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A Palm-Jacobaeus Loss Formula for Multi-Service Systems with Separated Resources

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Abstract: This article presents a method to determine the blocking probability (non-availability) for strictly determined multi-service resources that belong to a group of multi-service resources. The dependencies obtained during the process correspond to the Palm-Jacobaeus formula derived for the group of resources servicing single-service traffic. The approach to determining the blocking probability is based on the determination of the availability of resources at the occupancy level of allocation units. The analytical results are compared with the obtained results of the simulation experiments for a number of selected parameters of multi-service groups of resources. The results of the present study indicate a high accuracy of the proposed solutions. The elaborated extension of the Palm-Jacobaeus formula can be used in the modeling of separated wireless resources in cellular networks.

Keywords: Palm-Jacobaeus loss formula; blocking probability; multi-service traffic; separated resources; performance evaluation; cellular systems

1. Introduction

One of the main problems in service quality management in mobile networks is the problem of efficient resource allocation [1–3]. Resource allocation techniques are increasingly affecting the performance of subsequent cellular network technologies (3G, 4G, 5G) [4]. Among other things, this is directly related to two factors [5]. The first is the decreasing size of cells. This factor affects both the more frequent changes of cells due to user mobility and the increase in the number of cells (resources) that can handle a given request (call). The other factor, however, is the diversity of services provided in cellular networks in terms of, e.g., requested bit rates or acceptable delays.

In order to effectively use the resources of cellular networks, including in particular 4G and 5G multi-service networks, it became necessary to develop mechanisms that optimize the traffic distribution among neighboring cells. As a result of initial research [6], the concept of self-organizing (self-optimizing, self-configuring) networks (SONs) was developed, in which the load balancing of individual cells of cellular networks is a key element [7,8]. The developed concept of load balancing made it possible to take into account the changing load of individual cells over time—resulting, among others, from user mobility [9]. Such optimized traffic distribution in mobile networks allows mobile operators both to better use network resources and to improve service quality parameters (quality of experience) of multi-service traffic streams generated by end users [10,11].

Currently ongoing studies on self-optimizing cellular systems take into account not only the load-balancing criterion. The use of appropriate self-optimization techniques reduces the costs of the manual configuration of network elements, reduces the number of rejected connections, ensures better matching of resources to requests, and reduces energy costs [11]. SONs also support the implementation of the concept of end-to-end network slicing in 5G systems [12,13].

Regardless of the objective and technique used to optimize resource management, however, the area covered by a specific group of cells can be treated—from the point of view of traffic engineering—as a multi-service system in which multi-service requests are handled using "separated" resources (cells), creating network resources [10]. An analysis of the state of research on the analytical modeling of cellular systems indicates that multi-service (multi-rate) models are most often used to determine the traffic characteristics of such systems at the call level [14–20].

In most of the work on modeling cellular systems at the call level, only single cells [16–18,21] were taken into account. For the purpose of analyzing a group of cells (resources) as a single system, it is necessary to use more complex multi-service models, including the so-called limited-availability group model [22] and the non-full-availability group model [10]. In these studies [10,22], groups of cells with a connection handover mechanism for load balancing were analyzed. The developed methods take into account only information about the amount of occupied resources in a group of cells (resources), without the possibility of determining the occupancy states of specific (indicated) cells (resources) in the group. This assumption did not directly affect the determination of the probability of blocking in a group of cells (resources); however, it did not allow for taking into account separate call admission restrictions in individual cells.

In many new tasks posed to, among others, SON mechanisms and network slicing in 5G systems, the ability to determine the occupancy (non-availability) of precisely indicated (and not any) cells can be very important. The development of a general and effective analytical method for the determination of the non-availability of strictly determined/defined resources in a certain set of them for defined classes of traffic streams will make it possible for the method to be further parameterized, e.g., within the context of the aforementioned traffic distribution in a group of cells, the distribution of resources for network slicing, or the assessment of the number of necessary links in trunks (e.g., EtherChannels).

In the case of single-service systems, the Palm-Jacobaeus formula provided us with the ability to determine the probability of blocking the indicated resources within their group. A method for the determination of the blocking probability $H(x_d)$ of x_d strictly determined resources (from a set containing k resources) is proposed in [23], while in [24] this method is presented in the form of the following equation, known as the Palm-Jacobaeus loss formula:

$$H(x_d) = \frac{E_k(A)}{E_{k-x_d}(A)}, \tag{1}$$

where $E_k(A)$, the so-called Erlang B-formula, is the blocking probability in a single-service group composed of k resources to which traffic with an intensity of A Erlangs is offered. The parameter x_d defines the number of strictly determined resources to be investigated. In traffic engineering, Equation (1) has been used, directly or in the form of the so-called modified Palm-Jacobaeus loss formula [25,26], to model complex non-full-availability systems, single-service resource groups [25,27], and single-service multi-stage switching networks, e.g., [28–32].

