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Fuzzy Supplier Selection Method Based on Smaller-The-Better Quality Characteristic

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Abstract: Many important parts of tool machines all have the important smaller-the-better (STB) quality characteristics. The important STB quality characteristics will impact on the quality of the end-product. At the same time, supplier quality influences the quality and functionality of the end-product, so suppliers must be selected with caution. The six sigma quality index for the STB quality characteristics can directly reflect process quality levels. Besides, this index possesses a mathematical relationship with process yield. Nevertheless, the point estimation will cause the risk of misjudgment, due to sampling errors. As a result, this study applies the confidence interval of the index to a two-tailed fuzzy testing method, in order to select appropriate suppliers. Now that this method is on the basis of the confidence interval, the possibility of misjudgment caused by sampling errors will be reduced, while the precision of the selection will be enhanced. The method can help companies increase product quality, as well as the competitiveness of the industry chain as a whole. Finally, a numerical example is presented to show how to approach this method and its efficacy.

Keywords: fuzzy test method; quality characteristic; membership function; process quality level; process yield

1. Introduction

Many important parts of tool machines all have important smaller-the-better (STB) quality characteristics, such as roundness, concentricity and verticality of gears, bearings and axle centers, etc. The important smaller-the-better (STB) quality characteristics will decide the quality of the end product [1–3]. Process capability indices (PCIs) are commonly adopted by companies to measure process quality. PCIs not only help manufacturers evaluate process quality during production, but are also viewed as a useful tool of communication between engineers working for the same company. This has been confirmed by extensive research [4–10].

The project of six sigma quality improvement led by Motorola can effectively help companies enhance quality as well as reduce defect rates for production processes [11,12]. According to Linderman et al. [13], six sigma assessment helped Motorola achieve its quality-control target of only 3.4 defects per million opportunities. Thus, six sigma quality levels have also been widely applied to manufacturing, in order to measure process quality [14–18].

Many researchers, such as Chen et al. [19], Huang et al. [20], Wang et al. [21], and Yu and Chen [22], have attempted to derive relationships between capability indices and six sigma quality levels. This was

aimed at determining whether process quality meets specified requirements. In order to directly determine quality levels based on index values, Chen et al. [11] came up with the six sigma index of quality characteristics to assess the process quality of STB quality characteristics.

According to Prahalad and Hamel [23] and Grossman and Helpman [24], obtaining parts from suppliers has become a trend in business strategy to enhance competitiveness and operational flexibility. Chen and Chen [25] also pointed out that forming partnerships with suppliers can help companies increase product quality, as well as the competitiveness of the industry chain as a whole. Lei and Hitt [26] and Wu et al. [27] indicated that the quality of raw materials, components, and equipment should be taken into consideration when selecting suppliers, because they can affect end products' quality. The six sigma index can straightly display process quality levels as well as process yield. Consequently, we chose this index as our instrument for supplier selection in this study. Since this index contains unidentified parameters, sample data are required for estimation [28,29]. Numerous researchers have come up with fuzzy testing methods using confidence intervals to improve the precision of estimation and solve the problem of uncertainty found in the measured data [30,31]. A smaller size of the sample leads to a larger length of the interval, whereas a larger size of the sample results in a smaller length of the interval [28]. Consequently, applying confidence intervals to the statistical testing method can lead to inconsistencies or different sizes of the sample. Considering practicality, the sample size is rarely large. As a result, this study developed a two-tailed fuzzy testing method to select a proper supplier, which is more reasonable than the statistical testing method and more consistent with the discussions made in several studies [12,28–32]. The proposed method can help companies increase product quality, as well as the competitiveness of the industry chain as a whole.

The other sections of this paper are organized as follows. Section 2 is literature review. Section 3 is related to research design and methods including the confidence interval of the six sigma index and the two-tailed confidence-interval-based fuzzy testing method for the supplier selection. In Section 4, a numerical example is taken to illustrate the efficacy of the proposed method. Last but not least, conclusions are made in Section 5.

2. Literature Review

The supplier selection model developed by this study adopts the industrial division of labor in the industrial chain as a background and quality as a basis. At the same time, to solve the practical limitation that the sample is so small that the traditional statistical testing accuracy is insufficient, this study develops the fuzzy testing method based on the confidence interval. Subsequently, this study will review the literature according to the above description.

There are two important industrial chains in Taiwan, tool machines and electronics, of which the information and communication technology (ICT) has established itself as a crucial player in the global electronics industry [24,33]. In fact, Taiwan has formed a complete industrial ecological chain from IC design, wafer foundry, packaging and testing to production and assembly [34]. In addition, central Taiwan is an industrial settlement of tool machines. In the face of increasingly fierce global competition, enterprises have paid more and more attention to their specialized core technologies and gradually outsourced non-core components to other manufacturers, or purchased them from suppliers. This type of strategy has become a trend in business models [1,2,35].

