

Article



# **Power Flow Calculation Methods for Power Systems** with Novel Structure UPFC

# Jian Yang<sup>D</sup> and Zheng Xu \*<sup>D</sup>

Department of Electrical Engineering, Zhejiang University, Hangzhou 310027, China; yangjian\_zju@zju.edu.cn \* Correspondence: xuzheng007@zju.edu.cn; Tel.: +86-571-8795-2074

Received: 28 June 2020; Accepted: 24 July 2020; Published: 25 July 2020



**Abstract:** Latest unified power flow controller (UPFC) projects adopt novel device structures to meet the requirements of practical applications. Developing power flow calculation methods for these new UPFC structures is of great significance for the design and operation of related projects. To address this issue, this paper deduces an equivalent model of the general structure UPFC, and presents the modified power mismatch equations and Jacobian matrix of the system. Then, three power flow calculation methods suitable for the novel structure UPFC are proposed based on the Newton–Raphson algorithm. The main difference between the three methods is the processing method of UPFC equivalent power injections and the Jacobian matrix. The characteristics of these methods are compared by case studies. Results demonstrate the effectiveness of the three methods for different UPFC structures, and it is found that the improved method, which considers the impact of the Jacobian matrix modifications in UPFC equivalent power injections, can realize power flow calculation in existing calculation programs, and improve the convergence characteristics.

**Keywords:** flexible AC transmission systems (FACTS); unified power flow controller (UPFC); power flow calculation; Newton–Raphson algorithm; novel structure

## 1. Introduction

Flexible AC transmission systems (FACTS) are a collection of power electronic-based controllers [1,2], including the static var compensator (SVC), static synchronous compensator (STATCOM), static synchronous series compensator (SSSC), unified power flow controller (UPFC), etc. FACTS provide an effective means to enhance the controllability and stability of power systems, and a solution to improve the utilization of power equipment [3]. As the latest generation of FACTS devices, unified power flow controllers (UPFC) have comprehensive control capability [4]. While improving transmission capacity and stability of power grids, UPFC also has unique advantages such as accurate and flexible power flow control, centralized layout and little impact on short-circuit current [5]. Therefore, it has a good application prospect in modern power systems.

In recent years, UPFCs have been widely adopted in China. Three UPFC projects have been put into operation in the last five years. These projects have made some improvements in the device structure [6–9]. As shown in Figure 1, different from the early projects, the latest projects present the following characteristics in terms of the UPFC device structure: (1) As most of the 110 kV and above power grids in China adopt double-circuit lines, the UPFC series side may contain two converters that control the power flows of the double-circuit lines separately, in order to improve the overall transmission capacity; (2) In view of the low demand for reactive power in urban high-voltage grids, the UPFC shunt-side bus can be changed from the original bus to a lower voltage bus to save occupation and investment. These novel UPFC structures have been practically applied because of their technical and economic advantages, and also provide important references for further applications of



UPFC. The analysis, development and comparison of power flow calculation methods for these novel structures are of great significance to the design and operation of related projects.

**Figure 1.** The structures of unified power flow controller (UPFC) projects: (**a**) Schematic diagram of the conventional structure UPFC; (**b**) Schematic diagram of the UPFC project in Nanjing western grid; (**c**) Schematic diagram of the UPFC project in Suzhou southern grid.

A large amount of research has been conducted on the power flow calculation method for the power system with UPFC, and various UPFC models have been proposed. In terms of algorithms, the unified iteration method and alternating iteration method are the two most important algorithms. The unified iteration method iteratively calculates the UPFC state variables (such as the magnitude and phase angle of the converter output voltage) together with the system state variables; the alternating iterative method decouples the UPFC from the external system, and then it solves the state variables of the UPFC and the external system separately.

In terms of UPFC models, various UPFC models have been proposed, such as the decoupled model, the comprehensive model, the load injection model, the  $\pi$  load injection model, etc. [10]. The decoupled model is adopted in [11–14]. In [11], for the typical control strategy that the UPFC controls the line active and reactive power and the AC bus voltage, an additional UPFC bus is added to the system. Then, the two buses connected with the UPFC are regarded as PV bus and PQ bus, respectively. Using this method, the UPFC can be removed from the system, and the power flow calculation can be realized by conventional power flow calculation programs. In [12], the UPFC state variables and the system state variables are calculated by the alternating iterative method. In [13,14], the equivalent circuit of the UPFC-embedded line is deduced according to the UPFC control targets, and thus the output voltages of the UPFC can be determined directly. Then, the modifications of power mismatch equations and the Jacobian matrix are presented, and a Newton–Raphson algorithm-based calculation method is proposed. In [15], the load injection model is adopted, and the UPFC is equivalent to an impedance and two injected loads at UPFC terminals. In [16], the  $\pi$  load injection model is adopted, and the UPFC is modeled with the  $\pi$  equivalent circuit and injected loads. Theoretically, when the load injection model (or the  $\pi$  load injection model) is adopted, the relevant elements of the Jacobian matrix could be modified to improve the convergence characteristic, but in practice, for the convenience of program implementation, the modifications to the Jacobian matrix are usually ignored. The above methods essentially solve the state variables of the UPFC and the external system separately, and thus belong to the category of the alternating iteration method. Different from these methods, the literature [17–20] adopts the comprehensive model. The series and shunt sides of the UPFC are equivalent to a series circuit of a controllable voltage source and an equivalent impedance, respectively. The voltage magnitudes and phase angles of the controllable voltage sources are regarded as new state variables, and solved together with the rest state variables of the system.

In general, no matter what model is used, the convergence characteristic of the power flow calculation is better when the unified iteration method is adopted. However, on the other hand, the alternating iterative method is more convenient and flexible in use, and requires fewer changes to traditional AC power flow calculation methods. Therefore, some articles try to integrate UPFC into some emerging methods to obtain the advantages of the above two methods. In [10], a new UPFC model based on current injection is developed to integrate the UPFC into the Newton–Raphson current injection load flow method (NR-CIM). In [21], the power/current injection mismatches (PCIM) load flow method is proposed, and the UPFC model is implemented in the developed load flow method. In addition to the research on power flow calculation methods and modeling methods, the constraints handling approach of the UPFC has been analyzed in [22,23]. Although the above articles are mainly applicable to the UPFC of the conventional structure, and have not considered the novel structure UPFC.

