



Spacing Allocation Method for Vehicular Platoon: A Cooperative Game Theory Approach

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Featured Application: The proposed method is applied to spacing allocation of the vehicles in platoon.

Abstract: Recently, spacing policies of the vehicular platoon have been widely developed to enhance safety, traffic efficiency, and fuel consumption. However, the integrated spacing policies aim to maximum overall benefit, and the distributed spacing policies intense to get optimal monomer benefit. Ignoring the fairness of the benefit allocation of each vehicle will reduce the motivation to constitute the platoon. To fill this critical gap, this study proposes a spacing allocation method by treating spacing decisions as cooperative games. A flock's model which is used to be the payoff function is introduced based on bionic motion principles. We present a characteristic function of the platoon for the cooperative game model considering the specific structure of the platoon. The τ value, Shapley value, and average lexicographic value are introduced and applied to allocate the spacing fairly. Proposed methods are compared with constant distance policy in some typical situations. The simulation results demonstrate that the spacing policy based on cooperative game theory improved the stable time for consistency control and the convergence of longitudinal following error.

Keywords: spacing allocation; platoon control; cooperative game theory; motion control

1. Introduction

Cooperative control of multi-vehicles in platoon and its extended application in intelligent transportation technology are considered as efficient measures to solve nuisance to motorists, consumes less fuel, and likely causes less accidents [1]. The main control objective of the platoon is to maintain the desired space and form a pre-specified formation called spacing policy. The studies on related field reveal that the profit of vehicular platoon will be significantly affected by spacing policy [2].

Spacing policy aims to ensure a proper distance between the nearest vehicles and prevent malicious insertion of the vehicles outside the platoon. Current spacing policies, including constant distance (CD) policy, constant time headway (CTH) policy, and non-linear distance (NLD) policy are widely used to analyze the macro performance of the platoon combined with their distributed controllers [3–5]. Due to the complex physical environment and difficulties in modelling for platoon, the researches on



spacing policy mainly focus on the string ability and attempt to attenuate and recover from a speed disturbance [6]. According to the use of global information or local information to make decisions, the spacing strategy can be divided into integrated spacing policies and distributed spacing policies. The integrated spacing policies aim to maximum overall benefit, and the distributed spacing policies intense to get optimal monomer benefit. However, ignoring the fairness of the benefit allocation of each vehicle will reduce the motivation to constitute the platoon.

Besides, the desired profit is difficult to be modelled as explicit functions, and the solving process of the nonlinear optimal problem makes it even harder to achieve a reliable performance.

Therefore, the theory of multiple-agents-system (MAS) is introduced to decoupling the interaction of the vehicles [7]. The MAS is widely used in the platoon control which derived from the research of the bionics on fish, birds and flocks [8]. Bionic motion principles are concluded from the investigation of the flocks, including collision avoidance, velocity matching and center gathering, with which the motion of the agents will achieve consistent control performance. Considering the consistency control of MAS as the optimization problem of spacing policies and fairly allocate the profit to vehicles in platoon is an effective method to address aforementioned research gap.

To fill the research gap, we attempt to propose a fair spacing policy based on cooperative game theory instead of a distributed space policy. The main contributions of this paper are as follows:

- (a) To evaluate the behaviors of the vehicles in platoon, we propose a flock's model according to the bionic motion principles with the information from the nearest vehicle, which can be used to achieve a longitudinal consistency control performance for the vehicle platoon.
- (b) A general expression of the motion controller is derived from the proposed flock's model which can ensure the local stability, string stability and traffic flow stability.
- (c) By modifying and applying the Sharpley value, τ value and the average lexicographic value, the inter-vehicle spacing is fairly allocated to the vehicles in the platoon with proposing characteristic function specified by platoon's structure.

The rest of the paper is organized as follows. We summered the relative literature of spacing policy and the cooperative game theory used in transportation, based on which, the limitations and difficulties of current methods are concluded in Section 2. Section 3 presents a flock's model based on bionic behaviors. In Section 4, the proposed model is considered as a profit function, and then the methodology of spacing allocation for vehicle platoon is presented based on cooperative game model. Besides, a general motion controller is derived to satisfy local stability, string stability, and traffic stability. The simulation experiments and results analysis are shown in Section 5. We conclude the paper in Section 6.

2. Literature Review

2.1. Spacing Policy

To completely take advantages of platoon, several spacing policies are proposed which can be roughly divided into three categories.

The desired spacing in CD policy between vehicle nodes is a preset constant which won't be affected by driving state as shown in Equation (1) and the d_{cd} denotes the preset space in platoon [9].

$$J_{ct} = d_{cd} \tag{1}$$

CTH policy is suggested as a safe practice for human drivers and is largely used in adapted cruise control [10]. The desired spacing determined by CTH is a function of velocity v_i and a preset time headway t_{τ} , as shown in Equation (2).

$$J_{cth} = d_{cth} + t_{\tau} v_i \tag{2}$$

where d_{cth} denotes the minimum safe space between the vehicles in standstill in CTH policy.

