

Overview of Thermo-Hydro-Mechanical Constitutive Models for Saturated Cohesive Soils

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Abstract

The need to mathematically model and numerically simulate the effect of temperature on the hydro-mechanical behavior of soils, driven by the increased interest in energy geostructures and sundry other geothermal systems, has necessitated the development of constitutive models that can realistically simulate the thermo-hydro-mechanical (THM) behavior of such materials. Many such models, possessing varying degrees of sophistication, have been developed since the 1970s. A detailed overview and critical assessment of such models has not, however, been presented. In order to address this shortcoming and to better understand the models' relative strengths and limitations, a comprehensive overview and assessment of constitutive models that have been developed to simulate the THM behavior of saturated cohesive soils is presented in this paper.

Keywords: thermo-hydro-mechanical; constitutive modeling; saturated cohesive soils; elastoplasticity; viscoplasticity; temperature



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1. Introduction

The need to mathematically model and numerically simulate complex multi-physics problems such as energy storage using geo-structures, radioactive waste disposal in deep underground or offshore repositories, and high-temperature oil recovery from reservoirs has led to the development of constitutive models that realistically simulate the macroscopically observed thermo-hydro-mechanical (THM) behavior of saturated cohesive soils. Indeed, many such models, possessing varying levels of sophistication, have been developed over the past fifty or so years to simulate the behavior of such geomaterials.

Missing from such development, however, has been a detailed overview and critical assessment of such constitutive models that would facilitate a better understanding of their relative strengths and limitations. To address this shortcoming, a comprehensive overview and assessment of the constitutive models that have been developed to simulate the THM behavior of saturated cohesive soils is given in this paper. This overview expands and updates the earlier compilation of Mashayekhi [1].

2. Methods

Information related to the modeling and simulation of the THM behavior of saturated cohesive soils has been available in the literature since the 1970's. Consequently, some older references related to this subject were only available in hardcopy form. More recent references were accessed from various electronic databases.

The review of pertinent subject matter included some doctoral dissertations and, to a lesser degree, masters theses. Since easy access to such documents is not necessarily afforded by all academic institutions, relatively few such dissertations and theses are cited herein; those cited do not constitute a complete collection of such references.

In preparing the current overview, only material related to saturated cohesive soils (i.e., clays and silty clays) was considered. Other fine-grained soils (e.g., silts and clayey silts) were not included in this overview. This was done for reasons of brevity and because less information is available regarding the THM behavior of such materials [2].

The present overview of constitutive models was organized in a systematic fashion. It begins with the discussion of relatively simple models that are incapable of simulating the inelastic, time-dependent THM behavior of saturated cohesive soils. This is followed by the discussion of progressively more robust elastoplastic models that account for the aforementioned inelastic behavior. The overview culminates with the discussion of the most functionally complex models which account for the time-dependent inelastic response of saturated cohesive soils.

3. General Observations

Ideally, the aforementioned constitutive models should be capable of realistically simulating the experimentally observed macroscopic THM behavior of saturated cohesive soils. Since a detailed overview of such behavior has been given in a recent publication [2], it is not discussed further herein.

When extending an isothermal model for saturated cohesive soils to include the effect of temperature, some or all of the following issues need to be considered: (1) the effect of temperature on mechanical parameters characterizing the soil (e.g., the elastic and plastic modulus, compression and swell/re-compression indices, etc.); (2) the effects of temperature and pressure on the thermoelastic parameters such as the coefficient of volumetric thermal expansion; (3) the interaction between the liquid and solid phases under non-isothermal conditions; (4) the effect of temperature on the volumetric or pore pressure and strength response of normally consolidated (NC) and overconsolidated (OC) soils; and (5) the effect of temperature on the time and rate-dependent behavior of the soils.

Classes and sub-classes of constitutive models developed to simulate the THM behavior of saturated cohesive soils are next discussed, beginning with the simplest formulations. Figure 1 presents an overview of these designations. Although it facilitates the present discussion, such a classification of constitutive models is, of course, not unique.

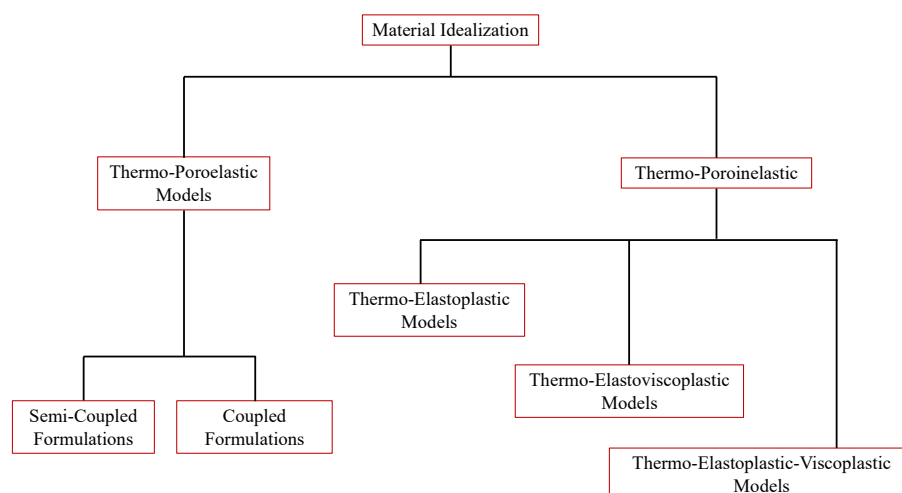


Figure 1. Overview of classes and sub-classes of constitutive models developed to simulate the THM behavior of saturated cohesive soils.

4. Thermo-Poroelastic Models

The first class of constitutive models developed to simulate the THM behavior of saturated cohesive soils consists of poroelastic formulations that have, in some manner, been extended to account for non-isothermal conditions. The governing equations for such models are derived based on the self-consistent theory of Biot [3,4].

4.1. Semi-Coupled Formulations

The simplest way in which to simulate such behavior is to combine the equations that idealize the soil skeleton as a linear thermo-poroelastic continuum with those governing the flow of fluid through a porous medium (e.g., Darcy's law). In such models, the mechanical contribution to the energy balance equations and convective terms are either partially accounted for or are neglected altogether. The temperature field is uncoupled from the determination of the displacements, strains, and excess pore pressures, although the increment in thermoelastic volumetric strain ($\Delta \varepsilon_{kk}^T$) is dependent upon changes in the temperature (ΔT) through the relation

$$\Delta \varepsilon_{kk}^T = \beta^d \Delta T, \quad (1)$$

where β^d is the so-called “drained” coefficient of isotropic volumetric thermal expansion.

Such semi-coupled thermo-poroelastic models have typically been used to simulate the thermally induced consolidation of saturated fine-grained soils [5–11].

The benefits of semi-coupled thermo-poroelastic models include simplicity in numerical implementation and relatively low computational costs. Their biggest drawbacks are the failure to account for the time and rate-dependent, inelastic nature of soils, and the fact that they only approximate the interaction between thermal and mechanical effects. Consequently, semi-coupled thermo-poroelastic models have a somewhat limited range of applicability.

4.2. Coupled Formulations

The next level of model complexity involves fully coupled thermo-poroelastic formulations. In such models, the soil is once again idealized as a linear thermo-poroelastic continuum. Now, however, a complete coupling between mechanical and thermal processes is assumed [12,13]. The importance of such coupling in simulating the process consolidation has been discussed by Bai and Abousleiman [14].

