

Article

A Thermodynamic Closure Model for Titan's Surface Temperature: Its Long-Term Stability Anchored to Methane's Triple Point

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Abstract

We develop a minimal thermodynamic model to predict Titan's surface temperature based on radiative–convective equilibrium and the principle of maximum entropy production (MEP). The model retains only the essential atmospheric constituents: gaseous methane, which absorbs both longwave and near-infrared radiation, and stratospheric haze, which scatters and absorbs solar flux. Subject to Clausius–Clapeyron scaling of methane vapor pressure together with energy balances at the surface, tropopause, and stratopause, the model links the convective flux to the surface temperature, which exhibits a pronounced maximum due to competing radiative effects of tropospheric methane. As the surface warms, enhanced greenhouse effect would strengthen the convection, whereas the rising anti-greenhouse effect would suppress convection. The resulting convective peak corresponds to MEP, which thus selects a surface temperature slightly above methane's triple point. To assess its long-term evolution, we consider a 20% dimmer early Sun and a hypothetical 20% enrichment of the oceanic methane. Even in combination, they only cool the surface by ~2 K, in sharp contrast to the ~20 K cooling inferred in studies that prescribe haze abundance. This study suggests a critical role of self-adjusting haze in providing the internal degree of freedom necessary for MEP closure, thereby stabilizing Titan's temperature.

Keywords: Titan temperature; long-term evolution; methane triple point; anti-greenhouse effect; self-adjusting haze; maximum entropy production

1. Introduction

Titan's atmosphere shares several important similarities with Earth's. Like water vapor on Earth, methane is the dominant thermal absorber driving an active hydrological cycle, and analogous to terrestrial clouds, Titan hosts abundant organic haze that scatters and absorbs solar radiation. Any attempt to predict Titan's temperature must therefore account for the haze amount, but doing so from first principles remains impractical as haze formation involves complex photochemical and microphysical processes with effectively unbounded degrees of freedom [1,2]. Consequently, most existing models treat the haze abundance as a tunable parameter to match observed temperature [3,4], a diagnostic approach that offers only limited explanation of haze amount and surface temperature.

Recognizing this common shortfall in the thermodynamic closure of a climate system, researchers increasingly view the generic climate state as a macroscopic manifestation of a nonequilibrium thermodynamic system hence subject to the principle of maximum



Academic Editor: Mirko Severi

Received: 24 December 2025

Revised: 10 February 2026

Accepted: 19 February 2026

Published: 22 February 2026

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entropy production (MEP) [5,6]. Although initial application of MEP to climate studies was on empirical grounds [7], its physical foundation has continually strengthened through the years, including its recent connection to the fluctuation theorem [8,9], which has received experimental validation in laboratory settings [10,11]. Moreover, eddy-resolving calculations of horizontal convection [12,13] have reproduced a signature MEP feature—the formation of a mid-latitude front [14]—that is absent in coarse-grained numerical solutions [15].

Building on these developments, this author has previously applied MEP to Earth's climate system and showed that clouds may self-adjust to stabilize the surface temperature near water's triple point, possibly resolving the faint young Sun paradox (FYSP) [16,17]. In the present study, I extend this framework to Titan's atmosphere and demonstrate that its surface temperature is similarly constrained to lie just beyond methane's triple point. Specifically, this stabilization is the outcome of competing (longwave) greenhouse and (shortwave) anti-greenhouse effects of the tropospheric methane. The predicted surface temperature and climate properties align closely with current observation, thus providing further support of MEP as a testable working hypothesis.

To further assess long-term stability of the modeled temperature, we have considered its perturbation when Sun is 20% dimmer in early Titan and the oceanic methane can be enriched by 20%, yet the modeled surface temperature is only ~2 K lower, in sharp contrast to ~20 K cooling previously inferred under prescribed haze abundance [18]. The study thus underscores a possibly critical role of self-adjusting haze in enabling the MEP closure that stabilizes the surface temperature.

The paper is organized as follows. I introduce in Section 2 the minimal physical configuration of Titan's atmosphere and formulate energy balances linking key climate properties to the surface temperature. I then apply MEP to constrain a surface temperature anchored on methane's triple point, and further test its stability against changing external conditions. I provide additional discussion in Section 3 and conclude the paper in Section 4.

2. The Model

2.1. Physical Configuration

We first deduce the minimal physical configuration of our model, as sketched in Figure 1, in which the surface, tropopause and stratopause are designated as levels 0, 1 and 2, respectively. For convenience, all symbols and parameter values are listed in Appendix A, and as a general convention, dependent variables are global means, which are starred for current surface (reference) values, hatted when nondimensionalized by these reference values, and subscripted 0/1/2 for model levels.

In past discussions of Titan's tropospheric temperature, it is often assumed to be in radiative equilibrium but with a lapse rate bounded above by an adiabat through convective adjustment [19–21]. This interpretation is conceptually misleading because radiative fluxes depend on mole fraction of the thermal absorber, which is itself redistributed by convection throughout the troposphere [22]. A more logical reasoning would be to begin with the inevitable convection as solar radiation warms the surface. Owing to the mixed methane–ethane ocean [23], the rising air in convective plumes is initially undersaturated (Raoult's law) [18,24] and cools dry adiabatically until it condenses at the lifting condensation level (LCL). Above the LCL, convection is sustained by buoyancy released by condensation, and its highest reach during episodic convective storms defines the effective tropopause.

With convection being initiated at the surface, the methane mole fraction is uniform below the LCL, but decreases with height above the LCL, as methane condenses, up to the tropopause, beyond which it again becomes uniform in the absence of convection [25]. Despite its low concentration in the stratosphere, methane remains the primary producer

of organic haze through photolysis, a prominent feature of the Titan's atmosphere. Since its particle size overlaps visible wavelengths, the haze scatters sunlight efficiently [26], and its stratospheric aggregate also absorbs the shortwave flux in comparable measure [27], which I take to be its intrinsic property. As haze is thermally inactive [3], the absorbed solar energy must be balanced by optically thin hence near-isotropic thermal emission by the stratospheric methane [2]. As such, about half of the shortwave flux absorbed by haze is reradiated downward in the longwave band through the tropopause [27]. The shortwave flux entering the troposphere is further attenuated by near-infrared methane absorption [28] before ultimately absorbed by the ocean, the origin of the anti-greenhouse effect of the tropospheric methane.

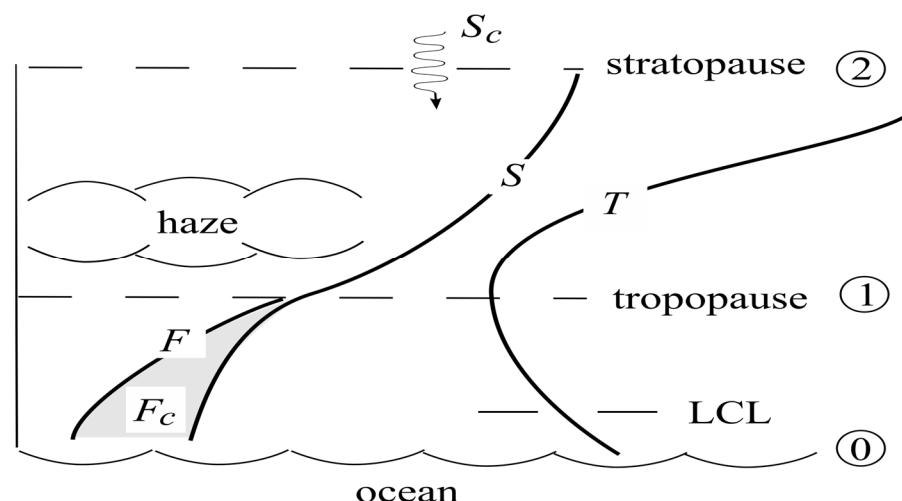


Figure 1. Minimal physical configuration of the model. S_c is the solar constant, S is the shortwave flux, which equals the net longwave flux (F) in the stratosphere, but exceeds it in the troposphere with the excess being the convective flux (F_c , shaded). T is the temperature and levels 0, 1 and 2 are surface, tropopause, and stratosphere, respectively.

