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Modulation Code for Reducing Intertrack Interference on Staggered Bit-Patterned Media Recording

Seongkwon Jeong and Jaejin Lee *

School of Electronic Engineering, Soongsil University, 369 Sangdo-ro, Dongjak-gu, Seoul 06978, Korea; seongkwon@ssu.ac.kr

* Correspondence: zlee@ssu.ac.kr; Tel.: +82-2-820-0901

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Abstract: A bit-patterned media recording (BPMR) system is a type of ultrahigh-capacity magnetic storage system that can extend to an areal density of 1 terabit per square inch or higher. However, because the space between islands in the down- and cross-track directions is reduced to extend the areal density, the effect of two-dimensional interference is increased. However, using a staggered array, which is one of the possible island distributions for BPMR, helps to decrease intertrack interference. A 7/10 modulation code for a staggered BPMR is proposed to avoid the effect of two-dimensional interference and provide distance among nonidentical codewords for improving the correcting capability.

Keywords: modulation code; bit-patterned media recording; intertrack interference

1. Introduction

For most conventional magnetic storage systems, the superparamagnetic limit is a significant obstacle to increasing the areal density (AD). To overcome the problem and extend the AD to more than 1 terabit per square inch (Tb/in²), bit-patterned media recording (BPMR) has become a candidate for the next generation of magnetic storage systems [1]. In addition, BPMR has such advantages as improved thermal stability, a decreased nonlinear transition shift, and reduced transition noise [2]. Because of these advantages, BPMR can satisfy the demand for storing a tremendous quantity of data in the information age. However, as the distances of down-track bit period T_x and cross-track pitch T_z for achieving high AD become closer, two-dimensional (2D) interference, which comprises intersymbol interference (ISI) and intertrack interference (ITI), is increased [3,4]. In addition, BPMR has unavoidable problems, such as track misregistration (TMR) and media noise caused by imperfect fabrication. According to the lithography method adopted, bit-patterned media (BPM) structures can be placed in a regular or staggered array BPM layout, as shown in Figure 1. When the islands are placed hexagonally in the staggered array, the bit error rate (BER) performance when the staggered array is used is better than when the regular array is used, because of reduced ITI [5,6].

To eliminate the 2D interference that degrades system performance, various schemes have been proposed for BPMR, such as signal detection methods, error control codes, and modulation codes. To address the 2D interference problem and help detect the input data, the partial response maximum likelihood (PRML) method has been employed [7]. This is applied to data storage systems where the channel response is equalized to a partial response (PR) pulse shape and a maximum likelihood (ML) sequence detector. For 2D data storage systems, a 2D equalizer and detection schemes have been proposed [6,8]. To ensure the reliability of data storage systems, error control codes, such as the low-density parity check (LDPC) code, are required. They considerably improve system performance [9,10]. In a previous

study [10], a proposed product code that consists of inner and outer code using a LDPC exhibited better BER performance than the LDPC code alone. To prevent error patterns that cause 2D interference, such modulation codes as 5/6 and 9/12 modulation have been proposed for BPMR [11,12]. Because data storage systems cannot retransmit data, unlike typical communication systems with such retransmission schemes as automatic repeat requests, strict requirements for a low probability of decoding failure and a high code rate should be met [9].

Figure 1. Bit-patterned media (BPM) structures of regular and staggered array.

In this work, a 7/10 modulation code for staggered BPMR is proposed. To reduce ITI effectively, the proposed code focuses on the ITI problem rather than the ISI problem, because the effect of ITI is greater than that of ISI in the BPMR [13]. Moreover, because the Hamming distance among codewords is at least 2 or more, the decoding capability is improved.

The remainder of this paper is organized as follows. In Section 2, the staggered BPMR channel model and PRML detection for staggered BPMR are explained. In Section 3, the proposed 7/10 modulation code is introduced. The simulation and results are discussed in Section 4. Section 5 provides conclusions.

