

Article

Modeling, Comparative Investigation and Compensation for Hysteresis Response of Actuator Using Nonlinear Transformation

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Abstract

Aiming at problems such as the reduced positioning accuracy and insufficient dynamic response caused by the inherent hysteresis nonlinearity of piezoelectric actuators, this paper proposes a hysteresis modeling and compensation method for piezoelectric actuators based on nonlinear transformation. The proposed NT hysteresis model utilizes the nonlinear characteristics of activation functions to describe the complex nonlinearity in the hysteresis response of piezoelectric actuators. On this basis, a feedforward compensation controller is further designed based on the proposed inverse NT hysteresis model. Moreover, a composite controller is constructed by combining it with PID feedback to improve the dynamic response speed and trajectory tracking accuracy of piezoelectric actuators. Based on experimental data, the fitting performance of the proposed model is compared with that of several common hysteresis models using evaluation indicators including *RMSE*, *MAPE* and *SMAPE*. Finally, the performance of the proposed control method is verified through step response and sinusoidal trajectory tracking experiments. Experimental results show that the proposed NT hysteresis model performs best in characterizing the hysteresis characteristics of piezoelectric actuators, and the feedforward compensation controller constructed based on its inverse model exhibits superior control performance.

Keywords: piezoelectric actuator; hysteresis modeling; compensation; nonlinear transformation; performance evaluation



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1. Introduction

Piezoelectric actuators (PZTs) have been widely used due to their simple driving principle, compact structure, and high displacement resolution [1–3]. However, the inherent hysteresis between input voltage and displacement response of piezoelectric actuators can seriously affect positioning accuracy and further hinder engineering applications. Therefore, establishing a model that can accurately describe hysteresis behavior is of great engineering significance for improving system control accuracy, reducing system debugging costs, enhancing model generality, and supporting adaptive control design.

Generally, hysteresis modeling of piezoelectric actuators falls into two main categories: physical models and phenomenological models. Compared with physical models,

phenomenological models do not consider natural physical properties and can directly use numerical formulations characterizing the hysteretic nonlinear input–output relationship, thus finding greater application. Therefore, phenomenological models such as the Dahl model [4,5], LuGre model [6,7], Prandtl–Ishlinskii (PI) model [8–10], Bouc–Wen (BW) model [11–13] and four-parameter viscoelastic (FP) model [14,15] have found broad application in the modeling of hysteresis nonlinearities, which are mainly especially time and rate dependent [16,17], of piezoelectric actuators. These models each have their own advantages and can describe different materials' hysteresis effects and dynamic characteristics. For example, the Dahl model is simple to construct and easy to understand and implement; the LuGre model has high precision when simulating the hysteresis phenomenon of piezoelectric actuators; the PI model features high computational efficiency and easy inverse computation; the BW model enjoys broad popularity and application in simulating hysteresis phenomena in structural mechanics, magnetorheological, and seismic isolation devices; and the FP model ensures computational accuracy through avoiding the computation of nonlinear equations. However, these models also have some shortcomings, such as a large number of model parameters, difficulty in solving nonlinear differential equations, challenges in determining intermediate functions, and low modeling accuracy, which pose certain obstacles to practical applications. In addition, there is a lack of comprehensive comparative analysis of these models in terms of their ability to characterize the hysteresis properties of piezoelectric actuators, leaving a gap in the evaluation of their fitting performance. In all, an ideal hysteresis model should feature a small number of unknown parameters, the absence of highly nonlinear differential equations, no need to determine intermediate functions, and ease of solution, as well as maintaining high fitting accuracy.

Based on the modeling idea of nonlinear transformation, a novel nonlinear transformation (NT) model is proposed in this paper. Through using the nonlinearity of activation functions to replace and characterize the nonlinear hysteresis response of piezoelectric actuators, the proposed NT model is simple to construct, featuring fewer unknown parameters and the absence of highly nonlinear differential equations in its expression, which means no need to determine intermediate functions, thereby avoiding the shortcomings of existing common hysteresis models that involve nonlinear differential equations and low solution efficiency. Subsequently, based on the Particle Swarm Optimization (PSO) algorithm, the fitting performance of the proposed NT model and existing hysteresis models is comprehensively analyzed using experimental data under various loading conditions. All six hysteresis models above are compared using multiple indicators, including Root Mean Square Error (*RMSE*), Mean Absolute Percentage Error (*MAPE*), Symmetric Mean Absolute Percentage Error (*SMAPE*), and the Coefficient of Determination (R^2). Quantitative analysis of the model's fitting errors is also conducted. The results show that the proposed NT model outperforms the other five general models in all indicators. Additionally, it exhibits significant advantages over other existing models in terms of maximum fitting error, indicating that the proposed NT model performs best in characterizing the hysteresis properties of piezoelectric actuators. In addition, a discrete inverse NT model is derived to support feedforward compensation and motion-control applications.

The main contributions of this paper are threefold. First, a compact phenomenological hysteresis modeling framework based on nonlinear transformation is established for piezoelectric actuators, enriching the modeling methodology beyond conventional differential-equation-based or operator-based formulations. Second, the proposed NT model is systematically compared with five representative hysteresis models under the same experimental datasets and evaluation criteria, thereby filling a gap in comprehensive performance benchmarking. Finally, the control applicability of the model is further vali-

dated by deriving a discrete inverse model and conducting sinusoidal trajectory-tracking and step-response experiments with feedforward compensation.

2. Hysteresis Modeling

2.1. Modeling Idea

The nonlinear hysteresis effect of piezoelectric actuators can be characterized by a hysteresis loop with an upper convex and lower convex shape, as shown in Figure 1. It exhibits a lower convex characteristic during the displacement rising phase (as shown by the curve from point 0 to point A in Figure 1) and an upper convex characteristic during the displacement falling phase (as shown by the curve from point A back to point 0 in Figure 1). To describe nonlinear characteristics, nonlinear differential equations or intermediate variables are often introduced in existing research, which leads to complex model structures, difficulty in equation solving, and low computational accuracy.

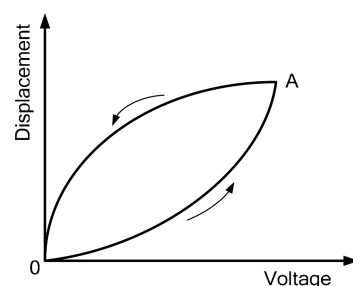


Figure 1. Schematic diagram of hysteresis effect.

The activation function features an S-shaped curve, similar to the hysteresis loop. Based on the idea of nonlinear transformation, the activation function is introduced to describe the nonlinear hysteresis loop in this paper. The idea of nonlinear transformation is illustrated in Figure 2. The S-shaped curve of the activation function shows an upper convex characteristic in the first quadrant and a lower convex characteristic in the third quadrant (as shown in Figure 2a). By transforming the negative half of the activation function located in the third quadrant to the first quadrant, and introducing corresponding shape parameters to adjust the upper and lower convex curves, it is possible to approximate the upper and lower convex characteristics of the hysteresis loop (as shown in Figure 2b), making it feasible to describe the displacement-voltage nonlinearity of piezoelectric actuators.