NLD policy [5,11] consider the host vehicle as a non-linear function. Human range policy is extracted from the test database and is found to be in the form of a quadratic curve, as shown in Equation (3). Thus, the NLD is transformed into an optimal problem.

$$J_{nld} = d_{nld} + Tv_i + Gv_i^2 \tag{3}$$

where d_{nld} denotes the minimum safe space between the vehicles in standstill in NLD policy, *T* and *G* are constant coefficients.

Through the mathematic analysis of different spacing policy, the CTH policy can guarantee the string stability without the acceleration information from the reference vehicle. However, the traffic flow stability and road capability are weakened in comparison with the CD policy because that CTH policy can't maintain the traffic flow stability. NLD policy has potential to improve traffic stability and capability through the optimization of the control parameters. According to the current researches, we may find that the space policy will strongly affect the following performance:

- (1) The choices of the desired spacing will influence the traffic capability and traffic stability [12].
- (2) The aerodynamic drafting effect will result in the significant reduction of the fuel economy [13–15]. It requires that the spacing in platoon should be small enough.

2.2. Game Theory Application in Transportation Research

In the field of transportation research, non-cooperative games are widely used to describe the interaction of the drivers [16]. However, it is keenly aware that the game model constructed is different from the actual situation due to the unpredicted human driver behaviors, which creates complexity and randomness in traffic flow models. With the development of autonomous driving technology in recent years, it seems feasible to consider the local motion controller of autonomous vehicles as a rational participant, and the game theory will play an important role in the analysis of the transportation behavior, especially the cooperative control of the vehicles which can be considered as agents [17]. In the control of automated vehicles, scholars have explored the application of non-cooperative game theory concepts to behavioral decision-making methods. For example, game theory is used to solve the security problem with the analysis of the information flow topology in the vehicle platoons, and a Nash equilibrium is adapted to select the appropriate security improvement strategy by Basiri et al. [18]; Gattami et al. combined optimal control with game theory to calculate the optimal spacing required to ensure safety, given the conflict between safety and fuel consumption in the platoon [19]; and a congestion game is proposed to explore the motivation to form a platoon by Farokhi [20]. While these studies have concentrated on game theory in cooperative platoon control, others have concerned the conflict between vehicles while lane-changing. For instance, Yu et al. proposed a novel game-theory-based model, using interactions with surrounding drivers based on turn signals and lateral movement to simulate the lane-changing behavior of human drivers [21]. Similarly, Elhenawy developed a game-theory-based algorithm for autonomous vehicles at uncontrolled intersections, with simulations demonstrating that the algorithm could reduce travel time and delay efficiently [22]. Ding proposed a coordinated multi-vehicle strategy for mandatory lane changes that improved the average speed of the vehicles and driving stability in simulations [23], while Kang developed a repeat-game-theoretical decision-making model with an updated payoff function for human drivers [24]. In addition to the non-cooperative games, cooperative games also have potential value in the study of vehicle platoons.

Compared with non-cooperative games, cooperative games pay more attention to the method of the profit's allocation [25,26]. After enough trainees, the result of the non-cooperative game will lead to the result of the cooperative game. We can reasonably speculate that the vehicles driving on the road are fully trained and rational participants. Therefore, the greatest economic benefits which will be assigned to all vehicles in platoon can be achieved through cooperation and competition.

3. Flock's Model for Platoon

The connected automated vehicles (CAVs) in platoon are always considered as agents in multiple-agent systems which can use bionic concepts to describe the group behaviors. In this section, a novel flock's model which is suitable for cooperative control of platoon is introduced according to the consistency control principles of the MAS. The principles can be expressed as follows when applied in platoon control.

Collision Avoidance: avoid collisions with neighboring vehicles. Velocity matching: attempt to match velocity with leading vehicle or vehicle in front. Center gathering: attempt to stay close to the center of the neighboring vehicles.

Collision avoidance serves to establish the minimum required separation distance while velocity matching tends to maintain it. Center gathering is used to ensure the traffic flow stability.

The relationship of the CAVs in platoon has obvious differences with normal self-driving vehicle. For examples, the CAVs in platoon are more like to cooperate than competition. With reasonable rules, they can achieve consistent motion pattern and avoid harmful effects caused by irrational competition. Therefore, the cooperative game theory is suitable for solving the cooperative control of the CAVs in platoon.

In this section, a spacing policy and the performance function are proposed referring to the aforementioned distributed model based on flocks. In addition to several general spacing policies, the relationship between car follower model (CFM) and traffic oscillations has been extensively studied [27,28]. The traditional model considers that the traffic flow is in a steady state, and the desired spacing between vehicles is related to the velocity. However, the later cellular automaton (CA) model believed that the randomization enables the models to depict the spontaneous formation of jams [29]. In actual traffic flow, driving behavior is closely related to time difference, random behavior, etc. [30]. Therefore, a reasonable modeling model should be able to consider the relationship between distance and speed, and have resistance to the impact of random interference fluctuations on traffic flow. With reference to the three principles in the previously mentioned content, the desired spacing will be calculated with positions X_i , and velocities \dot{X} of the vehicles. ω_v , ω_c are denote the weight coefficients of velocity match principle and center gathering principle respectively.