2. Staggered BPMR Channel Model and PRML Detection

2.1. Staggered BPMR Channel Model

Figure 2 is a block diagram of the proposed system model. Before passing through the staggered BPMR channel, the binary user data $a_k \in \{0, 1\}$ are encoded by a modulation encoder to encode the 2D data array $c_{p,q}$, and $c_{p,q}$ is magnetized to record data $d_{p,q} \in \{-1, 1\}$. The analytical 2D Gaussian island pulse response $P(z, x)$ without media noise and write errors is given in [14].

$$P(z, x) = A \cdot \exp \left\{ -\frac{1}{2c^2} \left[\left(\frac{z}{PW_z} \right)^2 + \left(\frac{x}{PW_x} \right)^2 \right] \right\}, \quad (1)$$

where z and x are the indices in the cross- and down-track directions, respectively, A is the normalized peak amplitude, c represents the relationship between the standard deviation of a Gaussian function and PW50 (a parameter of the pulse width at half of the peak amplitude), and PW_z and PW_x are the PW50 of the cross- and down-track pulses, respectively. In this study, $A = 1$, $c = 1/2.3548$, $PW_z = 24.8$ nm, and $PW_x = 19.4$ nm. The BPMR 2D channel island pulse response $h_{m,n}$ is calculated by sampling the 2D Gaussian island pulse response as follows:

$$h_{m,n} = P(mT_z + \Delta_{TMR}, nT_x), \quad (2)$$

where m and n are the indices of bit islands for the cross- and down-track directions, respectively, T_z and T_x denote track pitch and bit period, respectively, and Δ_{TMR} is the read head offset, which is

generated when the recording heads cannot remain at the center of the main data track. The Δ_{TMR} is expressed as follows:

$$\Delta_{TMR} = \frac{TMR_z \times T_z}{100}, \quad (3)$$

where TMR_z is the percentage of the TMR. The readback signal $r_{p,q}$ corrupted by electronic noise in staggered array BPMR is given by

$$\begin{aligned} r_{p,q} = & \sum_{n=-N}^N d_{p,q+n} \cdot h_{0,n} + \sum_{m=0}^{\lfloor \frac{N-1}{2} \rfloor} \sum_{n=-N+m+1}^{N-m} d_{p-(2m+1),q+n} \cdot h_{-(2m+1),n-\frac{1}{2}} + \\ & \sum_{m=0}^{\lfloor \frac{N-1}{2} \rfloor} \sum_{n=-N+m+1}^{N-m} d_{p+(2m+1),q+n} \cdot h_{(2m+1),n-\frac{1}{2}} + \sum_{m=1}^{\lfloor \frac{N}{2} \rfloor} \sum_{n=-N+m}^{N-m} d_{p-2m,q+n} \cdot h_{-2m,n} + \\ & \sum_{m=1}^{\lfloor \frac{N}{2} \rfloor} \sum_{n=-N+m}^{N-m} d_{p+2m,q+n} \cdot h_{2m,n} + n_{p,q} \end{aligned} \quad (4)$$

where N is the length of interference from neighboring islands, $\lfloor x \rfloor$ is a floor function, which is the function that takes as input a real number x and gives as output the greatest integer less than or equal to x , and $n_{p,q}$ is electronic noise modeled as additive white Gaussian noise with variance σ^2 and zero mean. Since the interference from neighboring islands in $N = 2$ is relatively negligible, we set $N = 1$ for simplicity.

Figure 2. Block diagram of the proposed system model.

2.2. PRML Detection

The 2D interference that occurs when the readback signal is affected by surrounding bits is usually equalized to target the response by PRML, which is usually employed in data storage systems. The PRML detector consists of a PR equalizer and a ML channel decoder based on a Viterbi algorithm. A PR equalizer reshapes the channel response to the PR pulse shape according to the PR target. Thus, a suitable PR target for channel response is important for achieving a better performance. However, when an unsuitable PR target is used, the equalizer output can be an inaccurate value because of noise enhancement. The received data $r_{p,q}$ influenced by the BPMR channel and noise are entered into the 2D equalizer. The equalizer output $e_{p,q}$ is calculated by