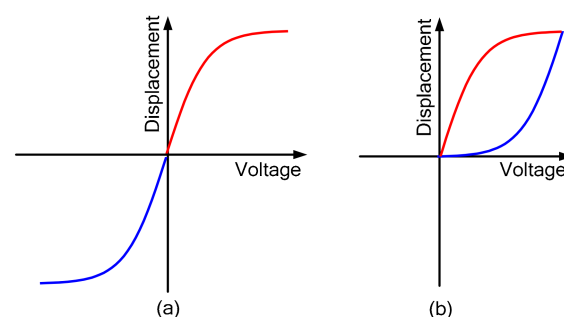


Figure 2. Schematic diagram of nonlinear transformation. (a) Original curve. (b) Transformed curve.

As shown in Figure 2, after nonlinear transformation, the nonlinear hysteresis loop of the piezoelectric actuator can be converted into an S-shaped curve of the activation function. The lower convex characteristic during the displacement rising phase (blue curve) and the upper convex characteristic during the displacement falling phase (red curve) can be described by the curves of the activation function in the third and first quadrants, respectively.

Among various activation functions, the Softsign function has a smoother asymptote compared to the Tanh and Sigmoid functions, and it has higher computational efficiency than the Softplus function, while also possessing a smooth derivative. In this paper, the Softsign function is used as the nonlinear transformation function for the hysteresis characteristics of piezoelectric actuators, thereby avoiding the disadvantages of introducing nonlinear differential equations or intermediate variables. The Softsign function used in this study is shown as follows:

$$\text{Softsign}(t) = \frac{u(t)}{1 + |u(t)|} \quad (1)$$

It is necessary to explicate how the Softsign function fundamentally represents hysteresis evolution and the memory effects of actuators. The Softsign function inherently encodes two critical physical constraints: (1) First, the approach to saturation. As $u \rightarrow \pm\infty$ is input, Softsign asymptotically approaches ± 1 , naturally replicating the saturation hysteresis. More importantly, its polynomial saturation rate provides a better fit for certain materials. (2) Next, the reversible component at zero. Unlike piecewise linear models, Softsign is smooth at the origin with a non-zero, finite derivative. This mathematically guarantees a continuous transition from reversible to irreversible domain wall motion, a prerequisite for physically sound hysteresis representation.

Because Softsign is a strictly monotonic, bounded function with a Lipschitz constant of 1, it acts as a contraction map that stabilizes the state evolution. This stabilization is essential: it prevents the internal state from diverging and guarantees that the fading memory property is consistently maintained. The state reflects a weighted, bounded recollection of past extrema, which is the very essence of memory in hysteretic systems.

2.2. Hysteresis Modeling Based on Nonlinear Transformation

A piezoelectric actuator can be approximately seen as a spring-damper system. In addition to characterizing the nonlinear hysteresis loop, it is also necessary to consider the stiffness and damping of the system. Based on this, the stiffness component and damping component that vary with the driving voltage is further introduced, thereby leading to the proposed NT model, as shown in Equation (2).

$$d(t) = k_0 u(t) + c_0 \dot{u}(t) + k_1 \frac{k_2 u(t)}{1 + |k_2 u(t)|} \quad (2)$$

where $u(s)$ denotes the driving voltage and $d(s)$ represents the output displacement. To establish a rigid boundary between structural dynamics and material science, the parameters are categorized into two distinct classes based on their physical–mathematical roles.

The parameters k_0 and c_0 govern the fundamental structural dynamics of the actuator assembly. Specifically, k_0 represents the baseline elastic structural stiffness of the piezoelectric stack, dictates the linear voltage-to-displacement conversion under quasi-static conditions, and remains highly stable across varied frequencies. The parameter c_0 represents the linear viscous damping factor, capturing the physical energy dissipation within the mechanical joints and material interfaces during cyclic motions.

Conversely, k_1 and k_2 serve as phenomenological coefficients mapped directly to the microscopic ferroelectric dipole polarization kinetics. Under an alternating electrical excitation, the reorientation and switching behaviors of the internal micro-domains introduce a distinct nonlinear lag, resulting in an asymmetric upper and lower convex hysteresis loop. Within this context, k_1 acts as an amplitude coefficient that scales the maximum bounds of the hysteretic trajectory, which is tied to the saturation polarization capacity of the material. The parameter k_2 acts as a shape parameter regulating the voltage saturation threshold

within the Softsign activation framework. As the operating frequency scales upward, the microscopic polarization reorientation lags further behind the external electric field. This physical polarization mismatch modifies the geometry of the hysteresis loop, which is successfully captured by the condition-specific evolution of the curve-fitting coefficients k_1 and k_2 .

The construction process of the proposed NT hysteresis model is illustrated in Figure 3. First, the model employs the nonlinear characteristics of the activation function to directly characterize the inherent nonlinear components in the hysteresis loop of piezoelectric actuators, thereby effectively avoiding the problems of complex derivation, high-order coupling, and numerical solution difficulties encountered by traditional nonlinear differential equations. Furthermore, a stiffness and a damping factor that can reflect the dynamic characteristics of the system are introduced. The overall structure of the constructed NT model is concise and intuitive, with a small number of unknown parameters to be identified. It does not involve strongly nonlinear differential equations, nor does it require the introduction and determination of complex intermediate state variables, which significantly reduces model complexity and online computational burden. Compared with traditional hysteresis models such as Prandtl–Ishlinskii and Bouc–Wen, this model maintains high characterization accuracy while being more convenient for parameter identification, numerical solution, and practical engineering implementation, providing a sound foundation for subsequent inverse-model construction and real-time compensation control.

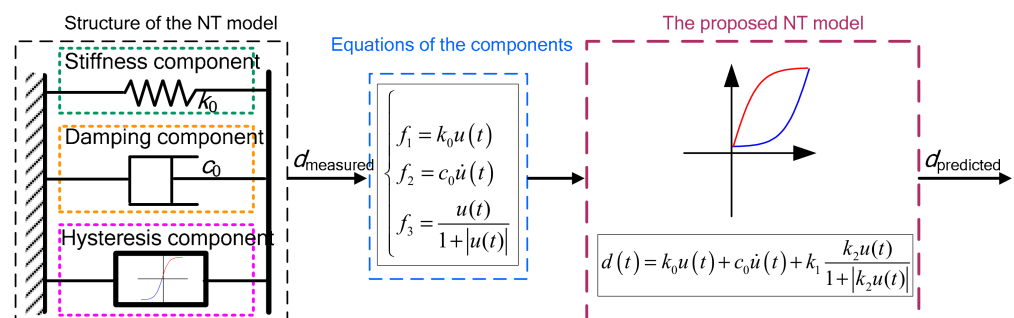


Figure 3. Block diagram of the developed NT model.

2.3. Inverse NT Model Derivation

To further demonstrate the applicability of the proposed NT model in precision positioning and feedforward control, a discrete inverse model is derived.