As seen in Figure 1, the shortwave flux (S) equals the net longwave flux (F) in the stratosphere in maintaining radiative equilibrium [2] but because the shortwave optical depth is much smaller than the longwave one in the troposphere (Section 2.5), S exceeds F with the excess representing the convective flux (F_c , shaded). Although the convective flux is small compared with the solar constant, it remains commensurate with radiative fluxes [20] and hence cannot be neglected. More fundamentally, convective flux is the primary source of irreversible entropy production and thus plays a central role in our model closure.

2.2. Vertical Profiles

We next consider vertical profiles of climate properties (Figure 2) necessary for later derivation of the optical depth. The modeled ones for the current surface temperature of $T_0 = 94$ K are shown in solid lines alongside the observed ones in solid dots taken from [29] (their Table 1). The observed partial pressure (e) is calculated as the product of the methane mole fraction (mf) with the total pressure (p).

To interpret the temperature profile, we note that although rising air in convective plumes cools dry adiabatically until it reaches the LCL, convective plumes are episodic on Titan [22], as seen in the noisy lapse rate [30] (their Figure 6), which nonetheless averages to about half the (nitrogen) dry adiabatic rate. Above the LCL, convective motion is propelled by self-induced condensation but, because of shallowness of the LCL, only occasional convective storms may reach the tropopause, whose marking is discussed later (Section 2.4).

Then the slow planetary rotation impedes the tropical convergence of vapor that propels the deep updraft—the Intertropical Convergence Zone (ITCZ)—of Earth, which has been attributed as the driver of its moist-adiabatic lapse rate [31]. Sans these terrestrial provisions, the tropospheric lapse rate simply decreases monotonically with height in Titan, displaying no imprint of adiabatic motion, and given the simple form of the observed temperature profile, we approximate it by an exponential function, which is then uniquely specified by the temperature (T_0) and lapse rate (γ_0) at the surface

$$T = T_0 e^{-\gamma_0 z / T_0} \quad (1)$$

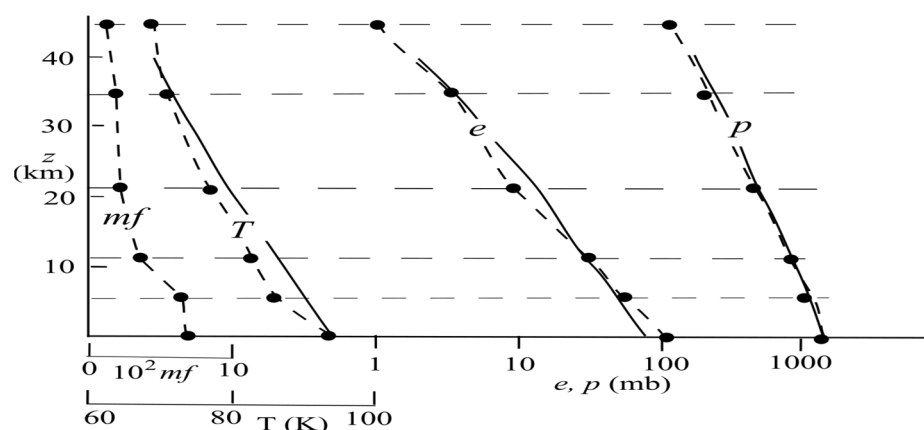


Figure 2. Observed vertical profiles (solid dots) of methane mole fraction (mf), temperature (T), partial pressure (e) and total pressure (p) derived from Huygens GCMS and HASI measurements [29] (their Table 1). Solid lines are model-derived profiles, which match closely the observed ones.

The modeled temperature profile is for $\gamma_0 = 0.7$ K/km (about half the dry adiabatic lapse rate), the latter a property that is independent of the surface temperature. It is seen that the modeled temperature closely matches the observed one, in support of the approximation (1).

For the observed mole fraction, it is homogenized below the LCL by convective mixing, but decreases with height above the LCL due to condensation, and again tends to a constant in the stratosphere in the absence of convection [29,32]. For a model derivation of the partial pressure, we note that the observed relative humidity inferred from Huygens measurements lies in mid-tenths, so for simplicity, we adopt a single representative value $U = 0.5$, which is seen to reproduce the observed partial pressure profile under Clausius–Clapeyron (C–C) scaling [29]. This approximation represents an empirical closure given the episodic nature of the convection. With the uniform relative humidity, the partial pressure satisfies

$$\frac{de}{e} = \frac{L_m}{R_m} \cdot \frac{dT}{T^2} \quad (2)$$

where L_m is the latent heat of evaporation of methane and R_m is its gas constant. Integrating vertically from the surface yields

$$e = e^* \exp[\alpha^* (1 - T^*/T)] \quad (3)$$

where

$$\alpha^* \equiv L_m (R_m T^*)^{-1} \approx 10.4 \quad (4)$$

and the reference vapor pressure is defined as

$$e^* \equiv Ue_s^* \quad (5)$$

with e_s^* being the saturated vapor pressure at the reference temperature T^* . The partial pressure (3), shown in the solid line, reproduces the observed profile, thus supporting the assumed relative humidity. The sensitivity of the model climate to its chosen value will be discussed later in Section 2.7. Since Titan's atmosphere is a dilute, N_2 -dominated mixture [1], we assume hydrostatic balance and ideal gas law so the total pressure satisfies

$$\begin{aligned} \frac{dp}{p} &= \frac{g}{RT} dz \\ &= \frac{g}{R\gamma T} dT \text{ (by definition)} \\ &= \left(\frac{gT_0}{R\gamma_0} \right) \frac{dT}{T^2} \text{ (Equation (1))} \\ &= \left(\frac{R_m g T_0}{R L_m \gamma_0} \right) \frac{de}{e} \text{ (Equation (2))} \end{aligned} \quad (6)$$

Since the bracketed term is relatively unvarying compared with the partial pressure, we obtain a power law

$$p/p_0 \approx (e/e_0)^n \quad (7)$$

where

$$n = \frac{R_m g T_0}{R L_m \gamma_0} \approx 0.62 \quad (8)$$

for the present climate. One may regard this power law as intrinsic to the Titan's troposphere since the surface lapse rate (γ_0), being proportional to the dry adiabatic lapse rate, is unvarying, and the surface temperature (T_0) is seen later to be constrained by MEP to be near methane's triple point. In support of this power-law dependence, the modeled total pressure is seen to match the observed one almost exactly.

To recap, we have assumed, as intrinsic to Titan's troposphere, an exponentially decaying lapse rate and a uniform 50% relative humidity, which have reproduced the observed vertical profiles of temperature, partial pressure and total pressure, in support of the model approximations.

2.3. Thermal Optical Depth

Nondimensionalizing the partial pressure (hatted) by its reference value (starred) and applying the Stefan–Boltzmann (SB) law, (3) becomes

$$\hat{e} = \exp \left[\alpha^* \left(1 - \hat{B}^{-1/4} \right) \right] \quad (9)$$

where $\hat{B}$ is the hemispheric blackbody flux. This linkage of the partial pressure to the local temperature is plotted in Figure 3 in the solid line, which we shall show next, may double as the thermal optical depth.

The differential optical depth is

$$d\tau = k_m \rho_m dz \quad (10)$$

where k_m is the gray absorption coefficient of methane in the longwave band and ρ_m is its density. Applying the ideal gas law, following the same steps as in (6), and integrating from the atmospheric top of zero optical depth and partial pressure, we derive

$$\tau = \nu e \quad (11)$$

where

$$\nu \equiv \frac{k_m T_0}{L_m \gamma_0} \quad (12)$$

is a parameter gauging the longwave absorption coefficient. While this coefficient is, in principle, measurable in the laboratory [33], its value in a climate setting remains too uncertain [2] to be extractable for our model. As discussed following (6), we regard ν as intrinsic to Titan's atmosphere, so k_m can be diagnosed from the current surface optical depth ($\tau^* = 5$) [3,34] and vapor pressure (5), rendering

$$\tau^* = \nu e^* \quad (13)$$

Diving (11) by (13), it is nondimensionalized to

$$\hat{\tau} = \hat{e} \quad (14)$$

The vapor pressure curve in Figure 3 thus doubles as the optical depth. To aid the later analytical solution, this optical depth is further approximated by a logarithmic function (dashed)

$$\hat{B} = \mu_1 \ln(1 + \mu_2 \hat{\tau}) \quad (15)$$

for which $\mu_1 = 0.43$ and $\mu_2 = 9.07$ are chosen so as to match both value and gradient of the two curves at the surface ($\hat{B} = 1$). Significantly, we have produced an explicit link of the optical depth to the local temperature when subject to C–C scaling—necessary for calculating radiative fluxes. This is a major gap in previous studies, which often assume certain power-law dependence between the optical depth and local temperature without addressing varying absorber density, hence incomplete [34].

