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Effective Compressive Strengths of Corner and Edge Concrete Columns Based on an Adaptive Neuro-Fuzzy Inference System

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Abstract: In the current design codes, the effective compressive strength can be used to reflect decrease in load-transfer performance when upper/lower columns and slabs have different concrete compressive strengths. In this regard, this study proposed a method that can accurately estimate the effective compressive strengths by using an adaptive neuro-fuzzy inference system (ANFIS). The ANFIS is an algorithm that introduces a learning system that corrects errors into a fuzzy theory and has widely been used to solve problems with complex mechanisms. In order to constitute the ANFIS algorithm, 50 data randomly extracted from 75 existing test datasets were used in training, and 25 were used for verification. It was found that analysis using the ANFIS model provides a more accurate evaluation of the effective compressive strengths of corner and edge columns than do the equations specified in the current design codes. In addition, parametric studies were performed using the ANFIS model, and a simplified equation for calculating the effective compressive strength was proposed, so that it can be easily used in practice.

Keywords: ANFIS; fuzzy; neuro-fuzzy; effective compressive strength; high strength concrete; slab; column; reinforced concrete

1. Introduction

High-strength concrete (HSC) has excellent compression resistance and is very effective for column members subjected to axial loads [1,2]. Meanwhile, in terms of economic efficiency, normal-strength concrete (NSC) is mainly used for flexure-dominant members, such as slabs. However, when the HSC column and the NSC slab are used together, the load-transfer performance of the column is affected by the low concrete compressive strength of the slab. The current design codes (ACI 318-19; CSA A23.3-14) suggest that the effective compressive strength (f'_{ce}) of the column can be used in such cases [3,4]. ACI 318-19 [3] prescribes that when the concrete compressive strength (f'_{cc}) of the corner or edge column exceeds 1.4 times the slab concrete compressive strength (f'_{cj}), f'_{cj} should be used as the design compressive strength of the column. On the other hand, CSA A23.3-14 [4] specifies that if f'_{cc} exceeds f'_{cj} , f'_{cj} should be used in the corner column, whereas $1.4 f'_{cj}$ should be used in the edge column.

Many experimental studies have been conducted on slab-columns with different concrete compressive strengths. Bianchini et al. [5] performed a compressive-strength test using the compressive-strengths ratio of the column and the slab, and the confinement conditions of the

slab (i.e., interior, edge, and corner columns) as variables. Based on the test results, the issue of the load-transfer performance of the HSC column, which is reduced by the NSC slab, was raised, and this research was reflected in ACI 318-63 [6]. After this, Choi et al. [1], Gamble and Klinar [7], Kayani [8], Lee and Mendis [9], Ospina and Alexander [10], Shin et al. [11], Shu and Hawkins [12], and Urban and Goldyn [13] carried out experimental and analytical research and reported the effects of the compressive strength of the column and the slab (f'_{cc} and f'_{cj}), the column width (c), and the slab thickness (h) on the effective compressive strength of the column (f'_{ce}). The effective compressive strengths specified in the current design codes (ACI 318-19; CSA A23.3-14) have often yet failed to properly reflect the effects of the many variables, and compared to experimental test data, they provide very conservative results with large scatters. Therefore, in this study, ANFIS (adaptive neuro-fuzzy inference system) [14], a combination of fuzzy theory and an artificial neural network, was introduced to propose an algorithm that can evaluate the effective compressive strength of the column with high accuracy. The ANFIS is a system that can solve complex problems in science, finance and engineering, which are hard to analyze in a numerical manner. It is advantageous in that it can provide an output with a high accuracy by considering the correlations between various variables. However, it can also suffer from overfitting and overtraining, so adequate datasets are required for training the ANFIS algorithm [15–17]. In this study, a simplified equation was developed by means of extensive parametric studies and dimensional analysis based on the ANFIS evaluation results. The accuracy of the proposed model was verified by comparison with the design codes and empirical equations proposed by other researchers.