In control applications, the desired displacement $d_r(t)$ is typically given, and the corresponding driving voltage $u(t)$ needs to be generated. Because the NT model contains both the derivative term $\dot{u}(t)$ and the nonlinear fractional term with an absolute value, direct continuous-time inversion is inconvenient for real-time implementation. Therefore, a discrete inverse formulation is adopted. Let the sampling period be T_s and approximate the voltage derivative by backward difference:

$$\dot{u}(t) \approx \frac{u_n - u_{n-1}}{T_s} \quad (3)$$

Then, the model at the n th sampling instant becomes

$$d_n = k_0 u_n + c_0 \frac{u_n - u_{n-1}}{T_s} + k_1 \frac{k_2 u_n}{1 + |k_2 u_n|} \quad (4)$$

Further, we can obtain

$$d_n + \frac{c_0}{T_s} u_{n-1} = \left(k_0 + \frac{c_0}{T_s} \right) u_n + k_1 \frac{k_2 u_n}{1 + |k_2 u_n|} \quad (5)$$

Define

$$A = k_0 + \frac{c_0}{T_s}, B_n = d_n + \frac{c_0}{T_s} u_{n-1} \quad (6)$$

Then, we can get

$$B_n = Au_n + k_1 \frac{k_2 u_n}{1 + |k_2 u_n|} \quad (7)$$

Because of the absolute-value term, Equation (7) is solved piecewise.

When $u_n \geq 0$, $|k_2 u_n| = k_2 u_n$, and therefore

$$B_n = Au_n + k_1 \frac{k_2 u_n}{1 + k_2 u_n} \quad (8)$$

By multiplying both sides by $(1 + k_2 u_n)$ and rearranging elements, we can obtain

$$Ak_2 u_n^2 + (A + k_1 k_2 - B_n k_2) u_n - B_n = 0 \quad (9)$$

When $u_n < 0$, $|k_2 u_n| = -k_2 u_n$, then

$$B_n = Au_n + k_1 \frac{k_2 u_n}{1 - k_2 u_n} \quad (10)$$

Similarly, we can obtain

$$Ak_2 u_n^2 + (-A - k_1 k_2 - B_n k_2) u_n + B_n = 0 \quad (11)$$

Therefore, the inverse problem of the NT model in discrete form is converted into a piecewise quadratic equation with respect to u_n . At each sampling instant, candidate solutions from the positive and negative branches are calculated, and the physically admissible solution is selected according to the sign constraint and continuity with the previous input u_{n-1} . This inverse-model formulation avoids complex iterative computation and is convenient for real-time feedforward implementation.

3. Model Identification and Evaluation Metrics

3.1. Model Identification

In recent years, swarm intelligence algorithms have been widely applied in the parameter recognition of piezoelectric actuators, such as Genetic Algorithms (GAs) [18–21], the Artificial Bee Colony (ABC) algorithm [22–24], Ant Colony Optimization (ACO) [25–27], Cuckoo Search (CS) [28–30] and Particle Swarm Optimization (PSO) [31–33]. Each of these algorithms has its own advantages. For example, GA, ABC, and ACO possess global search capabilities, strong adaptability, multi-mode search, and ease of implementation. However, they suffer from slow convergence speed and a tendency to get trapped in local optima when searching for the optimal solution. The CS algorithm performs better than GA and ABC with regard to convergence behavior and optimization accuracy in high-dimensional nonlinear optimization, but it has higher computational complexity, lower computational efficiency, and is more sensitive to noise. Compared with GA, ABC, and ACO, the PSO algorithm has advantages of simplicity, high convergence efficiency, and ease of application. In contrast to the CS algorithm, the PSO algorithm has fewer parameters and can be combined with other algorithms. Many researchers [34–38] have demonstrated the effectiveness of the PSO algorithm in multiple fields, including piezoelectric systems. Thereby, the PSO algorithm is used for hysteresis nonlinearity fitting in this paper.

$$\begin{cases} v_i^{k+1} = \omega_i v_i^k + c_1 \text{rand}_1 (pbest_i - x_i^k) + c_2 \text{rand}_2 (gbest - x_i^k) \\ x_i^{k+1} = x_i^k + v_i^{k+1} \end{cases} \quad (12)$$

where ω is the inertia weight, which controls the degree to which the particle maintains its original direction; c_1 and c_2 are learning factors; r_1 and r_2 are random numbers within the interval $[0, 1]$; and $x_i(t)$ and $v_i(t)$ are the position and velocity of the i th particle at time t .

The parameter identification process for the hysteresis nonlinearity of the piezoelectric actuator is essentially a problem of finding a global minimum. Defining a proper objective function is essential for the optimization problem. Herein, the *RMSE* between the measured displacement and the model output over a sampling period is taken as the optimization target. During the iteration process, the model parameters are continuously updated to minimize the *RMSE* value, thereby determining the optimal solution. Its mathematical formula is as follows:

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N [d_i^{\text{model}}(t) - d_i^{\text{exp}}(t)]^2} \quad (13)$$

where N is the number of samples in one sampling frequency, and the value varies for different frequencies, and $d_i^{\text{exp}}(s)$ and $d_i^{\text{model}}(s)$ represent the measured value and the model value, respectively.

To ensure the high-fidelity representation capability of the proposed NT model, separate parameter sets are identified independently for each distinct loading condition (voltage and frequency). This condition-specific tuning strategy is physically justified by the inherent rate-dependent and amplitude-dependent nature of piezoelectric actuators, where internal polarization dynamics and micro-domain switching lags shift under different excitation envelopes. By executing the PSO algorithm independently for every experimental dataset, we ensure a fair and rigorous structural benchmarking process, comparing the maximum mathematical capacity of each of the six competing model architectures under identical optimization conditions. In practical precision engineering implementations, these condition-specific optimal parameter sets are seamlessly integrated into real-time controllers via a multi-dimensional calibration look-up table or a gain-scheduling framework mapped to the operating frequency and voltage.

3.2. Evaluation Metrics

To perform a multi-dimensional performance comparison, several commonly used evaluation metrics are also chosen in order to conduct a comprehensive analysis. In addition to *RMSE*, the Mean Absolute Percentage Error (*MAPE*), the Symmetric Mean Absolute Percentage Error (*SMAPE*), and the Coefficient of Determination (R^2) are introduced as evaluation metrics so as to provide a comprehensive evaluation of the six hysteresis models from multiple dimensions.

MAPE is an indicator used to measure the accuracy of a predictive model and has found wide application in areas such as time series forecasting and regression analysis. Its mathematical expression is as follows:

$$MAPE = \frac{1}{N} \sum_{i=1}^N \left| \frac{d_i^{\text{model}}(t) - d_i^{\text{exp}}(t)}{d_i^{\text{exp}}(t)} \right| \quad (14)$$

SMAPE is often chosen to evaluate predictive accuracy, especially suitable for time series forecasting or regression problems. Unlike *MAPE*, the relative size of the actual value and the predicted value during calculation is taken into consideration in *SMAPE*. Its mathematical expression is as follows:

$$SMAPE = \frac{1}{N} \sum_{i=1}^N \frac{|d_i^{\text{model}}(t) - d_i^{\text{exp}}(t)|}{(d_i^{\text{model}}(t) + d_i^{\text{exp}}(t))/2} \quad (15)$$

R^2 is an indicator used in statistics to measure the goodness of fitting. It ranges from 0 to 1, where a value of 0 usually indicates poor fitting performance and a value of 1 indicates that the model perfectly fits the data. Its mathematical expression is as follows:

$$R^2 = 1 - \frac{\sum_{i=1}^N (d_i^{\text{exp}}(t) - d_i^{\text{model}}(t))^2}{\sum_{i=1}^N (d_i^{\text{exp}}(t) - \bar{d}_i^{\text{exp}}(t))^2} \quad (16)$$

These four evaluation metrics can cover different dimensions of assessment needs. *RMSE* measures the absolute error between the predicted value and the true value, with units consistent with the target variable; *MAPE* measures the relative error ratio between the predicted value and the true value, facilitating comparisons across different datasets; *SMAPE* avoids the division-by-zero issue that occurs in *MAPE* when the true value is zero; and R^2 measures the ability of the model to explain the variability in the data, reflecting the overall fitting performance of the model. Generally speaking, for the first three indicators, the lower the value of the evaluation metric, the better the model performance. For R^2 , the higher the value of the evaluation metric, the better the model performance.