Avoiding-collision profit J_a is a constant space to ensure a safe zone between two vehicles which is shown in Equation (4) and Figure 1.

$$J_a = d \tag{4}$$

where *d* denotes the constant distance in stationary state which is related to L_i , and the L_i demonstrates the length of the vehicle *i*.



Figure 1. Avoid collision principle.

Velocity-match profit J_v , the velocity of the host vehicle X_i should match with the average velocity of the nearest vehicles, as shown in Equation (5) and Figure 2. When the time gradually increases and the platoon enters a steady state, the velocities of all vehicles will tend to be equal.

$$J_{v} = \omega_{v} \left(\dot{X}_{i} - \frac{\dot{X}_{i-1} + \dot{X}_{i+1}}{2} \right)$$
(5)



Figure 2. Velocity match principle.

Considering that the last vehicle in platoon does not have two reference nearest vehicles, the equation should be modified as Equation (6).

$$J_v = \omega_v \left(\dot{X}_i - \dot{X}_{i-1} \right) \tag{6}$$

Center-gathering profit J_c , the vehicles should be located in the middle of its previous vehicle and following vehicle as shown in Equation (7). As shown in Figure 3, the vehicle *i* should decelerate to reach the center point of vehicle *i* – 1 and *i* + 1.

$$J_{c} = \omega_{c} \left(X_{i} - \frac{X_{i-1} + X_{i+1}}{2} \right)$$
(7)



Center point of vehicle i-1 and i+1

Figure 3. Center gathering principle.

Obviously, the host vehicle has not center point while i = n which means that $J_c = 0$. Combining Equations (4), (5) and (7), we may obtain the whole desired spacing from Equation (8).

$$J_{i} = J_{ai} + J_{vi} + J_{ci}$$

= $d + \omega_{v} \Big(\dot{X}_{i} - \frac{\dot{X}_{i-1} + \dot{X}_{i+1}}{2} \Big) + \omega_{c} \Big(X_{i} - \frac{X_{i-1} + X_{i+1}}{2} \Big)$
= $-\frac{\omega_{c}}{2} X_{i-1} - \frac{\omega_{v}}{2} \dot{X}_{i-1} + \omega_{c} X_{i} + \omega_{v} \dot{X}_{i} - \frac{\omega_{v}}{2} \dot{X}_{i+1} - \frac{\omega_{c}}{2} X_{i+1} + d$ (8)

Considering J_i as the desired spacing between vehicle i - 1 and vehicle i, the desired position X_{desi} of the vehicle i can be calculated by Equation (9).

$$X_{desi} = X_{i-1} - J_i \tag{9}$$

The principles described above reflect the purposes of consistency control for platoon. When avoiding-collision principle is met, the safety of the vehicles in platoon can be guaranteed. Velocity-matching and center-gathering principles express the information flow in platoon that the action of the neighbors' action will affect the spacing decision of the host vehicle. The traditional information flow is used for acceleration decision-making to achieve the desired spacing and improve the response speed and error convergence accuracy; in contrast, the spacing decided by proposed method reflects a movement trend, and additional motion control is required to realize it.

4. Methodology Based on Cooperative Game Model for Platoon

The cooperative game theory focusses on the method of allocating benefits to participants according to their marginal contribution. When giving the headways of vehicles in platoon, it could be considering as a spacing allocation which make cooperative game model sometimes preferable.

In this section, a cooperative game-based platoon spacing policy is proposed. The cooperative game model is directly relative to the information follow topology of the platoon, which is shown in Figure 4. The information of the leading vehicle (vehicle 0) is available for all following vehicles (vehicle 1~n). Additionally, the following vehicles can obtain the position, velocity and acceleration of the nearest vehicles. The vehicles will determine their desired spacing with its previous vehicle according to the information.



Figure 4. Information flow topology of the platoon.

To simplify the problem, the following assumptions are made:

- (1) CAVs in platoon are assumed to be able to achieve autonomous acceleration and deceleration;
- (2) CAVs in platoon are assumed to have wireless communication capabilities.
- (3) For the controller design, it was assumed that there is no time delay in the process of the communication system.
- (4) The cooperative game of the platoon is a balanced cooperative game, which mean that the core of the problem is non-empty.
- (5) The vehicles in platoon are arranged and cannot skip a vehicle to form a separate coalition, which means that the available coalition only should be $\{0\}, \{0,1\}, \dots, \{0, 1, \dots, i, \dots, n-1, n\}$.

The profit of cooperative game here is the desired spacing which should be distributed to the participants. The allocation of spacing is considered as a transferable utility (*TU*) game, and the definition is shown as followed.

Cooperative game of spacing allocation for platoon can be described as a pair (N, v) consisting of a non-empty and finite set of vehicles N and coalition function $v: 2^N \rightarrow R, v(\emptyset) = 0$. Subsets of N are called coalitions, and v(K) is called the worth of coalition K.