The selection of suppliers is built on the basis of suppliers' performances. Weber et al. [36] used a questionnaire designed by Dickson [37] to investigate the frequency of 23 criteria appearing in different literature and sorted out the most commonly adopted selection criteria in order—delivery time, quality, and capacity. Patton [38] pointed out that product quality, price, delivery time, sales support, equipment and technology, order status, and financial status are seven criteria for the supplier evaluation. According to Verma and Pullman [39], the actual transaction data demonstrated that "quality", "price" and "delivery time" of the product are the key factors in selecting suppliers. Besides, many studies have also suggested that a supplier's process quality is very important, as it will affect the quality and functionality of the end-product. Therefore, to ensure the quality of the end-product,

suppliers must be selected with caution [12,25,27]. Furthermore, nowadays the industry emphasizes the professional division of labor, and the relationship between enterprises and suppliers has evolved from a purely transactional relationship to a strategic partnership. Enterprises can help suppliers improve their production capacity and performance through the partnership of co-prosperity and co-existence, in order to ensure the product quality of all components, as well as the quality of the end-products [40].

The process capability index (PCI) adopts a method of unit-free value quantization to evaluate the process quality level [41–43]. Plenty of scholars are exploring the relationship between the process capability index and the six sigma quality level to facilitate industrial applications [5,7,10,21,22]. Chen et al. [11] proposed a six sigma quality index, based on the relationship between PCI and the six sigma quality level. This index can directly reflect the process quality level, as well as the process yield. When the quality level is higher, the process yield is higher as well.

Since the evaluation indicators usually contain unknown parameters, the research, which must estimate, by means of sample data, indicated that the sample size is usually not enormous, due to the cost and timeliness considered by the industry [28,29]. The fuzzy selection model developed based on the confidence interval can solve the inconsistencies of evaluations [28–31]. In addition, Aloini et al. [44] applied the business procurement knowledge base to establish a fuzzy decision support system (DSS) to carry out a more objective supplier selection. According to the concepts of many relevant studies, not only can the fuzzy evaluation model evaluate the process quality, but it can also identify key quality characteristics that need to be enhanced and provide suggestions on improvement [5,10,21].

3. Research Design and Methods

This section first introduces research design, as well as explaining the research content and the main axis with a flowchart. Next, related research methods are proposed, including the confidence interval of six sigma index Q_{PU} and developing a fuzzy supplier selection model.

3.1. Research Design

As mentioned earlier, the quality of the supplier will affect the quality and functionality of the end-product. Therefore, to ensure the quality of the end-product, the supplier must be cautiously selected. Firstly, this study discovered a fact from the literature review that the six sigma index is an appropriate tool for the quality-based supplier selection, whose advantage is that it can directly reflect the one-to-one relationship between the six sigma process quality level and process yield. Secondly, based on the consideration of cost and timeliness in practice, the sample size can hardly be enormous. On the basis of the traditional statistical evaluation rules, this study develops the fuzzy supplier selection method. Finally, fuzzy evaluation rules are established to provide practical applications. The flow of the entire research design is shown in Figure 1:
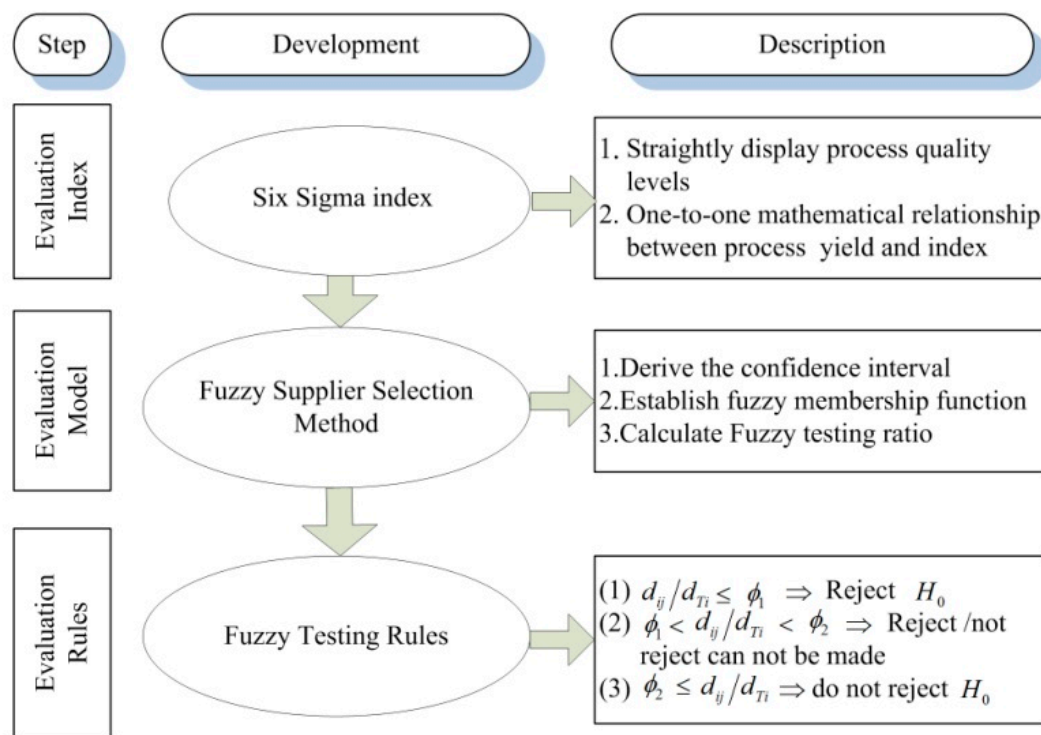
Figure 1. Research design flowchart.