This paper deduces an equivalent model of the general structure UPFC, and proposes three power flow calculation methods suitable for different UPFC structures. The rest of this paper is organized as follows: the general structure UPFC and its equivalent model are presented in Section 2. Three power flow calculation methods are proposed in Section 3. The characteristics of these methods are validated by case studies in Section 4. At last, conclusions are drawn in Section 5.

## 2. The General Structure UPFC and Its Equivalent Model

#### 2.1. The General Structure of UPFC

As shown in Figure 1, the differences between the novel UPFC structures and the conventional structure include: (1) the shunt side of the UPFC may be connected to a different bus with the series side; (2) the series side of the UPFC may contains multiple converters. Considering the adaptability to the conventional structure and the convenience of explanation, this paper takes the structure shown in Figure 2a as the general structure of UPFC for analysis. Other structures will be discussed on the basis of this general structure.

When the decoupled model is adopted, the UPFC and its connected line can be equivalent to the power injections of corresponding buses (which are  $P_{lm} + jQ_{lm}$ ,  $P_{ml} + jQ_{ml}$  and  $P_{ne} + jQ_{ne}$ ), as shown in Figure 2b. The equivalent circuit of Figure 2a is shown in Figure 2c. In Figure 2c,  $\underline{u}_l$ ,  $\underline{u}_k$ ,  $\underline{u}_m$  and  $\underline{u}_n$  are the voltages of bus l, bus k, bus m and bus n,  $\underline{i}_1$ ,  $\underline{i}_2$ , and  $\underline{i}_n$  are the currents of corresponding lines.  $r_{lm}$ ,  $x_{lm}$  and  $b_{lm}$  are the resistance, reactance, and susceptance of the transmission line.  $\underline{u}_B$  and  $\underline{u}_E$  are the equivalent voltage sources of the series converter and the shunt converter,  $x_B$  and  $x_E$  are the reactance of the series transformer and the shunt transformer, respectively.  $P_{dc}$  is the active power exchanged between the series side and the shunt side of the UPFC.

According to Figure 2c, the equivalent power injections of the UPFC under different control strategies can be deduced as follows.

Shunt





Transmission

Line

 $\frac{m}{m}$ 

Series

**Figure 2.** The general structure of UPFC and its equivalent model: (**a**) the general structure of UPFC; (**b**) the equivalent model of the general structure UPFC; (**c**) the equivalent circuit of the general structure UPFC.

## 2.2. Power Flow of the UPFC Series Side

There are multiple control strategies for the UPFC series converter, mainly including: power flow regulation, series reactive compensation, phase angle shifting and terminal voltage regulation [24,25]. When these control strategies are adopted, the power flows of the UPFC series side can be calculated by the following method:

(1) Power flow regulation

Under this strategy, the active power flow and the reactive power flow of the transmission line are controlled. If we assume  $P_{ml} + jQ_{ml}$  is controlled to  $P_c + jQ_c$  ( $P_c$  and  $Q_c$  are the targets of power flow regulation), then according to Figure 2c we have:

$$\underline{i}_2 = \left(\frac{P_c + jQ_c}{\underline{u}_m}\right)^* - \underline{u}_m \times jb_{lm} \tag{1}$$

$$\underline{u}_k = \underline{u}_m - (r_{lm} + jx_{lm}) \times \underline{i}_2 \tag{2}$$

where \* denotes the conjugate of a complex number.

(2) Series reactive compensation

Under this strategy, the series side of the UPFC is controlled to be an equivalent series impedance. If we assume the equivalent impedance is  $\underline{Z}_c$ , then according to Figure 2c we have:

$$\underline{u}_{k} = \frac{\underline{Z}_{c} \times \underline{u}_{l} + \underline{Z}_{l} \times \underline{u}_{m}}{\underline{Z}_{c} + \underline{Z}_{l} + jb_{lm}\underline{Z}_{l}\underline{Z}_{c}}$$
(3)

where:

$$\underline{Z}_l = r_{lm} + jx_{lm} \tag{4}$$

(3) Phase angle shifting

Under this strategy, the voltage magnitude of bus *k* is the same as that of bus *l*. If we assume the desired phase angle difference between bus *k* and bus *l* is  $\theta$ , then we have:

$$\underline{u}_k = u_k \angle \theta_k = u_l \angle (\theta_l + \theta_c) \tag{5}$$

where:

$$\underline{u}_l = u_l \angle \theta_l \tag{6}$$

(4) Terminal voltage regulation

Under this strategy, the phase angle of bus k is the same as that of bus l. If we assume the desired voltage magnitude of bus k is  $u_c$ , then we have:

$$\underline{u}_k = u_k \angle \theta_k = u_c \angle \theta_l \tag{7}$$

According to Equations (2), (3), (5), and (7), it can be found that  $\underline{u}_k$  can always be represented by  $\underline{u}_l$ ,  $\underline{u}_m$  and the control targets of UPFC, no matter what control strategy is adopted. Then, according to Figure 2c, the power flows of the UPFC series side can be calculated by:

$$P_{lm} + jQ_{lm} = \underline{u}_l \times \left(\underline{u}_k \times jb_{lm} + \frac{\underline{u}_k - \underline{u}_m}{r_{lm} + jx_{lm}}\right)^*$$
(8)

$$P_{ml} + jQ_{ml} = \underline{u}_m \times \left(\underline{u}_m \times jb_{lm} - \frac{\underline{u}_k - \underline{u}_m}{r_{lm} + jx_{lm}}\right)^*$$
(9)

$$P_{km} + jQ_{km} = \underline{u}_k \times \left(\underline{u}_k \times jb_{lm} + \frac{\underline{u}_k - \underline{u}_m}{r_{lm} + jx_{lm}}\right)^*$$
(10)

When the UPFC has multiple series converters connected in series with multiple lines, the power flows of each line can be calculated according to Equations (8)–(10).

## 2.3. Power Flow of the UPFC Shunt Side

The shunt converter of the UPFC is used to control the DC voltage. When the active loss of the UPFC is ignored, the active power exchanged between the shunt converter and the power grid is:

$$P_{ne} = P_{km} - P_{lm} \tag{11}$$

When there are multiple series converters, we have:

$$P_{ne} = \sum_{i} (P_{kmi} - P_{lmi}) \tag{12}$$

where  $P_{kmi}$  and  $P_{lmi}$  are the active power flows of the *i*th series converter.