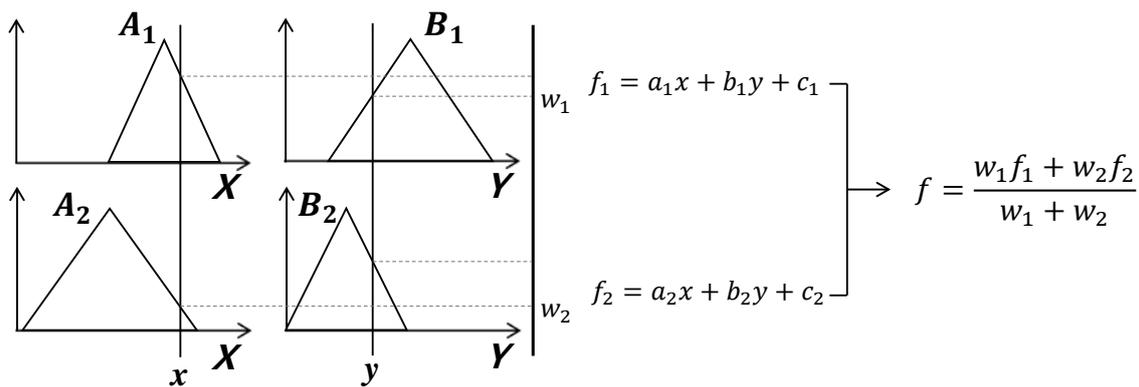
2. ANFIS Structure

Fuzzy theory has been widely used in many academic and industrial fields, since it is very effective at solving problems that deal with variables that are difficult to express quantitatively, such as language [14,18]. In particular, it has mainly been used to solve engineering problems, which pose difficulties in numerical analysis, because of a huge uncertainty of input and output variables and complex interaction mechanisms [19–21]. In order to predict highly accurate results by means of the fuzzy theory, it is of utmost importance to properly configure fuzzy sets and rules. In this study, as shown in Figure 1, fuzzy rules are configured based on the Sugeno fuzzy model and ANFIS with a back-propagation algorithm for error correction, that is used to more accurately evaluate the effective compressive strengths of the column [22–25].

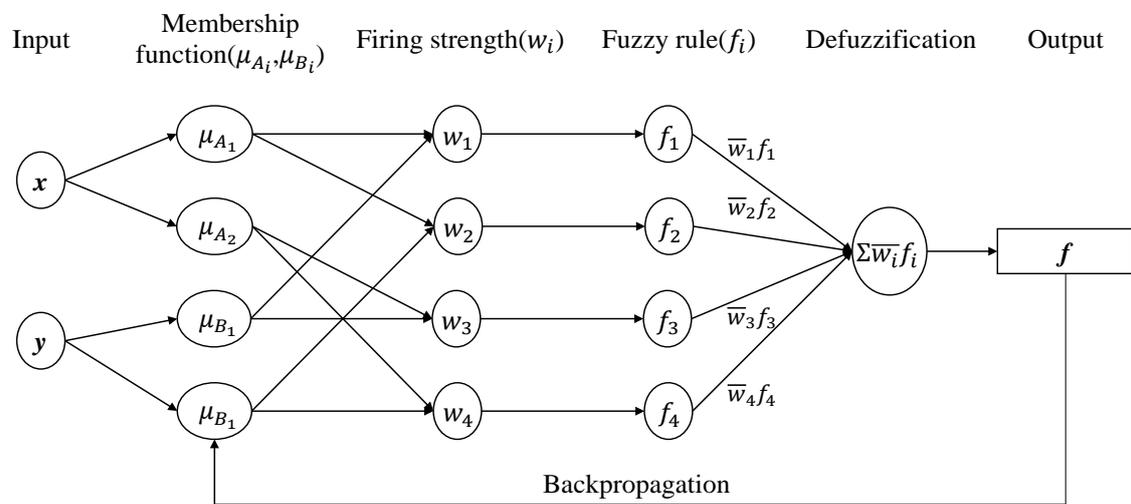
In Figure 1a, x and y are the input variables, A_i and B_i are the fuzzy sets of input variables x and y corresponding to the i -th fuzzy rule, f_i is the output variable of the i -th fuzzy rule, a_i , b_i and c_i are the consequent parameters of the i -th fuzzy rule, and f is the final result obtained from the defuzzification of f_i , where a_i , b_i and c_i are calculated using the least-squares estimator. In the ANFIS structure shown in Figure 1b, μ_{A_i} and μ_{B_i} are the membership functions of fuzzy sets A_i and B_i , and w_i is the i -th firing strength and represents the membership degree of the i -th rule, which is the minimum value among the degrees of membership calculated according to the membership functions μ_{A_i} and μ_{B_i} . When all firing strengths are calculated, the final result (f) is estimated using the normalized firing strength (\bar{w}_i) and the output variable of the fuzzy rule, where \bar{w}_i is calculated as follows:

$$\bar{w}_i = \frac{w_i}{\sum w_i} \quad (1)$$

The error is then calculated by comparing the final result with the experimental data, and generated errors are reduced by means of the backpropagation, as shown in Figure 1b.



