F_a and T_t/F_a-F_e/F_a curves (previously shown in Figure A) can be approximated by parabolic curves (in grey) with the following Equations:

$$\left(\frac{T_h/F_a}{\mu_h \cdot r_h}\right)^2 + \frac{F_e/F_a}{\mu_h} = 1 \quad \text{(vi)}$$

$$\left(\frac{T_t/F_a}{\mu_t \cdot \frac{r_t}{\gamma \cdot cos\alpha}}\right)^2 + \frac{F_e/F_a}{\frac{\mu_t}{\gamma \cdot \delta}} = 1 \quad \text{(vii)}$$

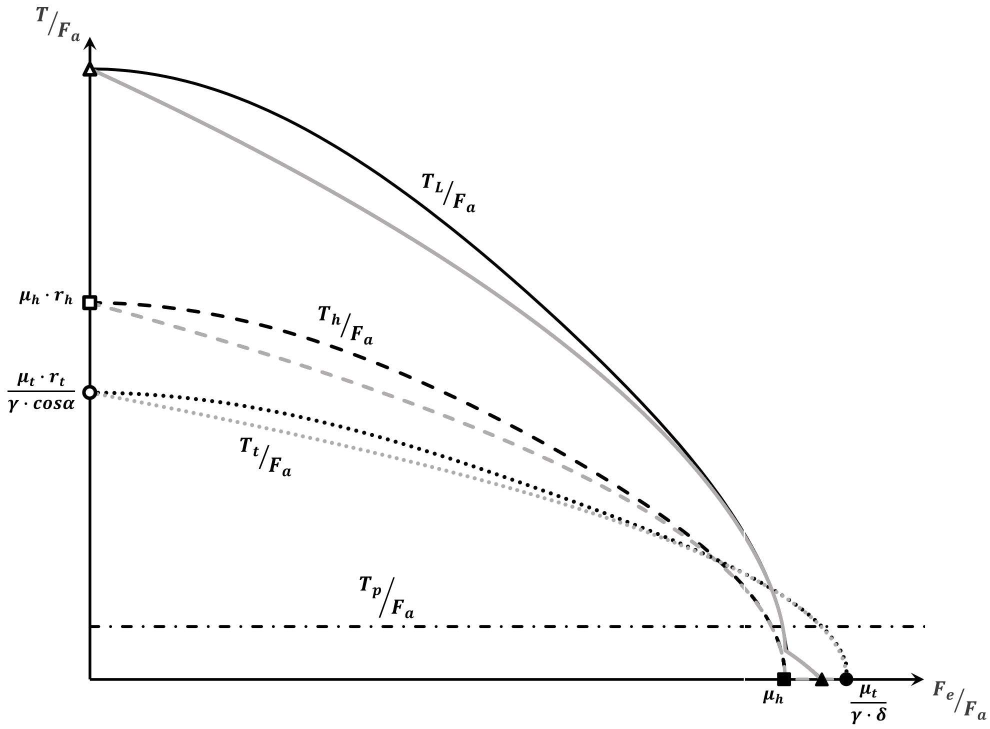

Figure B. Slip limit curves: Equations (i)–(iv) (black) and new Equations (vi)-(vii) (grey).

Even though other geometrical curves, such as a rotated ellipse, provide a better overall approximation, the use of a parabola has two main advantages. First, its formulation is quite simple, and leads to simple mathematical expressions for the estimation of screw self-loosening. Second, even though the parabola does not fit well for low values of F_e/F_a, the error for high values of F_e/F_a where self-loosening is expected to occur is relatively small since there is a good fit in that zone as shown in Figure B. A wide parameter range was used in both models obtaining a maximum difference of 1.4%.

Equations (vi)-(vii) can be rewritten as:

$$T_h = \mu_h \cdot F_a \cdot r_h \cdot \sqrt{1 - \frac{F_e}{\mu_h \cdot F_a}} \tag{viii}$$

$$T_t = \mu_t \cdot F_a \cdot \frac{r_t}{\gamma \cdot cos\alpha} \cdot \sqrt{1 - \gamma \cdot \delta \cdot \frac{F_e}{\mu_t \cdot F_a}} \tag{ix}$$

Thus, replacing expressions (v), (viii) and (ix) in Equation (2), the expression (8) presented in the manuscript is reached.

$$T_L = \mu_h \cdot F_a \cdot r_h \cdot \langle 1 - \frac{F_e}{\mu_h \cdot F_a} \rangle^{1/2} + \mu_t \cdot F_a \cdot \frac{r_t}{\gamma \cdot cos\alpha} \cdot \langle 1 - \gamma \cdot \delta \cdot \frac{F_e}{\mu_t \cdot F_a} \rangle^{1/2} - \left(F_a \cdot \frac{tan\beta}{\gamma} \cdot r_t \right)$$