A counterpart (corresponding equivalent) of the group of single-service resources is the group of separated multi-service resources—the limited availability group (LAG). The LAG is a system composed of k independent separated resources, each with the capacity of f adopted (assumed) AUs (allocation units). It is important to underline that the resources are independent of one another and are separated, which means that the LAG can service a given call only when at least one resource, from among all k resources, has sufficient resources needed to provide service to this call. Such a definition of service excludes any possibility of a division of resources demanded by a call between a given number of different resources.

The LAG is analyzed, for example, in [33–35]. In [36], a simple approximate LAG model for resources with different capacities is proposed, while [37] presents a LAG model for different resource capacities. In multi-service traffic engineering, the LAG is used to model and optimize mobile networks, cf. [10], as well as output groups of links (resources) in multi-service switching networks, e.g., [38–41].

We see the main application of the elaborated extension of the Palm-Jacobaeus formula in modeling of separated wireless resources in cellular networks. However, preliminary studies undertaken by us led to the conclusion that the determination of the blocking probability of strictly determined resources in the resource group will make it possible to simplify the modeling of a large number of complex multi-service systems in mobile and optical networks, including network nodes and switching networks.

This article is structured as follows. Section 2 presents the model of a group with limited availability. Section 3 includes a proposal of a method to determine the blocking probability for strictly defined resources. Section 4 provides a comparison of the results obtained in the analytical modeling with the results of the simulation experiments for a number of selected structures of multi-service resource groups. The last section summarizes the most important results of the study.

2. Model of the Group with Limited Availability

Consider a system called LAG (limited-availability group) in the literature, cf. [36,37]. The group is composed of k identical component resources, each with the capacity of f AUs (allocation units) (Allocation unit is a universal term describing the unit of resources required in a given system. An example of calculating the allocation unit in 5G systems with OFDM (orthogonal frequency division multiplexing)-based cells is presented in [42]), therefore defining the capacity of the system to be equal to $V = kf$ AUs. The system services a call only when the call can be serviced by the available resources of one of the component resources. In traffic engineering, AU is defined as the greatest common divisor of the bitrates of all the call classes offered to the system [43–47].

The assumption is that the LAG services a mixture M of independent Erlang traffic streams with the following intensities: $A_1, A_2, \ldots, A_M$. A call of class i requires t_i AUs to set up a connection. A multi-dimensional Markov process in the LAG can be approximated by a one-dimensional Markov chain and can be described by the following equation [48]:

$$n\left[P(n)\right]_V = \sum_{i=1}^{M} A_i t_i \xi_i(n-t_i)\left[P(n-t_i)\right]_V, \tag{2}$$

where $[P(n)]_V$ is the probability that the LAG is in a state of n busy AUs in the system, whereas $\xi_i(n)$ is the conditional transition probability for a traffic stream of class i between the adjacent (neighboring) states of the process. The blocking probability E_i for a stream of class i in the LAG model can be determined by the following equation:

$$E_i = \sum_{n=0}^{V} \left[P(n)\right]_V \left[1 - \xi_i(n)\right]. \tag{3}$$

The conditional transition probability $\xi_i(n)$ in the LAG model [48] can be approximated by the following dependency:

$$\xi_i(n) = \frac{F(V-n,k,f,0) - F(V-n,k,t_i-1,0)}{F(V-n,k,f,0)}, \tag{4}$$

where $F(x,k,f,t)$ is the number of possible arrangements of x objects in k identical boxes, each of which is able to accommodate f objects. A further assumption is that there are at least t objects (from among all the x objects to be arranged) in each of the boxes:

$$F(x,k,f,t) = \sum_{i=0}^{\left\lfloor \frac{x-kt}{f-t+1} \right\rfloor} (-1)^i \binom{k}{i} \binom{x-k(t-1)-1-i(f-t+1)}{k-1}. \tag{5}$$

Note that Equation (4) applies to the arrangements of free (unoccupied) AUs in the LAG and can be interpreted as the ratio between the number of favorable arrangements, i.e., those in which at least one resource that has t_i free AUs can be found, and the number of all possible arrangements $V - n$ of free AUs.

3. Non-Availability Probability of Strictly Defined Resources

The non-availability probability $H(i, x_d)$ of x_d strictly determined resources determines the probability of an event in which each resource from the x_d selected resources does not have enough t_i free AUs to set up a connection of class i. This probability can be determined with respect to the occupancy of AUs, i.e., directly at the occupancy level of AUs. Thus, we have:

$$H(i, x_d) = \sum_{n=(f-t_i+1)x_d}^{V} H(i, x_d|n)[P(n)]_V, \tag{6}$$

where $H(i, x_d|n)$ is the conditional non-availability probability of a selected number of x_d resources for a call of class i, determined under the assumption that the total number of busy AUs in the LAG is equal to n. The lower limit of the sum in Equation (6) determines the minimum number of AUs that can induce non-availability for a selected number of x_d resources.