Furthermore, the comparative investigation strategy is illustrated in Figure 4, which is mainly composed of three components: the piezoelectric system, signal processing and comparative investigation. The individual prediction is performed by each hysteresis model, and thereby the prediction error compared with the measured experiment values can be obtained. Through comparison of prediction error, the advantage of the proposed NT model can be determined.

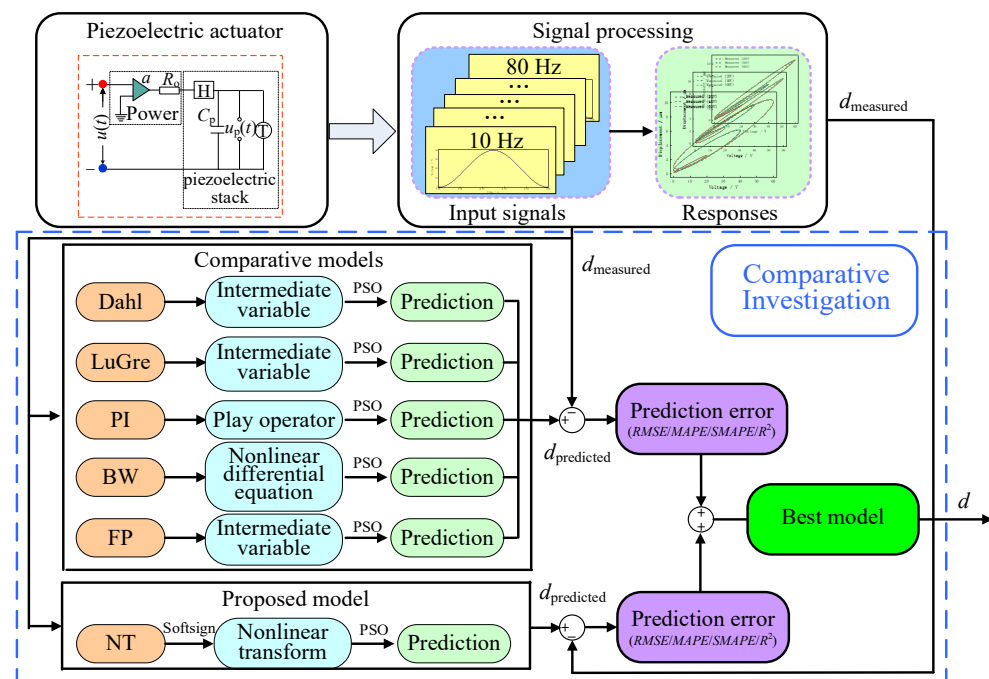


Figure 4. Comparative investigation strategy of evolution based on the developed model.

4. Results and Analysis

4.1. Experimental Setup

The experimental configuration for displacement and voltage measurement of the piezoelectric actuator (installed inside a micro-positioning stage) is shown in Figure 5. The resonant frequencies of the micro-positioning stage under no-load conditions are 104.4, 138.4 and 120 Hz in the x , y , and θ_z direction, respectively. The system includes a DSP

card (Manufacture: Texas Instruments, Type: TMS320C6747, Frequency: 375 MHz), a piezoelectric actuator (Manufacture: PANT; Type: PTJ1501010601, shown in Table 1), a capacitive displacement sensor (Manufacture: Micro-Epsilon, Type: CSE1, Resolution: 0.02 μm , installed inside micro-positioning stage), a computer, a driver power supply, auxiliary measurement brackets, and a three-dimensional manual adjustment platform. The measurement procedure is described as follows. The multifunctional data acquisition (DAQ) card converts the digital signal generated by the computer into an analog signal, which is subsequently transmitted to the power drive module of the piezoelectric actuator. The output voltage from the drive module is applied to the piezoelectric actuator, thereby driving the device to generate micro-displacement, which further results in a displacement of the micro-positioning stage. Finally, the resulting displacement is collected by a non-contact capacitive displacement sensor and finally collected into the computer via a network port.

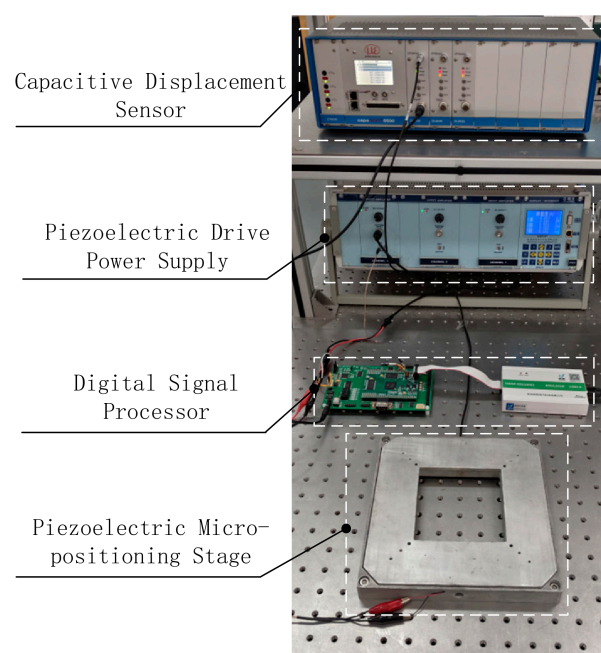


Figure 5. Experiment setup for measuring hysteresis displacement.

Table 1. Parameters of piezoelectric actuator.

Model	d_{33} (10^{-12} C/N)	Density (g/cm^3)	Quality Factor	Elasticity Tensor (10^{-12} m^2/N)	Stiffness ($\text{N}/\mu\text{m}$)
PTJ1501010601	≥ 650	7.9	45	14.3	66

To further explore the hysteretic nonlinear behavior of piezoelectric actuators, three groups of sinusoidal excitation voltages (0–20 V–0, 0–40 V–0 and 0–60 V–0) are imposed on the actuator. The excitation frequency is set from 10 Hz to 80 Hz with an interval of 10 Hz, and repeated experiments are carried out to record the output responses. Accordingly, the corresponding hysteresis correlation between input voltage and output displacement is acquired. To guarantee experimental stability, the sampling duration for all tests is configured to five operating cycles. Both voltage and displacement responses can be directly acquired from sensor measurements, whereas the voltage change rate is calculated by taking the derivative of voltage with respect to time.

4.2. Fitting Analysis of Hysteresis Model

In Figures 6–8, the hysteresis responses of six hysteresis models under different loading conditions at frequencies of 10 Hz, 40 Hz, and 80 Hz with voltages of 20 V, 40 V, and 60 V are compared, respectively. The results show that the proposed NT model, along with the LuGre and FP models, exhibits a relatively good match between the measured values and the model values, effectively characterizing the hysteresis characteristics of the piezoelectric actuator under different loading conditions. With regards to the PI model, some deviation can be observed in fitting the hysteresis curves under the loading conditions of 20 V and 10 Hz, 20 V and 80 Hz, and 60 V and 80 Hz. Meanwhile, significant deviations and distortions under most frequency and voltage conditions can also be found as in the Dahl and BW models, indicating clear shortcomings in their ability to characterize the hysteresis characteristics of the piezoelectric actuator.