Besides, the shunt converter of the UPFC also controls  $Q_{ne}$  or the voltage magnitude of bus n. When the shunt converter controls  $Q_{ne}$ ,  $Q_{ne}$  is equal to the set value. When the shunt converter controls the voltage magnitude of bus n,  $Q_{ne}$  is determined after the power flow calculation is solved.

## 2.4. Equivalent Model of Conventional Structure UPFC

The equivalent model of the conventional structure UPFC is shown in Figure 3. The conventional structure UPFC can be regarded as a special case of the general structure UPFC, and its equivalent model corresponds to the situation where bus *l* and bus *n* are the same bus in Figure 2b. Therefore, other formulas can be kept unchanged, except that  $P_{lm} + P_{ne}$  and  $Q_{lm} + Q_{ne}$  are taken as the active and reactive power exchanged between the UPFC and bus *l* (*n*).



Figure 3. The equivalent model of the conventional structure UPFC.

#### 3. Three Power Flow Calculation Methods

## 3.1. Power Mismatch Equations and Jacobian Matrix

After the UPFC is considered, the power mismatch equations and the Jacobian matrix need to be modified. For the general structure UPFC, after the UPFC equivalent power injections are considered, the power mismatch equations of bus *l*, bus *m* and bus *n* in polar coordinates become:

$$\begin{cases} \Delta P_{l} = P_{ls} - P_{lm} - u_{l} \sum_{i \in l} u_{i} (G_{li} \cos \theta_{li} + B_{li} \sin \theta_{li}) \\ \Delta Q_{l} = Q_{ls} - Q_{lm} - u_{l} \sum_{i \in l} u_{i} (G_{li} \sin \theta_{li} - B_{li} \cos \theta_{li}) \\ \Delta P_{m} = P_{ms} - P_{ml} - u_{m} \sum_{i \in m} u_{i} (G_{mi} \cos \theta_{mi} + B_{mi} \sin \theta_{mi}) \\ \Delta Q_{m} = Q_{ms} - Q_{ml} - u_{m} \sum_{i \in m} u_{i} (G_{mi} \sin \theta_{mi} - B_{mi} \cos \theta_{mi}) \\ \Delta P_{n} = P_{ns} - P_{ne} - u_{n} \sum_{i \in n} u_{i} (G_{ni} \cos \theta_{ni} + B_{ni} \sin \theta_{ni}) \\ \Delta Q_{n} = Q_{ns} - Q_{ne} - u_{n} \sum_{i \in n} u_{i} (G_{ni} \sin \theta_{ni} - B_{ni} \cos \theta_{ni}) \end{cases}$$
(13)

For bus *l*, bus *m* and bus *n*, there are three items on the right side of their power mismatch equations. The first items ( $P_{ls}$ ,  $Q_{ls}$ ,  $P_{ms}$ ,  $Q_{ms}$ ,  $P_{ns}$  and  $Q_{ns}$ ) are the nodal injection power without the UPFC line. The second items ( $P_{lm}$ ,  $Q_{lm}$ ,  $P_{ml}$ ,  $Q_{ml}$ ,  $P_{ne}$  and  $Q_{ne}$ ) are the UPFC equivalent power injections. The third items are the power injected by the nodes into the network, which are the same as those of the system without the UPFC line.

Except these UPFC buses (bus *l*, bus *m* and bus *n*), the power mismatch equations of other buses are the same as those of the system without the UPFC line.

According to Equations (8)–(11),  $P_{lm}$ ,  $Q_{lm}$ ,  $P_{ml}$ ,  $Q_{ml}$  and  $P_{ne}$  are only related to the voltages of UPFC buses.  $Q_{ne}$  is either a fixed value (when the shunt converter controls  $Q_{ne}$ ) or does not need to participate in the iterations (when the shunt converter controls the voltage magnitude of bus n). Therefore, the Newton–Raphson matrix equations can be expressed as follows:

$$\begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \Delta Q_1 \\ \Delta Q_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} & N_{11} & N_{12} \\ H_{21} & H_{22} + H_U & N_{21} & N_{22} + N_U \\ J_{11} & J_{12} & L_{11} & L_{12} \\ J_{21} & J_{22} + J_U & L_{21} & L_{22} + L_U \end{bmatrix} \begin{bmatrix} \Delta \theta_1 \\ \Delta \theta_2 \\ \Delta U_1 / U_1 \\ \Delta U_2 / U_2 \end{bmatrix}$$
(14)

where  $P_1$ ,  $Q_1$ ,  $\theta_1$  and  $U_1$  are variables of non-UPFC buses,  $P_2$ ,  $Q_2$ ,  $\theta_2$  and  $U_2$  are variables of UPFC buses.  $H_{ij}$ ,  $N_{ij}$ ,  $J_{ij}$  and  $L_{ij}$  (i = 1, 2; j = 1, 2) in the Jacobian matrix are the same as those of the system without the UPFC.  $H_U$ ,  $N_U$ ,  $J_U$  and  $L_U$  are the modifications of the Jacobian matrix required after considering the UPFC line.

For the general structure UPFC,  $P_2$ ,  $Q_2$ ,  $\theta_2$  and  $U_2$  are:

$$\Delta P_{2} = \begin{bmatrix} \Delta P_{l} & \Delta P_{m} & \Delta P_{n} \end{bmatrix}^{T}$$

$$\Delta Q_{2} = \begin{bmatrix} \Delta Q_{l} & \Delta Q_{m} & \Delta Q_{n} \end{bmatrix}^{T}$$

$$\Delta \theta_{2} = \begin{bmatrix} \Delta \theta_{l} & \Delta \theta_{m} & \Delta \theta_{n} \end{bmatrix}^{T}$$

$$\Delta U_{2}/U_{2} = \begin{bmatrix} \Delta u_{l}/u_{l} & \Delta u_{m}/u_{m} & \Delta u_{n}/u_{n} \end{bmatrix}^{T}$$
(15)

where  $\theta_l$ ,  $\theta_m$  and  $\theta_n$  are the phase angles of  $\underline{u}_l$ ,  $\underline{u}_m$ ,  $\underline{u}_n$ . Further,  $u_l$ ,  $u_m$ ,  $u_n$  are the voltage magnitudes of  $\underline{u}_l$ ,  $\underline{u}_m$ ,  $\underline{u}_m$ ,  $\underline{u}_n$ .