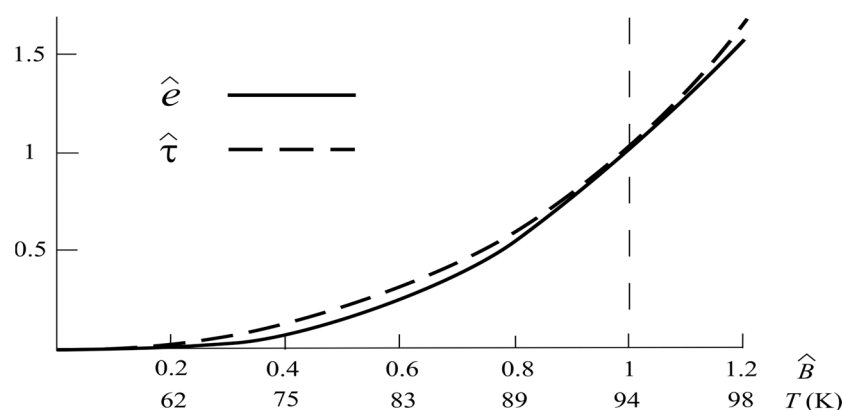


Figure 3. Partial pressure ($\hat{e}$, solid line) and the approximate optical depth ($\hat{\tau}$, dashed line) plotted against the local temperature.

2.4. Longwave Fluxes

The upward longwave flux is governed by the Schwarzschild equation

$$dF^\uparrow/d\tau = F^\uparrow - B \quad (16)$$

which has the solution

$$F^\uparrow = B_0 e^{-(\tau_0 - \tau)} + \int_{\tau}^{\tau_0} B' e^{-(\tau' - \tau)} d\tau' \quad (17)$$

The two terms on the right-hand side represent transmission of emissions from the surface and interim gas, respectively. Equation (15) allows the integration in (17) to render a dimensionless solution

$$\hat{F}^\uparrow = \hat{B} + \mu_1 e^{(\tau + \tau^*/\mu_2)} [E_1(\tau + \tau^*/\mu_2) - E_1(\tau_0 + \tau^*/\mu_2)] \quad (18)$$

where E_1 is the standard exponential integral. With the optical depth linked to the local temperature via (15), the upward longwave flux can be calculated as a function of the local temperature hence it is uniquely specified by the surface temperature (1). As an example, it is plotted in Figure 4 (upper solid line) for the present surface temperature ($T_0 = 94$ K). Expectedly, it decreases upward (moving to the left) as attenuation of the surface emission overrides the emission by the interim methane gas. This longwave flux is subjected to the following boundary condition at the tropopause (vertical dashed line).

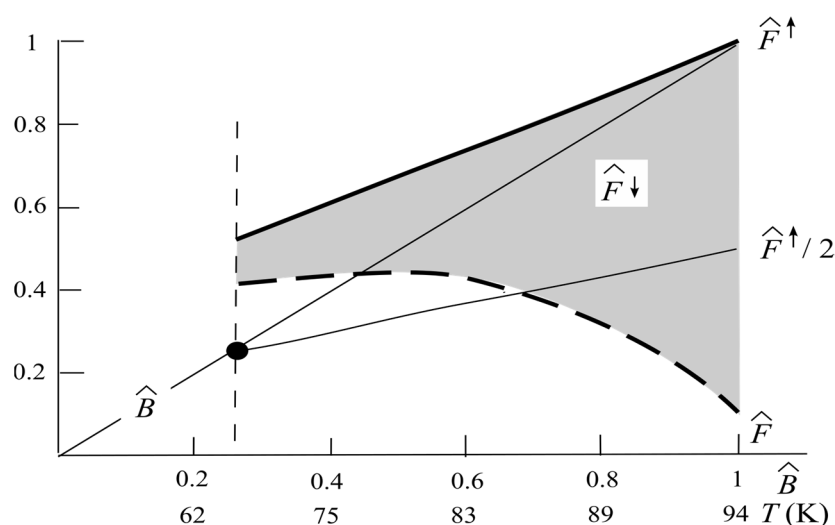


Figure 4. Vertical profiles of upward ($\hat{F}^\uparrow$, thick solid) downward ($\hat{F}^\downarrow$, shaded) and net ($\hat{F}$, thick dashed) longwave fluxes for the present surface temperature. The solid circle marks the intersection of the local temperature ($\hat{B}$) with the OLR skin temperature ($\hat{F}^\uparrow/2$), which defines the tropopause (vertical dashed line).

Because of the small optical depth of the stratosphere [32], the air temperature just beyond the tropopause should be at the skin temperature of the upward longwave flux (OLR, $\hat{F}_1^\uparrow$), and since the temperature is continuous across the tropopause, it sets the boundary condition that

$$\hat{B}_1 = \hat{F}_1^\uparrow/2 \quad (19)$$

As shown graphically in Figure 4, it is the intersection of the above two (thin) lines, which then specifies the tropopause temperature (vertical dashed line). Mathematically, substituting (19) into (18), the tropopause temperature satisfies

$$\hat{B}_1 = \mu_1 e^{x_1} [E_1(x_1) - E_1(x_0)] \quad (20)$$

where

$$x_0 \equiv \tau_0 + \tau^*/\mu_2 \quad (21)$$

and

$$x_1 \equiv \tau_1 + \tau^*/\mu_2 \quad (22)$$

Equating the tropopause temperature with (15) applied at the tropopause, we obtain an equation governing the tropopause optical depth τ_1 ,

$$\ln x_1 - \ln(\tau^*/\mu_2) - e^{x_1}[E_1(x_1) - E_1(x_0)] = 0 \quad (23)$$

For a given surface temperature, hence optical depth τ_0 , one may adjust τ_1 until this equation is satisfied. With (20) and (19), one then obtains the tropopause temperature and OLR. For the current surface temperature, the modeled tropopause has an optical depth of 0.45 and temperature of 68 K, both consistent with observations [30,32,34]. One should note that the close match of the modeled and observed temperatures is not due to tuning of the optical depth (15) since it involves no consideration of radiative fluxes; rather, it underscores the strong constraint the skin temperature (19) places on the energy balance of Titan.

Likewise, the downward longwave flux satisfies the Schwarzschild equation

$$dF^\downarrow/d\tau = -F^\downarrow + B \quad (24)$$

with the solution

$$F^\downarrow = F_1^\downarrow e^{-(\tau-\tau_1)} + \int_{\tau_1}^{\tau} B' e^{-(\tau-\tau')} d\tau' \quad (25)$$

where $F_1^\downarrow$ is the downward longwave flux at the tropopause to be discussed later (Section 2.5). Subject to nondimensionalization and (15), the downward longwave flux becomes

$$\hat{F}^\downarrow = \hat{B} - (\hat{B}_1 - \hat{F}_1^\downarrow) e^{-(\tau-\tau_1)} - \mu_1 e^{-(\tau+\tau^*/\mu_2)} [E_i(\tau + \tau^*/\mu_2) - E_i(\tau_1 + \tau^*/\mu_2)] \quad (26)$$

where E_i is the exponential integral. The downward longwave flux for the present surface temperature is plotted in Figure 4 (shaded) for the case of $\hat{F}_1^\downarrow = 0.11$, a value derived later for the current climate (Section 2.5). It is seen that the downward longwave flux increases strongly toward the surface (to the right) due to both increasing temperature and thermal opacity. The net longwave flux

$$\hat{F} = \hat{F}^\uparrow - \hat{F}^\downarrow \quad (27)$$

thus is as shown in the thick dashed line. It increases upward (to the left), signifying longwave cooling of the troposphere by the greenhouse effect. At the surface, in particular, the net longwave flux is from (27) and (26),

$$\begin{aligned} \hat{F}_0 &= \hat{B}_0 - \hat{F}_0^\downarrow \\ &= (\hat{B}_1 - \hat{F}_1^\downarrow) e^{-(\tau_0-\tau_1)} + \mu_1 e^{-x_0} [E_i(x_0) - E_i(x_1)] \end{aligned} \quad (28)$$

which enters the energy balance considered next.