$$\begin{aligned} e_{p,q} = & \sum_{n=1}^L r_{p,q-\lceil \frac{L}{2} \rceil + n} \cdot c_{\lceil \frac{L}{2} \rceil, n} + \sum_{m=1}^{\lceil \frac{L}{2} \rceil} \sum_{n=m+1}^{L-m+1} r_{p-(2m-1),q-\lceil \frac{L}{2} \rceil + n} \cdot c_{\lceil \frac{L}{2} \rceil - (2m-1), n} + \\ & \sum_{m=1}^{\lceil \frac{L}{2} \rceil} \sum_{n=m+1}^{L-m+1} r_{p+(2m-1),q-\lceil \frac{L}{2} \rceil + n} \cdot c_{\lceil \frac{L}{2} \rceil + (2m-1), n} + \sum_{m=1}^{\lceil \frac{L}{4} \rceil} \sum_{n=-N+m}^{L-m+1} r_{p-2m,q-\lceil \frac{L}{2} \rceil + n} \cdot c_{\lceil \frac{L}{2} \rceil - 2m, n} + \\ & \sum_{m=1}^{\lceil \frac{L}{4} \rceil} \sum_{n=-N+m}^{L-m+1} r_{p+2m,q-\lceil \frac{L}{2} \rceil + n} \cdot c_{\lceil \frac{L}{2} \rceil + 2m, n'} \end{aligned} \quad (5)$$

where $c_{m,n}$ is the equalizer coefficient, L is the equalizer length, and $\lceil x \rceil$ is a ceiling function, which maps x to the least integer greater than or equal to x . In this study, $L = 5$ was set. When $L = 5$, the equalizer coefficients are as follows:

$$\begin{bmatrix} & c_{1,2} & c_{1,3} & c_{1,4} & \\ c_{2,1} & c_{2,2} & & c_{2,3} & c_{2,4} \\ c_{3,1} & c_{3,2} & c_{3,3} & c_{3,4} & c_{3,5} \\ c_{4,1} & c_{4,2} & c_{4,3} & c_{4,4} & \\ & c_{5,2} & c_{5,3} & c_{5,4} & \end{bmatrix}. \quad (6)$$

The least mean square algorithm was used for updating equalizer coefficients.

$$c_{m,n}^{k+1} = c_{m,n}^k + \mu(e_{p,q} - \sum_{n=-1}^1 d_{p,q-n} \cdot f_{n+2}) \cdot r_{p,q}, \quad (7)$$

where $c_{m,n}^{k+1}$ and $c_{m,n}^k$ are updated and current equalizer coefficients, respectively, μ is an adaptation gain, and f_n is a PR target coefficient in the down-track direction. To calculate the reliability or soft value of the input data, a soft output Viterbi algorithm (SOVA) was used. The equalizer output is input to the one-dimensional (1D) SOVA detector for the down-track direction. The branch metric of 1D SOVA is calculated using the following equation:

$$\lambda_{p,q}(s_i, s_j) = \left\{ e_{p,q} - (f_1 \cdot \hat{a}_{p,q-1}(s_i) + f_2 \cdot \hat{a}_{p,q}(s_i) + f_3 \cdot a_{p,q+1}(s_j)) \right\}^2, \quad (8)$$

where s_i and s_j are the current and next state, and $\hat{a}(s_j)$ and $a(s_k)$ are decisions at s_i and s_j , respectively.

3. Proposed 7/10 Modulation Code for Mitigating ITI

3.1. Encoding Scheme

The modulation coding schemes, such as run-length limited code and maximum transition run code, make transmission that is suitable for a channel possible using constraints. Normally, modulation codes are used for preventing error patterns, timing recovery, and DC balance in data storage systems. To accommodate specific constraints, a lookup table, a finite state machine, and so on are utilized in the modulation encoder. In general, in the modulation coding schemes, the performance is excellent when the code rate is low.