3.2. Confidence Interval of Six Sigma Index Q_{PU}

Assume that X is normally distributed; i.e., $X \sim N(\mu, \sigma^2)$, where μ represents the process average and σ represents the standard deviation of the process. Then, the six sigma index of STB quality characteristics can be illustrated below:

$$Q_{PU} = \frac{USL - \mu}{\sigma} + 1.5 \tag{1}$$

where *USL* refers to upper specification limit. Chen et al. [11] suggested that the target value of STB quality characteristics is 0. Nevertheless, due to cost considerations and technical issues, the process average is not likely to approach this target value. So, when $USL - \mu \geq (k - 1.5)\sigma$, then the process is considered to have reached the $k - sigma$ quality level. That is

$$Q_{PU} \geq Q_{PU}(k) = \frac{(k - 1.5)\sigma}{\sigma} + 1.5 = k. \tag{2}$$

It is obvious that when the process quality level is $k - sigma$, then the value of Q_{PU} is equal to at least k. In addition, it is assumed that there is a one-to-one mathematical relationship between index Q_{PU} and process yield p, expressed as

$$p = p\{X \leq USL\} = p\{Z \leq Q_{PU} - 1.5\} = \Phi(Q_{PU} - 1.5) \tag{3}$$

where $\Phi(\cdot)$ is a cumulative distribution function of standard normal distribution. For example, when $Q_{PU} = 4.5$, then we can derive the process yield $p = \Phi(3.0) = 99.865\%$.

Suppose $(X_1, X_2, \cdots, X_n)$ is a random sample of X, then the estimator of six sigma index Q_{PU} can be written below:

$$Q_{PU}^* = \frac{USL - \overline{X}}{S} + 1.5, \tag{4}$$

where $\overline{X} = (n^{-1}) \times \sum_{i=1}^{n} X_i$ and $S = \sqrt{(n^{-1}) \times \sum_{i=1}^{n} (X_i - \mu^*)^2}$ are the maximum likelihood estimates (MLEs) of μ and σ, respectively. Under the assumption of normality, we let

$$Z = \frac{\overline{X} - \mu}{\sigma} \text{ and } K = \frac{nS^2}{\sigma^2}.$$

Then Z and K are distributed as $N(0,1)$ and χ^2_{n-1}, respectively. As a result,

$$p\left\{-Z_{\alpha'/2} \leq Z \leq Z_{\alpha'/2}\right\} = \sqrt{1-\alpha} \text{ and } p\left\{\chi^2_{\alpha'/2;n-1} \leq K \leq \chi^2_{1-(\alpha'/2);n-1}\right\} = \sqrt{1-\alpha}$$

where $\alpha' = 1 - \sqrt{1-\alpha}$, $Z_{\alpha'/2}$ is the upper $\alpha'/2$ quantile of the standardized normal distribution, and $\chi^2_{a,n-1}$ is the lower a quantile of the chi-square distribution with $n-1$ degrees of freedom, $a = \alpha'/2$ or $1 - \alpha'/2$. $\overline{X}$ and S^2 both are independent, so are Z and K. Resulting from these relationships, the following equation is expressed as

$$p\left\{-Z_{\alpha'/2} \leq Z \leq Z_{\alpha'/2}, \chi^2_{\alpha'/2;n-1} \leq K \leq \chi^2_{\alpha'/2;n-1}\right\} = 1 - \alpha. \tag{5}$$

Equivalently,

$$p\left\{\overline{X} - \left(Z_{0.5-\sqrt{1-\alpha}/2}\right)\frac{\sigma}{\sqrt{n}} \leq \mu \leq \overline{X} + \left(Z_{0.5-\sqrt{1-\alpha}/2}\right)\frac{\sigma}{\sqrt{n}}, \sigma_D \leq \sigma \leq \sigma_U\right\} = 1 - \alpha \tag{6}$$

where

$$\sigma_D = \sqrt{\frac{n}{\chi^2_{0.5+\sqrt{1-\alpha}/2;n-1}}} S \text{ and } \sigma_U = \sqrt{\frac{n}{\chi^2_{0.5-\sqrt{1-\alpha}/2;n-1}}} S.$$

Therefore, the confidence region can be shown as

$$CR = \left\{(\mu,\sigma) \middle| \overline{X} - \left(Z_{0.5-\sqrt{1-\alpha}/2}\right)\frac{\sigma}{\sqrt{n}} \leq \mu \leq \overline{X} + \left(Z_{0.5-\sqrt{1-\alpha}/2}\right)\frac{\sigma}{\sqrt{n}}, \sigma_D \leq \sigma \leq \sigma_U\right\}.$$

Obviously, the six sigma index Q_{PU} is the function of (μ,σ). Based on Chen et al. [11], the object function is $Q_{PU}(\mu,\sigma)$ and the confidence region (CR) is the feasible solution area. First, to find the lower confidence limit of Q_{PU}, an appropriate mathematical programming model is expressed as follows:

$$\begin{cases} LQ_{PU} = Min \{(USL - \mu)/\sigma + 1.5\} \\ subject\ to\ (\mu,\sigma) \in CR \end{cases} \tag{7}$$