If the shunt converter controls  $u_n$ ,  $\Delta u_n$  does not need to participate in the iterations. If the shunt converter controls  $Q_{ne}$ ,  $\Delta u_n$  needs to participate in the iterations, and according to Equations (8)–(11),  $H_u$ ,  $N_u$ ,  $J_u$  and  $L_u$  can be calculated as:

$$H_{\mathcal{U}} = -\begin{bmatrix} \frac{\partial P_{lm}}{\partial \theta_l} & \frac{\partial P_{lm}}{\partial \theta_m} & 0\\ \frac{\partial P_{ml}}{\partial \theta_l} & \frac{\partial P_{ml}}{\partial \theta_m} & 0\\ \frac{\partial P_{ne}}{\partial \theta_l} & \frac{\partial P_{ne}}{\partial \theta_m} & 0 \end{bmatrix}, N_{\mathcal{U}} = -\begin{bmatrix} u_l \frac{\partial P_{lm}}{\partial u_l} & u_m \frac{\partial P_{ml}}{\partial u_m} & 0\\ u_l \frac{\partial P_{me}}{\partial u_l} & u_m \frac{\partial P_{ml}}{\partial u_m} & 0\\ u_l \frac{\partial P_{ne}}{\partial u_l} & u_m \frac{\partial P_{me}}{\partial u_m} & 0\\ \frac{\partial Q_{ml}}{\partial \theta_l} & \frac{\partial Q_{lm}}{\partial \theta_m} & 0\\ 0 & 0 & 0 \end{bmatrix}, L_{\mathcal{U}} = -\begin{bmatrix} u_l \frac{\partial Q_{lm}}{\partial u_l} & u_m \frac{\partial Q_{lm}}{\partial u_m} & 0\\ u_l \frac{\partial Q_{ml}}{\partial u_l} & u_m \frac{\partial Q_{lm}}{\partial u_m} & 0\\ u_l \frac{\partial Q_{ml}}{\partial u_l} & u_m \frac{\partial Q_{ml}}{\partial u_m} & 0\\ 0 & 0 & 0 \end{bmatrix}.$$
(16)

# 3.2. Handling Conventional Structure UPFC

For the conventional structure UPFC, according to Figure 3, the power mismatch equations of UPFC buses (bus *l* and bus *m*) are:

$$\begin{cases} \Delta P_{l} = P_{ls} - (P_{lm} + P_{ne}) - u_{l} \sum_{i \in l} u_{i} (G_{li} \cos \theta_{li} + B_{li} \sin \theta_{li}) \\ \Delta Q_{l} = Q_{ls} - (Q_{lm} + Q_{ne}) - u_{l} \sum_{i \in l} u_{i} (G_{li} \sin \theta_{li} - B_{li} \cos \theta_{li}) \\ \begin{cases} \Delta P_{m} = P_{ms} - P_{ml} - u_{m} \sum_{i \in m} u_{i} (G_{mi} \cos \theta_{mi} + B_{mi} \sin \theta_{mi}) \\ \Delta Q_{m} = Q_{ms} - Q_{ml} - u_{m} \sum_{i \in m} u_{i} (G_{mi} \sin \theta_{mi} - B_{mi} \cos \theta_{mi}) \end{cases} \end{cases}$$
(17)

The Jacobian matrix of the system still can be represented by Equation (14). If we assume:

$$\begin{cases} \Delta P_{2} = \begin{bmatrix} \Delta P_{l} & \Delta P_{m} \end{bmatrix}^{\mathrm{T}} \\ \Delta Q_{2} = \begin{bmatrix} \Delta Q_{l} & \Delta Q_{m} \end{bmatrix}^{\mathrm{T}} \\ \Delta \theta_{2} = \begin{bmatrix} \Delta \theta_{l} & \Delta \theta_{m} \end{bmatrix}^{\mathrm{T}} \\ \Delta \theta_{2} = \begin{bmatrix} \Delta \theta_{l} & \Delta \theta_{m} \end{bmatrix}^{\mathrm{T}} \\ \Delta U_{2}/U_{2} = \begin{bmatrix} \Delta u_{l}/u_{l} & \Delta u_{m}/u_{m} \end{bmatrix}^{\mathrm{T}} \end{cases}$$
(18)

Then, the modifications of the Jacobian matrix are:

$$H_{U} = -\begin{bmatrix} \frac{\partial (P_{lm} + P_{ne})}{\partial \theta_{l}} & \frac{\partial (P_{lm} + P_{ne})}{\partial \theta_{m}} \\ \frac{\partial P_{ml}}{\partial \theta_{l}} & \frac{\partial P_{ml}}{\partial \theta_{m}} \end{bmatrix}, N_{U} = -\begin{bmatrix} u_{l} \frac{\partial (P_{lm} + P_{ne})}{\partial u_{l}} & u_{m} \frac{\partial (P_{lm} + P_{ne})}{\partial u_{m}} \\ u_{l} \frac{\partial P_{ml}}{\partial u_{l}} & u_{m} \frac{\partial P_{ml}}{\partial u_{m}} \end{bmatrix}, J_{U} = -\begin{bmatrix} u_{l} \frac{\partial (Q_{lm} + Q_{ne})}{\partial u_{l}} & u_{m} \frac{\partial (Q_{lm} + Q_{ne})}{\partial u_{m}} \\ u_{l} \frac{\partial (Q_{lm} + Q_{ne})}{\partial u_{l}} & \frac{\partial (Q_{lm} + Q_{ne})}{\partial \theta_{m}} \end{bmatrix}, L_{U} = -\begin{bmatrix} u_{l} \frac{\partial (Q_{lm} + Q_{ne})}{\partial u_{l}} & u_{m} \frac{\partial (Q_{lm} + Q_{ne})}{\partial u_{m}} \\ u_{l} \frac{\partial Q_{ml}}{\partial u_{l}} & u_{m} \frac{\partial (Q_{lm} + Q_{ne})}{\partial u_{m}} \end{bmatrix}.$$
(19)

Compared with Equation (16), it can be observed that because the number of UPFC buses decreases, the items in the modifications decrease. Especially, when the UPFC series converter controls  $P_{ml} + jQ_{ml}$ ,

and the UPFC shunt converter controls the voltage magnitude of bus *l*, only the partial derivatives of  $P_{lm} + P_{ne}$  need to be calculated in Equation (19). At the same time, according to Equations (1), (2) and (8)–(11),  $P_{lm} + P_{ne}$  can be calculated as:

$$P_{lm} + P_{ne} = P_{km} = -P_c + \frac{\left(S_1^2 + S_2^2\right)r_{lm}}{u_m^2}$$
(20)

where:

$$\begin{cases} S_1 = Q_c \times \cos(\theta_l - \theta_m) + P_c \times \sin(\theta_l - \theta_m) + u_m^2 \times \cos(\theta_l - \theta_m) b_{lm} \\ S_2 = Q_c \times \sin(\theta_l - \theta_m) - P_c \times \cos(\theta_l - \theta_m) + u_m^2 \times \sin(\theta_l - \theta_m) b_{lm} \end{cases}$$
(21)

According to Equation (20), only the partial derivative of  $P_{lm} + P_{ne}$  to  $u_m$  is not zero. Therefore, at this time, only one item in the Jacobian matrix needs to be modified with the partial derivative of  $P_{lm} + P_{ne}$  to  $u_m$  in **Nu**.