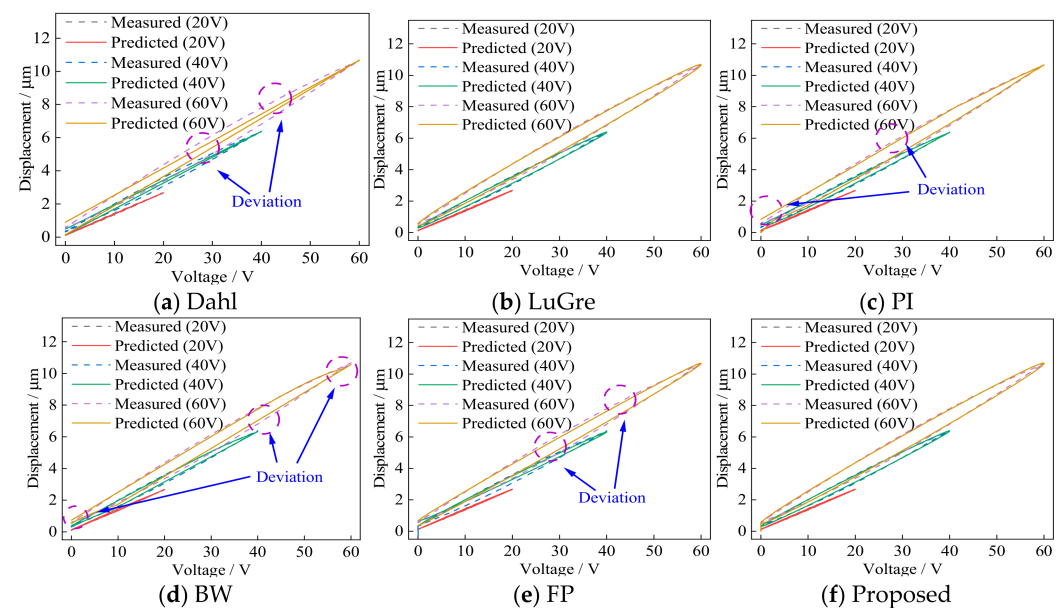


Figure 6. Comparison between measured value and model value of 10 Hz case.

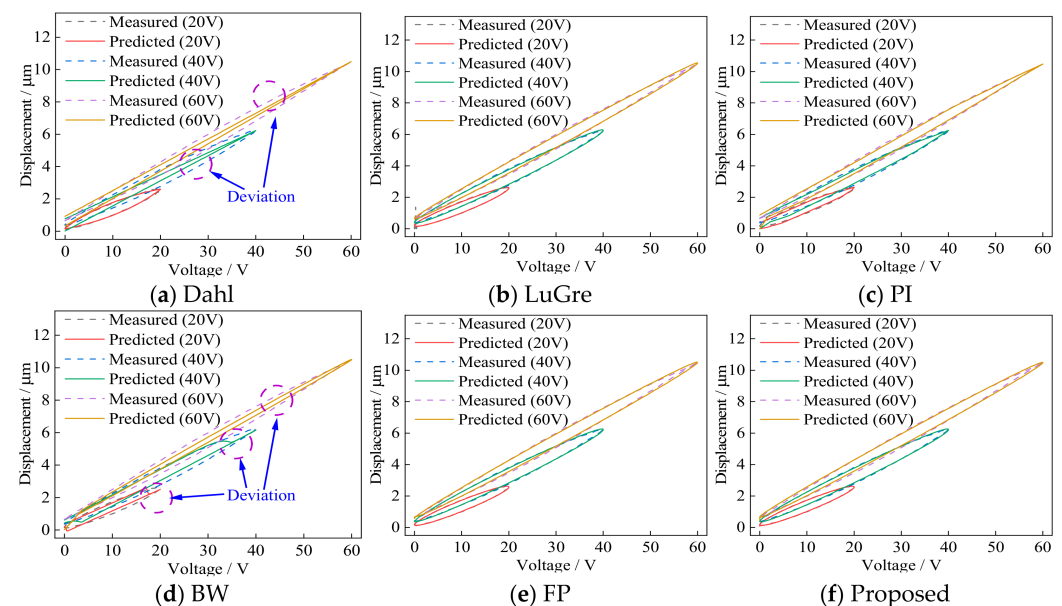


Figure 7. Comparison between measured value and model value of 40 Hz case.

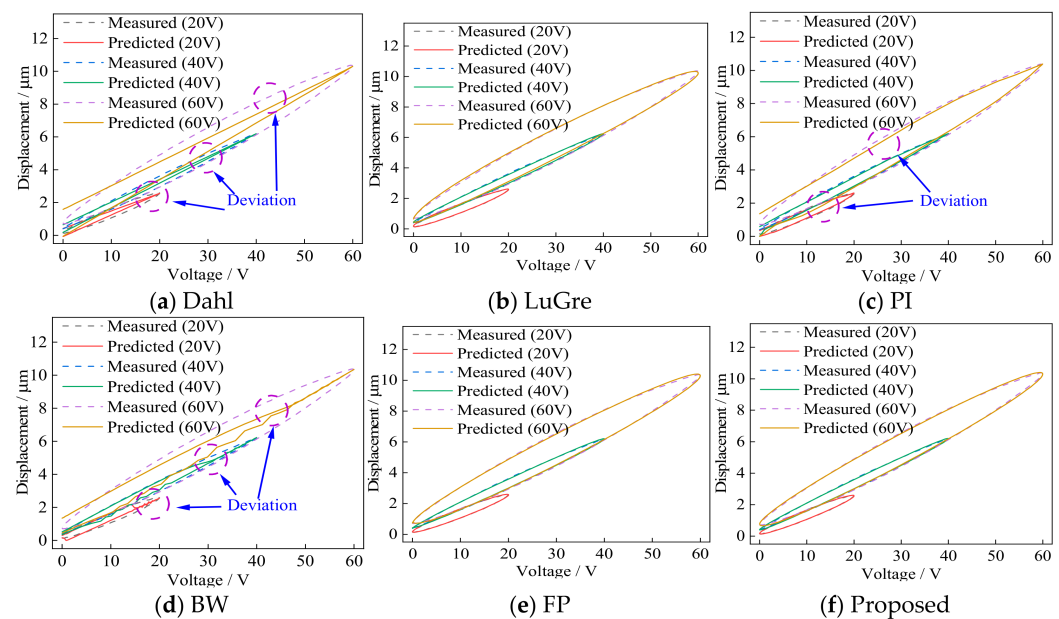


Figure 8. Comparison between measured value and model value of 80 Hz case.

In summary, compared to the proposed NT model, LuGre, and FP models, the PI, Dahl, and BW models show a certain problem of deviation in fitting performance, indicating their deficiencies in characterizing the hysteresis nonlinear characteristics. The proposed NT model, along with the LuGre and FP models, demonstrates better fitting performance under various loading conditions, indicating superior performance in describing the hysteresis nonlinear characteristics of the piezoelectric actuator.