Most coupled thermo-poroelastic models acknowledge that THM analyses of soils are complicated by the particulate nature of these materials. To further complicate matters, the thermal properties of geomaterials are not as well known as those for other materials such as metals. Nevertheless, some formulations account for the effect of pressure and temperature on the density of the pore fluid and the solid phase [15]. For the fluid phase, the dynamic viscosity (μ^*) is known to decrease with increases in temperature. Although existing analytical expressions for μ^* apply for free water, it is less evident that they also apply to saturated clays, as clay–water interactions are known to be sensitive to changes in temperature [16]. In addition, clays exhibit an apparent or “structural” viscosity [17] that is larger close to the clay particles and decreases with distance from the particle surface. The intrinsic permeability is independent of the thermo-mechanical load path that is applied to a sample [18]. Since changes in the pore fluid density are typically negligible, increases in permeability of a sample with temperature are only attributed to the decrease in μ^* for the pore water.

Coupled thermo-poroelastic constitutive models, while accounting for the interaction between thermal and mechanical effects, still fail to account for the aforementioned time-

and rate-dependent, inelastic nature of soils. As such, these models tend to be only marginally better than semi-coupled thermo-poroelastic ones.

5. Thermo-Elastoplastic Models

The second general class of constitutive models developed to simulate the THM behavior of saturated cohesive soils consists of time and rate-independent elastoplastic formulations that have, in some way, been extended to account for non-isothermal conditions. Figure 2 presents an overview the classes and sub-classes of thermo-elastoplastic models developed to simulate the THM behavior of saturated cohesive soils.

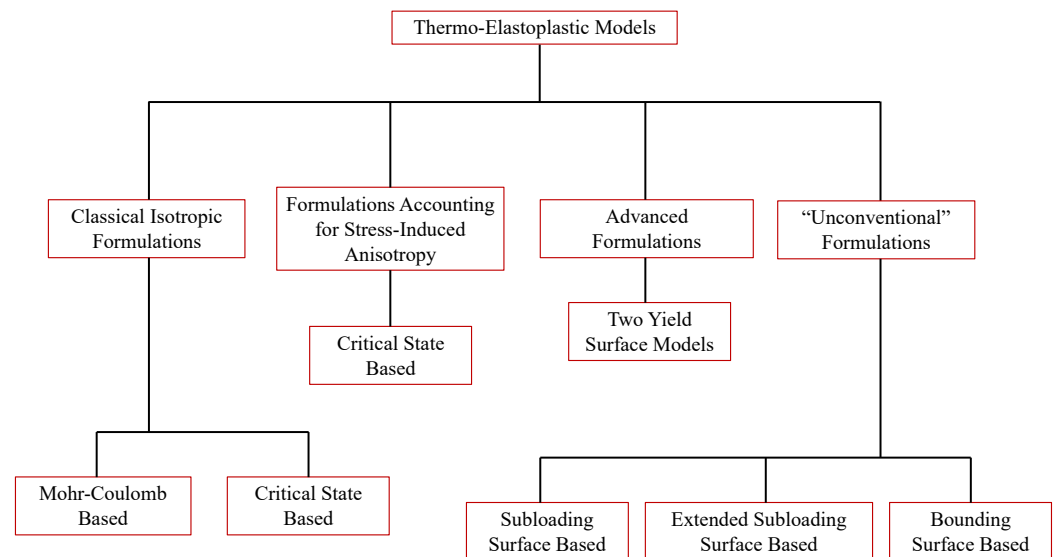


Figure 2. Overview of classes and sub-classes of thermo-elastoplastic models developed to simulate the THM behavior of saturated cohesive soils.

5.1. Classical Isotropic Formulations

Several thermo-elastoplastic models have been developed that overcome the shortcomings associated with semi- and fully coupled thermo-poroelastic formulations. In such models, the equations governing fluid flow through a porous medium are combined with constitutive equations that idealize the soil skeleton as a time and rate-independent elastoplastic continuum. The elastoplastic behavior is typically described either by a temperature-independent yield function, by the Mohr-Coulomb yield criterion, or by a yield surface based on Critical State soil mechanics [19] that is a function not only of effective mean normal stress (p') and the deviator stress (q), but also temperature (T). The preconsolidation pressure (p'_c), which describes the isotropic hardening in such models, is also typically a function of the temperature. Finally, varying degrees of coupling between mechanical and thermal effects are assumed.

Perhaps the earliest thermo-elastoplastic model for fully saturated soils was that developed by Lewis and Karahanoglu [20]. In this formulation, which was based on standard continuum mechanics, the assumption of thermal equilibrium between the solid and fluid phases was taken into account, while cross-transport phenomena were neglected. The elastoplastic behavior of the soil skeleton was assumed to be characterized either by a temperature-independent yield function, by the Mohr-Coulomb criterion, or by a critical state model. Lewis and Schrefler [15] subsequently developed a slightly modified form of this model.

The need to rationally analyze clay formulations subjected to thermal loading associated with nuclear waste disposal led to the development of several other classical

thermo-elastoplastic models. Initially, such models were extensions of Prager's thermoplasticity theory for metals [21]. For example, Borsetto et al. [22] presented a rather complex formulation that accounted for elastoplastic deformation, heating, and flow of groundwater. Subsequent models [23,24], that were based on the theory of mixtures and a yield surface based on Critical State soil mechanics [19], tended to be simpler. Besides being somewhat cumbersome to use, these models were also subject to criticisms concerning some of the postulates underlying mixture theories [25].

In an effort to improve upon earlier formulations, Hueckel and Borsetto [26] developed a model that employed a thermoplastic version of the Modified Cam Clay (MCC) model [27] and a non-associative flow rule. In this model, thermal effects were accounted for by varying the size of the elastic domain that is enclosed by the yield surface.

Modified versions of the Hueckel and Borsetto [26] model, in which relatively minor changes were made to the temperature dependence of p'_c (and thus the yield surface), were subsequently used to simulate the THM behavior of saturated clays [28–31]. Representative of such changes are the expressions proposed for p'_c , through which the functional form of the isothermal yield surface is rendered a function of temperature. For example, Hueckel and Pellegrini [29] proposed the following three-parameter expression:

$$p'_c = 2 \left\{ \bar{a} \exp \left\{ \frac{1}{\lambda - \kappa_T} [e_1 + (1 + e_0) \varepsilon_p^{T_p}] \right\} + a_1 \Delta T + a_2 \text{sign}(\Delta T) [\Delta T]^2 \right\}, \quad (2)$$

where λ is the negative of the slope of the virgin compression line in the space of void ratio (e) versus the natural logarithm of p' , $\varepsilon_p^{T_p}$ is the thermoplastic volumetric strain, κ_T is the temperature-dependent "logarithmic bulk modulus", e_0 is the initial value of e , e_1 is the value of e associated with $p' = 10$ kPa, and $\bar{a}$, a_1 , and a_2 are model parameters (the latter two of which are negative for clays). It is timely to note that the model of Hueckel and Pellegrini [29] was also noteworthy in that it accounted for the expansion of interstitial water in low porosity clays [32].

In a subsequent model, Del Olmo et al. [30] also proposed a similar three-parameter expression for p'_c ; i.e.,

$$p'_c = p'_{c0} \exp \left\{ \frac{1}{\lambda - \kappa} \left[(1 - a_0 \Delta T) (1 + e_0) \varepsilon^{ir} \right] \right\} + 2 \left[a_1 \Delta T + a_2 (\Delta T)^2 \right], \quad (3)$$

where p'_{c0} is the initial value of p'_c , λ and e_0 are as previously defined, κ is the negative of the slope of the swell/re-compression line in $e - \ln p'$ space, and a_0 , a_1 , and a_2 are model parameters (with a_1 and a_2 being negative).