The conditional distribution $H(i, x_d|n)$ can be determined on the basis of the following reasoning. The selected x_d resources are unavailable to a call of class i if n_d AUs ($n_d \leq n$) are arranged in these x_d resources in such a way that none of them has t_i free AUs. On the basis of Equation (5), the number of such arrangements is $F(n_d, x_d, f, f - t_i + 1)$. The remaining $n - n_d$ busy AUs can be arranged (accommodated) in any way within the remaining $k - x_d$ resources. The number of such arrangements is $F(n - n_d, k - x_d, f, 0)$. Since the total number of arrangements of n AUs in k resources is $F(n, k, f, 0)$, the probability $H(i, x_d|n)$ can be determined in the following way:

$$H(i, x_d|n) = \sum_{n_d=(f-t_i+1)x_d}^{\min(n, f x_d)} \frac{F(n_d, x_d, f, f - t_i + 1)F(n - n_d, k - x_d, f, 0)}{F(n, k, f, 0)}. \tag{7}$$

Equation (6) derived in the present article for multi-service systems is the counterpart of the Palm-Jacobaeus loss formula (Equation (1)) derived for single-service systems [24]. The original Palm-Jacobaues formula is a substantial extension of the Erlang model for the full-availability group (Erlang B-formula [49]): The Erlang B-formula makes it possible to determine the blocking probability in a system with a capacity of k resources, whereas the Palm-Jacobaeus loss formula [24] allows us to determine the probability $H(x_d)$ of occupancy (non-availability) of x_d strictly determined resources (from a set containing k resources). In a similar way, Equation (6), developed in the present article, is a substantial extension of the model proposed in [48]. Equations (2) and (3) allow us to determine the occupancy distribution and the blocking probability in a multi-service system with a capacity of $V = kf$ AUs. The model proposed in the article makes it possible to determine the probability of an event in which the selected x_d (from among all the k resources) resources of a system can serve a call of a given class. Equation (6) proposed in the article is then in the same relation to the LAG model as the Palm-Jacobaeus formula is to the basic Erlang model and constitutes a substantial extension of the practical capabilities of the model in engineering applications.

4. Results and Discussion

4.1. General Assumptions

In the model that allows us to determine the probability of the non-availability of strictly determined resources for requests/calls (flows/streams) of particular traffic, we used the approach, generally accepted in the literature on the subject, according to which variable bitrates of real packet streams are replaced with constant bitrates [19,46,50,51]. Such an approach much simplifies the process of modeling, since it allows multi-rate (multi-service) systems to be analyzed in a Markovian way at the flow/stream/session level (call level). There are two approaches to the replacement of variable bitrates of packet streams with their equivalents with constant bitrates: They can be chosen on the basis of the maximum bitrates [42,51] of particular packet streams or, alternatively, on the basis of the so-called equivalent bandwidth determined for each packet stream [45,52]. An analysis at the flow level is the only approach to dimensioning, designing, and optimizing whole networks, which is much appreciated by network operators [19,46].

After the determination of constant bitrates (volume of required resources) for calls of individual classes serviced in the system, it is possible to determine the allocation unit (AU) for a given system. The AU is defined as the highest common divisor of the capacity of the system and all bitrates (equivalent bandwidths, volume of required resources) allocated to calls of individual classes. Subsequently, both the capacity of the system and the amount of resources required for a call of a given class to be serviced, are expressed in the number of AUs.

The value of a single AU can also depend on the applied technological solutions. Besides the convention of expressing AUs in bits per second, dominant in the literature, commonly used expressions include the so-called interference (noise) unit, used in WCDMA systems (wideband code division multiplexing) [19,53]. To maintain the maximum versatility of the developed model and taking the constant development of 5G systems for granted, we assume in our further research that it will be possible to express the volume of demands and the capacity of the system in the multiplicity of a certain AU, without specifying precisely the method for their determination. A suitable example of the determination of demands of individual traffic classes in 5G systems with OFDMA multiplying can be found in [42].

The proposed method for the determination of the non-availability probability of strictly determined resources is an approximate method, since the LAG model it uses is an approximate model for any analysis at the call level. It is possible to develop more accurate models, although an accurate and precise solution for the service process in the considered system would, however, involve its analysis at the level of the so-called microstates (a microstate is defined by the number of serviced calls of individual classes) whose sheer number would almost entirely prevent its solution. Such an approach is impractical and totally ineffective from an engineering point of view. Therefore, an approximation of real service processes with the LAG model (an analysis at the macrostate level, where the macrostate is determined by the number of occupied AUs) requires the validity of the adopted assumptions to be verified by simulation experiments.