4.3. Quantitative Analysis of Model Fitting Performance

4.3.1. RMSE Analysis

The RMSE performance of the six hysteresis models is compared, with driving voltages of 20, 40, and 60 V and working frequencies ranging from 10 to 80 Hz at intervals of 10 Hz. To intuitively display the numerical comparison of the RMSE of each model, satellite maps are used, as shown in Figures 9–11.

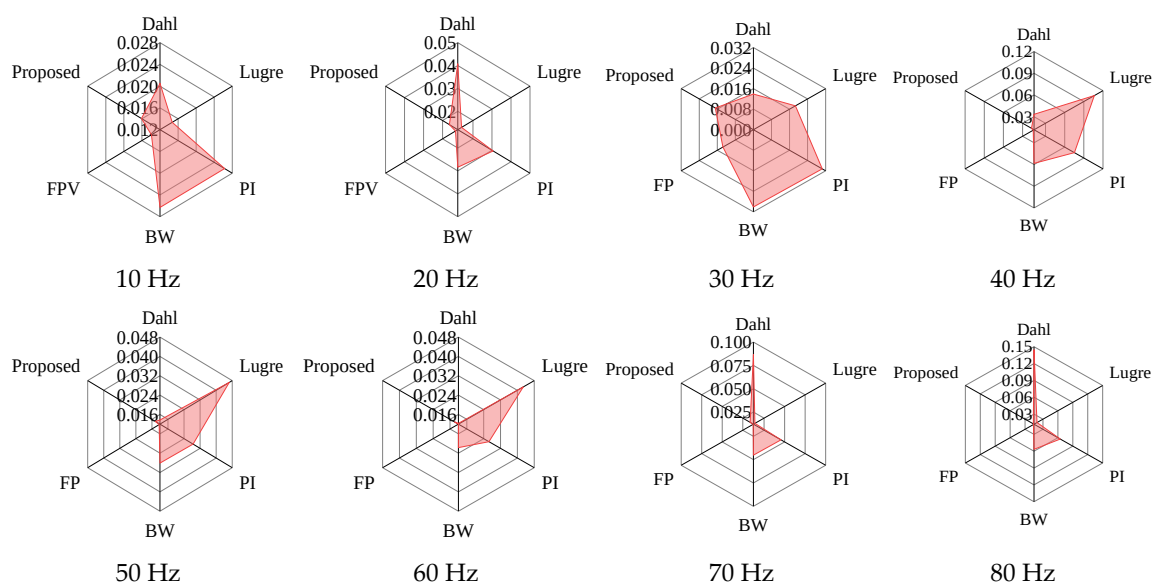


Figure 9. Evaluation of RMSE among the six models of the 20 V case.

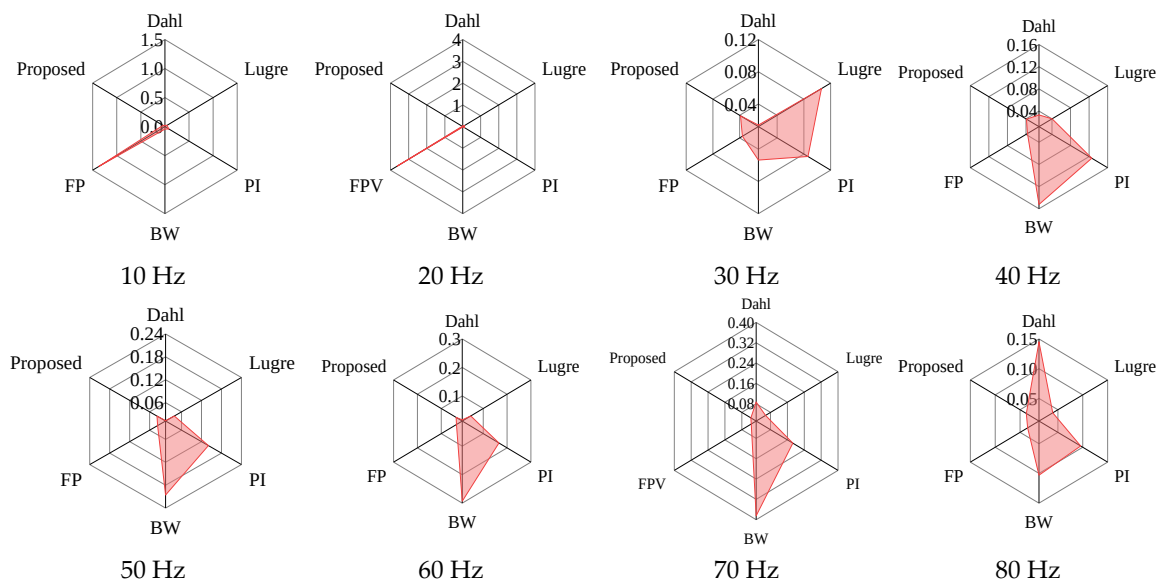


Figure 10. Evaluation of RMSE among the six models of the 40 V case.

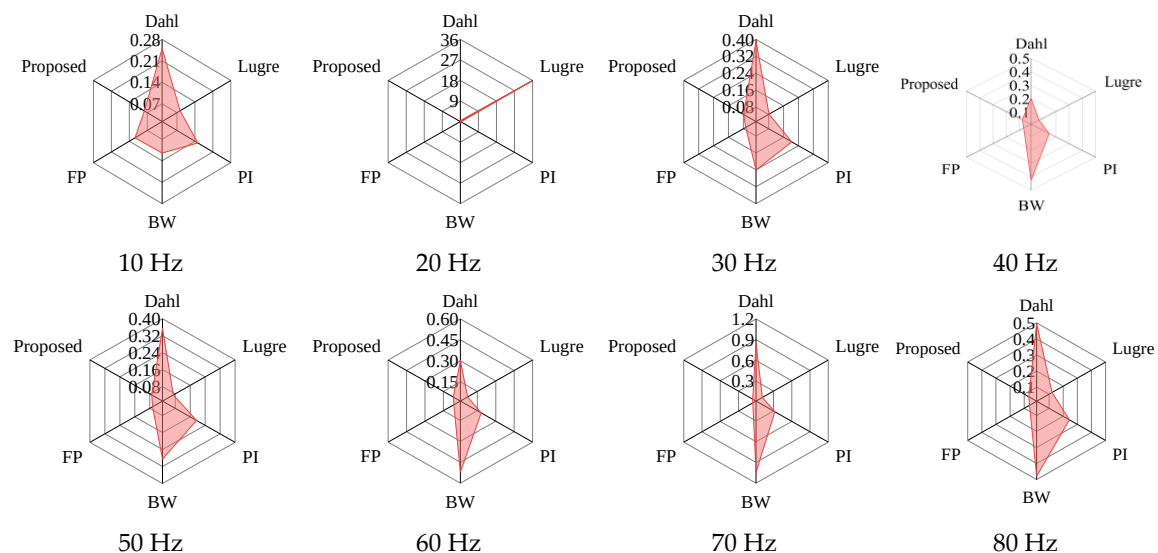


Figure 11. Evaluation of RMSE among the six models of the 60 V case.