Hueckel et al. [33] extended the earlier model of Hueckel and Borsetto [26] to include the effect of temperature on the internal friction angle. In this model, the size and shape of yield surface was assumed to be a function of temperature. François et al. [34] and Hueckel [35] subsequently used this extended model to address the role played by the model parameters when reaching failure.

Additional thermo-elastoplastic models were developed by Laloui and co-workers [36–38]. Each of these models introduced modifications to the hardening rule used for the yield surface. In a related development, Laloui and Cekerevac [39] combined the isotropic thermo-elastoplastic model of Laloui and Cekerevac [37] with a deviatoric cyclic mechanism so as to simulate the cyclic behavior of cohesive soils under non-isothermal conditions.

A thermo-elastoplastic model that combined the MCC model with a temperature-dependent associative flow rule was developed by Hamidi et al. [40]. This model represented an improved version of the earlier work of Hamidi and Khazaei [41], which was more focused on the reduction of void ratio under non-isothermal conditions. The model of Hamidi et al. [40] included two additional model parameters that directly control the

shape of the yield surface. Since an associative flow rule is used, these parameters also influence the plastic strain rate. This model also accounted for the effect of temperature on p'_c .

Classical thermo-elastoplastic models represent a relatively refined approach for simulating the THM response of saturated cohesive soils. In general, such models adequately simulate the thermo-elastoplastic behavior of NC and lightly OC cohesive soils. Inherent in such models is an abrupt change from elastic to elastoplastic response once the current stress state reaches the yield surface. This is, however, counter to experimental results [27,42] which indicate that loaded cohesive soils continuously develop both elastic and inelastic deformations without a distinct yield state. Such an abrupt change in response manifests itself in the inability to accurately simulate the behavior of moderately to heavily OC cohesive soils. The discussion of remedies proposed to alleviate such abrupt changes is deferred until Section 5.4.

5.2. Formulations Accounting for Stress-Induced Anisotropy

The effect of temperature on stress-induced anisotropy was first investigated by Hueckel and Pellegrini [29] and then subsequently by Hueckel and Pellegrini [43]. In the latter model, the difference between the isothermal mechanical and thermo-mechanical anisotropy was interpreted through the deformation history of the clay microstructure. The model of Hueckel and Pellegrini [43] combined the rotational hardening rule of Hueckel et al. [44] with the aforementioned model of Hueckel and Borsetto [26] and accounted for elastoplastic coupling. The inherent anisotropy was accounted for by the initial rotation of the yield surface from the p' axis. The isotropic hardening of the yield surface was assumed to depend on the plastic volumetric strain and on temperature. By contrast, the rotational hardening of this surface was assumed to be a function of the deviatoric and volumetric plastic volumetric strain but to be independent of temperature.

Rojas and Garnica [45] also developed a critical state based elastoplastic model for soils that accounted for inherent and stress-induced anisotropy. This model differed from the aforementioned model of Hueckel and Pellegrini [29] in that it was developed based on two general equations: the free energy and a dissipation function, which represent the elastic and plastic behavior, respectively. To account for stress-induced anisotropy, the yield surface was assumed to undergo rotational hardening. Plastic strain increments were computed assuming a non-associative flow rule.

The model of Hueckel and Pellegrini [43] was subsequently used by Wang et al. [46], who incorporated temperature effects in an isotropic hardening variable, and temperature-dependent scaling of the yield surface through the thermal dependence of the preconsolidation pressure. Despite the anisotropic nature of the model, rotational hardening was not employed. Consequently, the simulation of thermal and stress-induced anisotropy was precluded, as the inclination (in stress space) of the yield surface remained constant during both mechanical and thermal loading.

More recently, Shah et al. [47] extended the SANICLAY model of Dafalias et al. [48] to account for non-isothermal conditions. The extended model assumed a temperature-dependent evolution of stress anisotropy and isotropic hardening. Thermoelastic behavior was modeled using temperature-independent linear elasticity.

In summarizing this sub-section, although thermo-elastoplastic models that account for stress-induced anisotropy were more general than isotropic ones, relatively little progress has been made in understanding the effect of temperature on stress-induced anisotropy. This is largely due to the lack of suitable experimental results from studies that investigate the effect of temperature on stress-induced anisotropy.

5.3. Advanced Formulations

In order to overcome some of the aforementioned shortcomings associated with the classical thermo-elastoplastic models, a series of more advanced formulations was developed in which temperature is defined as an additional “mechanism” that is typically active within a second yield surface. This additional surface is typically defined in accordance with the MCC model [27] and facilitates the simulation of OC cohesive soils in that plastic strains are predicted for stress states lying in between an outer yield surface and the inner surface.

In the first such advanced thermo-elastoplastic model, Robinet et al. [49] defined a second (“thermal softening”) yield surface within the usual “mechanical” surface. The second yield surface was defined by a suitable “threshold” temperature (i.e., the highest temperature experienced by a soil), above which plastic strains are activated. This causes the surface to contract and produces thermally-induced volumetric strains that are a function of the overconsolidation ratio (OCR). By contrast to Equations (2) and (3), Robinet et al. [49] proposed the following two-parameter, double exponential expression for p'_c :

$$p'_c = p'_{c0} \exp(\beta_m \varepsilon_{mv}^p) \exp(\beta_T \varepsilon_{Tv}^p), \quad (4)$$

where p'_{c0} is as previously defined, β_m and β_T are coefficients of elastoplastic and thermo-plastic compressibility, respectively, and ε_{mv}^p and ε_{Tv}^p are the mechanical and thermomechanical plastic volumetric strains, respectively.

Subsequent advanced thermo-elastoplastic models [50,51] also used a two yield surface formulation consisting of a “thermal yield” and a “loading yield” surface. These models focused on the volumetric response, as opposed to the thermally-induced stress-strain or strength behavior, of saturated cohesive soils. One of these models [51] was extended by employing a non-associative flow rule and including temperature-dependent distortional hardening to more accurately simulate the response under axisymmetric triaxial (two-invariant) stress states [52]. More recently, advanced thermo-elastoplastic models have been proposed in which the two yield surfaces are coupled via induced thermal and mechanical plastic volumetric strains [53].

Golchin et al. [54] proposed a hyperelastic rate-independent thermo-mechanical model that ensures thermodynamic consistency by deriving all constitutive equations from two potential functions; i.e., a Gibbs-type free energy potential and a dissipation potential. This ensures energy consistency and incorporates both plastic hardening and thermal softening. This model employed a temperature-dependent preconsolidation pressure with a stress ratio parameter.

In a somewhat related development, Zhao et al. [55] used non-equilibrium thermodynamics to model creep and cyclic behavior, with the goal of describing the thermal creep, strain accumulation, and cyclic stabilization of both clays and sands. This formulation, which is an extension of the so-called TTS model [56] based on the theory of non-equilibrium thermodynamics [57], uses a hyperelastic relation and considers time-dependent inelastic behavior as energy dissipation determined by Onsager’s reciprocal principle. The granular entropy is used instead of plastic strain to describe the irreversible process.

Although advanced formulations improved on certain aspects associated with classical thermo-elastoplastic models, they still exhibit the aforementioned abrupt change from elastic to elastoplastic response. To overcome such a shortcoming requires the use of so-called “unconventional” models, which are discussed next.