#### 3.3. Three Power Flow Calculation Methods

Using the power mismatch equations and the Jacobian matrix given above, the power flow of the power systems with novel structure UPFC can be solved by the Newton–Raphson algorithm. However, when the above method (which is referred to as the "original method") is used for power flow calculation, the power mismatch equations need to be recalculated according to Equation (13) (or Equation (17)), and the Jacobian matrix needs to be modified according to Equation (14). Notably, because the Jacobian matrix needs to be modified in each step of iterations it is difficult to implement this method in existing power flow calculation programs.

Therefore, in practice, the Jacobian matrix is usually not modified, but the equivalent power injections are updated during iterations. The power mismatch equations of this method (which is referred to as the "simplified method") are the same as those of the original method, so the same calculation accuracy as the original method can be achieved. However, due to the simplification of the Jacobian matrix, the convergence characteristics of the power flow calculation will deteriorate. Under the same condition, the simplified method may require more iterations or even may not converge.

To solve this problem, under the condition that the Jacobian matrix is not changed during the iterations, the impact of the Jacobian matrix modifications can be considered in the UPFC equivalent power injections. According to Equation (14), we have:

$$\begin{bmatrix} \Delta P_{1} \\ \Delta P_{2} \\ \Delta Q_{1} \\ \Delta Q_{2} \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & H_{U} & 0 & N_{U} \\ 0 & 0 & 0 & 0 \\ 0 & J_{U} & 0 & L_{U} \end{bmatrix} \begin{bmatrix} \Delta \theta_{1} \\ \Delta \theta_{2} \\ \Delta U_{1}/U_{1} \\ \Delta U_{2}/U_{2} \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} & N_{11} & N_{12} \\ H_{21} & H_{22} & N_{21} & N_{22} \\ J_{11} & J_{12} & L_{11} & L_{12} \\ J_{21} & J_{22} & L_{21} & L_{22} \end{bmatrix} \begin{bmatrix} \Delta \theta_{1} \\ \Delta \theta_{2} \\ \Delta U_{1}/U_{1} \\ \Delta U_{2}/U_{2} \end{bmatrix}$$
(22)

The right side of Equation (22) is the Jacobian matrix used in the simplified method. Therefore, if the power mismatch equations of the UPFC buses are modified so that the system power deviations have the form of the left side of Equation (22), the power flow calculation process without the Jacobian matrix modifications can have the same convergence characteristics as the original method.

To achieve this goal, the second items of the left side of Equation (22) can be added to the equivalent power injections of the UPFC. Then, the new equivalent power injections of the UPFC become:

$$\begin{bmatrix} P_{lm}' \\ P_{ml}' \\ P_{ne'}' \\ Q_{lm'}' \\ Q_{ml'} \\ Q_{ne'} \end{bmatrix} = \begin{bmatrix} P_{lm} \\ P_{ml} \\ P_{ne} \\ Q_{lm} \\ Q_{ml} \\ Q_{ml} \\ Q_{ne} \end{bmatrix} + \begin{bmatrix} H_{U} & N_{U} \\ J_{U} & L_{U} \end{bmatrix} \begin{bmatrix} \Delta \theta_{2} \\ \Delta U_{2}/U_{2} \end{bmatrix}$$
(23)

And the power mismatch equations of UPFC buses become:

$$\begin{cases} \Delta P_{l}' = P_{ls} - P_{lm}' - u_{l} \sum_{i \in l} u_{i} (G_{li} \cos \theta_{li} + B_{li} \sin \theta_{li}) \\ \Delta Q_{l}' = Q_{ls} - Q_{lm}' - u_{l} \sum_{i \in l} u_{i} (G_{li} \sin \theta_{li} - B_{li} \cos \theta_{li}) \\ \Delta P_{m}' = P_{ms} - P_{ml}' - u_{m} \sum_{i \in m} u_{i} (G_{mi} \cos \theta_{mi} + B_{mi} \sin \theta_{mi}) \\ \Delta Q_{m}' = Q_{ms} - Q_{ml}' - u_{m} \sum_{i \in m} u_{i} (G_{mi} \sin \theta_{mi} - B_{mi} \cos \theta_{mi}) \\ \Delta P_{n}' = P_{ns} - P_{ne}' - u_{n} \sum_{i \in m} u_{i} (G_{ni} \cos \theta_{ni} + B_{ni} \sin \theta_{ni}) \\ \Delta Q_{n}' = Q_{ns} - Q_{ne}' - u_{n} \sum_{i \in n} u_{i} (G_{ni} \sin \theta_{ni} - B_{ni} \cos \theta_{ni}) \end{cases}$$

$$(24)$$

Then, the system power deviations can be represented by the left side of Equation (22).

It should be noted that, in principle, at the *i*th iteration,  $\Delta \theta_2$  and  $\Delta U_2/U_2$  in Equation (23) should be the results of this iteration. To calculate the new equivalent power injections of the UPFC in explicit form,  $\Delta \theta_2$  and  $\Delta U_2/U_2$  are estimated as follows [26]:

$$\begin{bmatrix} \Delta \theta_2^i \\ \Delta U_2^i / U_2^i \end{bmatrix} \approx \lambda \begin{bmatrix} \Delta \theta_2^{i-1} \\ \Delta U_2^{i-1} / U_2^{i-1} \end{bmatrix}$$
(25)

where  $\lambda$  is a scale factor, and the value range of  $\lambda$  is [0,1]. At the first iteration,  $\Delta \theta_2$  and  $\Delta U_2/U_2$  are both equal to **0**.