2.5. Energy Balances

To aid the following discussion, we show a schematic of heat fluxes in Figure 5. Their dimensionless values are calculated for the present surface temperature based on expressions derived below. To obtain dimensional fluxes, multiply the dimensionless values with the reference value $B^* = 4.43 \text{ Wm}^{-2}$.

The solid line is the incoming shortwave flux, which is scattered by stratospheric haze to cross the stratopause (level 2) at

$$\hat{S}_2 = \hat{S}_c(1 - rh) \quad (29)$$

where $\hat{S}_c (= 0.85)$ is the present solar constant, r is the haze reflectance, and h is the unknown haze cover. Since the haze scatters and absorbs the solar radiation in comparable measure (Section 2.1), equating the gray absorption coefficient (a) with the reflectance (r), the downward shortwave flux entering through the tropopause (level 1) is

$$\begin{aligned}\hat{S}_1 &= \hat{S}_c(1 - rh - ah) \\ &= \hat{S}_c(1 - 2ah)\end{aligned}\quad (30)$$

This flux is further attenuated by methane absorption in the near-infrared before reaching the surface [35,36]. Although haze may extend into the troposphere through sedimentation [24], particle growth and aging have greatly diminished its shortwave absorptive efficiency [35], which moreover has no intrinsic temperature dependence and hence can be folded into the anti-greenhouse effect of the overlying stratospheric haze. As such, the observed tropospheric attenuation of the shortwave flux must be attributed primarily to absorption by the gaseous methane [27,35]. Let k be the ratio of methane's gray shortwave to longwave absorption coefficients, then the shortwave flux reaching the surface is

$$\hat{S}_0 = \hat{S}_1 e^{-k(\tau_0 - \tau_1)} \quad (31)$$

where τ 's pertain to the longwave optical depths discussed in Section 2.3. For the observed shortwave attenuation of 30% [27], it implies a shortwave optical depth of $k(\tau_0 - \tau_1) = 0.36$, and using longwave optical depths of $\tau_0 = 5$, $\tau_1 = 0.45$ (Section 2.4), it yields $k = 0.079$, an intrinsic property of the gaseous methane. This equation entails the anti-greenhouse effect of the tropospheric methane; the latter thus plays the dual roles of (longwave) greenhouse and (shortwave) anti-greenhouse effects in the troposphere.

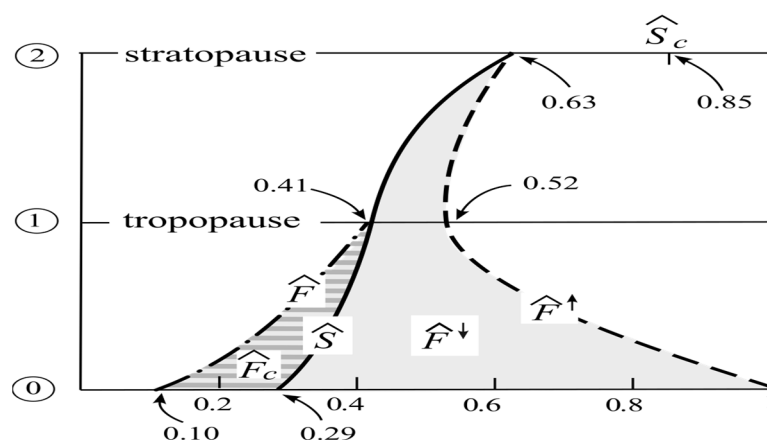


Figure 5. A schematic of heat fluxes. The dimensionless values are calculated for the present surface temperature based on model derivations (multiplying by $B^* = 4.43 \text{ Wm}^{-2}$ to obtain dimensional values). The solid line is the downward shortwave flux ($\hat{S}$), dashed line is the upward longwave flux ($\hat{F}^\uparrow$), shaded (including the overlaid stripes) is the downward longwave flux ($\hat{F}^\downarrow$), the dash-dotted line (continuing with stratospheric solid line) is the net longwave flux ($\hat{F}$), and stripped area is the convective flux ($\hat{F}_c$).

The absorbed solar heat ($\hat{S}_c ah$) by the stratospheric haze can be expended only by the longwave emission of the stratospheric methane, and its low optical depth promotes isotropic emission, so we assume half the solar absorption is reradiated downward through the tropopause or

$$\hat{F}_1^\downarrow = \hat{S}_c ah / 2 \quad (32)$$

We recognize that the present stratospheric longwave emission might have 6/4 partition in favor of the upward one [27], a possible discrepancy considered minor for our crude model. The energy balance at the tropopause states

$$\hat{S}_1 + \hat{F}_1^\downarrow = \hat{F}_1^\uparrow \quad (33)$$

or the combined downward shortwave and longwave fluxes equal the OLR. Because of the isotropic stratospheric emission, this energy balance at the tropopause assures the same at the stratopause. Substituting (30) and (32) into (33), we derive the haze absorption

$$ah = 2(1 - \hat{F}_1^\uparrow / \hat{S}_c) / 3 \quad (34)$$

which is thus a function of both the surface temperature (via OLR) and solar constant. Qualitatively, higher OLR or lower solar constant would be countered by less haze that let in more sunlight to maintain the tropopause energy balance. Substituting (34) into (30), the incident SW flux through the tropopause is

$$\hat{S}_1 = (4\hat{F}_1^\uparrow - \hat{S}_c) / 3 \quad (35)$$

which thus declines with lower OLR or higher solar constant. As seen in (31), this shortwave flux is further attenuated in the troposphere by near-infrared methane absorption, which represents a critical anti-greenhouse effect in addition to that of the stratospheric haze.

In the stratosphere, there is radiative equilibrium, so the shortwave flux ($\hat{S}$) overlaps the net longwave flux ($\hat{F}$). In the troposphere, small k implies that the shortwave flux is less attenuated than the net longwave flux (dash-dotted), the excess representing the convective flux $\hat{F}_c$ (striped)

$$\hat{F}_c = \hat{S} - \hat{F} \quad (36)$$

The troposphere, by definition, thus is in convective–radiative equilibrium, so the convective flux may not be neglected even if it is locally small, as sometimes assumed [4]. More significantly, however, this convective flux is the source of the irreversible entropy production hence critical to the model closure, as discussed next.

2.6. Maximum Entropy Production

In the above, we have linked climate properties to the surface temperature, which are plotted in Figure 6, referred to as the regime diagram. This diagram encapsulates the core physics of the model leading to the MEP solution, as expounded next. As the surface warms, OLR ($\hat{F}_1^\uparrow$) increases with the surface emission initially, but plateaus when the greenhouse effect rapidly approaches saturation after the triple point (the left vertical dashed line) [37]. To counter the rising OLR, haze absorption (ah) would decrease to let in more solar flux through the tropopause ($\hat{S}_1$), which plateaus like the OLR. This shortwave flux is attenuated in the troposphere due to near-infrared absorption by methane, so the shortwave flux reaching the surface ($\hat{S}_0$) would reverse the above rising trend and decline with the surface temperature; this is the anti-greenhouse effect exerted by the tropospheric methane, in addition to that by the stratospheric haze.

In the meantime, the rapidly increasing greenhouse effect that asymptotes the OLR has strongly depressed the net surface longwave flux ($\hat{F}_0$), which undergoes inflection near the triple point (the left vertical dashed line) before it decays to zero. For convenience, we shall refer to the surface temperature above the triple point as “saturated greenhouse” range. The deficit of $\hat{F}_0$ against the absorbed shortwave flux $\hat{S}_0$ must be accommodated by the surface convective flux ($\hat{F}_{c,0}$, shaded), which thus exhibits a bulge as the surface warms

from the saturating greenhouse to the still-rising anti-greenhouse effects. We shall next show that this bulge also manifests in the irreversible entropy production ($\hat{E}$).