5.4. “Unconventional” Formulations

The desire to circumvent the aforementioned sharp delineation between elastic and inelastic response led to the development of “unconventional” isothermal elastoplastic con-

stitutive models that were based on the concept of a subloading surface [58], an extended subloading surface [59], or a bounding surface in stress space [60]. The essence of such models is the notion that inelastic deformations can occur for stress states either within or on an outer loading or bounding surface. Thus, unlike classical elastoplastic formulations, inelastic states are not restricted only to those lying on a yield surface, which represents the boundary of the elastic domain. Consequently, unconventional models more accurately simulate the response of moderately to heavily OC cohesive soils, without compromising the accurate simulation of NC and lightly OC soils. Not surprisingly, several unconventional thermo-elastoplastic bounding surface models, as well as sundry double-hardening models, were subsequently developed.

An example of such an unconventional thermo-elastoplastic model is the double-hardening formulation developed by Liu and Xing [61]. In this model, the hardening is characterized by two separate hardening parameters.

This was followed by the development of models based on the subloading concept [62,63]. The model of Zhang et al. [62] is noteworthy in that it added only one parameter, a thermal expansion coefficient, to an isothermal formulation. In this model, the concept of a temperature-deduced equivalent stress is used in conjunction with a subloading surface proposed by Hashiguchi and Ueno [58] for OC states. The temperature-deduced stress is the equivalent (imaginary) stress that produces the same elastic strain during heating at constant isotropic stress.

In a related development, Yao and Zhou [64] extended the isothermal unified hardening (UH) model of Yao et al. [65] to account for non-isothermal conditions. The isothermal UH model is based on the subloading concept and uses one more parameter than the MCC model.

A thermo-elastoplastic model based on the concept of a bounding surface [66] in stress invariant space was developed by Zhou and Ng [67]. In this model, the size of the bounding surface was assumed to be a function of temperature through the preconsolidation pressure. The shape of this surface also varied with temperature.

Hong et al. [68] compared the performance of the thermo-elastoplastic models of Cui et al. [50], Abuel-Naga et al. [51], and Laloui and François [38]. Based on the findings of this comparison, Hong et al. [69] proposed an improved thermo-elastoplastic bounding surface model. This model combined the two-yield surface model of Cui et al. [50] with the bounding surface concept in order to improve the simulated transition from elastic to elastoplastic states.

In a related development, Cheng et al. [70] proposed a two-surface model to simulate the thermal cyclic behavior of saturated cohesive soils. In this model, which was based on the aforementioned model of Hong et al. [69], a law was introduced in order to unify the loading and thermal yield limits. This law was combined with isotropic and progressive hardening laws to more accurately simulate thermal cycling response. The model was extended to undrained condition by applying the approach of Coussy [71] to the elastoplastic state.

More recently, Mašín [72] developed a hypoplastic THM model that extended a double-structure model [73] that separated the hydro- and mechanical responses at the macro and microstructural levels [74]. In this formulation, the microstructural level is considered fully saturated, with mechanical response governed by Terzaghi's effective stress. The response at each structure level is coupled through a double structure function. The model of Mašín [72] assumed the position and slope of the isotropic compression lines to be a function of temperature, and included a suitable coupling function with a temperature-dependent asymptotic state boundary.

Zhou et al. [75] used a double-surface model to simulate thermal cycling of saturated cohesive soils. The model included a bounding surface, which isotropically hardens via preconsolidation pressure, and a memory surface with a similar shape to the bounding surface. The memory surface was assumed to pass through the maximum stress state experienced by the soil. The model integrated thermal effects into the yield function, the definition of compressibility, and into the expression for the plastic modulus.

Using the model of Zhou et al. [75] as a basis, Ng et al. [76] proposed a “unified” bounding surface based model for non-isothermal conditions that was purported to simulate the behavior of both sands and clays. The model employed three surfaces, namely a loading surface, a bounding surface, and a memory surface. The loading surface represents the current stress state, the bounding surface defines the ultimate yield limit, and the memory surface tracks the historical maximum stress state. The memory surface and non-linear elasticity together capture hysteresis loops and accumulation of plastic strain during thermal and mechanical cycles. The model of Ng et al. [76] included a new thermal softening relationship, a temperature-dependent critical state line and plastic modulus, in conjunction with a state-dependent dilatancy (for improved simulation of the behavior of sands and highly overconsolidated clays).

Since they eliminate the aforementioned abrupt change from elastic to elastoplastic response, “unconventional” models represent an improvement over both classical and advanced thermo-elastoplastic models. The only shortcomings of such models is that they do not account for the time and rate-dependent behavior of saturated cohesive soils. To account for such behavior requires the use of thermo-elastoviscoplastic models, which are discussed next.

6. Thermo-Elastoviscoplastic Models

None of the aforementioned thermo-elastoplastic models can account for the time- and rate-dependent THM behavior of saturated cohesive soils. In the past, such an omission was understandable, as accounting for such behavior in a thermal-mechanical formulation was a fairly challenging endeavor. Due to the more recent advances in isothermal models [77], the consideration of such behavior in the THM simulation of such materials is now better understood. Consequently the consideration of time- and/or rate-dependent behavior in THM simulations of saturated cohesive soils can now conceivably be realized in a practical manner.

Figure 3 presents an overview of the classes and sub-classes of thermo-elastoviscoplastic models developed to simulate the THM behavior of saturated cohesive soils.

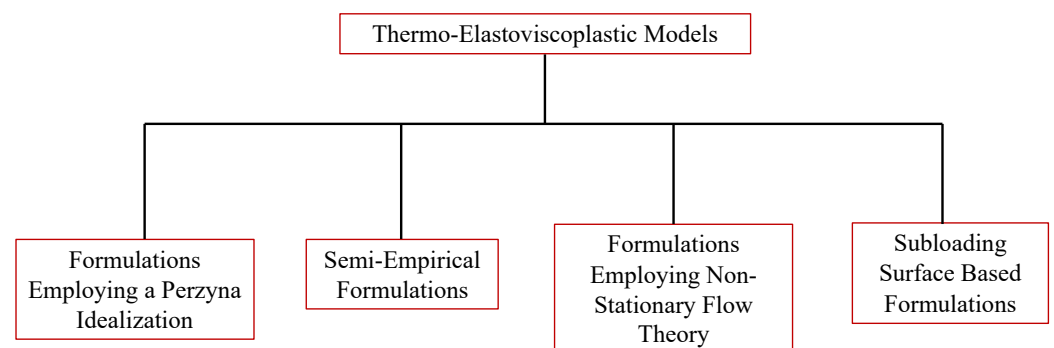


Figure 3. Overview of classes and sub-classes of thermo-elastoviscoplastic models developed to simulate the THM behavior of saturated cohesive soils.

6.1. Formulations Employing a Perzyna Idealization

To model the rate-dependent behavior of metals, Perzyna [78,79] developed the so-called “overstress” model, which is considered to be a generalization of the earlier models of Hohenemser [80] and Prager [81]. In particular, the original yield condition in these models was replaced by the more general yield condition associated with the concept of the overstress.

The first thermo-elastoviscoplastic model based on a Perzyna idealization appears to be that proposed by Modaressi and Laloui [82]. This model combined a bounding surface formulation with a Perzyna [78] viscoplastic idealization, thus essentially extending the earlier isothermal model of Kaliakin and Dafalias [83]. It is timely to note that the aforementioned thermo-elastoplastic models of Laloui et al. [36], Laloui and Cekerevac [37], and Laloui and François [38] were essentially simplified time- and rate-independent forms of the Modaressi and Laloui [82] model.

In the improved thermo-elastoviscoplastic model subsequently proposed by Laloui et al. [84], the thermo-elastoplastic formulation of Laloui and Cekerevac [37] was combined with the unique effective stress–strain rate relationship proposed by Leroueil et al. [85]. At about the same time, Laloui and Cekerevac [86] improved the functional form describing the temperature dependence of the preconsolidation pressure (and thus the definition of the yield surface) associated with the aforementioned model of Modaressi and Laloui [82].