The spacing profit of a coalition could be obtained from proposed spacing policy based on three principles. Assuming that the key indicators ω_v and ω_c are the same to simplify the model. While the vehicles are non-cooperative, they won't care velocity and center gathering. Therefore, if a coalition only has one participant, what they care is avoiding collision. Besides, the last following vehicle in platoon cannot follow the centering position of two vehicles. Thus, the characteristic function of the game model is given in Equation (10). The profits of all other coalitions are zeros.

$$\begin{cases} v[\varnothing] = 0 \\ v[\{1\}] = J_{a1} + J_{v1} \\ v[\{1,2\}] = J_{a1} + J_{v1} + J_{c1} + J_{a2} + J_{v2} \\ v[\{1,2,3\}] = J_{a1} + J_{v1} + J_{c1} + J_{a2} + J_{v2} + J_{c2} + J_{a3} + J_{v3} \\ & \cdots \\ v[N] = \sum_{i=1}^{n} J_{ai} + \sum_{i=1}^{n} J_{vi} + \sum_{i=1}^{n-1} J_{ci} \end{cases}$$
(10)

where *N* denotes the grand coalition composed of *n* vehicles.

Definition 1. The game $v \in G^N$ is monotonous, if all coalition S and T satisfy $S \subset T$, and there is $v(S) \leq v(T)$.

Definition 2. The game $v \in G^N$ is super-additivity, if all coalition S and T satisfy $S \cap T = \emptyset$, and $v(S \cup T) \ge v(S) + v(T)$.

Definition 3. The game $v \in G^N$ is convex, if all coalition S and T, and $v(S) + v(T) \le v(S \cup T) + v(S \cap T)$.

The proposed cooperative game of platoon is convex. The proof is shown in Equation (11).

$$v(N) - [v(N/n) + v(n)]$$

$$= \sum_{i=1}^{n} J_{ai} + \sum_{i=1}^{n} J_{vi} + \sum_{i=1}^{n-1} J_{ci} - \left[\sum_{i=1}^{n-1} J_{ai} + \sum_{i=1}^{n-1} J_{vi} + \sum_{i=1}^{n-2} J_{ci} + J_{an} + J_{vn} \right]$$

$$= J_{cn-1} \ge 0$$

$$(11)$$

4.1. τ Value and Shapley Value for the Cooperative Game

 τ value is defined within the quasi-balanced game, which can be obtain from the upper vector *M* and lower vector *m* [30].

The upper vector M(N, v) denotes the marginal contribution and the biggest desired profit in grand coalition. It can be obtained from Equation (12).

$$M_i^K(v) = v(K \cup i) - v(K) \tag{12}$$

The lower vector m denotes the minimum right profit, otherwise, it is not individual rational for i to join in the coalition. The vector m could be calculated by Equation (13).

$$m_i = J_{ai} + J_{vi} \tag{13}$$

Definition 4. The game theory is quasi-balanced if

- (1) $m(v) \le M(N, v)$
- (2) $\sum_{i=1}^{n} m(v) \le v(N) \le \sum_{i=1}^{n} M_i(N, v)$

For the first condition, it can be easily proved. Besides, the aforementioned definition will lead to the conclusion as shown in Equation (14).

$$v(N) = \sum_{i=1}^{n} M_i(N, v)$$
 (14)

For all games where $v \in G^N$, the τ value could be obtain from Equation (15).

$$\tau(v) = \alpha m(v) + (1 - \alpha)M(N, v) \tag{15}$$

where α is a coefficient belong to [0, 1], and is uniquely decided by Equation (16).

$$\sum_{i \in N} \tau_i(v) = v(N) \tag{16}$$

According to the Equation (16), the τ value of game v is calculating with Equation (17).

$$\tau_{1} = \alpha m_{1} + (1 - \alpha) M_{1} = J_{a1} + J_{v1}$$

$$\tau_{2} = (1 - \alpha) J_{c1} + J_{a2} + J_{v2}$$

$$\tau_{3} = (1 - \alpha) J_{c2} + J_{a3} + J_{v3} \longrightarrow \sum_{i=1}^{n} \tau_{i} = \sum_{i=1}^{n} J_{ai} + \sum_{i=1}^{n} J_{vi} + (1 - \alpha) \sum_{i=1}^{n-1} J_{ci}$$
(17)

$$\dots$$

$$\tau_{n} = (1 - \alpha) J_{cn-1} + J_{an} + J_{vn}$$

where $\alpha \in [0, 1]$.

Combined with Equation (13), we can know that $\alpha = 0$. This result reveals that every participant in platoon will get their maximum available profit with τ value which can satisfy the individual rational and group rational.

The order of the platoon is fixed while each vehicle is numbered according to the location which cannot be randomly arranged. Let the sequence of the platoon consist of a leading vehicle and *n* following vehicle as $N = \{0, 1, 2, ..., i, ..., n - 1, n\}$. The Shapley value of the participant is equal to the average of the marginal contribution in different coalition [31].