(a) Sugeno fuzzy inference concept.



(b) Adaptive neuro-fuzzy inference system (ANFIS) structure.

Figure 1. Concept of fuzzy inference and ANFIS structure.

3. Proposed Model Using ANFIS

Several researchers [1,7–13] have investigated the effects of the concrete compressive strengths of the column and the slab (f'_{cc} and f'_{cj}), the column width (c), and the slab thickness (h) on the effective compressive strength of the column (f'_{ce}). According to the test results conducted by Lee and Mendis [9] and Shu and Hawkins [12], the effective compressive strength of the column (f'_{ce}) increases as the aspect ratio of slab thickness to column width (h/c) decreases. Based on these results, the concrete compressive strength of the column and slab (f'_{cc} and f'_{cj}), and the aspect ratio of slab thickness to column width (h/c) were chosen as the input variables for estimating the effective compressive strength of the column (f'_{ce}). In addition, a total of 75 test results were collected from the existing literature, as shown in Table 1 [2,5,7,9,12,26,27]. Although the authors have put all their efforts in to collect all the test results available up to date, it should be noted that the test data are not yet sufficient enough.

Among the collected test specimens, 50 specimens were randomly sampled and used in the training of the ANFIS algorithm, and the remaining 25 specimens were used for verification. As shown in Figure 2, the bell-shaped function was applied as the shape of the fuzzy set, and the membership function of the bell-shaped fuzzy set is calculated as follows:

$$\text{membership function} = \frac{1}{1 + \left| \frac{x-x_c}{x_w} \right|^{2x_q}} \tag{2}$$

where x is the input variable, and x_c , x_w and x_q are the shape factors of the fuzzy set, which determine the center, width, and slope parameters of the fuzzy set, respectively.

Table 1. Summary of collected test specimens.

Researcher	Number of Specimens	f'_{cc} (MPa)	f'_{cj} (MPa)	f'_{ce} (MPa)*	h/c
Lee et al. [2]	1	88.33	46.89	53.3	0.6
Bianchini et al. [5]	22	15.8–46.9	8.8–24.8	12.8–29	0.6
Gamble and Klinar [7]	6	79.3–97.9	15.9–45.5	15.8–46.9	0.5–0.7
Lee and Mendis [9]	3	77.4–83.9	17.5–28.2	31.7–33.7	0.3–0.8
Shu and Hawkins [12]	41	38.6–50.8	6.9–39.2	8.6–43.6	0.2–3
Shah et al. [26]	1	84	29	42.3	0.6
McHarg et al. [27]	1	80.7	30	43.8	0.7

* Note: $f'_{ce} = \frac{P_{test} - f_y A_s}{\alpha(A_g - A_s)}$ where P_{test} is the axial force measured in the experiment, α is 0.85 in ACI 318 and 0.85–0.0015 f'_c in CSA A23.3, f'_c is the compressive strength of the concrete, A_g is the gross sectional area of the column, and A_s and f_y are the sectional area and yield strength of the longitudinal reinforcement in the column, respectively.