In this paper, a 7/10 modulation code that prevents serious ITI is proposed. The proposed modulation code using a lookup table and one-to-one mapping encodes the 7 bits of user data sequence $a = [a_0, a_1, a_2, a_3, a_4, a_5, a_6]$ to the $5 \times 2 (= 10 \text{ bits})$ array of coded data sequence $c = [c_0, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, c_9]$, as shown in Figure 3. To improve the performance of the proposed 7/10 modulation coding scheme, the codeword selection process was divided into Step 1 for removing error patterns causing the ITI effect, and Step 2 for providing enough Hamming distance among codewords to improve the correcting capability. The two islands on the upper track and the other two on the lower track affect one island on the main track in a staggered array BPMR. Therefore, a constraint for four neighboring islands is necessary to effectively reduce the ITI effect. However, heavy constraints on the islands cause the code rate to decrease. Thus, to increase the code rate, the proposed code restricts two pixels on the neighboring tracks not containing error patterns, which cause the ITI effect, such as $[1, 0, 1]^T$ and $[0, 1, 0]^T$.

For Step 1, Figure 4 shows the available patterns for each column ($[c_0, c_1, c_2, c_3, c_4]^T$ or $[c_5, c_6, c_7, c_8, c_9]^T$). The number of combinations in one column is 32 ($=2^5$). Out of 32 patterns, only 16 patterns that do not have patterns of $[1, 0, 1]^T$ and $[0, 1, 0]^T$ were selected for making codewords. Thus, 256 ($=16 \times 16$) codewords can be included by combining 16 codewords obtained in each column.

In Step 2, codewords having a distance of at least 2, which is the Hamming distance among nonidentical codewords, were found. The distance among the codewords enables the original codeword from the received sequence in the decoding process to be recovered correctly. Table 1 shows a list of 128 codewords obtained through Steps 1 and 2. For instance, the user data sequence of $a = [0, 0, 0, 0, 0, 0, 1]$ is encoded to $c_1 = [0, 0, 0, 0, 0, 0, 0, 0, 1, 1]$ by one-to-one mapping.

Figure 3. One-to-one mapping and demapping for the proposed modulation code.

$\begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix}$	or	$\begin{bmatrix} c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \end{bmatrix}$	$[0,0,0,0,0]^T$	$[0,0,1,1,1]^T$	$[1,0,0,0,0]^T$	$[1,1,0,0,1]^T$
			$[0,0,0,0,1]^T$	$[0,1,1,0,0]^T$	$[1,0,0,0,1]^T$	$[1,1,1,0,0]^T$
			$[0,0,0,1,1]^T$	$[0,1,1,1,0]^T$	$[1,0,0,1,1]^T$	$[1,1,1,1,0]^T$
			$[0,0,1,1,0]^T$	$[0,1,1,1,1]^T$	$[1,1,0,0,0]^T$	$[1,1,0,0,1]^T$

Figure 4. Available patterns for each column of codeword.

Table 1. Codeword list of the proposed 7/10 modulation code.