The weigh coefficient given to the coalition *S* is the probability that the participant who is in front of the participant happens to be a member of *S* in a random order. However, the particular sequence of the platoon has decided the number of the random orders and the essence of the Shapley value is the mathematical expectation of marginal contribution. In this game, the Shapley value is equal to τ value.

4.2. The Average Lexicographic Value

The average lexicographic (AL) value utilizes the vector which is consisted of the lexicographically optimal points [32]. For a normal game, the AL(v) demonstrates the average value of the biggest vector in all possible lexicography. It is normally expressed in Equation (18).

$$AL(v) = \frac{1}{n!} \sum_{\sigma \in \pi(N)} L^{\sigma}(v)$$
(18)

where *n*! represents the possibility of one sequence σ , and the L^{σ} denotes its corresponding lexicographically optimal points.

However, the sequence of a vehicle platoon is decided by physical environment which means that it is a balanced simplex game shown in Equation (19).

$$\sum_{S \in 2^N \setminus \{\varnothing\}} \lambda(S) v(S) \le v(N)$$
(19)

Definition 5. The game is a balanced simplex game, if its core C(v) is equal to non-empty imputation set i(v).

Theorem 1. AL(v) = CIS(v) if v is a balanced simplex game. The center of invert set (CIS) can be obtained from the Equation (20).

$$CIS(v) = \frac{1}{n} \sum_{k=1}^{n} f^{k}(v)$$
(20)

where $f^k(v) = (v(1), v(2), \dots, v(k) + v(N) - \sum_{i=1}^n v(i), \dots, v(n)).$

AL value satisfy the individual rationality, efficiency, core selectivity and symmetry. Note that the lexigraphy maximum of the core C(v) is an extreme point of the core for this ordering. Similarly, AL value also meet the virtual participants.

4.3. Motion Controller

In this section, a more general control law which is suitable for most of the spacing policy will be proposed. To describe the movement of the platoon, the dual-integrator model is used which is shown in Equation (21).

$$V_i = X_i$$

$$\dot{A}_i = \dot{V}_i$$
(21)

where X_i , V_i and A_i represent the position, velocity and acceleration of vehicle *i* respectively.

Assuming that the Δi denote the actual relative position between vehicle i - 1 and vehicle i, and the actual space is given by Equation (22).

$$\Delta_i = \dot{X}_{i-1} - \dot{X}_i \tag{22}$$

Existing spacing policy can be written as the form of Equation (23), which is a function of the relative position and relative speed between host vehicle and nearest neighbors.

$$R_{des} = R(\Delta_i, \Delta_{i+1}, \dot{\Delta}_i, \dot{\Delta}_{i+1})$$
(23)

Notice that we can get the dynamic model of the following error with Equation (24).

$$e = \Delta_i - R_i$$

$$\dot{e} = \dot{\Delta}_i - \dot{R}_i$$
(24)

where $\dot{R}_i = \frac{\partial R}{\partial \Delta_i} \dot{\Delta}_i + \frac{\partial R}{\partial \Delta_{i+1}} \dot{\Delta}_{i+1} + \frac{\partial R}{\partial \dot{\Delta}_i} \ddot{\Delta}_i + \frac{\partial R}{\partial \dot{\Delta}_{i+1}} \ddot{\Delta}_{i+1}$.

Let $\dot{e} = -\eta e$, and the control law can be obtained in Equation (25).

$$\ddot{\Delta}_{i} = \frac{-\eta[\Delta_{i} - R] + \dot{\Delta}_{i} - \left(\frac{\partial R}{\partial \Delta_{i}}\dot{\Delta}_{i} + \frac{\partial R}{\partial \Delta_{i+1}}\dot{\Delta}_{i+1} + \frac{\partial R}{\partial \dot{\Delta}_{i+1}}\ddot{\Delta}_{i+1}\right)}{\partial R/\partial \dot{\Delta}_{i}}$$
(25)

where η is a positive constant.

4.4. String Stability and Traffic Flow Stability

In the research of the platoon, the string stability is required to ensure that the error will not propagate with the upstream of the platoon. Refer to the [33–35], the string stability can be described as Equation (26).

$$|G(s)| = \left|\frac{e_i(s)}{e_{i-1}(s)}\right| = \left|\frac{V_i(s)}{V_{i-1}(s)}\right| \le 1$$
 (26)

where G(s) denotes the transfer function between the following errors.