4.2. Simulator

In order to determine the accuracy of the analytical method for determining the probability of the non-availability of strictly determined resources for calls of particular traffic classes, the results of the analytical calculations have been compared with the simulation data. The simulator was developed for the sole purpose of this article and was based on the process interaction approach, in line with the detailed and specific assumptions concerning modeling of multi-serve traffic streams presented in [54]. The simulator was implemented in C++. The input data for the simulator are

- the number k of resources,
- the capacity f of a single resource,
- the number M of traffic stream classes,
- the number t_i of demanded allocation units for a stream of class i ($1 \leq i \leq M$),
- the intensity μ_i of a service stream of class i,
- the proportions of offered traffic $A_1t_1 : A_2t_2 : \ldots : A_Mt_M = x : y : \ldots : z$, i.e., the values of the parameters $x, y, \ldots, z$,

- the average value a of traffic offered to a single allocation unit in the system, where:

$$a = \frac{\sum_{i=1}^{M} A_i t_i}{kf}. \tag{8}$$

On the basis of the parameters $x, y, \ldots, z$, which define the proportions of offered traffic $A_1t_1 : A_2t_: \ldots : A_Mt_M$, the simulator determines the values of the traffic intensity A_i for each of the offered traffic classes ($1 \leq i \leq M$). Then, on the basis of the definition of traffic intensity,

$$A_i = \frac{\lambda_i}{\mu_i}, \tag{9}$$

the simulator determines the value of the parameter λ_i, which defines the intensity of generating traffic streams of class i. The parameters λ_i and μ_i are given to exponential distribution generators used in the program to determine the time intervals between the arrival of a next traffic stream of class i and the service time of a given call of class i.

To determine the probability of the non-availability of strictly determined resources for calls of particular traffic classes, the condition for the termination of a simulation experiment is the amount of elapsed time (duration time) for individual series necessary to generate a predefined number of calls of the class that is least active (most frequently, this is the class with the biggest number of demanded allocation units). The average result is calculated on the basis of 10 series. In practice, to obtain confidence intervals that are not greater than 5% of the average value of the simulation, it is necessary to generate about 100,000,000 calls of the least active class.

4.3. Accuracy of the Model

In line with the information given in Section 4.1, the process of determining the non-availability probability in the system is based on an approximation of the Markov process in the system. The simulator developed for the purpose of this article (presented in Section 4.2) was used to evaluate the influence of this approximation on the accuracy of the proposed analytical model. The study carried out by us included a wide range of systems. The influence of the number of resources (k), their capacity (f), the number of classes of offered traffic streams (M), the volume of demanded resources (t_i), the proportion of offered traffic $A_1t_1 : A_2t_2 : \ldots : A_Mt_M$, and the intensity of service streams (μ_i) were all investigated. A a result of this study, three representative systems (from the perspective of the differences in the results) were chosen. The systems had the following parameters:

1. System 1: $k = 3$, $f = 20$ AUs, $V = 60$ AUs, $M = 3$, $t_1 = 1$ AU, $t_2 = 2$ AUs, $t_3 = 3$ AUs, $A_1t_1 : A_2 : t_2 : A_3t_3 = 1 : 1 : 1$, $\forall_{1 \leq i \leq M}\, \mu_i = 1$;
2. System 2: $k = 5$, $f = 20$ AUs, $V = 100$ AUs, $M = 3$, $t_1 = 1$ AU, $t_2 = 2$ AUs, $t_3 = 5$ AUs, $A_1t_1 : A_2 : t_2 : A_3t_3 = 1 : 1 : 1$, $\forall_{1 \leq i \leq M}\, \mu_i = 1$;
3. System 3: $k = 5$, $f = 50$ AUs, $V = 250$ AUs, $M = 3$, $t_1 = 1$ AU, $t_2 = 3$ AUs, $t_3 = 7$ AUs, $A_1t_1 : A_2 : t_2 : A_3t_3 = 1 : 1 : 1$, $\forall_{1 \leq i \leq M}\, \mu_i = 1$.

The obtained results of the non-availability for the strictly determined resources in the systems under investigation are presented in Figures 1–9. The determined confidence intervals for the simulation results are far lower than the markers used to indicate the results of the simulation experiments. For this reason they are not visible in the figures. In the specification of the examined systems, both the resources and the demands of the offered traffic streams are expressed in AUs. As a result, the notion of a small or large system is rather relative. If a system services call classes for which the AU is equal to 100 Mbps, then such a system can be considered to be large. If, however, the AU is 100 kbps, then the system can be regarded as small even though the number of demanded AUs is the same in both systems.