Raude et al. [87] presented a unified thermoplastic-viscoplastic model to describe the rate-independent and time-dependent response of clayey rocks under non-isothermal conditions. In this model, the total infinitesimal strain rate tensor was decomposed into the sum of two elastic components and two inelastic components. The two elastic components included a mechanical and a thermo-elastic strain rate. The two inelastic components consisted of a plastic strain rate based on the classical isothermal theory of plasticity and a viscoplastic strain rate based on a Perzyna [78] viscoplastic idealization. In computing the inelastic strain increments, a non-associative flow rule was used. In this model, the effect of temperature was incorporated in temperature-dependent yield surfaces and fluidity coefficients. The effect of temperature on fluidity coefficients was adopted to simulate increasing creep rate with temperature.

6.2. Semi-Empirical Formulations

Another subset of classical thermo-elastoviscoplastic models includes some semi-empirical (graphical) formulations. Kurz et al. [88] proposed such a model, which was based on concepts introduced by Yin et al. [89] and on the earlier viscoplastic formulation of Kelln et al. [90]. The model extended the MCC model to account for temperature effects on the creep rate. The infinitesimal strain rate tensor was decomposed into the sum of a recoverable (elastic) and non-recoverable (viscoplastic) parts. The elastic response was assumed to be independent of temperature. The functional form for temperature dependence of the creep rate coefficient was based on the work of Fox and Edil [91]. The general idea of including temperature effects in the creep rate coefficient was rooted in the similarity between volumetric response of cohesive soils under non-isothermal time-independent loading and under isothermal constant stress creep. Kurz et al. [88] concluded that the slope (κ) of the swell/rebound curve was time-dependent but temperature independent. The latter conclusion differs from the assumptions made by Graham et al. [92], who decomposed the plastic strain rate into mechanical and thermal contributions, included the temperature dependency of the yield surface through the preconsolidation pressure, and assumed a temperature-dependent critical state line in the space of void ratio (e) versus the natural logarithm of the mean normal effective stress (p').

The related thermo-elastoviscoplastic model developed by Hamidi [93] combined the effects of structure, strain rate, and temperature on the behavior of cohesive soils. The effect of structure was incorporated into through a modified version of the critical state based model of Liu and Carter [94]. To account for the strain rate effects, the aforementioned viscoplastic model of Kelln et al. [90] was used. Finally, the effect of temperature was included in the formulation by adopting the approach of Hamidi and Khazaei [41] according to which the normally consolidated line and the preconsolidation pressure are a function of temperature. The model of Hamidi [93] did not, however, assume any significant coupling between the soil structure, strain rate and temperature.

Fathalikhani et al. [95] subsequently modified the model of Kurz et al. [88] to improve predictions under thermal consolidation and cyclic loading. This was done by adjusting the volumetric formulation and separating elastic and viscoplastic paths during unloading-reloading through the addition of a term to the current specific volume to account for thermally induced viscous effects. These modifications were mainly to improve the simulations for thermal consolidation and unloading-reloading cycles. Similar to the model of Kurz et al. [88], this model considers temperature independent elastic response.

Chen and Yin [96] extended the one-dimensional EVP model [97,98] to account for thermal creep. The thermal strain was divided into elastic and irreversible parts. This model introduced temperature as a state variable and modeled the irreversible thermal creep using logarithmic formulations similar to mechanical creep.

Chen et al. [99] subsequently generalized the model of Chen and Yin [96] into a three-dimensional model with anisotropic critical state theory and time-dependent creep behavior. The model introduced a temperature-dependent yield surface via temperature-dependent preconsolidation pressure. The model used the Equivalent Time Concept [98] model to account for time-dependent creep behavior, the Perzyna overstress theory to define the visco-plastic strain rate, and the anisotropic critical state framework of Wheeler et al. [100]. The thermal dependency of the creep coefficient was also modeled using a temperature-dependent creep coefficient and an additional temperature-dependent power law function.

In a related development, Cheng and Yin [101] adopted a fractional-order viscoplasticity theory with temperature-dependent viscosity. In this formulation, progressive hardening and a smooth transition from elastic to viscoplastic behavior under non-isothermal conditions was introduced. The thermoplastic behavior was modeled via a temperature-dependent preconsolidation pressure in conjunction with the thermal proportional dependency law. The viscoplastic portion of the formulation was defined using a temperature-dependent creep coefficient.

6.3. Formulations Employing Non-Stationary Flow Surface Theory

Qiao and Ding [102] developed a thermo-elastoviscoplastic model that used the MCC yield surface with an associative flow rule and incorporated an extended version of the non-stationary flow surface theory of Qiao et al. [103]. In this theory, the yield surface flows with viscoplastic strain, governed by a time-independent hardening parameter (the viscoplastic strain) and a time factor (the viscoplastic strain rate). The viscoplastic strain rate is assumed to affect the preconsolidation pressure, which increases with increases in the viscoplastic strain rate. In addition, a threshold viscoplastic strain rate is assumed below which the strain rate dependency vanishes. The temperature dependence of the preconsolidation pressure in the model of Qiao and Ding [102] was defined in a manner similar to that used by Laloui and Cekerevac [37] and by Laloui and François [38].

The non-stationary flow surface theory of Qiao et al. [103] is noteworthy in that it represents an alternative to a Perzyna-type viscoplastic formulation [78]. Consequently, it overcomes some potential shortcomings associated with such a formulation [104–106].

The subsequent thermo-elastoviscoplastic model of Qiao and Ding [102] was based on the premise that strain rate and temperature effects influence the behavior of soils in a similar manner. Both are thus considered amendable for study through application of the theory of absolute reaction rates (rate process theory) [107,108]. This theory assumes that (1) the relative movement of “flow units” is constrained by energy barriers, (2) the flow units vibrate in their location at a free frequency, and (3) the new position of the flow units can be achieved by receiving sufficient activation energy (i.e., mechanical or thermal energy) to overcome the energy barriers. Since heating contributes to the activation energy, it enhances thermal vibration and weakens particle bonds.

6.4. Formulations Based on the Concept of a Subloading Surface

Another noteworthy thermo-elastoviscoplastic model is that developed by Maranha et al. [109]. This model, which was based on the purely viscoplastic model of Maranha et al. [110], extended the viscoplastic subloading soil formulation of Hashiguchi and Okayasu [111] to non-isothermal conditions by including reversible thermal expansion, thermal isotropic hardening, evolution of the center of homology, a temperature dependent viscosity function, and preconsolidation pressure for inelastic response.

Cheng et al. [112] extended a non-isothermal fractional order, two surface bounding/subloading, viscoplastic model for stiff clays. The model was developed by generalizing the plastic strain rate and modifying the isotach viscosity proposed by Cheng and Yin [101]. In this model, the total infinitesimal strain rate increment was decomposed into the sum of the mechanically-induced elastic, temperature-induced elastic, and viscoplastic parts. The yield surface was temperature dependent, with both isotropic and progressive hardening to account for thermal collapse, strain rate effects, and smooth transitions from elastic to viscoplastic behavior. The exponential relation for the variation in the creep coefficient with temperature as proposed by Fox and Edil [91] was likewise used in the formulation.

6.5. Other Formulations

Finally, in the model developed by Kong [113], the evolution of the preconsolidation pressure was realized using a time and temperature dependent hardening law. In this model, the critical state ratio was assumed to be temperature dependent, but the viscous response was considered to be independent of temperature.