According to Equation (8) and Equation (10), we can get the equation of the space policy Equation (27).

$$R = d + \frac{\omega_v}{2} (\dot{\Delta}_{i+1} - \dot{\Delta}_i) + \frac{\omega_c}{2} (\Delta_{i+1} - \Delta_i)$$

= $d - \frac{\omega_c}{2} \Delta_i - \frac{\omega_v}{2} \dot{\Delta}_i + \frac{\omega_c}{2} \Delta_{i+1} + \frac{\omega_v}{2} \dot{\Delta}_{i+1}$ (27)

Combing with the control law, we can get Equation (28).

$$-\eta \left[\dot{\Delta}_{i} - \left(-\frac{\omega_{c}}{2} \dot{\Delta}_{i} - \frac{\omega_{v}}{2} \ddot{\Delta}_{i} + \frac{\omega_{c}}{2} \dot{\Delta}_{i+1} + \frac{\omega_{v}}{2} \ddot{\Delta}_{i+1} \right) \right] \\ = \ddot{\Delta}_{i} - \left(-\frac{\omega_{c}}{2} \ddot{\Delta}_{i} - \frac{\omega_{v}}{2} \ddot{\Delta}_{i} + \frac{\omega_{c}}{2} \ddot{\Delta}_{i+1} + \frac{\omega_{v}}{2} \ddot{\Delta}_{i+1} \right)$$
(28)

Through the Laplace transform and letting the $L(\Delta_i) = \Delta V_i$, we may get Equation (29).

$$-\eta \Big[\Delta V_{i} - \left(-\frac{\omega_{c}}{2} \Delta V_{i} - \frac{\omega_{v}}{2} s \Delta V_{i} + \frac{\omega_{c}}{2} \Delta V_{i+1} + \frac{\omega_{v}}{2} s \Delta V_{i+1} \right) \Big]$$

$$= s \Delta V_{i} - \left(-\frac{\omega_{c}}{2} s \Delta V_{i} - \frac{\omega_{v}}{2} s^{2} \Delta V_{i} + \frac{\omega_{c}}{2} s \Delta V_{i+1} + \frac{\omega_{v}}{2} s^{2} \Delta V_{i+1} \right) \Leftrightarrow$$

$$\left[-\eta \left(1 + \frac{\omega_{c}}{2} + \frac{\omega_{v}}{2} s \right) - s - \frac{\omega_{c}}{2} s - \frac{\omega_{v}}{2} s^{2} \right] \Delta V_{i}$$

$$= \left[-\eta \left(\frac{\omega_{c}}{2} + \frac{\omega_{v}}{2} s \right) - \frac{\omega_{c}}{2} s - \frac{\omega_{v}}{2} s^{2} \right] \Delta V_{i+1} \qquad (29)$$

Finally, the transfer function G(s) is simplified in Equation (30).

$$G(s) = \frac{\Delta V_{i+1}}{\Delta V_i} = \frac{-\eta - s - \eta \left(\frac{\omega_c}{2} + \frac{\omega_v}{2}s\right) - \frac{\omega_c}{2}s - \frac{\omega_v}{2}s^2}{-\eta \left(\frac{\omega_c}{2} + \frac{\omega_v}{2}s\right) - \frac{\omega_c}{2}s - \frac{\omega_v}{2}s^2}$$
(30)

Substitute the s = jw into Equation (31), and the calculating progress is shown in Equation (30).

$$\begin{aligned} \left| \frac{-\eta - \eta \frac{\omega_{c}}{2} + \frac{\omega_{p}}{2} w^{2} - jw - \eta \left(\frac{\omega_{p}}{2} jw\right) - \frac{\omega_{c}}{2} jw}{-\eta \frac{\omega_{c}}{2} + \frac{\omega_{p}}{2} w^{2} - \eta \frac{\omega_{p}}{2} jw - \frac{\omega_{c}}{2} jw}{(-\eta - \eta \frac{\omega_{c}}{2} + \frac{\omega_{p}}{2} w^{2})^{2} + (-1 - \eta \frac{\omega_{p}}{2} - \frac{\omega_{c}}{2})^{2} w^{2}} \leq 1 \\ \eta^{2} - 2\eta \left(\eta \frac{\omega_{c}}{2} + \frac{\omega_{p}}{2} w^{2}\right) + (-\eta \frac{\omega_{p}}{2} - \frac{\omega_{c}}{2})^{2} w^{2} \leq 0 \\ - \left(\eta^{2} + \eta^{2} \omega_{c}\right) + [\eta \omega_{v} - 1 - (\eta \omega_{v} + \omega_{c})] w^{2} \leq 0 \end{aligned}$$

$$(31)$$

To make sure the inequality always true, the Equation (32) should be satisfied.

$$\begin{cases} \left(-\eta^2 + \eta^2 \omega_c\right) \le 0\\ -\eta \omega_v + 1 + (\eta \omega_v - \omega_c) \le 0 \end{cases}$$
(32)

Therefore, the sufficient condition of the string stability which is used to build the spacing policy is $\omega_c \ge 1$. String stability should be considerate to avoid the amplification effect of the following errors and we will further verify this conclusion.

The influence of headways policy on traffic flow can be generally shown in fundamental diagram, which reveals the relationship between traffic flow rate and traffic density [36]. A traffic flow is always stable when the traffic flow rate Q increase as the traffic density ρ increase [37]. The highest traffic density can be achieved by letting $\frac{\partial Q}{\partial \rho} = 0$.