When $\sigma_D \leq \sigma \leq \sigma_U$ and $\sigma \neq \sigma_U$, then $Q_{PU}(\mu,\sigma) > Q_{PU}(\mu,\sigma)$. Therefore, the mathematical programming model is rewritten below:

$$\begin{cases} LQ_{PU} = Min \{(USL - \mu)/\sigma + 1.5\} \\ subject\ to\ \mu_{LU} \leq \mu \leq \mu_{RU} \end{cases} \tag{8}$$

where

$$\mu_{LU} = \overline{X} - \frac{Z_{0.5-\sqrt{1-\alpha}/2}}{\sqrt{\chi^2_{0.5-\sqrt{1-\alpha}/2;n-1}}} \times S, \ \mu_{RU} = \overline{X} + \frac{Z_{0.5-\sqrt{1-\alpha}/2}}{\sqrt{\chi^2_{0.5-\sqrt{1-\alpha}/2;n-1}}} \times S.$$

Similarly, when $\mu_{LU} \leq \mu \leq \mu_{RU}$ and $\mu \neq \mu_{RU}$, then $Q_{PU}(\mu, \sigma) > Q_{PU}(\mu, \sigma)$. Therefore, the lower confidence limit LQ_{PU} is

$$
\begin{aligned}
LQ_{PU} \quad &= \frac{USL - \mu_{RU}}{\sigma_U} + 1.5 \\
&= \left(Q^*_{PU} - 1.5\right) \times \sqrt{\frac{\chi^2_{0.5-\sqrt{1-\alpha}/2;n-1}}{n}} - \frac{Z_{0.5-\sqrt{1-\alpha}/2}}{\sqrt{n}} + 1.5.
\end{aligned} \tag{9}
$$

Second, to find the upper confidence limit of Q_{PU}, an appropriate mathematical programming model is written as:

$$
\begin{cases}
UQ_{PU} = Max\left\{(USL - \mu)/\sigma + 1.5\right\} \\
subject\ to\ (\mu, \sigma) \in CR
\end{cases} \tag{10}
$$

When $\sigma_D \leq \sigma \leq \sigma_U$ and $\sigma \neq \sigma_D$, then $Q_{PU}(\mu, \sigma) < Q_{PU}(\mu, \sigma)$. Therefore, the mathematical programming model is rewritten as follows:

$$
\begin{cases}
UQ_{PU} = Max\left\{(USL - \mu)/\sigma + 1.5\right\} \\
subject\ to\ \mu_{LD} \leq \mu \leq \mu_{RD}
\end{cases} \tag{11}
$$

where

$$
\mu_{LD} = \overline{X} - \frac{Z_{0.5-\sqrt{1-\alpha}/2}}{\sqrt{\chi^2_{0.5+\sqrt{1-\alpha}/2;n-1}}} \times S,\ \mu_{RD} = \overline{X} + \frac{Z_{0.5-\sqrt{1-\alpha}/2}}{\sqrt{\chi^2_{0.5+\sqrt{1-\alpha}/2;n-1}}} \times S.
$$

Similarly, for any $\mu_{LD} \leq \mu \leq \mu_{RD}$ and $\mu \neq \mu_{LD}$, $Q_{PU}(\mu, \sigma) < Q_{PU}(\mu, \sigma)$. Therefore, the upper confidence limit UQ_{PU} is

$$
\begin{aligned}
UQ_{PU} \quad &= \frac{USL - \mu_{LD}}{\sigma_D} + 1.5 \\
&= \left(Q^*_{PU} - 1.5\right) \times \sqrt{\frac{\chi^2_{0.5+\sqrt{1-\alpha}/2;n-1}}{n}} + \frac{Z_{0.5-\sqrt{1-\alpha}/2}}{\sqrt{n}} + 1.5
\end{aligned} \tag{12}
$$

Thus, the $100(1 - \alpha)\%$ confidence interval of Q_{PU} is $CI = [LQ_{PU}, UQ_{PU}]$.

3.3. Developing a Fuzzy Supplier Selection Model

If we let $(x_{h1}, x_{h2}, \cdots, x_{hn})$ be the observed value of $(X_{h1}, X_{h2}, \cdots, X_{hn})$ for supplier h, then the observed value of Q^*_{PUh} is

$$
Q^*_{PUh0} = \frac{USL - \overline{x}_h}{s_h} + 1.5, \tag{13}
$$

where $\overline{x}_h = \left(n^{-1}\right) \times \sum_{i=1}^{n} x_{hi}$ and $s_h = \sqrt{\left(n^{-1}\right) \times \sum_{i=1}^{n} (x_{hi} - \overline{x}_h)^2}$. Therefore, the observed values of the lower and upper confidence limits of Q_{PUh} are functions of α, and can be shown as follows:

$$
LQ_{PUh0}(\alpha)\left(Q^*_{PUh0} - 1.5\right) \times \sqrt{\frac{\chi^2_{0.5-\sqrt{1-\alpha}/2;n-1}}{n}} - \frac{Z_{0.5-\sqrt{1-\alpha}/2}}{\sqrt{n}} + 1.5, \tag{14}
$$

$$
LQ_{PUh0}(\alpha)\left(Q^*_{PUh0} - 1.5\right) \times \sqrt{\frac{\chi^2_{0.5-\sqrt{1-\alpha}/2;n-1}}{n}} + \frac{Z_{0.5-\sqrt{1-\alpha}/2}}{\sqrt{n}} + 1.5. \tag{15}
$$