Input Sequence	Codeword	Input Sequence	Codeword	Input Sequence	Codeword	Input Sequence	Codeword
0000000	000000000	0100000	0011100001	1000000	1000000001	1100000	1100100001
0000001	0000000011	0100001	0011100111	1000001	1000000111	1100001	1100100111
0000010	0000000110	0100010	0011101110	1000010	1000001110	1100010	1100101110
0000011	0000001100	0100011	0011110000	1000011	1000010000	1100011	1100110000
0000100	0000001111	0100100	0011110011	1000100	1000010011	1100100	1100110011
0000101	0000010001	0100101	0011111001	1000101	1000011001	1100101	1100111001
0000110	0000011000	0100110	0011111100	1000110	1000011100	1100110	1100111100
0000111	0000011110	0100111	0011111111	1000111	1000011111	1100111	1100111111
0001000	0000100001	0101000	0110000000	1001000	1000100000	1101000	1110000001
0001001	0000100111	0101001	0110000011	1001001	1000100011	1101001	1110000111
0001010	0000101110	0101010	0110000110	1001010	1000100110	1101010	1110001110
0001011	0000110000	0101011	0110001100	1001011	1000101100	1101011	1110010000
0001100	0000110011	0101100	0110001111	1001100	1000101111	1101100	1110010011
0001101	0000111001	0101101	0110010001	1001101	1000110001	1101101	1110011001
0001110	0000111100	0101110	0110011000	1001110	1000111000	1101110	1110011100
0001111	0000111111	0101111	0110011110	1001111	1000111110	1101111	1110011111
0010000	0001100000	0110000	0111000001	1010000	1001100001	1110000	1111000000
0010001	0001100011	0110001	0111000111	1010001	1001100111	1110001	1111000011
0010010	0001100110	0110010	0111001110	1010010	1001101110	1110010	1111000110
0010011	0001101100	0110011	0111010000	1010011	1001110000	1110011	1111001100
0010100	0001101111	0110100	0111010011	1010100	1001110011	1110100	1111001111
0010101	0001110001	0110101	0111011001	1010101	1001111001	1110101	1111010001
0010110	0001111000	0110110	0111011100	1010110	1001111100	1110110	1111011000
0010111	0001111110	0110111	0111011111	1010111	1001111111	1110111	1111011110
0011000	0011000000	0111000	0111100000	1011000	1100000000	1111000	1111100001
0011001	0011000011	0111001	0111100011	1011001	1100000011	1111001	1111100111
0011010	0011000110	0111010	0111100110	1011010	1100000110	1111010	1111101110
0011011	0011001100	0111011	0111101100	1011011	1100001100	1111011	1111110000
0011100	0011001111	0111100	0111101111	1011100	1100001111	1111100	1111110011
0011101	0011010001	0111101	0111110001	1011101	1100010001	1111101	1111111001
0011110	0011011000	0111110	0111111000	1011110	1100011000	1111110	1111111100
0011111	0011011110	0111111	0111111110	1011111	1100011110	1111111	1111111111

3.2. Decoding Scheme

To decode the received sequence $\hat{c} = [\hat{c}_0, \hat{c}_1, \hat{c}_2, \hat{c}_3, \hat{c}_4, \hat{c}_5, \hat{c}_6, \hat{c}_7, \hat{c}_8, \hat{c}_9]$, a minimum Euclidean distance was implemented in the demodulation process. The Euclidean distance d^l between the received sequence and codeword is calculated by

$$d^l(\hat{c}, c^l) = \sum_{i=0}^9 (\hat{c}_i - c_i^l)^2 \quad (9)$$

where $c^l = [c_0^l, c_1^l, c_2^l, c_3^l, c_4^l, c_5^l, c_6^l, c_7^l, c_8^l, c_9^l]$ is the l -th codeword. In the proposed code, there are 128 Euclidean distances. The smallest of all Euclidean distances, which is the minimum Euclidean distance, was selected. Finally, the codeword corresponding to the minimum Euclidean distance was determined by demapping.

4. Simulation and Results

In this work, 100 pages with a page size of 900×900 islands per page were simulated. The 1D PR target for the 2D equalizer was (0.1, 1.0, 0.1). The 1D SOVA was used for channel detection. The channel signal-to-noise ratio (SNR) was defined as $10\log_{10}(1/\sigma^2)$, where σ^2 is additive white Gaussian noise. Table 2 shows the read head and pulse response parameters [14].

Table 2. Read head and pulse response parameters.

Parameters	
Square island with length	11 nm
Square island with thickness	10 nm
Read head element thickness	4 nm
Read head element width	15 nm
Read head gap distance	6 nm
Read head fly height	10 nm

Figure 5 displays the BER performance of the PRML and the proposed 7/10 modulation code at the same AD. At $\text{BER} = 10^{-6}$, when the AD was 2.0 Tb/in^2 , the performance of the proposed code was 2 dB better than that of the PRML. In addition, when the AD was 3.0 Tb/in^2 , the BER curve of the proposed code exhibited better performance than that of PRML detection. Since the proposed 7/10 modulation code eliminated ITI error patterns and provided enough distance among the nonidentical codewords, the performance of the proposed codes showed a better performance.