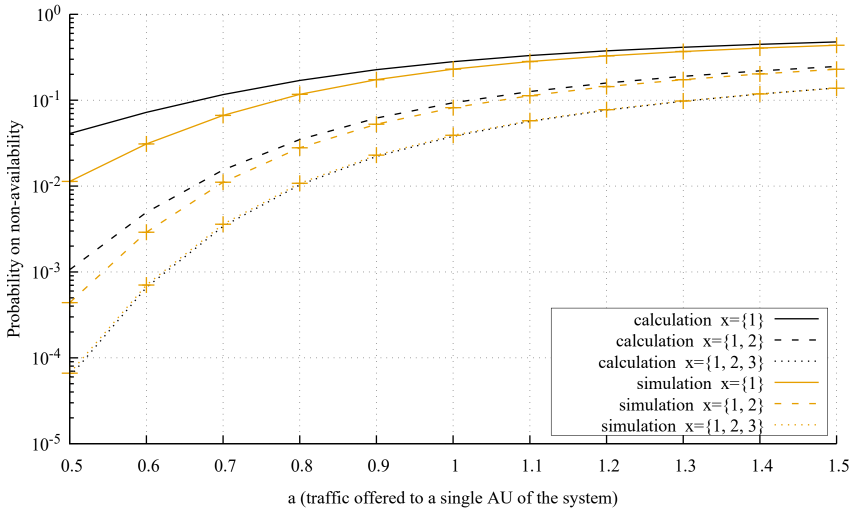

Figure 1. Probability of the non-availability of x strictly determined resources for class 1 calls ($t_1 = 1$) in System 1, where x is the set of strictly determined resources in the limited-availability group.

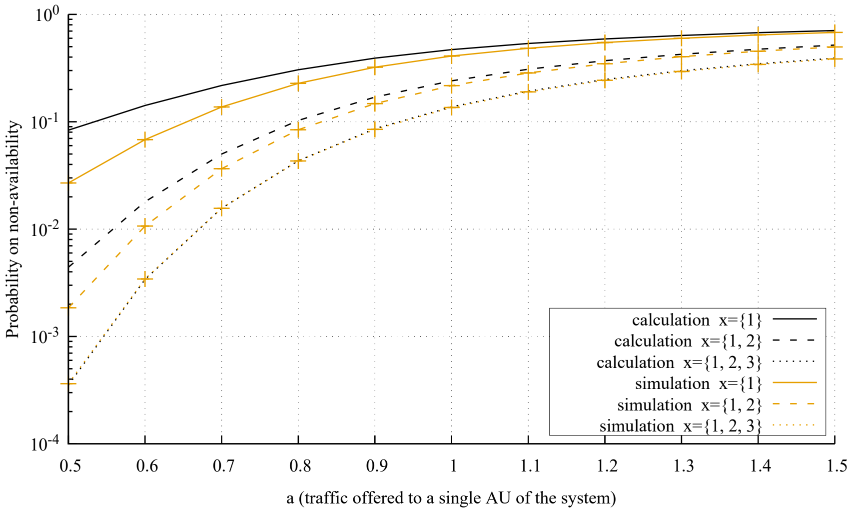

Figure 2. Probability of the non-availability of x strictly determined resources for class 2 calls ($t_2 = 2$) in System 1, where x is the set of strictly determined resources in the limited-availability group.

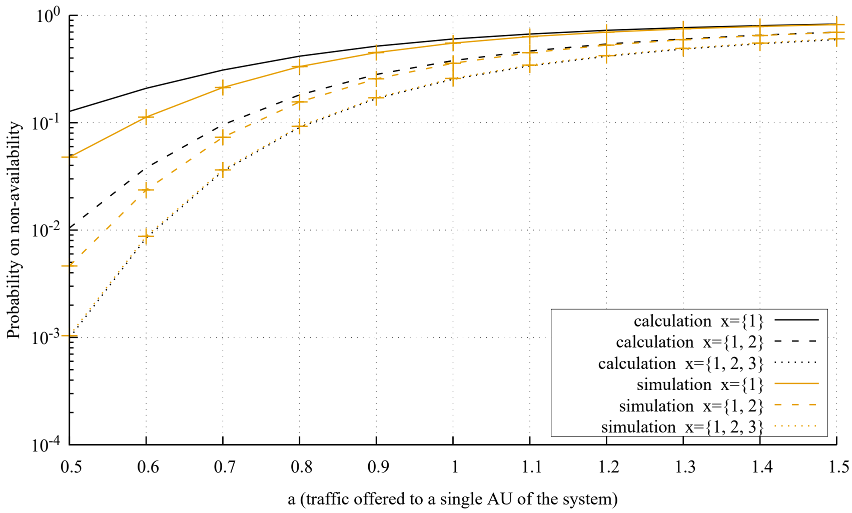

Figure 3. Probability of the non-availability of x strictly determined resources for class 3 calls ($t_3 = 3$) in System 1, where x is the set of strictly determined resources in the limited-availability group.