This study examined the statistical testing process before developing the fuzzy supplier selection model. Based on Chen et al. [12], when determining whether the quality levels of any two suppliers are equal, this study adopted the following null hypothesis $H_0:Q_{PUi} = Q_{PUj}$ versus the alternative hypothesis $H_1:Q_{PUi} \neq Q_{PUj}$, for any value of $i \neq j$. For applying confidence intervals $CI_i = [LQ_{PUi}, UQ_{PUi}]$ and $CI_j = \left[LQ_{PUj}, UQ_{PUj}\right]$ to the statistical testing method, the rules are made as follows:

(1)   If $UQ_{PUi} < LQ_{PUj}$, then H_0 is rejected, and $Q_{PUi} < Q_{PUj}$ is concluded.

(2)   If $CI_i \cap CI_j \neq \phi$, then H_0 is not rejected, and $Q_{PUi} = Q_{PUj}$ is concluded.

(3)   If $LQ_{PUi} > UQ_{PUj}$, then H_0 is rejected, and $Q_{PUi} > Q_{PUj}$ is concluded.

Next, this study developed the fuzzy supplier selection model based on the aforementioned statistical testing rules related to the fuzzy hypothesis testing method recommended by Chen et al. [12]. This study first examined the case of $Q^*_{PUj} \geq Q^*_{PUi}$. If $Q^*_{PUj} < Q^*_{PUi}$, then the decision will not be made until the inequality is reversed.

Applying the confidence interval CI in accordance with the method proposed by Chen et al. [12], the α−cuts of the triangular fuzzy number $\widetilde{Q}^*_{PUh0}$ are derived as follows:

$$\widetilde{Q}^*_{PUh0}[\alpha] = \begin{cases} [LQ_{PUh0}(\alpha), UQ_{PUh0}(\alpha)], & \text{for } 0.01 \leq \alpha \leq 1 \\ [LQ_{PUh0}(0.01), UQ_{PUh0}(0.01)], & \text{for } 0 \leq \alpha \leq 0.01 \end{cases}. \tag{16}$$

Thus, the triangular fuzzy number of Q^*_{PUh0} is $\widetilde{Q}^*_{PUh0} = \Delta(Q_{Lh}, Q_{Mh}, Q_{Rh})$, where

$$Q_{Lh} = \left(Q^*_{PUh0} - 1.5\right) \times \sqrt{\frac{\chi^2_{0.0025;n-1}}{n}} - \frac{Z_{0.0025}}{\sqrt{n}} + 1.5, \tag{17}$$

$$Q_{Mh} = \left(Q^*_{PUh0} - 1.5\right) \times \sqrt{\frac{\chi^2_{0.5;n-1}}{n}} + 1.5, \tag{18}$$

$$Q_{Rh} = \left(Q^*_{PUh0} - 1.5\right) \times \sqrt{\frac{\chi^2_{0.9975;n-1}}{n}} + \frac{Z_{0.0025}}{\sqrt{n}} + 1.5. \tag{19}$$

It is observed that the membership function of the fuzzy number $\widetilde{Q}^*_{PUh0}$ is

$$\eta_h(x) = \begin{cases} 0 & \text{if } x < Q_{Lh} \\ \alpha' & \text{if } Q_{Lh} \leq x < Q_{Mh} \\ 1 & \text{if } x = Q_{Mh} \\ \alpha'' & \text{if } Q_{Mh} < x \leq Q_{Rh} \\ 0 & \text{if } Q_{Rh} < x \end{cases}. \tag{20}$$

where α' and α'' are determined by

$$\left(Q^*_{PUh0} - 1.5\right) \times \sqrt{\frac{\chi^2_{0.5 - \sqrt{1-\alpha'}/2;n-1}}{n}} - \frac{Z_{0.5 - \sqrt{1-\alpha'}/2}}{\sqrt{n}} + 1.5 = x, \tag{21}$$

$$\left(Q^*_{PUh0} - 1.5\right) \times \sqrt{\frac{\chi^2_{0.5 - \sqrt{1-\alpha'}/2;n-1}}{n}} + \frac{Z_{0.5 - \sqrt{1-\alpha'}/2}}{\sqrt{n}} + 1.5 = x. \tag{22}$$