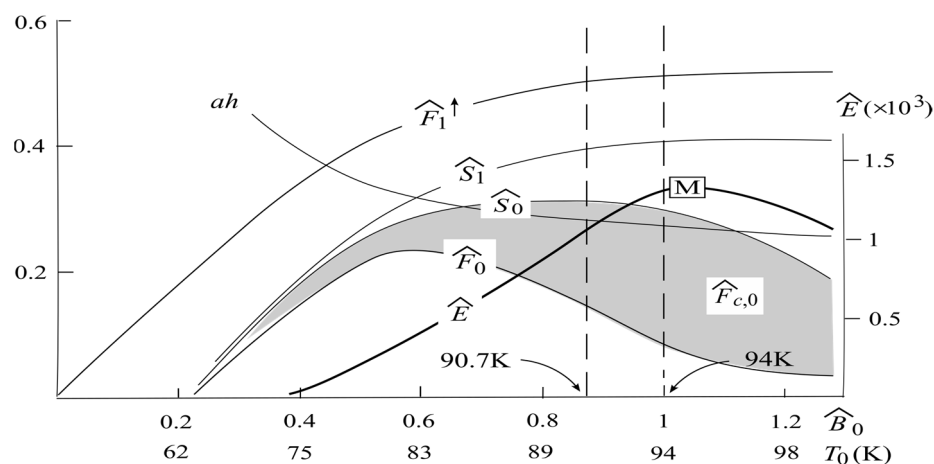


Figure 6. Regime diagram showing climate properties plotted against the surface temperature. The competition between the (longwave) greenhouse and (shortwave) anti-greenhouse effects of the tropospheric methane leads to a bulge in the surface convective flux ($\hat{F}_{c,0}$, shaded) where the entropy production ($\hat{E}$) is maximized (the open box labeled M for MEP). The corresponding surface temperature is 95 K, close to the observed one (94 K) and near methane's triple-point (90.7 K).

The irreversible entropy production (E) stems from convection in the troposphere (between levels 0 and 1) [38]

$$\begin{aligned} E &= \int_0^1 F_c d\left(\frac{1}{T}\right) \\ &= -\frac{F_{c,0}}{T_0} - \int_0^1 \frac{dF_c}{T} \end{aligned} \quad (37)$$

recognizing that convection terminates at the tropopause. Defining a “radiative” cooling temperature T_r by

$$\frac{F_{c,0}}{T_r} \equiv - \int_0^1 \frac{dF_c}{T} \quad (38)$$

Equation (37) becomes

$$E = \frac{F_{c,0}}{T_0} \eta \quad (39)$$

where

$$\eta \equiv \frac{T_0}{T_r} - 1 \quad (40)$$

is the thermodynamic efficiency of the atmospheric heat engine. To derive T_r and consistent with our minimal model, we approximate the vertical profile of the convective flux (striped area in Figure 5) as linear in $\hat{B}$, then (38) yields

$$\frac{T_r}{T_0} = \frac{3}{4} \frac{[1 - (T_1/T_0)^4]}{[1 - (T_1/T_0)^3]} \quad (41)$$

While the above linear assumption is crude, it is nonetheless better justified than the commonly used effective temperature T_e of the OLR. To see this, we plot in Figure 7 the tropopause temperature T_1 , the effective temperature T_e , the radiative cooling temperature T_r , as well as thermodynamic efficiencies η_e (based on T_e) and η (based on T_r) against the surface temperature T_0 over its plausible range. It is seen that T_r is quite higher than T_1 and it does not level off like T_e ; this difference is the reason that η asymptotes to around 15%, a value consistent with the current observation [39], whereas η_e increases indefinitely, clearly unphysical. In fact, as seen in [31], the radiative cooling temperature on Earth

is proportional to the surface temperature to render a thermodynamic efficiency that is relative unvarying, which should apply to Titan as well. The thick lines are thus the pertinent ones for calculating the entropy production (39) and with the levering-off of the thermodynamic efficiency and the minor dependence on the surface temperature in the saturated greenhouse range (above the triple point, shaded), one expects the entropy production to vary primarily with the surface convective flux to exhibit a maximum.

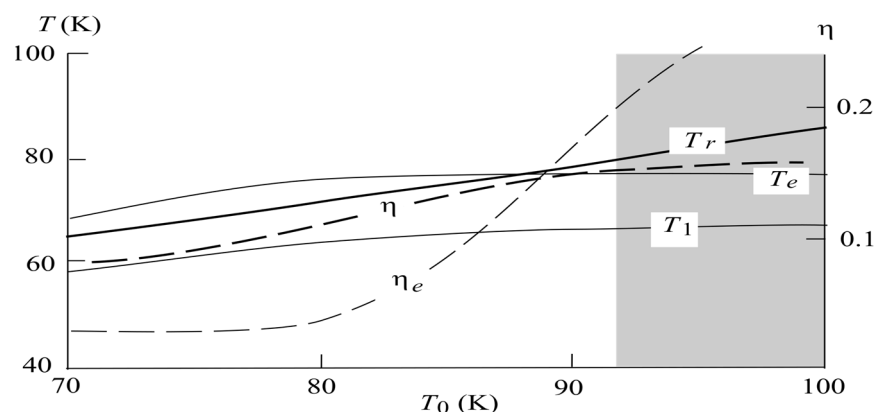


Figure 7. The tropopause temperature T_1 , the radiative cooling temperature T_r , the effective temperature T_e , as well as thermodynamic efficiencies η (based on T_r) and η_e (based on T_e), plotted against the surface temperature T_0 . Only thick lines are pertinent, which shows leveling-off of the thermodynamic efficiency (thick dashed) when the greenhouse effect is saturated (above the triple point, shaded).

This surface convective flux, as seen in Figure 5, is the deficit of the net surface longwave flux $\hat{F}_0$ against the shortwave flux absorbed by the ocean $\hat{S}_0$, or

$$\hat{F}_{c,0} = \hat{S}_0 - \hat{F}_0 \quad (42)$$

so its maximization

$$d\hat{F}_{c,0}/d\hat{B}_0 = 0 \quad (43)$$

yields

$$d\hat{S}_0/d\hat{B}_0 = d\hat{F}_0/d\hat{B}_0 \quad (44)$$

or equal downward slopes of these two curves in Figure 6. As such, the maximum must lie in the saturated greenhouse range where $\hat{F}_0$ slope is easing while $\hat{S}_0$ slope is steepening. Because the much weaker visible absorption than the thermal one due to small k (31), $\hat{S}_0$ curve is shifted to higher optical depth, which nonetheless represents only minor surface warming because of the C–C scaling (Figure 3). As such, we expect MEP, while above the triple point, to remain close to it. In Figure 6, the nondimensionalized entropy production ($\hat{E}$) is plotted in a thick solid line, which shows a maximum at around 95 K (the open box marked M for MEP), which is close to the observed 94 K (the right vertical dashed line), and both are indeed near the triple point, as we have reasoned above. Since the vertical profiles of climate properties shown in Figure 2 are based on the current surface temperature, they are likewise reproduced by MEP.

Being a minimal model, there are fewer tunable parameters compared with typical climate models, so the observational agreement offers an added support of MEP as a governing principle of Titan's climate. In addition, the predicted haze albedo (ah) is 0.26, close to the observed Bond albedo [40], which underscores the importance of the haze abundance in regulating the solar forcing. Our approach differs fundamentally from prevailing ones that prescribe the surface temperature to reproduce the observed climate

properties, including the haze abundance. While such diagnostic approach has validated the radiative–convective equilibrium, it does not offer a causal explanation of the observed haze amount or its coupled surface temperature, the stated aim of the present study.

2.7. Long-Term Stability

While MEP successfully predicts the present surface temperature, one naturally wonders how robust it is and how it might have evolved over geological time. Previous studies have explored Titan’s thermal history, but typically without accounting for potential negative feedback of the haze amount. Since self-adjusting haze layer plays a key role in stabilizing our surface temperature, we expect a sharp contrast in our projected long-term change compared with earlier works.

One well-documented forcing change pertains to the solar constant, which has increased by about 20% since the late heavy bombardment (LHB) about four billion years ago. A similar scenario was experienced by Earth [41], yet early Earth could be as warm as today. This is the well-known FYSP [42], which can be resolved by MEP when self-adjusting cloud buffers the solar forcing [16,17]. By analogy, Titan’s haze may serve a similar regulatory role, mitigating the impact of solar variability.