To verify the performance in accordance with the modulation coding scheme, the PRML (uncoded system) and the 4/6 [15], 8/10 [16], and the proposed 7/10 modulation codes were compared. For a fair comparison, the user density (UD), which is defined by $\text{UD} = \text{AD} \times \text{code rate}$, should be considered. The code rate of the PRML was 1 because of the uncoded system. The code rates of the 4/6, 8/10, and 7/10 modulation codes were 0.66, 0.8, and 0.7, respectively.

Figure 6 shows the BER comparison with respect to the modulation coding scheme at $\text{UD} = 1.4 \text{ Tb/in}^2$. The AD of the PRML and the 4/6, 8/10, and 7/10 modulation codes were 1.4, 2.1, 1.75, and 2.0 Tb/in^2 , respectively. The track pitch T_z and bit period T_x for each AD are shown in Table 3. For example, when the UD was 1.4 Tb/in^2 and 7/10 modulation code was used, the AD was 2.0 Tb/in^2 . To achieve an AD of 2.0 Tb/in^2 , T_z and T_x were 18 nm. In this simulation, the 8/10 modulation code had the worst performance. At a BER of 10^{-6} , the proposed 7/10 modulation code provided performance gains of approximately 0.4 and 1.6 dB over the 4/6 modulation code and the PRML detector, respectively. The proposed scheme has a higher code rate and better performance than the 4/6 modulation code, because the patterns that generate ITI are effectively removed. The 4/6 modulation code has the advantage of providing enough distance among the nonidentical codewords, and the 8/10 modulation code has the advantage of removing the ITI pattern. The proposed code showed good performance, because it combines these two advantages.

Figure 5. The bit error rate (BER) performance of the partial response maximum likelihood (PRML) and the proposed 7/10 modulation code at the same areal densities (AD).

Figure 6. BER comparison with respect to modulation coding scheme at the user density (UD) = 1.4 Tb/in^2 .

Table 3. The track pitch T_z and bit period T_x for each AD.

	T_z, T_x	T_z, T_x
PRML	21.5 nm (1.4 Tb/in^2)	17.5 nm (2.1 Tb/in^2)
4/6 modulation code	17.5 nm (2.1 Tb/in^2)	14.1 nm (3.15 Tb/in^2)
8/10 modulation code	19.2 nm (1.75 Tb/in^2)	15.7 nm (2.62 Tb/in^2)
7/10 modulation code	18 nm (2.0 Tb/in^2)	14.5 nm (3.0 Tb/in^2)

Figure 7 illustrates the BER performance according to the modulation coding scheme at $\text{UD} = 2.1 \text{ Tb/in}^2$. For a fair comparison, the suitable T_z and T_x for each AD are as presented in Table 2. At a BER of 10^{-3} , the proposed modulation code performed approximately 0.8 and 2.3 dB better than the 8/10 and 4/6 modulation codes, respectively. In this simulation, the performance of the 4/6 modulation code was the poorest. The reason is that the 4/6 modulation code proposed for holographic data storage is not suitable for BPMR. Most importantly, however, the PRML detector provided the best performance in all cases because the ITI effect dramatically increased, because of the fair comparison. Therefore,

when the AD is high and interference is severe, one must consider whether a modulation code should be used.

Figure 7. BER performance according to modulation coding scheme at UD = 2.1 Tb/in².

Figure 8 shows the BER performance depending on TMR from 0 to 30% when the UD was 1.4 Tb/in². In a situation with some reasonable error factors, the proposed 7/10 modulation code showed the best performance at UD = 1.4 Tb/in².

Figure 8. BER performance depending on TMR from 0% to 30% when UD = 1.4 Tb/in².

5. Conclusions

A 7/10 modulation code is proposed for staggered BPMR. To improve the performance of the proposed 7/10 modulation code, error patterns that cause ITI are eliminated, and enough distance among the nonidentical codewords is provided for improving the correcting capability. At a UD of 1.4 Tb/in², the proposed 7/10 modulation code has the best performance. However, at a UD of 2.1 Tb/in², the PRML detector achieves the best performance because increasing the AD for fair comparison causes a significant ITI problem. Thus, if the UD is high, one must consider whether a modulation code should be used.

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