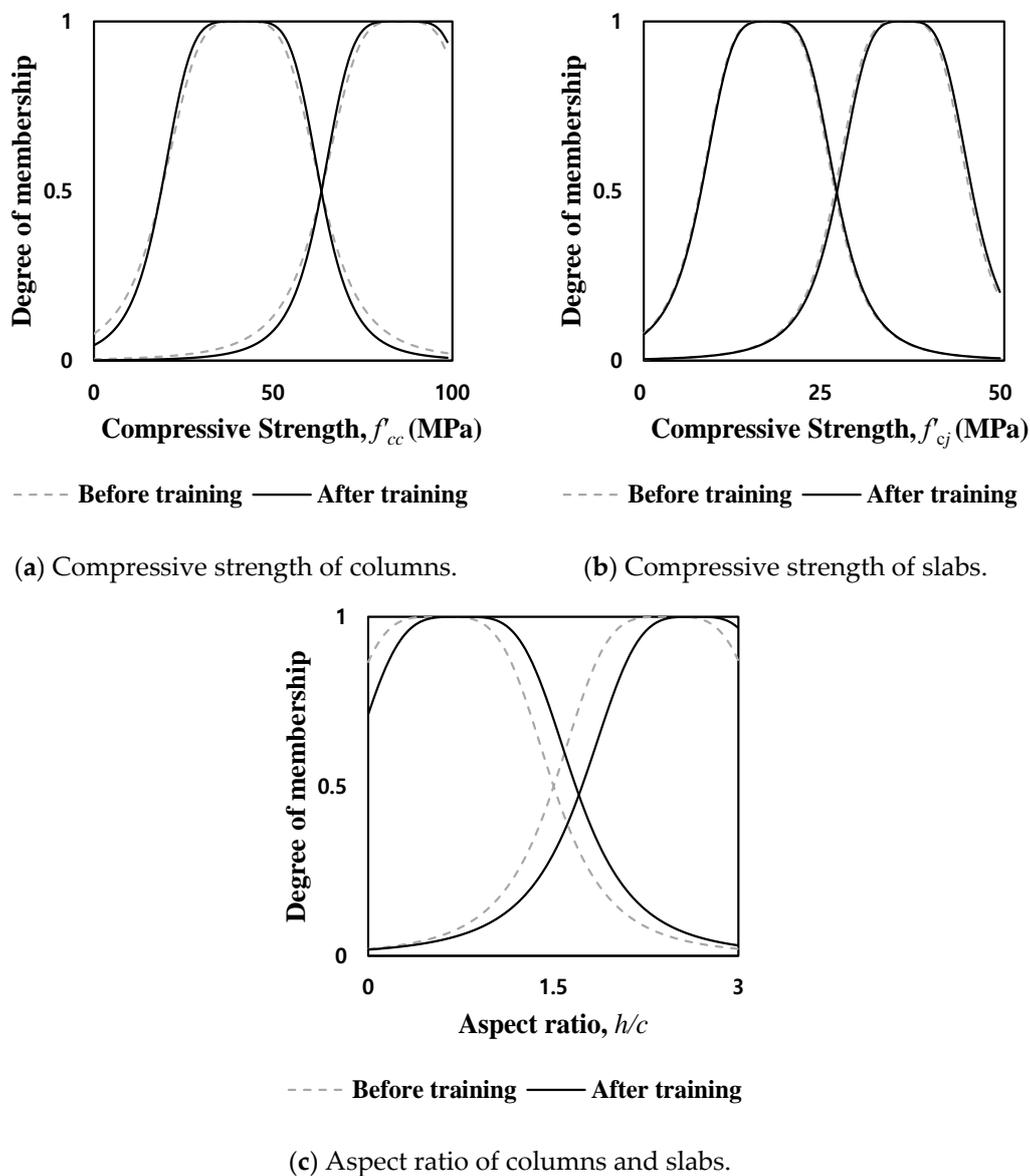


Figure 2. Comparison of membership functions before and after training.

For the fuzzy theory, including ANFIS, the accuracy of the results greatly depends on the configuration of the fuzzy set. Therefore, it is of utmost importance to configure the fuzzy set appropriately. In the general fuzzy theory, the fuzzy set is configured by referring to the existing codes and experts' consensus. However, in the ANFIS, the initial fuzzy set is assumed based on the collected data, and the optimized fuzzy set is then configured by means of repetitive training. In this study, the shape factors of the initial fuzzy set were assumed, using the K-means clustering technique proposed by Macqueen [28]. Figure 2 compares the initial membership function and the final membership function determined after the ANFIS training, which are denoted by a dotted line and a solid line, respectively. As shown in Figure 1, the ANFIS derives the results by means of rules, and the number of fuzzy rules is determined by the number of variables and fuzzy sets. In the algorithm constituted in this study, since there are three variables and two fuzzy sets for each variable, a total of 8 (=2³) rules are generated, and the effective compressive strength of the *i*-th rule ($f'_{ce,i}$) can be calculated as follows:

$$f'_{ce,i} = a_i f'_{cc} + b_i f'_{cj} + c_i h/c + d_i \tag{3}$$

where a_i , b_i , c_i and d_i are the consequent parameters determined according to the input variable and fuzzy rule, and the final result f'_{ce} can be derived by defuzzification, as shown below:

$$f'_{ce} = \sum_i^n \bar{w}_i f'_{ce,i} \tag{4}$$

where the defuzzification is the process of converting a fuzzy value into a crisp value. In the fuzzy theory including ANFIS, an operation is carried out using a fuzzy value obtained from the fuzzification of a crisp value. Therefore, the result value of the rule is also a fuzzy value, which should be converted into a crisp value for quantitative representation. In this study, defuzzification was performed using a centroid method.