With this method (which is referred to as the "improved method"), after the power flow calculation converges,  $\Delta \theta_2$  and  $\Delta U_2$  both approach **0**, so that the new equivalent power injections of the UPFC approach the original power injections. Therefore, the power flow obtained by the improved method is the same as that of the original method. Besides, because the improved method uses the same Jacobian matrix as the simplified method, it can be easily implemented in existing power flow calculation programs. Since the improved method will have better convergence characteristics than the simplified method.

## 4. Case Studies

#### 4.1. WSCC 9-Bus Test System

In order to compare the characteristics of the three power flow calculation methods (the original method, the simplified method, and the improved method), the Western System Coordinating Council (WSCC) 9-bus test system is modified [27]. As shown in Figure 4, a general structure UPFC is added in the system. The series converter is connected in line 4–5, and controls the power flow of the line. The shunt converter is connected with bus 6, and controls the voltage of bus 6 at 1 per unit (pu).





Figure 4. The Western System Coordinating Council (WSCC) 9-bus test system with the general structure UPFC.

The three power flow calculation methods are adopted, respectively, under an Intel(R) Core(TM) i7-5600U CPU personal laptop. All the calculations are initialized with a flat start, and terminated when the largest voltage magnitude change and the largest voltage phase angle change are both smaller than  $10^{-8}$  or the maximum iteration count (which is 50) is reached.

When  $P_c + jQ_c = -30$  MW-j30 MVAr, the three calculation methods can all converge. The power flow calculation results of the three methods are the same, which are shown in Tables 1 and 2. It is shown that the results of the power flow calculations are reasonable, and the control targets of the UPFC are achieved. The accuracy of the simplified method and the improved method are verified.

Table 1. Bus voltage results.

Bus No.	Voltage/pu	Phase Angle/°
1	1.000	0.000
2	1.000	9.285
3	1.000	4.147
4	0.979	-2.428
5	1.001	-4.959
6	1.000	1.292
7	0.983	0.114
8	0.994	3.403
0	0.052	4 524

9	0.952	-4.53
8	0.994	3.403
/	0.905	0.11-

From Bus No.	To Bus No.	P/MW	Q/MVAr
1	4	72.08	37.62
2	8	163.00	17.80
3	6	85.00	2.12
4	5	28.53	14.94
4	9	43.55	18.87
5	6	-60.00	0.00
6	7	21.71	4.15
7	8	-78.37	-11.00
8	9	84.09	-0.03

Table 2. Power flow results.

Furthermore, in order to compare the convergence characteristics of these methods,  $P_c$  is changed from -210 MW to 120 MW. The iteration numbers required under different  $P_c$  are shown in Table 3 Table 3 and Figure 5.

and Figure 5. For the improved method, when the value of  $\lambda$  changes from 0 to 1 in steps of 0.05, the minimum number of iterations under different  $\lambda$  and the corresponding value of  $\lambda$  are given in Table 3. Besides, when  $\lambda$  is 0.1, the iteration number of the improved method is also presented in

	Original Mathad	Simplified Method	Improved Method		
$P_c/NIVV$	Original Method		$\lambda = \lambda_{\min}$	$\lambda_{\min}$	$\lambda = 0.1$
120	6	47	17	0.15	25
90	6	19	12	0.1	12
60	5	11	9	0.05	10
30	5	8	9	0.05, 0.1	9
0	5	8	9	0.05	10
-30	5	8	9	0.05	10
-60	5	8	8	0.05, 0.1	8
-90	5	11	9	0.05	10
-120	5	15	11	0.1	11
-150	5	22	13	0.1	13
-180	6	32	16	0.15	19
-210	6	>50 (not converge)	17	0.15	28

**Table 3.** Iteration numbers of the three methods under different  $P_c$ .



**Figure 5.** The comparison of iteration numbers under different  $P_c$ .

According to Table 3 and Figure 5, the following conclusions can be drawn:

- The number of iterations required by the original method is always the least. Under different control targets of the UPFC, the iteration numbers of the original method are always kept at about 5, and the change is very small.
- (2) When the power flow control target is close to the natural power flow (which is about -30 MW), the number of iterations required by the simplified method is about 10, which is slightly more than the number required by the original method. However, when the regulation effect of the UPFC on power flow increases (for example, when  $P_c < -100$  MW), the number of iterations required by the simplified method increases significantly.
- (3) The improved method requires more iterations than the original method to achieve convergence; but compared with the simplified method, when the UPFC power flow regulation effect increases, the improved method has significantly fewer iterations than the simplified method. The number of iterations can be effectively reduced by simply using the improved method with  $\lambda = 0.1$ . When the UPFC power flow regulation effect is small, the iteration number of the improved method is

not much different from the simplified method; both methods can reach convergence in about 10 iterations.

In addition, for each situation in Table 3, when the original method or the improved method is used, the computation time is about 0.01 s. In most cases, the computation time of the simplified method is also around 0.01 s, except when  $P_c$  is 120 MW or -210 MW, the computation time is 0.02 s. This is because the iterations of the three methods are all based on the Newton–Raphson algorithm, only the calculation method of the UPFC equivalent power injections and the Jacobian matrix are different. The calculation of the UPFC equivalent power injections and the modification of the Jacobian matrix do not need to be iterated, so their computation time is very short, and can almost be ignored. Therefore, the difference in the computation time of the three methods is mainly related to the number of iterations. When  $P_c$  is 120 MW or -210 MW, the number of iterations of the simplified method increases significantly, so the computation time also increases.

When  $P_c$  is -150 MW, the variations of the largest voltage magnitude change and the largest voltage phase angle change of the three methods are shown in Figure 6. As shown in Figure 6, the original method maintains the square-convergence characteristic of the Newton–Raphson algorithm, while the convergence characteristic of the simplified method is approximately linear convergence. The convergence characteristic of the improved method is between the original method and the simplified method.



**Figure 6.** The iteration process of the three methods when  $P_c$  is -150 MW. (**a**) The largest voltage magnitude change; (**b**) the largest voltage phase angle change.

For the system shown in Figure 4, if the general structure UPFC is changed to a conventional structure UPFC with the shunt converter connected to bus 4, the iteration numbers of the three methods under different UPFC control targets are shown in Figure 7. As shown in Figure 7, the iteration numbers are all about 6. According to the analysis in Section 3.2, the main reason for the above result lies in: for the conventional structure UPFC, if the series converter controls the active power and reactive power of the line, and the shunt converter controls the AC bus voltage, only one item is needed to be modified in the Jacobian matrix. The addition of the UPFC has little effect on the Jacobian matrix, so the iteration numbers of the three methods are very close.