In summarizing this section, it is timely to point out that, since they account for time and rate-effects, thermo-elastoviscoplastic models extended the simulative capabilities of thermo-elastoplastic models. The major drawback associated with such formulations is the need to determine values for model parameters that may be functions of both time and temperature. The availability of experimental results required to determine such values is often difficult to ensure.

7. Thermo-Elastoplastic-Viscoplastic Formulations

Models in this category are based on the work of Dafalias [114–116], who presented the theoretical formulation for an isothermal time-dependent version of the bounding surface model for isotropic cohesive soils. In this formulation, the total infinitesimal strain rate is decomposed into the sum an elastic, plastic and viscoplastic rate. Coupling between the latter two rates is realized though a suitably defined scalar loading index [114,116]. Dafalias' formulation was subsequently refined and implemented numerically [117], and then formally presented [83] and verified [118].

A coupled thermo-elastoplastic-viscoplastic formulation that was based on the concept of a bounding surface in stress invariant space was described by Kaliakin [119]. Lack of

suitable experimental data precluded further development of this model at the time of its inception. Subsequent experimental activity has, however, largely alleviated such limitations [2].

The most robust aspects of isothermal bounding surface models for saturated cohesive soils have been synthesized through the development of the Generalized Bounding Surface Model (GBSM) [120]. In its most general form, the GBSM is a fully three-dimensional, time-dependent model that accounts for both inherent and stress induced anisotropy with the microfabric-inspired rotational hardening rule. To better simulate the behavior of cohesive soils exhibiting softening, the model employs a non-associative flow rule. The GBSM improves upon many aspects of previous bounding surface models and expands the scope of the formulation to facilitate its specialization for THM simulations. A limited number of such simulations, based on the work of Mashayeki [1], have been realized [120].

8. Trends in Characterizing the Elastic Response

Having presented an overview of classes of THM constitutive models for saturated cohesive soils, it is timely to next investigate specific elastic idealizations that have been used in such models. The present discussion is divided into two parts. The first part presents past functional forms that have been proposed for the “drained” coefficient of thermal volume expansion (β^d). The second part summarizes function forms that have been proposed for the elastic moduli associated with THM models for saturated cohesive soils. Such models have almost exclusively assumed elastic isotropy. The two parameters associated with isotropic elastic material idealizations are typically the bulk modulus (K) and either the shear modulus (G) or Poisson’s ratio (ν). With minor exceptions, these parameters are assumed to be independent of the plastic internal variables that are used to quantify the hardening or softening of the cohesive soil.

8.1. Drained Coefficient of Volumetric Thermal Expansion

Many analytical expressions, possessing different levels of sophistication, that have been proposed to describe β^d . In general, these expressions differ in the functional representation of the coefficient that is assumed and its specific dependency on temperature, stress, and stress history [1].

In general, the rate of thermoelastic volumetric strain ($\dot{\epsilon}_{kk}^T$) is related to the temperature increment ($\dot{T}$) in the following manner (recall Equation (1)):

$$\dot{\epsilon}_{kk}^T = \beta^d \dot{T}, \quad (5)$$

where β^d is again the “drained” coefficient of isotropic volumetric thermal expansion. For a given temperature increment, this coefficient is used to produce thermoelastic strain with no change in stress.

In a general sense, Palciauskas and Domenico [121] defined β^d as the fractional increase in volume of a porous medium at constant stress and fluid pressure as follows:

$$\beta^d = (1 - n)\beta^s + n\beta^p = \beta^s + n(\beta^p - \beta^s), \quad (6)$$

where n is the porosity, and β^s and β^p are the coefficients of volumetric thermal expansion for the solid phase and pore space, respectively. The latter accounts for all effects arising from the internal pore geometry and stress fields that develop in response to changes in temperature. In practice, β^p also contains nonreversible phenomena such as microfracture generation due to the differential thermal expansion of the minerals within rocks or microstructure changes in soils.

In the first robust thermo-elastoplastic model, Hueckel and Borsetto [26] proposed the following four-parameter expression for β^d :

$$\beta^d(\mathcal{T}, p') = \alpha_0^* + 2\alpha_2\mathcal{T} + (\alpha_1 + 2\alpha_3\mathcal{T}) \ln\left(\frac{p'}{p'_g}\right), \quad (7)$$

where p' is again the mean normal effective stress, α_0^* , α_1 , α_2 and α_3 are model parameters, and p'_g is the isotropic component of geostatic stress corresponding to a state of zero elastic strain.

In their model, Robinet et al. [49] proposed the following expression for the “drained” coefficient of volumetric thermal expansion:

$$\beta^d(\mathcal{T}, p') = n\alpha^w + (1 - n)\alpha^s, \quad (8)$$

where α^s is the coefficient of thermal expansion for the soil skeleton, α^w is the coefficient of thermal expansion for water, and n is the porosity. Following the approach used by Baldi et al. [32], Robinet et al. [49] related α^w to $\mathcal{T}$ and p' through the six-parameter expression proposed by Juza [122]; i.e.,

$$\alpha^w(\mathcal{T}, p') = \alpha_0 + (\alpha_1 + \beta_1\mathcal{T}) \ln(c p') + (\alpha_2 + \beta_2\mathcal{T}) [\ln(c p')]^2, \quad (9)$$

where the model parameters have the following values: $\alpha_0 = (4.505 \times 10^{-4}) \text{ }^\circ\text{C}^{-1}$, $\alpha_1 = (9.156 \times 10^{-5}) \text{ }^\circ\text{C}^{-1}$, $\alpha_2 = (6.381 \times 10^{-6}) \text{ }^\circ\text{C}^{-1}$, $\beta_1 = -(1.2 \times 10^{-6}) \text{ }^\circ\text{C}^{-2}$, $\beta_2 = -(5.766 \times 10^{-6}) \text{ }^\circ\text{C}^{-2}$, and $c = 0.15 \text{ kbar}^{-1}$.

In several models developed by Laloui and co-workers [36–38,82], the isotropic thermal expansion coefficient of the solid skeleton (β'_s) was assumed to vary with temperature and pressure according to the following two-parameter expression:

$$\beta'_s = (\beta'_{s0} + \zeta\mathcal{T})\xi, \quad (10)$$

where ξ is the ratio between the critical state pressure for the initial state (p_{c0}) and the value of p' at ambient temperature. In Equation (10), β'_{s0} is the volumetric thermal expansion coefficient at ambient or reference temperature and ζ (units of $^\circ\text{C}^{-2}$) corresponds to the slope of the variation in β'_s with respect to present temperature $\mathcal{T}$ at $\xi = 1$. Laloui [123] suggested using a value of $\zeta = \beta'_{s0}/100 \text{ }^\circ\text{C}$.

In a similar manner, Liu and Xing [61] assumed the isotropic thermal expansion coefficient of the solid skeleton to be a function of the OCR according to the following two-parameter expression:

$$\beta'_s = \beta'_{s0} \left[3.5 - 0.2(\text{OCR}) \right] (1 + \varsigma\mathcal{T})(\text{OCR}), \quad (11)$$

where β'_{s0} is the volumetric thermal expansion coefficient at a reference temperature and ς corresponds to the slope of the variation in β'_s with respect to present temperature $\mathcal{T}$.

Cui et al. [50] assumed a one-parameter relation between thermoelastic volumetric strain and the increment in temperature. In this relation, $\beta^d = \alpha_2$, where α_2 is similar to the parameter used in Equation (7). Similar to Cui et al. [50], Zhang et al. [62] adopted a one-parameter thermal expansion coefficient.

Yao and Zhou [64], assumed a slightly more general relation between thermoelastic volumetric strain and the increment in temperature than the one-parameter expressions proposed by Cui et al. [50] and Zhang et al. [62]. In this relation, β^d was assumed to be

a function of stress and temperature. Having presented this relation, Yao and Zhou [64] nevertheless used a constant value for β^d .