Figure 3 and Table 2 compare the results of effective compressive strengths calculated using the ANFIS algorithm presented in this study, existing codes [3,4] and empirical equations [8,10] proposed by existing researchers. The equation for effective compressive strength specified in ACI 318-19 [3] provided very conservative results with the average (AVG) of the ratio of the analysis value to the test value ($f'_{ce,analysis} / f'_{ce,test}$) being 0.79 and the COV being 0.32. It also showed a larger scatter than did the other models. The equation in CSA A23.3-14 [4] was about as accurate as the ACI 318-19 code equation, with the average of the ratio of the analysis value to the test value and the COV calculated at 0.81 and 0.28, respectively. The equation proposed by Kayani [8], and Ospania and Alexander [10] provided predictive values better than those provided by the current design codes, but still showed a large deviation between the test and analysis values. On the other hand, the ANFIS model proposed in this study provided more accuracy than did the other models, with the average of the ratio of the analysis value to the test value and the COV calculated at 1.05 and 0.13, respectively.

Table 2. Comparisons of proposed ANFIS model and other existing models.

	$f'_{ce,analysis} / f'_{ce,test}$				
	MEAN	STDEV	COV	MIN	MAX
ACI 318-19 [3]	0.79	0.25	0.32	0.19	1.27
CSA.A23.3-14 [4]	0.81	0.23	0.28	0.19	1.27
Kayani [8]	0.91	0.22	0.24	0.29	1.56
Ospania and Alexander [10]	0.9	0.25	0.27	0.23	1.53
Proposed ANFIS model	1.05	0.13	0.13	0.87	1.45
Simplified model using ANFIS	0.99	0.18	0.18	0.63	1.40

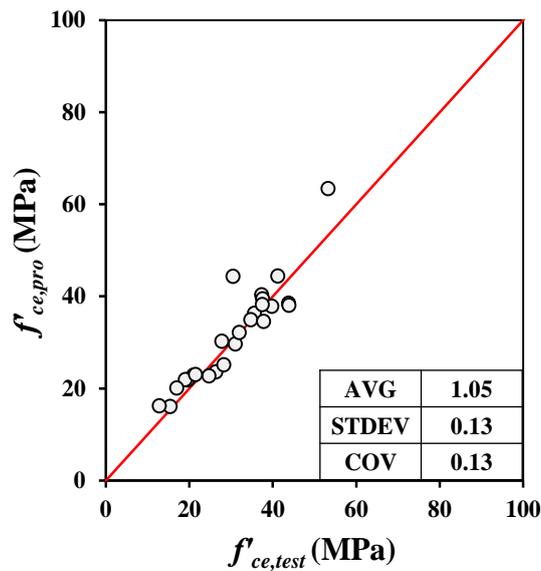


Figure 3. Verification of proposed ANFIS model.

4. Simplified Model Using ANFIS

The proposed ANFIS model provides more accuracy than do the other equations, but its practical application is difficult, because it consists of complex algorithms. Therefore, in this study, parametric studies of the virtual data were performed based on the previously constituted ANFIS algorithm, and a simplified equation for effective compressive strengths of corner and edge columns was proposed. Table 3 shows the range of variables to be assumed for the parametric study, and Figure 4 shows the tendencies in the effective compressive strengths of the column (f'_{ce}) according to the concrete compressive strength of the column and the slab (f'_{cc} and f'_{cj}), and the aspect ratio of slab thickness to column width (h/c). The parametric studies found that as the f'_{cc} and f'_{cj} increase, the f'_{ce} shows an overall tendency to increase. It was also found that the f'_{ce} tends to decrease non-linearly with an increase in the h/c . Based on the parametric studies, a dimensional analysis method [29] was used to derive the simplified equation for the effective compressive strength of the column. In this study, three dimensionless terms (π_1 , π_2 , and π_3) were set as shown below:

$$\pi_1 = \frac{f'_{ce,pro}}{f'_{cj}}, \pi_2 = \frac{h}{c}, \pi_3 = \frac{f'_{cc}}{f'_{cj}} \tag{5}$$

where $f'_{ce,pro}$ is the effective compressive strength of the column by the proposed ANFIS model. In Equation (5), π_1 is placed on the left side, while π_2 and π_3 are placed on the right side to constitute a relational expression as follows:

$$\frac{f'_{ce,pro}}{f'_{cj}} = f\left(\frac{h}{c}, \frac{f'_{cc}}{f'_{cj}}\right) \tag{6}$$