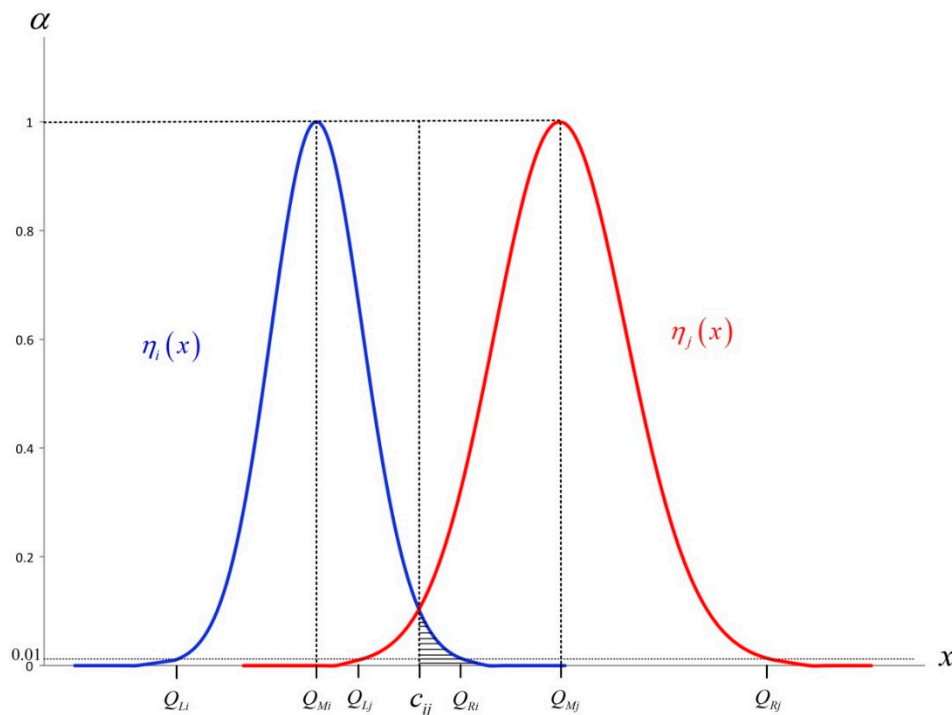Figure 2 is a diagram showing the membership functions of $\eta_i(x)$ and $\eta_j(x)$.

Figure 2. Membership functions of $\eta_i(x)$ and $\eta_j(x)$ with $Q^*_{PUj} \geq Q^*_{PUi}$.

This study calculates the total area a_{Ti} of $\eta_i(x)$ as follows:

$$a_{Ti} = \int\limits_{Q_{Li}}^{Q_{Ri}} \eta_i(x)\, dx. \tag{23}$$

Let $x = c_{ij}$, such that $\eta_i(c_{ij}) = \eta_j(c_{ij})$. Then, this study calculates the slashed area a_{ij} of $\eta_i(x)$ in the following:

$$a_{ij} = \int\limits_{c}^{Q_{Ri}} \eta_i(x)\, dx. \tag{24}$$

According to Buckley [45], this study used a_{ij} as the numerator and a_{Ti} as the denominator and then performed a fuzzy testing method using a_{ij}/a_{Ti}. Chen et al. [12] felt that calculating this ratio was difficult and thus replaced it with the ratio quotient of their base lengths. First, let $d_{ij} = Q_{Ri} - c_{ij}$, so d_{ij} is the base length of the slashed area of $\eta_i(x)$. Based on Equation (19), d_{ij} is shown as follows:

$$d_{ij} = \left(Q^*_{PUi0} - 1.5\right) \times \sqrt{\frac{\chi^2_{0.9975;n-1}}{n}} + \frac{Z_{0.0025}}{\sqrt{n}} - \left(c_{ij} - 1.5\right). \tag{25}$$

Correspondingly, the base length of $\eta_i(x)$ can be exhibited as $d_{Ti} = Q_{Ri} - Q_{Li}$. Based on Equations (17) and (19), d_{Ti} can be shown as follows:

$$d_{Ti} = \left(Q^*_{PUi0} - 1.5\right) \times \left(\sqrt{\frac{\chi^2_{0.9975;n-1}}{n}} - \sqrt{\frac{\chi^2_{0.0025;n-1}}{n}} \right) + 2\frac{Z_{0.0025}}{\sqrt{n}}. \tag{26}$$

For the sake of convenience, we followed Chen et al. [12] and replaced a_{ij}/a_{Ti} with d_{ij}/d_{Ti} for fuzzy testing. The fuzzy testing ratio d_{ij}/d_{Ti} is defined as

$$d_{ij}/d_{Ti} = \frac{\left(Q^*_{PUi0} - 1.5\right) \times \sqrt{\frac{\chi^2_{0.9975;n-1}}{n}} + \frac{Z_{0.0025}}{\sqrt{n}} - \left(c_{ij} - 1.5\right)}{\left(Q^*_{PUi0} - 1.5\right)\left(\sqrt{\frac{\chi^2_{0.9975;n-1}}{n}} - \sqrt{\frac{\chi^2_{0.0025;n-1}}{n}}\right) + 2 \times \frac{Z_{0.0025}}{\sqrt{n}}}. \tag{27}$$

If we let $0 < \phi_1 < \phi_2 < 0.5$, in accordance with Buckley [45] and Chen et al. [12], where ϕ_1 and ϕ_2 are two expert coefficients, will be taken into consideration in the following rules:

(1) If $d_{ij}/d_{Ti} \leq \phi_1$, then H_0 is rejected, and $Q_{PUi} < Q_{PUj}$ is concluded.
(2) If $\phi_1 < d_{ij}/d_{Ti} < \phi_2$, then the decision of whether to reject/not reject cannot be made.
(3) If $\phi_2 \leq d_{ij}/d_{Ti}$, then H_0 is not rejected, and $Q_{PUi} = Q_{PUj}$ is concluded.