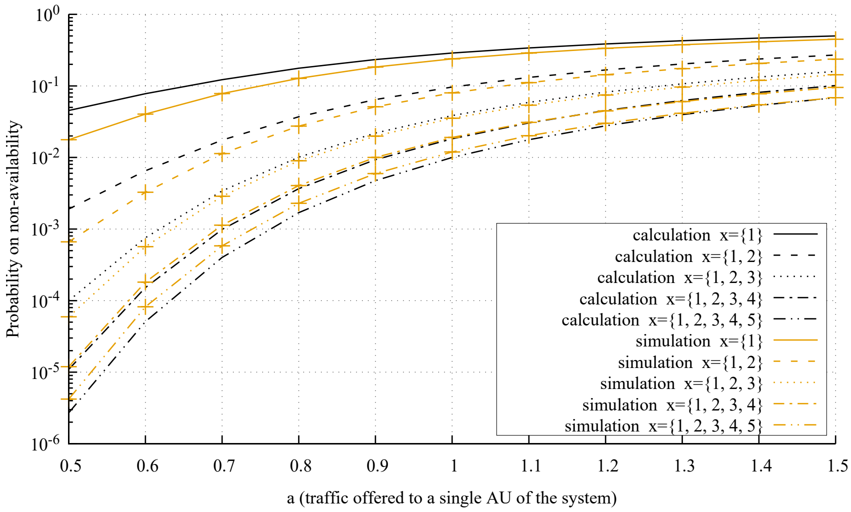

Figure 4. Probability of the non-availability of x strictly determined resources for class 1 calls ($t_1 = 1$) in System 2, where x is the set of strictly determined resources in the limited-availability group.

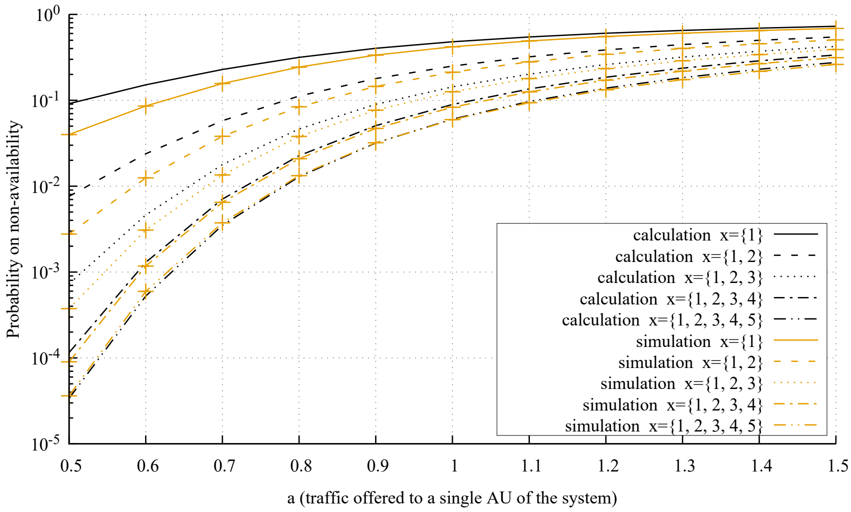

Figure 5. Probability of the non-availability of x strictly determined resources for class 2 calls ($t_2 = 2$) in System 2, where x is the set of strictly determined resources in the limited-availability group.