In the CTH policy, the traffic density at stead state is given in Equation (33).

$$\rho = \frac{1}{d_{cth} + t_{\tau}v} \tag{33}$$

The equation between *Q* and ρ can obtained in Equation (34).

$$Q = \rho v = \frac{1 - \rho d_{cth}}{t_{\tau}} \tag{34}$$

where d_{cth} is always positive and the slope $\frac{\partial Q}{\partial \rho}$ is negative which means that the traffic flow is unstable in the CTH policy.

Assuming that the platoon is driving in a stable state, where v_{i-1} is equal to v_i . Then, the traffic density at steady state is $\rho = \frac{1}{(d+L)}$, which is equal to the CD policy. Idealized CD policy has optimal traffic capacity and traffic flow stability. The desired spacing in proposed method and CD policy is independent of speed, which means that the flow rate is linear with velocity. Proposed spacing policy is same with CD policy in the form of mathematic expression so that it has similar traffic characteristics.

4.5. Model Validation

We are trying to validate the performance of the proposed method. It will take seconds to recover the stable state from the disturbance, and maintain a new stable state. However, the described progress is determined by the control parameters of spacing policy and motion controller. With the numerical simulation, the relationships between final stable states and parameters is shown in Figures 5–7.



Figure 5. The relationships of the stable states and control parameters in flock's model: (**a**) Spacing following error; and, (**b**) The velocity following error.



Figure 6. The relationships of the stable states and control parameters in Shapely value and τ value policies: (**a**) Spacing following error; and, (**b**) The velocity following error.

From these images, we can see that the final following error of space and velocity is directly influenced by the control parameters. The color denotes the value of the error which is basically divided into 2 part according to ω_c . The error is near zero while $\omega_c > 1$, and hardly affected by other parameters. It is consistent with the analysis in previous section. However, Sharpley value leads to two other unstable regions, as shown in Figure 6. The stable region and unstable regions are separated by red rectangle which is not parallel to the planes of the coordinate system. It reveals that the unstable regions have a strong correlation with the control parameters: As η increases, the following effect passes from the unstable region to the transition zone and gradually reaches the stable region; as ω_c decreases, the final error also gradually decreases. Through numerical simulation, the results show that the theoretical analysis on stability is effective, and it has the significance of the parameter selection in the simulation.



Figure 7. The relationships of the stable states and control parameters in *AL* value policies: (**a**) Spacing following error; and, (**b**) The velocity following error.

5. Simulation Experiments and Analysis

5.1. Simulation Environment

The longitudinal motion of CAV platoon is simulated in this section, where the leading vehicle will conduct the accelerate and brake to verify the performance of the proposed method. Four methods will be compared, including spacing policy based on three principles, headways obtained from Shapley value, average lexicographic value, and constant space policy, which are abbreviated as Normal spacing policy, Shapley value, AL value and CD policy in the results of the experiments. To valid the performance of the various policy, we simulate two representative scenes, including initial velocity error situation and leading vehicle deceleration situation. The initial velocity error situation is used to simulate the self-adjusting ability of the platoon when where is an initial following error. In this situation, the error of stable speeds shall be within -5/+0 km/h according to the limitation in European new car assessment programme (Euro NCAP) [38]. The leading vehicle deceleration situation. In this platoon, serial number 1 denotes the leading vehicle while others indicate the following vehicles. The main parameters are shown in Table 1.

Control Parameters	Default Value	Other Parameters	Default Value
η	5	L	3
ω_v	10	d	5
ω_c	5	п	4
u_{upper}	6	U _{lower}	-6

Table 1. The key parameters of the model.

5.2. Performance of the Spacing Policy

5.2.1. Initial Velocity-Error Situation

In this section, a velocity jump will used to test the performance of the proposed method. The initial state of the vehicles is shown in Equation (35), where the velocity of the leading vehicle is different with the following ones. The following vehicles should cooperate to adapt the leading vehicle. The results of the simulation are shown in Figure 8.

$$\begin{cases} X = \begin{bmatrix} 24 & 16 & 8 & 0 \end{bmatrix}^T \\ V = \begin{bmatrix} 15 & 10 & 10 & 10 \end{bmatrix}^T$$
(35)





Figure 8. The performance of multiple policies in initial-velocity-error condition (**a**) Position; (**b**) Velocity; (**c**) Longitudinal space.

As shown in Figure 8a,b, the velocities of the vehicles in the normal spacing policy and AL value both overshoot after reaching the reference velocity to adjust to the preset spaces. Unlike them, the vehicles in Shapley value much more focus on the consistency of the velocity which is reflected in the quick convergence of the velocity. This performance has two reasons: (a) the vehicles have ensured their desired spacing with their previous vehicle; (b) the desired spacing calculated from the Shapley value has strong sensitivity with the change of velocity. Additionally, the velocities controlled with normal spacing policy, Sharpley value and AL value met the requirements of speed error referring to Euro NCAP while vehicles reached the stable state. In Figure 8c, we may find that the actual spaces between the vehicles tend to be stable except the CD policy. The disturbance is increasing with the upstream of the platoon which demonstrates that proposed methods can ensure the string stability while CD policy can't.