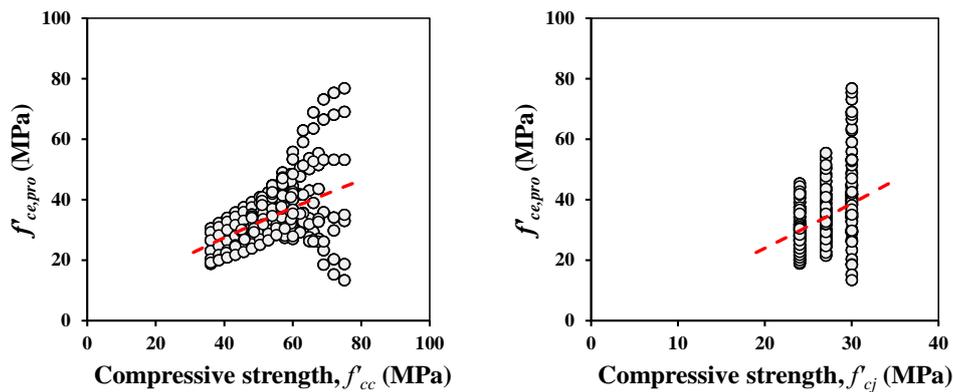
Table 3. Range of variables for parametric study.

	f'_{cc} (MPa)	f'_{cj} (MPa)	c (mm)	h (mm)
Parameter range	36–75	24–30	200–500	50–750

When it is represented by a dimensionless function for the h/c and the f'_{cc}/f'_{cj} , it can be expressed in the form of an exponent with respect to the dimensionless terms π_2 and π_3 , as follows:

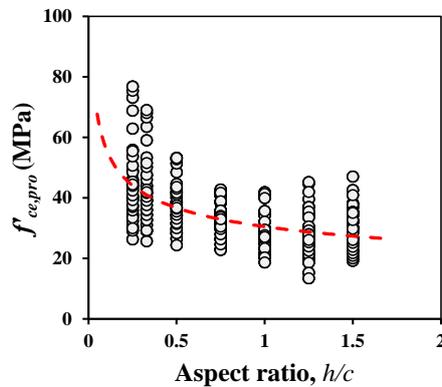
$$\frac{f'_{ce,pro}}{f'_{cj}} = K_1 \left(\frac{h}{c}\right)^\alpha \left(\frac{f'_{cc}}{f'_{cj}}\right)^\beta \tag{7}$$

where K_1 is the constant term, and α and β are the exponential terms. The test results conducted by Lee and Mendis [9] and Shu and Hawkins [12] showed that as the h/c increases, the f'_{ce} decreases. The results of the parametric studies shown in Figure 4c also confirmed that the value of h/c is inversely proportional to that of f'_{ce} .



(a) Compressive strength of columns.

(b) Compressive strength of slabs.



(c) Aspect ratio of columns and slabs.

Figure 4. Analysis results of parametric study.

The distribution of $f'_{ce,pro}/f'_{cj}$ with respect to c/h , which is the reciprocal of h/c , is shown in Figure 5a, and the equation of a trend line for c/h can be obtained. When the equation of a trend line for c/h is reflected in Equation (7), it can be represented as follows:

$$\frac{f'_{ce,pro}}{A(c/h)^{0.27} f'_{cj}} = K_2 \left(\frac{f'_{cc}}{f'_{cj}}\right)^\beta \tag{8}$$

where K_2 is the constant term. The distribution of $\frac{f'_{ce,pro}}{A(c/h)^{0.27} f'_{cj}}$ according to f'_{cc}/f'_{cj} is shown in Figure 5b, and the obtained equation of a trend line is reflected in Equation (8) and then represented as shown below:

$$\frac{f'_{ce,pro}}{(c/h)^{0.27} f'_{cj}} = 0.74 \left(\frac{f'_{cc}}{f'_{cj}} \right)^{0.63} \tag{9}$$