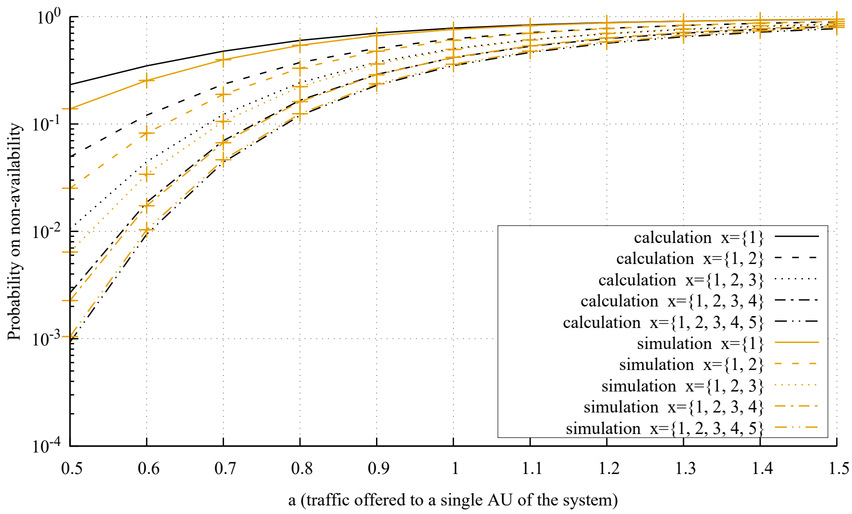

Figure 6. Probability of the non-availability of x strictly determined resources for class 3 calls ($t_3 = 5$) in System 2, where x is the set of strictly determined resources in the limited-availability group.

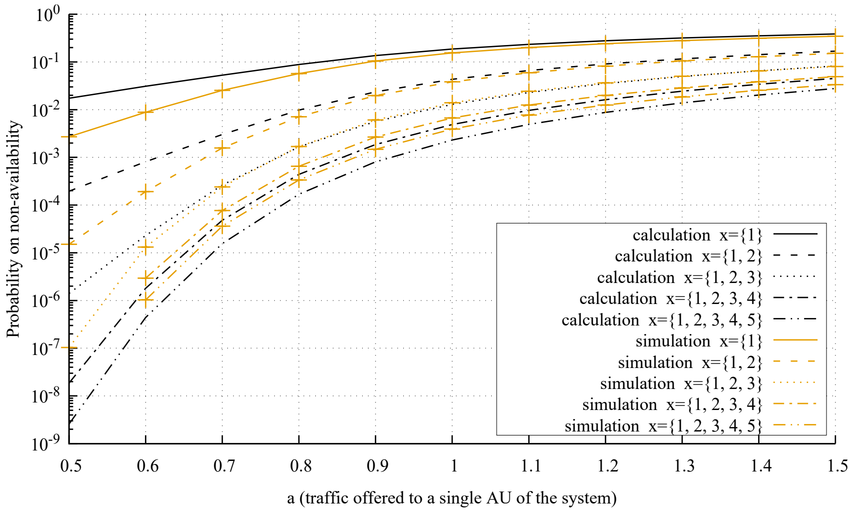

Figure 7. Probability of the non-availability of x strictly determined resources for class 1 calls ($t_1 = 1$) in System 3, where x is the set of strictly determined resources in the limited-availability group.

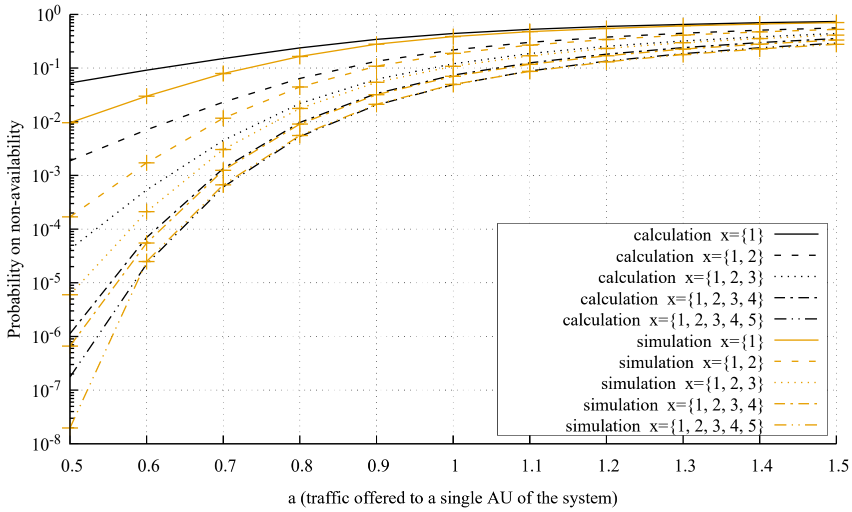

Figure 8. Probability of the non-availability of x strictly determined resources for class 2 calls ($t_2 = 3$) in System 3, where x is the set of strictly determined resources in the limited-availability group.