Besides the solar constant, another less certain external change is the composition of the mixed methane–ethane ocean as methane is converted to ethane over tens of million years [43], unless it is replenished by still-debated processes [44,45]. Then, even if methane evolution can be ascertained, translating it to temperature change encounters the same nonclosure of atmosphere models if the haze amount needs to be prescribed [18]. This shortfall is amended in our model by self-adjusting haze, hence a thermodynamical closure via MEP. To illustrate the temperature response to changing ocean composition, we consider a hypothetical 20% enrichment of the oceanic methane, which would increase the surface relative humidity by the same amount [46].

We first consider the simpler case of solar perturbation and plot in Figure 8 the evolution from the early Titan (dashed lines, $\hat{S}_c = 0.7$) to the present (solid lines, $\hat{S}_c = 0.85$), as indicated by arrows spanning the shaded differences. It shows surprisingly that the surface temperature has warmed only by 1 K. To interpret this result, we shall first derive an approximate MEP solution, which is of an analytical form to aid such understanding.

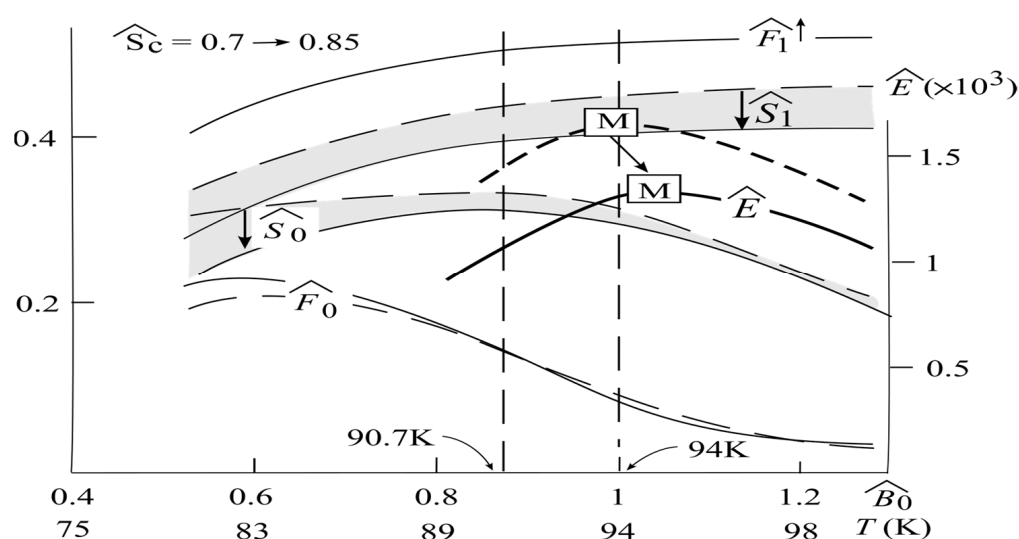


Figure 8. Regime diagram showing evolution (arrows) from early Titan (dashed lines, $\hat{S}_c = 0.7$) to present Titan (solid lines, $\hat{S}_c = 0.85$), with their differences shaded. Boxes labelled M (for MEP) show surface warming of about 1 K.

Since the perturbed MEP state remains in the saturated greenhouse range, both OLR ($\hat{F}_1^\uparrow$) and shortwave flux entering the troposphere ($\hat{S}_1$) have largely plateaued. Neglecting small terms in (28) based on Figures 4 and 5, the net surface longwave flux is approximately

$$\begin{aligned}\hat{F}_0 &\approx \mu_1 e^{-x_0} E_i(x_0) \\ &\approx \mu_1 / x_0\end{aligned}\quad (45)$$

Equation (44) implies, for the maximized surface convective flux,

$$d\hat{S}_0/d\tau_0 = d\hat{F}_0/d\tau_0 \quad (46)$$

Substituting from (31) and (45), and neglecting smaller terms, we derive

$$k\hat{S}_1 e^{-k(\tau_0 - \tau_1)} \approx \mu_1 \tau_0^{-2} \quad (47)$$

which states a balance between the anti-greenhouse (left-hand side) and greenhouse (right-hand side) effects. The anti-greenhouse efficiency depends on both the incoming shortwave flux ($\hat{S}_1$) and shortwave optical depth, which is much smaller than the longwave one, as entailed in the small k . The greenhouse efficiency on the other hand decays as the inverse square of the optical depth in the saturated greenhouse range. With the small k , (47) leads to an analytical expression for the surface optical depth of the MEP state

$$\tau_0 \approx (\mu_1/k)^{1/2} \hat{S}_1^{-1/2} \approx 3.64 \quad (48)$$

Subject to the linkage (15) and the SB law, the corresponding surface temperature is 91 K, which is about 4 K lower than that shown in Figure 6, a deviation that is not unexpected given the additional approximations, but since it has incorporated dominant terms, it may serve as a proxy of the full MEP state in assessing qualitative dependence and order-of-magnitude quantitative perturbations. Being in the saturated greenhouse range, this proxy MEP state has the following base values calculated from expressions derived in Section 2.5 and with the same reference values listed in Appendix A: $\hat{\tau}_1 = 0.09$, $\hat{B}_1 = 0.26$, $\hat{F}_1^\uparrow = 0.52$, $\hat{S}_c = 0.85$, $\hat{S}_1 = 0.41$, $\hat{S}_0 = 0.32$, $\hat{\tau}_0 = 0.72$, $\hat{B}_0 = 0.87$, and $\hat{T}_0 = 0.97$. These values are needed to calculate percentage perturbations.

Let primed variables denote perturbations and unprimed variables assigned above base values, we first consider climate response to the lower solar constant of $\hat{S}'_c = -0.15$ or $\hat{S}'_c/\hat{S}_c = -18\%$, which may represent the early Titan. The resulting percentage changes are listed in the top row of Table 1 based on following derivations.

Table 1. The climate response to lower solar constant S_c , higher relative humidity U , and their combination ($S_c + U$), expressed in percentage change from base values. The sensitivity is defined as the percentage change in the surface temperature divided by that of the external forcing.

	$F_1^\uparrow$	S_1	S_0	τ_0	B_0	T_0	Sensitivity
S_c (−18%)	0%	12%	14%	−6%	−2.6%	−0.7%	0.04
U (+20%)	−10%	−16%	−19%	8%	−5%	−1.4%	−0.07
$S_c + U$	−10%	−5%	−6%	2.5%	−8%	−2%	−0.1

The solar constant has no influence on OLR, so a weaker solar forcing is fully countered by less haze, and its reduced shortwave absorption implies smaller downward longwave flux hence greater shortwave flux entering the troposphere, so from (35),

$$\hat{S}'_1 = -\hat{S}'_c/3 \approx 0.05 \quad (49)$$

or

$$\hat{S}'_1 / \hat{S}_1 \approx 12\% \quad (50)$$

which is 1/3 smaller percentagewise than the forcing change. Since $\hat{S}_1$ anchors the anti-greenhouse effect, its increase requires smaller optical depth to balance the saturated greenhouse effect, as entailed in the proxy MEP solution (48), resulting in

$$\hat{\tau}'_0 / \hat{\tau}_0 = -(\hat{S}'_1 / \hat{S}_1) / 2 \approx -6\% \quad (51)$$

hence additional 1/3 reduction from the solar perturbation. From (31) and recognizing that $\tau'_1 = 0$ (OLR is unchanged)

$$\hat{S}'_0 / \hat{S}_0 = \hat{S}'_1 / \hat{S}_1 - k\tau'_0 \approx 14\% \quad (52)$$

which represents a slight increase in absorbed shortwave flux over the incoming one due to smaller optical depth. Since the optical depth $\hat{\tau}_0$ is linked to the surface emission $\hat{B}_0$ via C–C scaling (15), the latter is perturbed as

$$\hat{B}'_0 = \mu_1 \mu_2 (1 + \mu_2 \hat{\tau}_0)^{-1} \hat{\tau}'_0 = -0.022 \quad (53)$$

so that

$$\hat{B}'_0 / \hat{B}_0 \approx -2.5\% \quad (54)$$

which represents halving of the perturbed optical depth. The strongest reduction comes from a further four-fold reduction in the surface temperature on account of the SB law or

$$\hat{T}'_0 / \hat{T}_0 = (\hat{B}'_0 / \hat{B}_0) / 4 \approx -0.6\% \quad (55)$$

The surface temperature thus is lower by $T'_0 = 0.6$ K. All above perturbations of the proxy MEP state are consistent with that shown in Figure 8 based on the full solution, in support of our use of the proxy state in explaining the climate perturbations.