If Equation (9) is expressed with respect to the f'_{ce} , it can be represented as follows:

$$f'_{ce} = 0.74 \left(\frac{c}{h} \right)^{0.27} \left(\frac{f'_{cc}}{f'_{cj}} \right)^{0.63} f'_{cj} \tag{10}$$

Figure 6 shows the test and analysis results of the effective compressive strength estimated by Equation (10), as well as the current design codes and equations proposed by existing researchers. The analysis results using Equation (10) predicted the effective compressive strengths of specimens very accurately, with the AVG and the COVs calculated at 0.99 and 0.18, respectively. As shown in Figure 6 and Table 2, Equation (10) is slightly less accurate than the ANFIS model, but provides far more accurate analysis results than do the equations proposed by existing researchers and current design codes (ACI318-19, CSA.A23.3-14) [3,4].

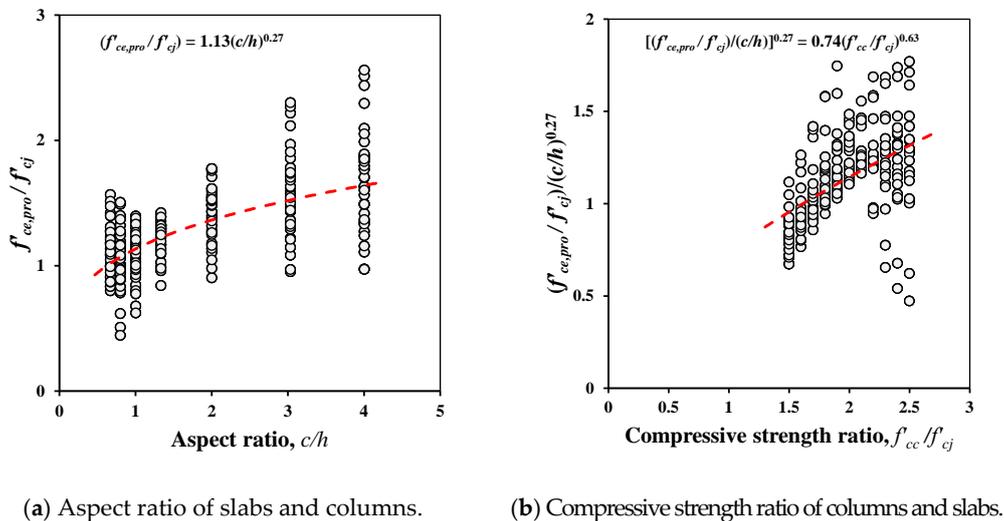


Figure 5. Dimensional analysis results.

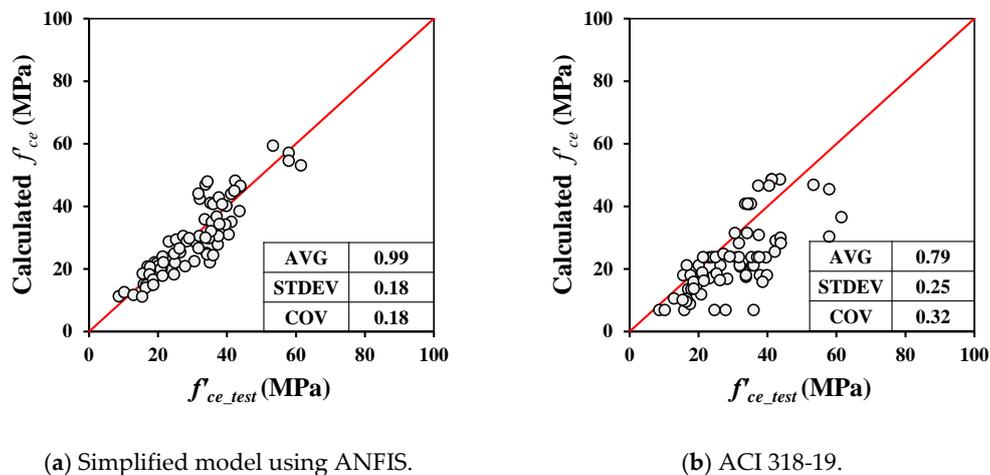


Figure 6. Cont.