In a somewhat similar approach, Hamidi et al. [40] assumed β^d to be a general function of temperature, soil mineralogy, and pressure. Nevertheless, citing the work of Bolzon and Schrefler [124], they also assumed a constant value of $\beta^d = 10^{-5} \text{ }^\circ\text{C}$.

In the semi-empirical (graphical) approach proposed by Graham et al. [92], the increment in thermoelastic volumetric strain is given by

$$\dot{\epsilon}_{kk}^{\mathcal{T}e} = \frac{\kappa_2 - \kappa_1}{1 + e_0} \ln \left\{ p'_{cons} \left[\frac{(p'_{c1})^{\kappa_1}}{(p'_{c2})^{\kappa_2}} \right]^{\frac{1}{\kappa_2 - \kappa_1}} \right\} = \frac{\kappa_2 - \kappa_1}{1 + e_0} \ln \left(\frac{OCR_{\mathcal{T}_0}}{OCR_{\mathcal{T}}} \right), \quad (12)$$

where κ_1 and κ_2 represent the slopes of the swelling line (on a plot of void ratio versus $\ln p'$) at temperatures $\mathcal{T}_1$ and $\mathcal{T}_2$, respectively, e_0 is the initial value of the void ratio, p'_{cons} is the pre-consolidation pressure, and p'_{c1} and p'_{c2} are values of p'_{cons} associated with temperatures $\mathcal{T}_1$ and $\mathcal{T}_2$, respectively.

In a somewhat different approach, Abuel-Naga et al. [51] expressed the incremental elastic volumetric strain as follows:

$$\dot{\epsilon}_{kk}^{\mathcal{T}e} = \beta^d \left(\frac{\dot{\mathcal{T}}}{\mathcal{T}} \right), \quad (13)$$

where β^d is determined using a back calculation technique. Since β^d also depends on the stress level, Abuel-Naga et al. [51] proposed the following relation for this task:

$$\frac{d\beta^d}{\beta^d} = b \frac{dp'}{p'}, \quad (14)$$

where the model parameter b is a function of microscopic and mineralogical constitution of the cohesive soil.

A somewhat different one-parameter expression for β^d was assumed by Zhou and Ng [67], i.e.,

$$\beta^d = -\frac{\alpha_s}{1 + e}, \quad (15)$$

where α_s is the constant isotropic thermal expansion coefficient of the soil skeleton, and e is the void ratio.

Constant idealizations for β^d were also assumed by Hong et al. [69] and by Qiao and Ding [102]. In the former case, the rationale for using such a constant value was the desire to keep the thermoelastic material idealization simple and load path independent.

More recently, Maranha et al. [109] assumed the following functional form for β^d :

$$\beta = \beta_0 + (\beta_{max} - \beta_0) \exp [-\xi_{\mathcal{T}} p'], \quad (16)$$

where β_0 is the average volumetric coefficient of thermal expansion for the solid phase, β_{max} is the coefficient of thermal expansion associated with $p' = 0$, and $\xi_{\mathcal{T}}$ is a material constant with units of stress^{-1} .

In summarizing this sub-section, it is interesting to note the trend followed in the functional forms proposed for β^d . Initially, some rather complex six- and four-parameter forms were proposed. This was followed by the adoption of simpler, two-parameter functional forms. More recently, many of the constitutive models assume a one-parameter functional form, some of which assume β^d to be constant. The trends described herein regarding β^d illustrate the well-known trade-off between predictive capabilities due to

model sophistication and practicality in determining the values of parameters associated with a particular model.

8.2. Elastic Bulk Modulus

In formulations based on Critical State soil mechanics [19], the elastic bulk modulus (K) is a nonlinear function of the mean normal effective stress (p'), i.e.,

$$K = \frac{(1 + e_0)}{\kappa} p', \quad (17)$$

where the critical state parameter κ is equal to the negative of the slope of the swell/recompression lines in a plot of void ratio (e) versus the natural logarithm of p' , and e_0 represents the initial value of the total void ratio (e) corresponding to the reference configuration with respect to which engineering strains are measured; for natural strains, e_0 would be the current total void ratio.

In many THM models for saturated cohesive soils [36–38,49–52,64,67,69,82,87,102], the variation in K with temperature is related to the hysteresis by thermal cycles or viscoplastic strain, which is acknowledged but neglected, or is assumed to be negligible. In constitutive models assuming a temperature-independent elastic bulk modulus, the functional form for K is then either given by Equation (17) or by the following two-parameter functional form:

$$K = K_{ref} \left(\frac{p'}{p'_{ref}} \right)^m, \quad (18)$$

where K_{ref} is the bulk elastic modulus at the reference pressure p'_{ref} (e.g., the atmospheric pressure) and m is the non-linear elasticity exponent [82]. In the model of Liu and Xing [61], the following empirical expression was adopted for m :

$$m = 0.028(\text{OCR})^2 - 0.51(\text{OCR}) + 2.07, \quad (19)$$

In a rather limited number of models, K has been assumed to be a function of temperature. For example, Hueckel and Borsetto [26] introduced temperature into Equation (17) by replacing κ with the following three-parameter empirical expression:

$$\kappa_T = \left[\frac{\kappa}{1 + e_0} + (\alpha_1 + \alpha_3 \Delta T) \Delta T \right] (1 + e_0), \quad (20)$$

where α_1 and α_3 are the model parameters (recall Equation (7)) and κ is as previously defined.

Graham et al. [92] adopted the following functional form, which also involved a reduction of κ with $\mathcal{T}$:

$$\kappa = \kappa_0 \left[1 + B \ln \left(\frac{\mathcal{T}}{\mathcal{T}_0} \right) \right], \quad (21)$$

where B is model parameter and κ_0 is the value of κ at reference temperature $\mathcal{T}_0$.

In a related development, Hamidi et al. [40] assumed that K varied according to a functional form very similar to Equation (18), i.e.,

$$K = K_0 \left(\frac{p'}{p'_0} \right)^a, \quad (22)$$

where p'_0 is the effective pre-consolidation pressure, and the parameter a allows for a non-linear dependence of K on p' . In particular,

$$K_0 = \frac{(1 + e_0)p'_0}{\kappa_T}, \quad (23)$$

where $\kappa_{\mathcal{T}}$ is essentially equal to κ from Equation (21).

In summarizing this sub-section, in many THM models for saturated cohesive soils, the variation in K with temperature is either related to the hysteresis by thermal cycles or viscoplastic strain, is acknowledged but neglected, or is assumed to be negligible. Functional forms proposed for K involve no more than three model parameters.

8.3. Elastic Shear Modulus

Since the elastic shear strain is widely accepted to be unaffected by temperature [26], the elastic shear modulus (G) has often been assumed to be independent of temperature [26,36–38,49,50,52,64,67,69,82,87,102]. The following two-parameter expression for G has thus been rather widely used [82]:

$$G = G_{ref} \left(\frac{p'}{p'_{ref}} \right)^m, \quad (24)$$

where G_{ref} is the elastic shear modulus at the reference pressure (p'_{ref}) and m is the non-linear elastic exponent. In the model of Liu and Xing [61], m was defined by Equation (19).

Zhou and Ng [67] assumed the elastic bulk and shear moduli to both be independent of temperature. The former was given by Equation (17); the latter was given by the following expression:

$$G = G_{ref} \frac{1}{(1+e)^3} \left(\frac{p'}{P_{atm}} \right)^{0.5}, \quad (25)$$

where G_{ref} is a reference value of G and P_{atm} is the atmospheric pressure. This functional form for G was also subsequently used by Zhou et al. [75].