From the figures, it can be seen that the *RMSE* values of the proposed NT model are within a small range under all loading conditions, and the fitting error is more stable and reliable compared to the other five existing hysteresis models. Although the FP model has smaller *RMSE* values than the proposed NT model under all working frequencies at 20 V, its *RMSE* performance is also superior to the proposed NT model under several working conditions such as 40 V and 30 Hz, 40 V and 50 Hz, 40 V and 60 Hz, 40 V and 80 Hz, 60 V and 30 Hz, 60 V and 40 Hz, and 60 V and 50 Hz. However, the FP model's *RMSE* reaches 1.3708 at 40 V and 10 Hz, which is 6686.83%, 3195.34%, 1681.96%, and 3194.63% higher than the other four existing models and 3319.13% higher than the proposed NT model, making it the largest error among all six models and indicating that its performance is not as reliable as the proposed NT model. Similar extreme fitting errors also exist in other existing hysteresis models, such as the Dahl model reaching its maximum *RMSE* of 0.8485 (proposed model: 0.0668) at 60 V and 70 Hz, the LuGre model reaching its maximum *RMSE* of 35.69 (proposed model: 0.0789) at 60 V and 20 Hz, the PI model reaching its maximum *RMSE* of 0.1859 (proposed model: 0.0371) at 40 V and 70 Hz, and the BW model reaching

its maximum *RMSE* of 1.0414 (proposed model: 0.0668) at 60 V and 70 Hz. In contrast, the proposed NT hysteresis model has a maximum *RMSE* of only 0.0812 at 60 V and 10 Hz.

In summary, the maximum *RMSE* of the proposed NT model under all loading conditions is only 0.0812, which is better than the other five hysteresis models. Thereby, it can be concluded that the proposed NT model has a clear advantage in fitting the hysteresis characteristics of the piezoelectric actuator.

4.3.2. MAPE Analysis

To further analyze and clarify the advantages and disadvantages of the six hysteresis models, the *MAPE* performance of all models is further analyzed. Considering the need to reduce the length of the text, only *MAPE* satellite maps for 20, 40, and 60 V driving voltages at 10, 30, 50, and 70 Hz are given, as shown in Figures 12–14.

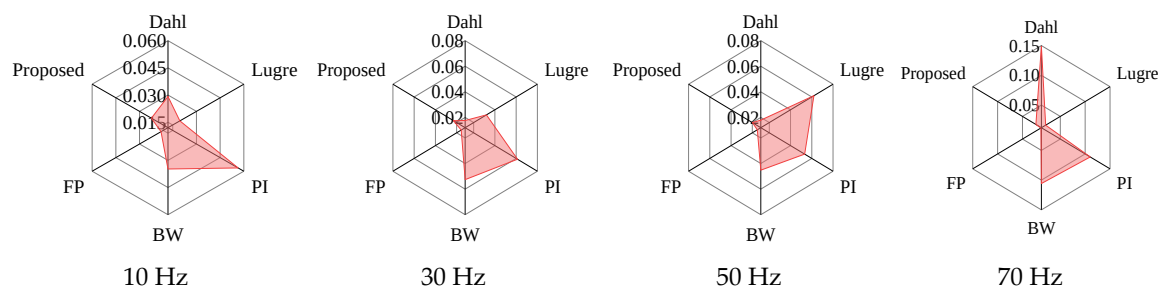


Figure 12. Evaluation of *MAPE* among the six models of the 20 V case.

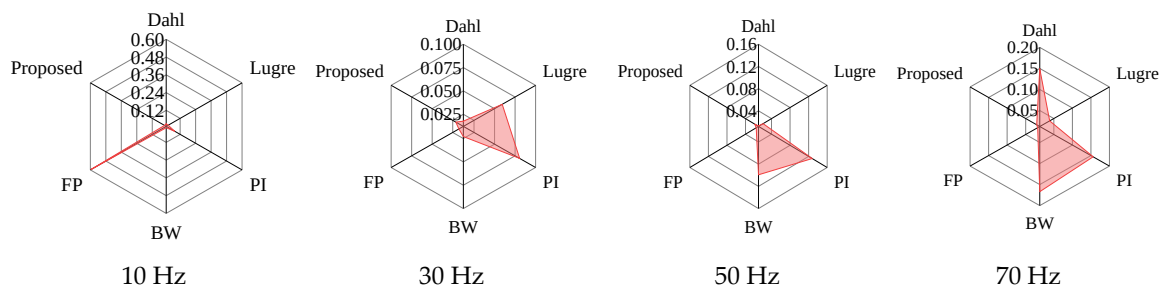


Figure 13. Evaluation of *MAPE* among the six models of the 40 V case.

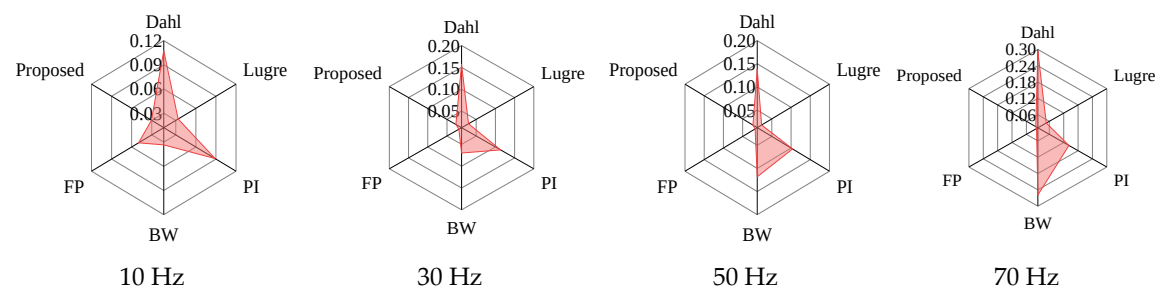


Figure 14. Evaluation of *MAPE* among the six models of the 60 V case.

From the figures, it can be seen that the *MAPE* of the proposed NT model is within a small range under all loading conditions, and its fitting accuracy is more reliable compared to the other five hysteresis models. Although the FP model has smaller *MAPE* values than the proposed NT model under all working frequencies at 20 V and 60 V, under other working conditions such as 40 V and 20 Hz, 40 V and 40 Hz, 40 V and 60 Hz, and 40 V and 80 Hz, its *MAPE* reaches 0.6044 and 0.5919 at 40 V and 10 Hz and 40 V and 20 Hz, respectively, which is 2437.1% and 2518.72% higher than the proposed NT model. Similar extreme fitting errors also exist in other conventional hysteresis models, such as

the Dahl model reaching its maximum *MAPE* of 0.2461 (proposed model: 0.0191) at 40 V and 80 Hz, the LuGre model reaching its maximum *MAPE* of 22.44 (proposed model: 0.0225) at 20 V and 40 Hz, the PI model reaching its maximum *MAPE* of 0.1694 (proposed model: 0.0225) at 20 V and 40 Hz, and the BW model reaching its maximum *MAPE* of 0.1841 (proposed model: 0.0224) at 60 V and 60 Hz. Overall, the proposed NT model and the FP model have an absolute advantage in fitting accuracy compared to other models. However, the proposed NT model performs more stably and reliably compared to the FP model, achieving satisfactory results in all loading conditions. The maximum *MAPE* of the proposed NT model under all loading conditions is only 0.045, demonstrating the reliability of the proposed model's performance.