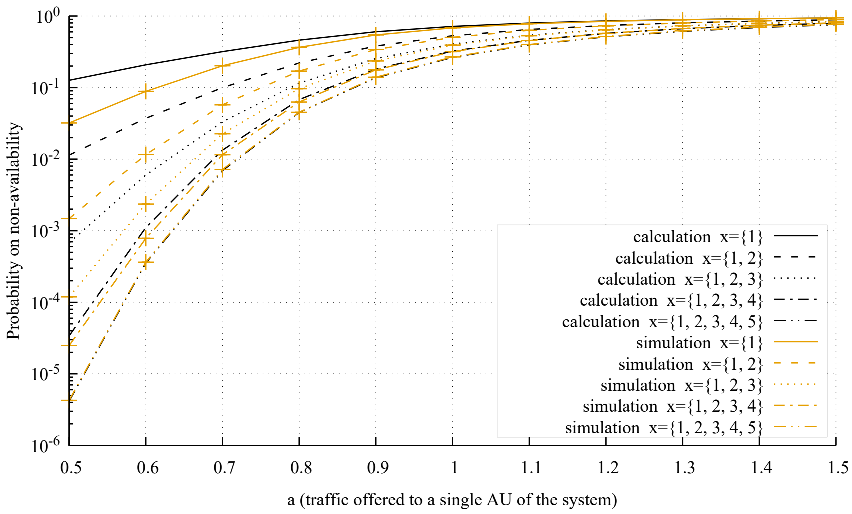

Figure 9. Probability of the non-availability of x strictly determined resources for class 3 calls ($t_3 = 7$) in System 3, where x is the set of strictly determined resources in the limited-availability group.

The comparative analysis of the obtained analytical and simulation results verifies that the developed analytical model, which allows us to determine the non-availability probability for strictly determined resources, is an approximate model. The main reasons for the inaccuracies, inherited from the basic model of the limited-availability group (which allows the probability to be determined in whole the system regardless of the occupancy distribution in individual resources), are the following:

- The analysis of the considered system is carried out from the microstate level (multi-dimensional Markov process) to the macrostate level (one-dimensional Markov process)—a detailed analysis of this problem is presented in [55].
- The conditional transition coefficients are determined in an approximate way (Equation (4)): in the process of determining the conditional transition probability (Equation (4)) in the occupancy distribution (Equation (2)) the division of n busy AUs between individual call classes is omitted.

On the basis of a large number of simulation experiments, it has been proven [48] that the influence of these two criteria on the inaccuracies in the obtained results stabilizes if the following condition is satisfied: $f > 5t_{\max}$, where $t_{\max}$ is the number of AUs required for a call with the maximum demands to be set up. In such cases, the basic LAG model offers the accuracy that is adequate for engineering applications.

The analysis of the results of the accuracy of the model proposed here shows that the accuracy of the model is not influenced by changes in the parameter μ_i, the proportions of the offered traffic, or the number of traffic classes. In addition, the influence of the number of demanded allocation units and the number of resources and their capacities is only slight. The results of the study, presented in Figures 1–9, allow us to evaluate the influence of any change in the value of these parameters. Any further increase in the number of resources, their capacity, and the demands of the individual traffic classes did not cause any increase in the inaccuracy of the systems under investigation. The main influence on the inaccuracy of the model is exerted by the number of resources x, whose non-availability is determined. The highest accuracy was obtained for a case in which we determined the non-availability of a large part of the resources, whereas the lowest accuracy was obtained when we determined the non-availability of only one resource. However, these dependencies are not critical for engineering applications related to the dimensioning of telecommunication networks, for which the possibility of servicing a given demand is critical. From the point of view of network operators, essential is also that the proposed method for the majority of traffic classes leads to overdimensioning of the system (i.e., the value of the non-availability probability is overestimated)—Figures 2, 3, 5, 6, 8 and 9—regardless of the number of unavailable resources. It is only for the traffic class with the lowest number of resources (Figures 1, 4 and 7) that the value of the non-availability probability is slightly underestimated. It is worthwhile to stress, though, that the classes with a larger number of required resources are mainly responsible for blocking.

To sum up, in the typical range of applications, i.e., for a traffic intensity per single AU between 0.7–1.0 Erlangs (Figures 1–9), the method ensures high accuracy and can be used in teletraffic engineering of multi-service communication systems.

Because the system is analyzed at the call (session) level, the direct determination of traffic parameters at the packet level is not possible. However, the possibility of expressing the amount of resources demanded at the packet level as the maximum bit rate or the equivalent bandwidth at the call level allows us to dimension the system (by determining its capacity) in such a way as to include, for example, acceptable packet delays, acceptable packet loss ratios, etc.

5. Conclusions

In this paper, a new method for calculating the probability of strictly determined multi-service resources in a group of resources has been proposed. The method allows us to elaborate a new formula for calculating the blocking probability of strictly determined resources in systems servicing multi-service traffic, e.g., in a group of cells in 4G and 5G systems. The new formula is an extension of the well-known Palm-Jacobaeus loss formula elaborated for systems with single-rate traffic. The new multi-service Palm-Jacobaeus formula opens new possibilities in the analytical modeling of communication systems with separated resources, such as multi-service switching networks [38,40,41,56–58], data centers [59–61], and systems with traffic overflow [62–65].

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