4. Numerical Example

This study aims to demonstrate the efficacy of our methodology, and develops the observed value $\left(x_{h,1}, x_{h,2}, \cdots, x_{h,36}\right)$ of random sample, with sample size $n = 36$ for each supplier. The upper specification limit for roundness of the parts is 0.02. According to Section 3, if we want to determine whether the quality levels of any two suppliers are identical, we should adopt the following null hypothesis $H_0: Q_{PUi} = Q_{PUj}$ versus the alternative hypothesis $H_1: Q_{PUi} \neq Q_{PUj}$ for any $i \neq j$. The observed value of Q^*_{PUi} and Q^*_{PUj} can be described as follows:

$Q^*_{PUi0} = 3.81$ and $Q^*_{PUj0} = 5.42$.

Thus, based on Equations (17) and (19), the triangular fuzzy numbers of Q^*_{PUi} and Q^*_{PUj} can be shown as follows:

$\Delta(Q_{Li}, Q_{Mi}, Q_{Ri}) = \Delta(2.852, 3.778, 4.755)$,
$\Delta(Q_{Lj}, Q_{Mj}, Q_{Rj}) = \Delta(4.047, 5.365, 6.771)$.

Besides, observed from Equation (20), the membership function of fuzzy number $\widetilde{Q}^*_{PUi}$ is

$$\eta_i(x) = \begin{cases} 0 \ if \ x < 2.852 \\ \alpha'_i \ if \ 2.852 \leq x < 3.778 \\ 1 \ if \ x = 3.778 \\ \alpha''_i \ if \ 3.778 < x \leq 4.755 \\ 0 \ if \ 4.755 < x \end{cases}.$$

Based on Equations (21) and (22), α'_i and α''_i are determined by

$$2.31 \times \sqrt{\frac{\chi^2_{0.5 - \sqrt{1-\alpha'_i}/2;n-1}}{n}} - \frac{Z_{0.5 - \sqrt{1-\alpha'_i}/2}}{\sqrt{n}} + 1.5 = x, \ and$$

$$2.31 \times \sqrt{\frac{\chi^2_{0.5 + \sqrt{1-\alpha''_i}/2;n-1}}{n}} + \frac{Z_{0.5 - \sqrt{1-\alpha''_i}/2}}{\sqrt{n}} + 1.5 = x.$$

Similarly, the membership function of fuzzy number $\widetilde{Q}^*_{PUj}$ is

$$\eta_j(x) = \begin{cases} 0 \ if \ x < 4.047 \\ \alpha'_j \ if \ 4.047 \leq x < 5.365 \\ 1 \ if \ x = 5.365 \\ \alpha''_j \ if \ 5.365 < x \leq 6.771 \\ 0 \ if \ 6.771 < x \end{cases}.$$

based on Equations (21) and (22), where α'_j and α''_j are determined by

$$3.92 \times \sqrt{\frac{\chi^2_{0.5+\sqrt{1-\alpha''_j}/2;n-1}}{n}} + \frac{Z_{0.5-\sqrt{1-\alpha''_j}/2}}{\sqrt{n}} + 1.5 = x, \text{ and}$$

$$3.92 \times \sqrt{\frac{\chi^2_{1-\alpha/2,35}}{36}} + \frac{Z_{\alpha/2}}{6} + 1.5 = x.$$

Obviously, when the confidence intervals $CI_i = [2.852, 4.755]$ and $CI_j = [4.047, 6.771]$, then $CI_i \cap CI_j \neq \phi$. According to statistical testing rules, H_0 is not rejected, and $Q_{PUi} = Q_{PUj}$ is concluded. Nevertheless, it is discovered that the observed values of $Q^*_{PUi0} = 3.81$ and $Q^*_{PUj0} = 5.42$. The quality levels of the two suppliers differ by at least one standard deviation, but the conclusions are not significantly different. This is because the small sample size ($n = 36$) resulted in a large error. Next, this study performed the fuzzy testing method using the fuzzy selection model suggested in Section 3. Figure 2 exhibits a graph related to the membership functions $\eta_i(x)$ and $\eta_j(x)$ with $Q^*_{PUi0} = 3.81$ and $Q^*_{PUj0} = 5.42$. In Figure 3, when $c_{ij} = 4.446$, such that $\eta_i(4.446) = \eta_j(4.446)$, the slashed area a_{ij} of $\eta_i(x)$ can be calculated.
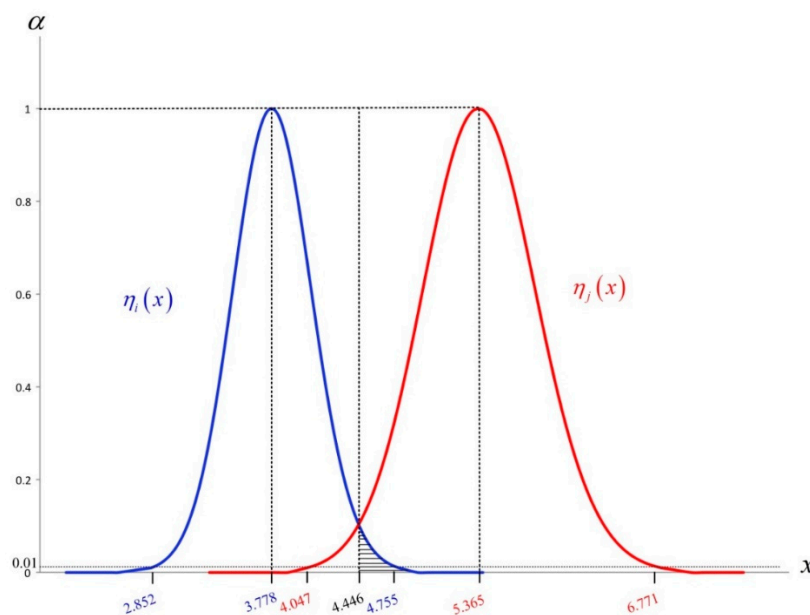