Figure 7. The iteration numbers of the test system with the conventional structure UPFC.

#### 4.2. IEEE 39-Bus Test System

In order to further verify the conclusions obtained by the foregoing test case, and to investigate the convergence characteristics of the three power flow calculation methods in a larger system, the Institute of Electrical and Electronics Engineers (IEEE) 39-bus test system is modified. As shown in Figure 8, the shunt converter is connected with bus 5 and the series converters are connected in the double-circuit lines between buses 4 and 14.



**Figure 8.** The Institute of Electrical and Electronics Engineers (IEEE) 39-bus test system with a novel structure UPFC.

Similar to the previous case, when the reactive power of the double-circuit lines is both controlled at 25 MVAr and the voltage of bus 5 is controlled at 1 pu, the iteration numbers of the three methods under different active power control targets are shown in Table 4 and Figure 9.

	Original Mathad	Simplified Method	Improved Method		
<b>P</b> <sub>C</sub> /1 <b>V1</b> VV	Oliginal Method	Simplined Method	$\lambda = \lambda_{\min}$	$\lambda = 0.1$	
1250	6	39	17	23	
1000	6	24	15	15	
750	5	17	13	13	
500	5	12	11	11	
250	5	8	9	9	
0	5	5	5	6	
-250	5	8	8	9	
-500	5	10	10	11	
-750	5	16	13	13	
-1000	5	23	15	15	
-1250	6	36	17	21	
-1500	6	>50 (not converge)	19	33	

**Table 4.** Iteration numbers of the three methods under different  $P_c$ .



**Figure 9.** The comparison of iteration numbers under different  $P_c$ .

As shown in Table 4 and Figure 9, the original method requires the least iterations. Under different  $P_c$ , the iteration numbers of the original method are always kept at about 5; When the power flow control target is close to the natural power flow (which is about –125 MW), that is,  $P_c$  changes in the range of –500 MW to 500 MW, the iteration numbers of the simplified method and the improved method are similar, both are about 10, but when  $P_c$  exceeds the above range, the number of iterations of the simplified method increases rapidly, and the number of iterations of the improved method is less than that of the simplified method, which verifies the effectiveness of the improved method.

In addition, for all situations in Table 4, when the number of iterations is less than 33, the computation time of the three methods is around 0.01 s; when the number of iterations is greater (equal to or more than 33), the computation time will reach 0.02 s. Overall, the computation time of the three methods is relatively short.

Compared with the WSCC 9-bus test case, we can also find that the number of iterations of the three methods is independent of system scale. After the system scale increases, the number of iterations of the original method can still be kept at about 5. When the UPFC power flow regulation effect is small, the iteration numbers of the simplified method and the improved method can also be kept at about 10.

Moreover, when the shunt converter of the UPFC is reconnected to bus 14 from bus 5, the iteration numbers of the three methods under different UPFC control targets are always about 6. The reason for this result is that the UPFC series converters are connected in double-circuit lines, and the shunt converter is connected to one terminal of the lines. Similar to the conventional structure UPFC,

when the series converters control the active power and reactive power of the lines, and the shunt converter controls the AC bus voltage, only one item in the Jacobian matrix needs to be modified, thus the addition of the UPFC has little effect on the Jacobian matrix. Therefore, the three power flow calculation methods are not much different.

## 5. Conclusions

This paper deduces a decoupled UPFC model based on the general structure UPFC. Three power flow calculation methods applicable to both the novel structure UPFC and the conventional structure UPFC are proposed, and the characteristics of these methods are analyzed and validated. The main conclusions are summarized as below:

- (1) Using the UPFC model proposed in this paper, the UPFC with different structures can be easily integrated in a power flow calculation.
- (2) The power flow calculation method ("original method"), which strictly corrects the power mismatch equations and the Jacobian matrix, can maintain good convergence characteristics under different control targets; however, because this method requires modifications of the Jacobian matrix, it is more complicated in terms of program implementation.
- (3) The simplified method, which only modifies the power mismatch equations based on the UPFC equivalent power injections and does not modify the Jacobian matrix, can realize the power flow calculation in existing calculation programs. However, when the UPFC has a great effect on power flow regulation, the iteration number of this method will increase significantly.
- (4) The improved method, which considers the impact of the Jacobian matrix modifications in the UPFC equivalent power injections, can also realize power flow calculation in existing power flow calculation programs, and this method is superior to the simplified method in terms of convergence characteristics.
- (5) For the conventional structure UPFC, if the series converter controls the active power and reactive power of the line, and the shunt converter controls the AC bus voltage, only one item is needed to be modified in the Jacobian matrix. The addition of the UPFC has little effect on the Jacobian matrix, thus the convergence characteristics of the three methods are very close.

The comparison of these methods is summarized in Table 5. The method with more stars ( $\bigstar$ ) is better. It should be noted that because all these methods are based on the Newton–Raphson algorithm, they have the same disadvantages, such as the need to update and factorize the Jacobian matrix in iteration. Various novel methodologies are developed to overcome these disadvantages [28,29]. In future works, the method to integrate the UPFC into these methodologies should be studied.

	Programming		Performance	
Method	Jacobian Matrix	UPFC Equivalence	Convergence Characteristic	Computing Time
Original method	*	***	***	***
Simplified method	***	***	*	**
Improved method	$\star\star\star$	**	**	$\star\star\star$

**Table 5.** Comparison of the proposed methods. " $\star$ " is used to score methods, the method with more stars ( $\star$ ) is better.

**Author Contributions:** Investigation, writing: J.Y.; Supervision, Z.X. All authors have read and agreed to the published version of the manuscript.

**Funding:** This research was funded by the Headquarters Research Projects of State Grid Corporation of China, grant number SGTYHT/15-JS-191.

Conflicts of Interest: The authors declare no conflict of interest.