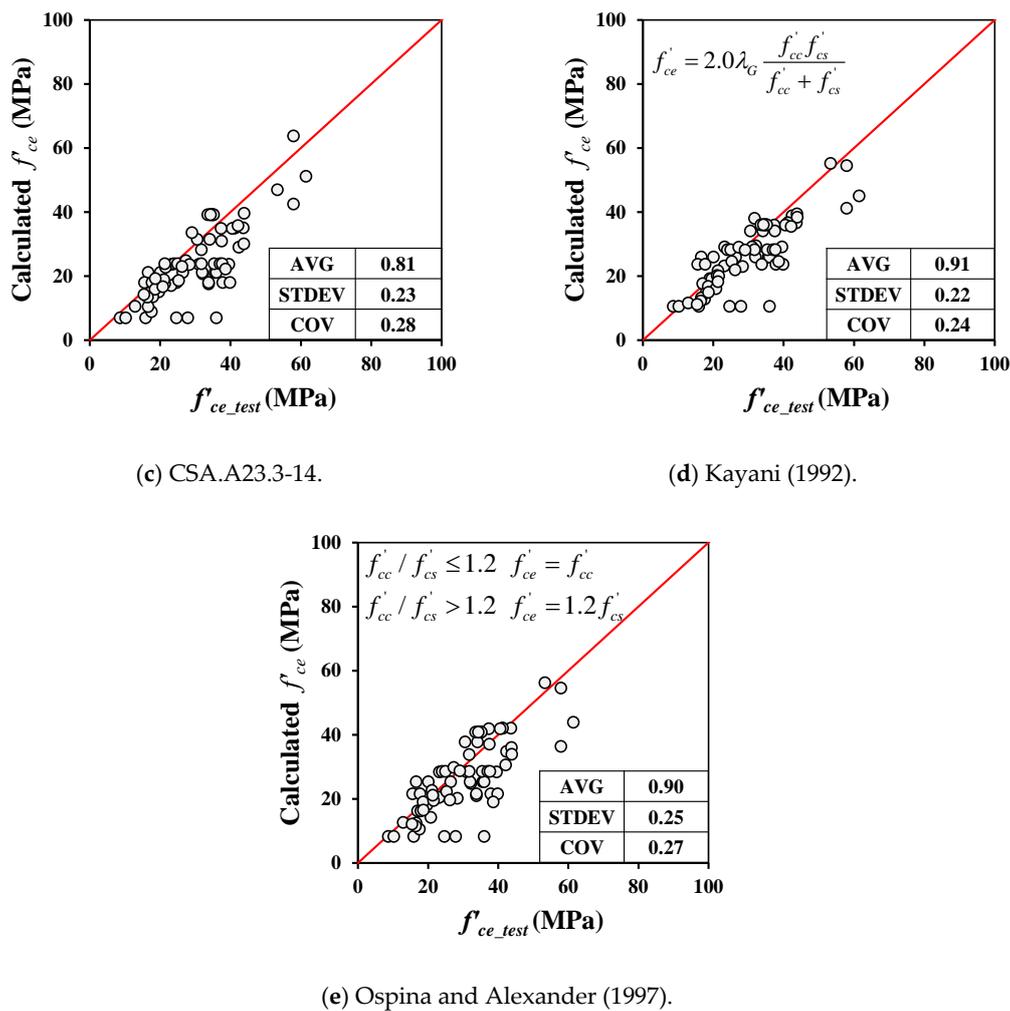


Figure 6. Comparison of simplified equation and other approaches.

5. Concluding Remarks

In this study, the effective compressive strength of a high-strength concrete (HSC) column intersected by normal-strength concrete (NSC) slabs was estimated by introducing the ANFIS algorithm. In addition, a simplified equation to facilitate practical applications was developed by means of extensive parametric studies and dimensional analyses. The accuracy of the ANFIS model and proposed equation was then verified by comparison with the collected test results. The following conclusions were obtained from the findings of this study.

- (1) The effective compressive strengths of HSC corner and edge columns intersected by NSC slabs were estimated using ANFIS, a type of neuro-fuzzy system. The results showed that the ANFIS model offers a very accurate evaluation of the collected test results, with the COV calculated at 0.13.
- (2) The equations for effective compressive strength specified in ACI 318-19 and CSA A23.3-14 provided very conservative results, but with relatively large scatters.
- (3) The parametric study was performed based on the ANFIS model, and a simplified equation for effective compressive strengths was proposed by means of a dimensional analysis. The simplified equation showed an accuracy which was very similar to that of the ANFIS model.

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