In relatively few instances, G has been assumed to be pressure and temperature dependent. For example, citing the work of Wroth and Houlsby [125], Graham et al. [92] adopted the following expression:

$$\left(\frac{G_{\mathcal{T}}}{p'_{cons}} \right)_{OC} = \left(\frac{G_{\mathcal{T}}}{p'_{cons}} \right)_{NC} [1 + C \ln(OCR)] [1 + D(\mathcal{T} - \mathcal{T}_0)], \quad (26)$$

where the subscripts “OC” and “NC” refer to OC and NC states, respectively, $\mathcal{T}_0$ is a reference temperature, and C and D are model parameters.

Hamidi et al. [40] also included the effect of temperature in the functional form of G . In particular, they adopted the following expression:

$$G_{\mathcal{T}(OC)} = G_{\mathcal{T}(0NC)} \left(\frac{p'}{p'_c} \right)^b \left[1 + D \ln \left(\frac{\mathcal{T}}{\mathcal{T}_0} \right) \right], \quad (27)$$

where $G_{\mathcal{T}(OC)}$ and $G_{\mathcal{T}(0NC)}$ are the shear modulus for overconsolidated soils at elevated temperature and the shear modulus of the normally consolidated soils at a reference temperature ($\mathcal{T}_0$), respectively, and D and b are dimensionless model parameters. It is interesting to note that, after proposing Equation (27), for some reason Hamidi et al. [40] did not actually use it in their model simulations.

In summarizing this sub-section, G is widely accepted to be unaffected by temperature [26]. Consequently, it has, with minor exceptions, often been assumed to be temperature-independent. As was the case for K , functional forms proposed for G involve no more than three model parameters.

9. Trends in Characterizing the Thermo-Elastoplastic and Thermo-Viscoplastic Response

Specific details associated with the various thermo-elastoplastic, thermo-elastoviscoplastic, and thermo-elastoviscoplastic-viscoplastic constitutive models for saturated cohesive soils are quite varied. As such, it is difficult to identify many general trends related to such models. One general characteristic can, however, be identified. In particular, when developing the formulations associated with such models, two general approaches have historically been followed.

In constitutive models employing the first approach, temperature is explicitly included as a state variable in the functional form of the yield or bounding surface, often through the functional form assumed for p'_c (recall Equations (2)–(4)). This approach is largely motivated by the results of laboratory experiments that have been performed to investigate the effect of temperature on the macroscopically observed THM behavior of saturated cohesive soils [2]. In constitutive models based on this approach [26,29,30,37,49–51,55,64,67,75,76,86,87,92,99,102,109,112], the thermoelastic response is accounted for through one of the analytical expressions for β^d discussed in Section 8.1. The temperature dependence of the yield or bounding surface then enters the formulation by making certain internal variables functions of temperature. As a result, thermally-induced inelastic strains do not need to be explicitly defined. Instead, the temperature dependence is included in the definition of an inelastic strain tensor, whose definition requires the definition of suitable plastic multipliers that are a function of stress invariants, as well as the temperature rate. This holds for both associative and non-associative flow rules. The primary difference between constitutive models based on this approach are the specific internal variables that are assumed to vary with temperature, the functional form used to account for the aforementioned temperature dependence, and the type of hardening rule assumed for the yield or bounding surface.

In constitutive models employing second general approach, the evolution of the yield or bounding surface is realized through an explicitly defined, thermally induced inelastic strain tensor. Consequently, in constitutive models based on this approach [1,120], temperature does not enter into the functional form of such a surface. In models based on this second approach, the thermoelastic response is again commonly defined by a suitably defined expression for β^d . The main difference between formulations based on the second approach is the functional form used to define the thermally induced inelastic strain tensor.

10. Conclusions

This paper presented a detailed overview and critical assessment of the constitutive models that have been developed to simulate the therm-hydro-mechanical (THM) behavior of saturated cohesive soils. Several conclusions reached in this overview are noteworthy. These are summarized below.

Beginning in the 1970s, constitutive models possessing varying levels of sophistication have been developed to simulate the THM behavior of saturated cohesive soils. Initially such models tended to be semi- and fully coupled thermo-poroelastic formulations. Not surprisingly, limitations associated with such models led to the development of a rather large number of thermo-elastoplastic models. Although initially such models were extensions of standard isothermal elastoplastic formulations, with time more advanced models were developed. These included “unconventional” formulations that were based on the concept of a subloading or a bounding surface in stress invariant space.

More recently, various thermo-elastoviscoplastic and thermo-elastoplastic-viscoplastic models that account for the time and rate-dependent behavior of saturated cohesive soils have been developed. Indeed, many of the more recent models [87,88,93,95,96,99,102,109,112]

have focused on modeling the time-dependent THM behavior of cohesive soils, mainly by incorporating temperature effects to augment of the functional form of the creep rate.

With relatively minor exceptions [29,43,45–47,93], relatively little progress has been made in understanding the effect of temperature on stress-induced anisotropy. This is largely due to the lack of suitable experimental results from studies that investigate the effect of temperature on stress-induced anisotropy. Without such results, it is difficult to advance the development of thermo-elastoplastic and thermo-elastoplastic constitutive models that rationally account for not only inherent, but also stress-induced anisotropy.

A few of the newer models [54,55] have been developed using a formulation that ensures thermodynamic consistency. Another important aspect in model development is the simulation of thermal cyclic effects [70,75,76]. Such models typically employ an advanced bounding surface plasticity framework.

Many of the thermo-elastoplastic and thermo-elastoplastic constitutive models, instead of being formulated in general three-dimensional stress space, have been developed under the specific assumption of an axisymmetric state of stress (e.g., critical state models) or in terms of a two-invariant stress space (e.g., Mohr-Coulomb based models which do not account for the effect of the intermediate principal stress). The shortcoming of such assumptions is that the stress state of actual soil deposits are more nearly three-dimensional or plane strain rather than those associated with axisymmetry or with a two-invariant stress space. This highlights the lack of suitable THM experimental results from plane strain or from true triaxial tests. Without such results, it is difficult to advance the development of truly general, three-dimensional thermo-elastoplastic and thermo-elastoplastic constitutive models.

In past THM constitutive models for saturated cohesive soils, various analytical expressions have been proposed for β^d . Initially, some rather complex six- and four-parameter forms were proposed. These were followed by simpler, two-parameter functional forms. More recently, many of the constitutive models assume a one-parameter functional form, some of which assume β^d to be constant.

In many THM models for saturated cohesive soils, the variation in the elastic bulk modulus (K) with temperature is either related to the hysteresis by thermal cycles or viscoplastic strain, is acknowledged but neglected, or is assumed to be negligible. Functional forms proposed for K typically involve no more than three model parameters.

Assumptions made regarding functional forms for the elastic shear modulus (G) have been influenced by the widely accepted observation that shear strains are unaffected by changes in temperature [26]. In light of this observation, G has, with minor exceptions, often been assumed to be temperature independent.

The aforementioned trends regarding β^d , K , and G illustrate the well-known trade-off between predictive capabilities due to model sophistication and practicality in determining the values of parameters associated with a particular model. This trade-off likewise applies to all other model parameters used in a given constitutive model that has been developed to simulate the THM behavior of saturated cohesive soils.

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Abbreviations

The following abbreviations are used in this manuscript:

GBSM	Generalized Bounding Surface Model
MCC	Modified cam clay
NC	Normally consolidated
OC	Overconsolidated
OCR	Overconsolidation ratio
THM	Thermo-hydro-mechanical

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