Tracking the perturbation sequence of Table 1 due to change in the solar constant (S_c), it is reduced 1/3 by haze (S_1) and methane absorption (S_0), additional 1/3 by the anti-greenhouse effect (τ_0), another halving by C–C scaling (B_0), and finally a four-fold reduction due to the SB law (T_0). Defining climate sensitivity by the percentage change in the surface temperature relative to that of the forcing, it is only 0.04. We project therefore that, if MEP applies, the early Titan is only about ~1 K cooler when Sun is 20% dimmer, a sharp contrast to ~20 K cooling projected previously when haze is fixed; the disparity of the prior works being deprived of haze feedback, the weaker solar flux needs to be wholly balanced by lowered surface emission, resulting in a much cooler surface. In our model, on the other hand, the solar perturbation is successively countered by stratospheric and tropospheric anti-greenhouse effect in facilitating MEP, resulting in only minor cooling of the surface.

We shall next consider the temperature response to 20% higher methane concentration in the ocean as reflected in the same increase in the relative humidity (Raoult's law) or $U' / U = 20\%$. The changing regimes are plotted in Figure 9, and the percentage changes in climate properties are listed in the middle row of Table 1.

Differing from the solar forcing, changing the relative humidity U would perturb the reference optical depth τ^* as they are linked through (5) and (13)

$$\tau^* = \nu U e_s^* \quad (56)$$

so that

$$\tau^{*}/\tau^{*} = U'/U \quad (57)$$

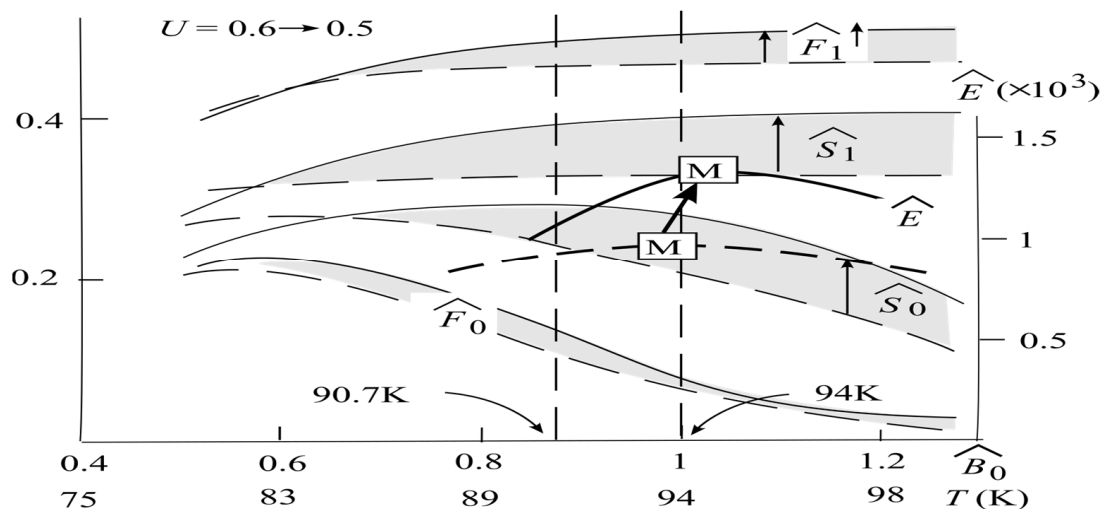


Figure 9. Regime diagram showing evolution (arrows) from higher relative humidity ($U = 0.6$, dashed lines) to lower relative humidity ($U = 0.5$, solid lines) with their differences shaded. Boxes labelled M (MEP) indicate surface warming of ~ 1 K.

Since by definition

$$\tau = \hat{\tau}\tau^{*} \quad (58)$$

we have

$$\tau'/\tau = \hat{\tau}'/\hat{\tau} + \tau^{*}/\tau^{*} \quad (59)$$

so the optical depth is perturbed by both temperature and relative humidity, the two terms on the right-hand side, respectively. Unlike the solar forcing, the changing relative humidity would alter the OLR governed by (23). Varying this equation, noting that

$$e^{x_1} dE_1/dx_1 = -1/x_1 \quad (60)$$

and neglect smaller terms, it is straightforward to derive

$$x_1'/x_1 = [2 - x_1 \ln(x_1) + x_1 \ln(\tau^{*}/\mu_2)]^{-1} \tau^{*}/\tau^{*} \approx 0.71 \tau^{*}/\tau^{*} \quad (61)$$

leading to

$$\tau_1'/\tau_1 \approx 0.33 \tau^{*}/\tau^{*} \quad (62)$$

and from (59),

$$\hat{\tau}_1'/\hat{\tau}_1 = \tau_1'/\tau_1 - \tau^{*}/\tau^{*} \approx -0.67 \tau^{*}/\tau^{*} \approx -0.13 \quad (63)$$

Applying (15), the tropopause temperature hence OLR would be perturbed as

$$\hat{B}_1' = \hat{F}_1^{\uparrow}/2 = \mu_1 \mu_2 (1 + \mu_2 \hat{\tau}_1)^{-1} \hat{\tau}_1' \approx 2.15 \hat{\tau}_1' \quad (64)$$

This OLR reduction would lessen the incoming shortwave flux as seen from (35), yielding

$$\hat{S}_1'/\hat{S}_1 \approx -16\% \quad (65)$$

and to boost the anti-greenhouse effect to balance the greenhouse effect, the optical depth would increase by, from (48),

$$\tau'_0/\tau_0 = -(\hat{S}'_1/\hat{S}_1)/2 \approx 8\% \quad (66)$$

With τ'_0 and τ'_1 known, (31) yields

$$\hat{S}'_0/\hat{S}_0 = \hat{S}'_1/\hat{S}_1 - k(\tau'_0 - \tau'_1) \approx -19\% \quad (67)$$

Again, the percentage change in shortwave flux absorbed by the surface is slightly amplified over the incoming one, so the anti-greenhouse effect due to the tropospheric methane is slightly enhanced. Applying (59), we derive

$$\hat{\tau}'_0/\hat{\tau}_0 = \tau'_0/\tau_0 - \tau^{*}/\tau^* \approx -12\% \quad (68)$$

That is, although the absolute optical depth increases (66), the scaled one actually decreases, which is what impacts the surface emission via C–C scaling (15), yielding

$$\hat{B}'_0 = \mu_1\mu_2(1 + \mu_2\hat{\tau}_0)^{-1}\hat{\tau}'_0 \approx -0.47 \quad (69)$$

so that

$$\hat{B}'_0/\hat{B}_0 \approx -5\% \quad (70)$$

and

$$\hat{T}'_0/\hat{T}_0 = (\hat{B}'_0/\hat{B}_0)/4 \approx -1.4\% \quad (71)$$

All the above changes are consistent with the regime change shown in Figure 9 based on the full solution, which thus is explained. In particular, the surface temperature is cooled by 1.3 K, commensurate with that seen in Figure 9, implying a sensitivity of -0.07 , which doubles that to the solar forcing, but remains small.

Tracking the perturbation sequence of the middle row of Table 1, the higher relative humidity ($U'/U = 20\%$), hence stronger greenhouse effect, would reduce OLR by 10%, which increases the haze amount and downward longwave radiation to reduce the short-wave flux entering the troposphere by 16%, and further for that reaching the surface. To boost the anti-greenhouse effect in countering the saturated greenhouse effect, the optical depth increases by 8%, but because of the higher relative humidity, the surface emission actually decreases by -5% , hence the surface temperature by -1.3% on account of the SB law. This cooling is of the same sign as that found by [46] when oceanic methane is enriched, but quantitatively the cooling is much smaller because of MEP. The sensitivity -0.07 doubles that to the solar change but remains very low.