5.2.2. Leading Vehicle Deceleration Situation

The safety is the most important performance of the platoon. In addition to the amplification impact of disturbance in platoon, the emergency braking of a vehicle can also cause huge safety risks. In this section, the initial state of the vehicles is shown in Equation (36). The leading vehicle will brake to stationary state to verify the safety of the proposed method. The deceleration is chosen to be -6 m/s^2 at 0 s. To easily observe the final spaces, the abscissa of the position-time curve is limited to $5\sim10 \text{ s}$. The results of the simulation are shown in Figure 9.



Figure 9. The performance of the multiple policies in deceleration condition: (**a**) Position; (**b**) Velocity; (**c**) Longitudinal space.

We can notice that the although the leading vehicle performs a strong braking, the proposed control methods ensures a stable space between the vehicles. Unlike the CD policy, there is no significant acting lag in the change of the velocity. Figure 9a,b shows that the vehicle controlled with the normal spacing policy and AL value are both basically equal to their nearest vehicles throughout the whole process. However, the space of vehicle 4 controlled with Shapley value cannot align with the previous vehicles. The reason for it is because the third vehicle's profit caused by center gathering principle is distributed to vehicle 4 as marginal contribution. The difference cannot be eliminated unless the error of the center gathering quickly drops to zero in the process of braking. In fact, the analysis is support by the performance shown in Figure 9c. The policy of the Shapley value makes the gradual enlargement of the space more obvious which is good for safety. When the vehicles stop under the emergency braking conditions, the increasing of the static interval instead of the decreasing can ensure that the safety will not be weaken as the number of vehicles in platoon increases. The performance of CD policy shows that the final behaviors of the platoon result from the following performance of the individual vehicle and the errors will increase with the accumulation of each vehicle upstream the platoon. Intuitively, the actual spaces will gradually decrease until a collision occurs.

5.3. Analysis of the Performance

In this section, the simulating data of initial velocity-error and leading vehicle deceleration is listed in Figure 10, including all situations and policies. According to the performance, we will analyze the practical significance of the proposed method and pay attention to the following error and stable time. We may find that the initial errors are different in each vehicles because of the difference in driving state of neighbors. In aforementioned control methods, the following errors are critical to the driving process and should be strictly managed. However, the following errors in the proposed method will be the fundament of the motion controller and reveal the changing tendency of the velocity. Besides, the normal spacing policy and the *AL* value both give good performance in average stable time. It is a strong proof that they have a great ability to prevent disturbance. In Figure 10b, the leading vehicle was assumed to deaccelerate from the stable state which means that the initial errors are equal to zeros. The decelerating progress will not stop until the vehicles reach the static state.



Figure 10. Longitudinal following error and stable time: (**a**) Initial velocity situation; (**b**) leading vehicle deceleration situation.

6. Conclusions

The cooperative control method for vehicle platoon has been widely verified to improve the traffic capability and stability, and fuel economy. The overall performance of the platoon is mainly influenced by the spacing policy and motion controller. To ensure the string stability, traffic flow stability, and local stability, the spacing policy tends to be more complex in comparison with the CD policy and CTH policy. This paper proposes a systematic spacing policy for platoon based on the cooperative game theory.

To transform the platoon performance into the profit function of the desired spacing, we propose a spacing policy based on the bionic motion principles. A characteristic function is used to describe the cooperative game model of the platoon, hence it is possible to obtain the desired spacing through the distribution methods of profit, like Shapley value, τ value, and *AL* value. Mathematical analysis proves that the proposed spacing policy with its motion controller can ensure string stability and traffic

(a) The initial errors with step input of the velocity error will be eliminated, and the platoon will form a new stable state with proposed method. The positions and velocities shown in the simulating results demonstrate that the vehicles can achieve consistency control for platoon in infinite time.

flow stability simultaneously compared with CD policy. To evaluate the effectiveness of the proposed

method, simulation experiments are conducted, and the results can be concluded as follows:

- (b) The platoon controlled with normal spacing policy, Shapley value and AL value can guarantee the local stability, string stability, and traffic flow stability. The results show that the errors will gradually converge and stop increasing upstream in platoon which agrees with the theoretical analysis.
- (c) Spacing is considered as the profit for distribution, and the platoon performance is related to the distribution methods based on the cooperative game theory. For example, *AL* value is an average distribution of the total profit on the basis of the minimum payoff which makes the vehicles drive in synchronization compared with other spacing policies.

To expand the research, the characteristic function can be used to evaluate the different behavioral decisions and achieve Pareto optimality in complex scenarios, like cut-in, cut-out, and no signal crossing. For examples, it can be used to decide the travel order while two platoon conflict with each other in no signal crossing, and it will help to decide the optimal position for a new vehicle while join the platoon, etc. Besides, the available structure of the game model is relative to the information flow of the platoon which means that the directed graph game model can describe the vehicle's confliction when they can communicate with others limitedly. The cooperative game model and the mathematical analysis proposed in this paper will provide a foundation for these future extensions.

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