## References

- Hingorani, N.G. High power electronics and flexible AC transmission system. *IEEE Power Eng. Rev.* 1988, *8*, 3–4. [CrossRef]
- 2. Georgilakis, P.S.; Hatziargyriou, N.D. Unified power flow controllers in smart power systems: Models, methods, and future research. *IET Smart Grid* **2019**, *2*, 2–10. [CrossRef]
- 3. Albatsh, F.M.; Mekhilef, S.; Ahmad, S.; Mokhlis, H.; Hassan, M.A. Enhancing power transfer capability through flexible AC transmission system devices: A review. *Front. Inf. Technol. Electron. Eng.* **2015**, *16*, 658–678. [CrossRef]
- 4. Gyugyi, L.; Schauder, C.D.; Williams, S.L.; Rietman, T.R.; Tprgerson, D.R. The unified power flow controller: A new approach to power transmission control. *IEEE Trans. Power Deliv.* **1995**, *10*, 1085–1097. [CrossRef]
- Tang, J.; Ma, H.; Yang, X.; Liu, X.; Wang, X.; Wu, J.; Li, D.; Liu, L. Research on AC transmission line fault ride-through control strategy of MMC-based unified power flow controller. *J. Eng.* 2017, 2017, 1580–1583. [CrossRef]
- Cai, H.; Yang, L.; Wang, H.; Song, P.; Xu, Z. Application of Unified Power Flow Controller (UPFC) in Jiangsu power system. In Proceedings of the 2017 IEEE Power & Energy Society General Meeting, Chicago, IL, USA, 16–20 July 2017; pp. 1–5.
- Li, P.; Wang, Y.; Feng, C.; Lin, J. Application of MMC-UPFC in the 500 kV power grid of Suzhou. J. Eng. 2017, 2017, 2514–2518. [CrossRef]
- 8. Wei, X.; Cui, Y.; Feng, Y.; Song, J.; Yu, Y.; Bao, W.; Guo, Q. Analysis of application effcet of 220 kV UPFC demonstration project in Shanghai Grid. *Power Syst. Prot. Control* **2018**, *46*, 136–142.
- 9. Yang, J.; Song, P.; Xu, Z.; Xiao, H.; Cai, H.; Xie, Z. Small-signal model of vector current-controlled MMC-UPFC. *IET Gener. Transm. Distrib.* **2019**, *13*, 4180–4189. [CrossRef]
- 10. Kamel, S.; Jurado, F.; Lopes, J.A.P. Comparison of various UPFC models for power flow control. *Electr. Power Syst. Res.* **2015**, *121*, 243–251. [CrossRef]
- 11. Nabavi-Niaki, A.; Iravani, M.R. Steady-state and dynamic models of unified power flow controller (UPFC) for power system studies. *IEEE Trans. Power Syst.* **1996**, *11*, 1937–1943. [CrossRef]
- 12. Xu, Y.; Li, N.; Wang, H.; Chen, H. Model of power flow control based on unified power flow controller. *Proc. EPSA* **1996**, *8*, 1–6.
- 13. Fang, W.L.; Ngan, H.W. Control setting of unified power flow controllers through a robust load flow calculation. *IEE Proc. Gener. Transm. Distrib.* **1999**, *146*, 365–369. [CrossRef]
- 14. Wang, X.; Wan, F.; Du, Z. *Modern Power System Analysis*; China Science Publishing & Media Ltd.: Beijing, China, 2003.
- 15. Haque, M.H.; Yam, C.M. A simple method of solving the controlled load flow problem of a power system in the presence of UPFC. *Electr. Power Syst. Res.* **2003**, *65*, 55–62. [CrossRef]
- 16. Alomoush, M.I. Exact Pi-model of UPFC-inserted transmission lines in power flow studies. *IEEE Power Eng. Rev.* **2002**, *22*, 54–56. [CrossRef]
- 17. Fuerte-Esquivel, C.R.; Acha, E. Unified power flow controller: A critical comparison of Newton–Raphson UPFC algorithms in power flow studies. *IEE Proc. Gener. Transm. Distrib.* **1997**, *144*, 437–444. [CrossRef]
- 18. Zhang, X. Modeling of FACTS in power flow and optimal power flow analysis. *Autom. Electr. Power Syst.* **2005**, *29*, 22–29.
- Zhang, X.; Godfrey, K.R. Advanced unified power flow controller model for power system steady state control. In Proceedings of the 2004 IEEE International Conference on Electric Utility Deregulation, Restructuring & Power Technologies, Hong Kong, China, 5–8 April 2004; pp. 228–233.
- 20. Jiang, X.; Fang, X.; Chow, J.H.; Edris, A.; Uzunovic, E.; Parisi, M.; Hopkins, L. A novel approach for modeling voltage-sourced converter-based FACTS controllers. *IEEE Trans. Power Deliv.* 2008, 23, 2591–2598. [CrossRef]
- Kamel, S.; Abdel-Akher, M.; Jurado, F.; Ruiz-Rodríguez, F.J. Modeling and analysis of voltage and power control devices in current injections load flow method. *Electr. Power Compon. Syst.* 2013, 41, 324–344. [CrossRef]
- 22. Ebeed, M.; Kamel, S.; Yu, J.; Jurado, F. Development of UPFC operating constraints enforcement approach for power flow control. *IET Gener. Transm. Distrib.* **2019**, *13*, 4579–4591. [CrossRef]
- 23. Ebeed, M.; Kamel, S.; Jurado, F. Constraints violation handling of GUPFC in Newton-Raphson power flow. *Electr. Power Compon. Syst.* **2017**, *45*, 925–936. [CrossRef]

- 24. Vural, A.M.; Tumay, M. Mathematical modeling and analysis of a unified power flow controller: A comparison of two approaches in power flow studies and effects of UPFC location. *Electr. Power Energy Syst.* 2007, 29, 617–629. [CrossRef]
- 25. Foerte-Esquive, C.R.; Acha, E.; Ambriz-Perez, H. A comprehensive Newton-Raphson UPFC model for the quadratic power flow solution of practical power networks. *IEEE Trans. Power Syst.* **2000**, *15*, 102–109. [CrossRef]
- 26. Yi, S.; Guo, Z. Power flow solution by improved mode additional injection model with power flow controllers. *Relay* **2004**, *32*, 5–10.
- 27. Anderson, P.M. Power system control and stability. IEEE Power Eng. Rev. 1995, 15, 40. [CrossRef]
- 28. Milano, F. Implicit continuous Newton method for power flow analysis. *IEEE Trans. Power Syst.* **2019**, *34*, 3309–3311. [CrossRef]
- 29. Tostado-Véliz, M.; Kamel, S.; Jurado, F. Power flow approach based on the S-iteration process. *IEEE Trans. Power Syst.* **2020**. [CrossRef]



© 2020 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/).