As a further illustration, we consider the combined perturbation of solar constant and relative humidity, with the regime diagram shown in Figure 10 and the percentage changes listed in the bottom row of Table 1. It is seen that the percentage changes, although sizable, remain linear and additive hence explainable by the foregoing discussion. It results in a surface cooling of about ~ 2 K, which is consistent with that shown in Figure 10 based on the full solution. This minor cooling departs sharply from the ~ 20 K cooling projected previously with fixed haze [18]. It is noted nonetheless that MEP peak is less distinct, so the surface temperature is more vulnerable to additional perturbations, and one may not preclude the possibility that the surface could cool to the triple point to render a vastly different early Titan of frozen ocean.

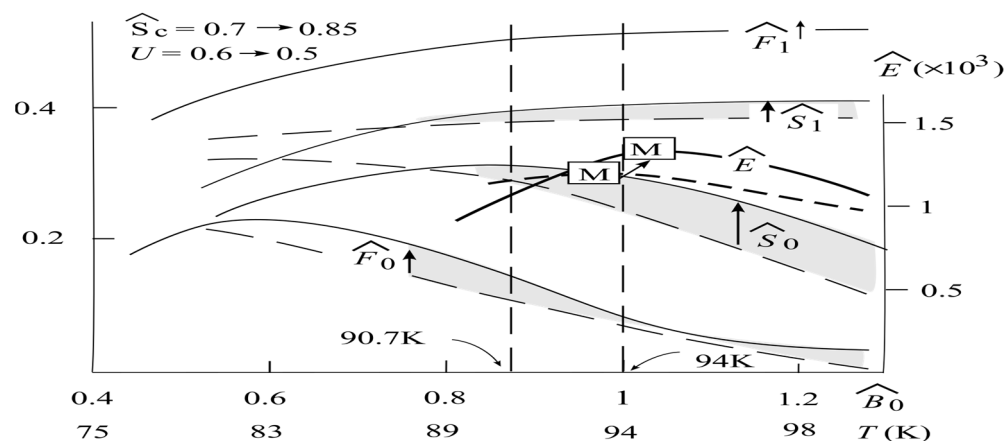


Figure 10. Regime diagram showing evolution (arrows) when forcings change from $\widehat{S}_c = 0.7, U = 0.6$ (dashed lines) to $\widehat{S}_c = 0.85, U = 0.5$ (solid lines) with their differences shaded. Boxes labelled M (MEP) indicate surface warming of ~2 K.

3. Discussion

Despite vast difference between Titan and Earth's atmospheres, our model points to a unifying thermodynamic tendency: in the presence of a dominant thermal absorber, the surface temperature is stabilized near and above the absorber's triple point through maximum entropy production (MEP), a thermodynamic principle not yet sufficiently acknowledged. Since it is the self-adjusting haze that provides the internal degree of freedom in facilitating this closure, previous assessments of Titan's long-term thermal evolution by prescribing a fixed haze abundance [18,46] may need re-evaluation.

Compared with Earth, clouds play only a minor role in Titan's energy balance. Low-level clouds associated with convective plumes cover only about 1% of Titan's surface [39,47], which are thus neglected in our minimal model, consistent with earlier studies [3]. In addition, Titan's slow rotation inhibits the tropical convergence of vapor that drives Earth's ITCZ and extensive high clouds [31]. The additional degree of freedom associated with Earth's high clouds in regulating the OLR necessitates a different MEP closure through their coupling to the surface dissipation and evaporation [16,48]. That is, while the triple point provides a common lower bound on surface temperature through the greenhouse effect, the upper bound is instituted by increasing evaporative cooling on Earth and by methane's anti-greenhouse effect on Titan.

We have not explicitly derived the stratospheric temperature, as it depends on a small and poorly constrained thermal optical depth, and in any event it exerts little influence on the surface climate. Nonetheless, its order of magnitude may be estimated from the modeled longwave emission ($S_{cah} \approx 0.62 \text{ W m}^{-2}$) and the observed stratospheric temperature range $T \in [70, 170] \text{ K}$. This range corresponds to blackbody fluxes $B \in [1.36, 47.36] \text{ W m}^{-2}$, and given the small optical depth hence nearly isotropic emission, the effective emission may be approximated by the median flux $\bar{B} = 24.36 \text{ W m}^{-2}$, which yields a column emissivity—approximately the optical depth—of

$$\varepsilon = S_{cah} / (2\bar{B}) \approx 0.013 \quad (72)$$

This value is much smaller than the optical depth (~ 0.45) inferred earlier from the tropopause energy balance (Section 2.4). The likely explanation is that the extremely low stratospheric pressure substantially weakens the collision-induced absorption, thereby suppressing the longwave opacity [32,33]. In contrast, the shortwave optical depth of the haze layer is of order unity [3,25], so the shortwave absorption by haze is two orders of magnitude more efficient than the longwave emission by methane. This disparity is

consistent with common assumptions [4,32] and offers a plausible explanation for Titan's exceptionally warm stratopause.

Although MEP has been successfully applied in some climate theories, its verification by climate models remains difficult since, by definition of a nonequilibrium thermodynamic system, they need to resolve microscopic elements, such as convective plumes, which obviously poses a daunting challenge. As such, one should regard MEP as a working hypothesis rather than a proven physical law, and the present study nonetheless further expands its utility in explaining Titan's surface temperature and its long-term stability.

4. Conclusions

We have developed a minimal thermodynamic model to predict Titan's surface temperature. The model retains only the essential climate constituents: gaseous methane, which absorbs both longwave and near-infrared radiation, and stratospheric haze, which scatters and absorbs incoming solar flux. By combining Clausius–Clapeyron scaling of methane vapor pressure with energy balances at the surface, tropopause, and stratopause, the model establishes a direct link between the surface temperature and key climatic properties, including the convective flux.

Notably, this convective flux exhibits a pronounced maximum arising from competing radiative effects of tropospheric methane. As the surface warms, the longwave greenhouse effect enhances radiative cooling and augments the convection, while the shortwave anti-greenhouse effect reduces the solar absorption by the surface and suppresses the convection. These opposing tendencies produce a distinct peak in convective flux and since this peak corresponds to maximum entropy production, the MEP thus selects a surface temperature near methane's triple point when the greenhouse effect rapidly approaches saturation, as is the current observation.

To assess the robustness of this temperature, we examine its perturbation by the dimmer early Sun and hypothetical enrichment of the oceanic methane. Even with 20% change in these forcings, the surface is cooled by only ~2 K, much smaller than the ~20 K cooling inferred from previous models of fixed haze abundance. In conclusion, the study underscores the possibly key role of self-adjusting haze in facilitating MEP that stabilizes Titan's surface temperature near methane's triple point.

Funding: This research received no external funding.

Data Availability Statement: No new data was created or analyzed in this study. Data sharing is not applicable since the article describes entirely theoretical research.

Acknowledgments: I want to thank anonymous reviewers for comments that have improved the paper.

Conflicts of Interest: The author declares no conflicts of interest.

Appendix A

a	Haze shortwave absorption coefficient
B	Hemispheric blackbody flux
B^*	Reference for heat fluxes ($= 4.43 \text{ Wm}^{-2}$)
e	Methane partial pressure
e_s	Saturation vapor pressure
e_s^*	Reference for e_s ($= 1.56 \times 10^2 \text{ mb}$)
E	Entropy production

F	Net longwave flux
$F^{\uparrow}$	Upward longwave flux
$F^{\downarrow}$	Downward longwave flux
F_c	Convective flux
g	Gravitation acceleration ($= 1.35 \text{ ms}^{-2}$)
h	Haze cover
k_m	Methane longwave absorption coefficient
L_m	Methane latent heat of evaporation ($= 5.1 \times 10^5 \text{ JKg}^{-1}$)
mf	Mole fraction
p	Atmospheric pressure
r	Haze shortwave reflectance
R	Nitrogen gas constant ($= 2.97 \times 10^2 \text{ JK}^{-1}\text{Kg}^{-1}$)
R_m	Methane gas constant ($= 5.18 \times 10^2 \text{ JK}^{-1}\text{Kg}^{-1}$)
S	Shortwave flux
S_c	Solar constant ($= 3.75 \text{ Wm}^{-2}$)
T	Temperature
T^*	Reference temperature ($=94 \text{ K}$)
T_r	Radiative cooling temperature
U	Relative humidity ($=0.5$)
γ	Lapse rate
η	Thermodynamic efficiency
ρ_m	Methane density
τ	Thermal optical depth
τ^*	Reference τ ($=5$)

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