Figure 3. Membership functions of $\eta_i(x)$ and $\eta_j(x)$. with $Q^*_{PUi0} = 3.81$ and $Q^*_{PUj0} = 5.42$.

Based on Figure 2 and Equations (25) and (26), the values of d_{ij} and d_{Ti} can be shown as follows:

$$d_{ij} = Q_{Ri} - c_{ij} = 4.755 - 4.446 = 0.309,$$

$$d_{Ti} = Q_{Ri} - Q_{Li} = 4.755 - 2.852 = 1.903.$$

Therefore,

$$d_{ij}/d_{Ti}$$

As noted by Chen et al. [12], ϕ_1 and ϕ_2 can be 0.2 and 0.4 in practice. Because $d_{ij}/d_{Ti} = 0.162 < \phi_1$, H_0 is rejected, and $Q_{PUi} < Q_{PUj}$ is concluded. Then, the differences between the fuzzy testing method proposed in this study and the traditional statistical testing method are displayed in Table 1:

Table 1. Comparison table of methods.

Method	Fuzzy Testing	Statistical Testing
Testing tool	Fuzzy testing ratio d_{ij}/d_{Ti}	Confidence intervals CI_i and CI_j
Calculation process	$d_{ij} = 0.309,$ $d_{Ti} = 1.903,$ $d_{ij}/d_{Ti} = 0.162.$	$CI_i = [2.585, 4.775]$ $CI_j = [4.047, 6.771]$ $CI_i \cap CI_j \neq \phi$
Results	Reject H_0	Do not reject H_0

Clearly, the small sample size ($n = 36$) produced a large error. Statistical testing would demonstrate that there is no major difference between the quality levels of the two suppliers, even though their quality levels differ by at least one standard deviation. In contrast, fuzzy testing using the proposed fuzzy selection model shows significant differences between the quality levels of the two suppliers; that is, $Q_{PUi} < Q_{PUj}$. It is thus clear that the proposed method is more accurate in practice.

5. Conclusions

Obtaining parts from suppliers has become a trend in business strategy to enhance the industries' competitiveness and operational flexibility. In order to make sure of the quality and functionality of the end product, this study took the roundness of parts, for example, to propose a fuzzy supplier selection model. For this product, the roundness of parts is a crucial STB quality characteristic. We selected the six sigma quality index as our selection tool. The index can directly reflect process quality levels and establish a one-to-one mathematical relationship with process yield. Then, mathematical programming was used to find the confidence interval of index Q_{PU}. In addition, this study proposed a two-tailed confidence-interval-based fuzzy testing method for supplier selection. Due to the fact that this method is based on the basis of confidence intervals, the risk probability of misjudgments caused by sampling errors will be lowered, while the precision of selection will be enhanced. Finally, a numerical example is explained in Section 4, to demonstrate the application and efficacy of this method. In this example, it is clear that the small sample size created a wide confidence interval; therefore, even though the quality levels of two suppliers differ significantly, statistical testing shows no significant differences. The proposed fuzzy selection model, however, can successfully identify the difference between the quality levels of the two suppliers, thereby demonstrating that the proposed approach is more accurate in practice. Furthermore, the proposed method can help companies increase product quality, as well as the competitiveness of the industry chain as a whole.

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Glossary

μ	process average
σ	process standard deviation
$\overline{X}$	estimator of μ
S	estimator of σ
$\overline{x}$	the observed value of $\overline{X}$
s	the observed value of S
α'	significance level
n	sample size

Q_{PU}	six sigma index
Q^*_{PU}	estimator of six sigma index
USL	upper specification limit
STB	smaller-the-better
$\Phi(\cdot)$	cumulative distribution function of standard normal distribution
CR	confidence region
LQ_{PU}	lower confidence limit of Q_{PU}
UQ_{PU}	upper confidence limit of Q_{PU}
Q^*_{PUh}	estimator of six sigma index for supplier h
Q^*_{PUh0}	the observed value of Q^*_{PUh}
$\eta_h(x)$	membership function of the fuzzy number $\widetilde{Q}^*_{PUh0}$

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