4.3.3. SMAPE Analysis

The six hysteresis models are further analyzed using *SMAPE*. To reduce the length of the text and to comprehensively analyze the model fitting performance under all loading conditions, as well as to complement the *MAPE* analysis above, only *SMAPE* satellite maps for 20, 40, and 60 V driving voltages at 20, 40, 60, and 80 Hz operating conditions are given in this paper.

The results show that although the FP model has lower *SMAPE* values than the proposed NT model in most conditions, its *SMAPE* value reaches an astonishing 1.2133 at 40 V and 20 Hz, which is 5055.42% higher than the proposed NT model. Overall, the maximum *SMAPE* values of the Dahl, LuGre, PI, BW, FP, and proposed hysteresis models are 0.539, 0.7143, 0.2195, 0.239, 1.2133, and 0.0456, respectively, occurring at 60 V and 70 Hz, 60 V and 20 Hz, 20 V and 40 Hz, 60 V and 60 Hz, 40 V and 20 Hz, and 20 V and 60 Hz, respectively, which are far higher than the maximum *SMAPE* value of the proposed NT model.

4.3.4. R^2 Analysis

R^2 results of the six hysteresis models are also analyzed under driving voltages of 20, 40, and 60 V, with frequencies ranging from 10 to 80 Hz at intervals of 10 Hz. It should be noted that the R^2 values of the Dahl, LuGre, PI, BW, and FP models all have results below 0.94, with the lowest R^2 values being 0.9399, 0.0116, 0.9921, 0.9095, and 0.2147, respectively (highlighted numbers in Table 2), occurring at 60 V and 70 Hz, 60 V and 20 Hz, 60 V and 70 Hz, 60 V and 70 Hz, and 60 V and 70 Hz. The lowest R^2 value of the proposed NT model is 0.9994 at 60 V and 10 Hz, significantly higher than the R^2 values of the other five hysteresis models.

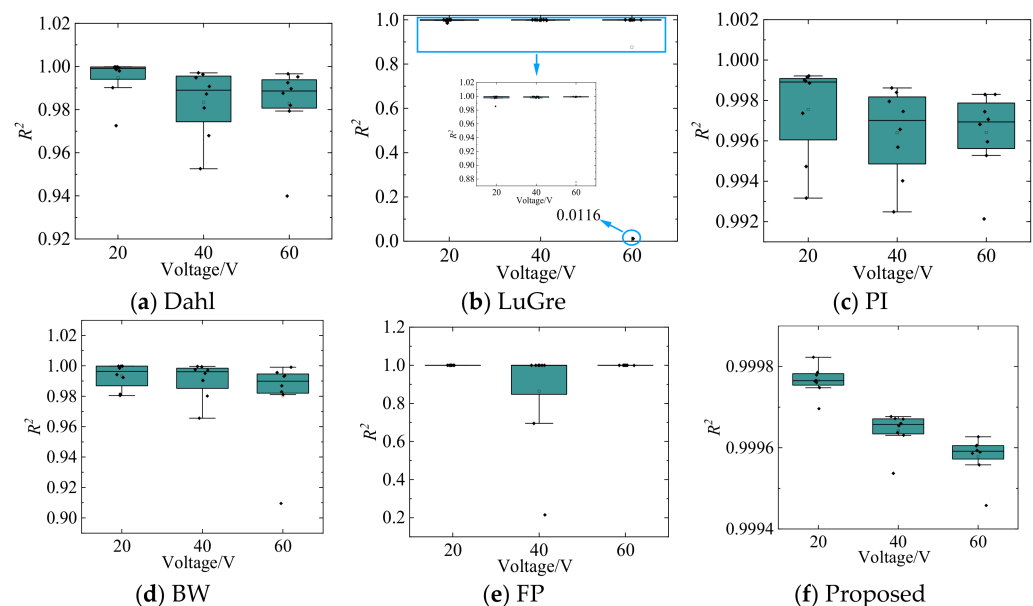
Table 2. Comparison of running time.

Model		10 Hz (s)	20 Hz (s)	30 Hz (s)	40 Hz (s)	50 Hz (s)	60 Hz (s)	70 Hz (s)	80 Hz (s)
Dahl	0-20 V-0	7.2841	5.2841	3.5871	6.2871	6.9415	5.9815	6.2871	10.3987
	0-40 V-0	8.5118	7.2819	8.2841	11.5719	7.5811	8.8418	6.9981	6.2871
	0-60 V-0	6.2155	7.2874	7.2884	7.8512	4.9952	8.0175	5.5215	9.2871
LuGre	0-20 V-0	9.5871	11.6587	8.2874	7.2871	6.5841	7.8501	9.5812	11.2841
	0-40 V-0	8.5844	9.8518	9.9951	11.3894	9.5274	8.5052	6.8418	20.6813
	0-60 V-0	8.0284	9.0284	7.8415	8.6841	9.8408	8.0768	5.9047	8.6512
PI	0-20 V-0	0.3857	0.6384	0.6121	0.7355	0.6358	0.5685	0.6015	0.5325
	0-40 V-0	0.5964	0.5981	0.6874	0.6428	0.6927	0.5027	0.5987	0.5027
	0-60 V-0	0.4287	0.5625	0.5897	0.5024	0.7268	0.5154	0.6657	0.4685

Table 2. Cont.

	Model	10 Hz (s)	20 Hz (s)	30 Hz (s)	40 Hz (s)	50 Hz (s)	60 Hz (s)	70 Hz (s)	80 Hz (s)
BW	0-20 V-0	0.5517	1.3551	1.3651	3.9517	17.2541	5.0287	11.2854	2.9418
	0-40 V-0	12.5521	4.1587	4.3514	5.6841	9.2541	10.2384	12.8941	6.9514
	0-60 V-0	5.9658	4.5871	4.9514	4.6811	23.5186	14.5187	9.9671	13.5417
FP	0-20 V-0	15.8224	9.5812	9.5521	6.8421	18.6418	8.6418	8.2541	8.2841
	0-40 V-0	11.5505	8.9587	7.5138	11.4179	11.3813	8.0017	7.2887	16.5712
	0-60 V-0	8.5128	8.3681	7.9928	16.8541	9.8571	7.5298	7.9817	9.5518
Proposed	0-20 V-0	1.6517	1.3641	2.3171	3.3544	4.7025	3.1821	4.3874	2.9517
	0-40 V-0	3.9517	1.6984	2.3351	3.4812	5.6651	5.2471	6.3514	4.7518
	0-60 V-0	5.8941	1.4173	2.0017	1.3254	3.9117	4.2541	5.9577	7.5281

The R^2 distribution map of the six hysteresis models under all loading conditions is further given in Figure 15. It can be seen that the R^2 distribution of the proposed NT model is the most concentrated, with the smallest errors.

Figure 15. Statistical distributions of R^2 among the six models.

4.3.5. Worst R^2 Performance

The relationship between the measured value and the model value when the six hysteresis models reach their own lowest R^2 values is further demonstrated in Figure 16. The dashed line represents the data fitting comparison line $y = x$. The closer the data is to the line $y = x$, the better the model fitting effect. The proposed NT model achieves perfect matching between the model value and the measured value when its lowest R^2 is 0.9994, fully demonstrating the good fitting effect of the proposed NT model on the experimental data.

4.3.6. Running Time Performance

As can be seen from the Table 2, the overall computation time of the proposed NT hysteresis model is satisfactory and ranks at a good level among the six hysteresis models.

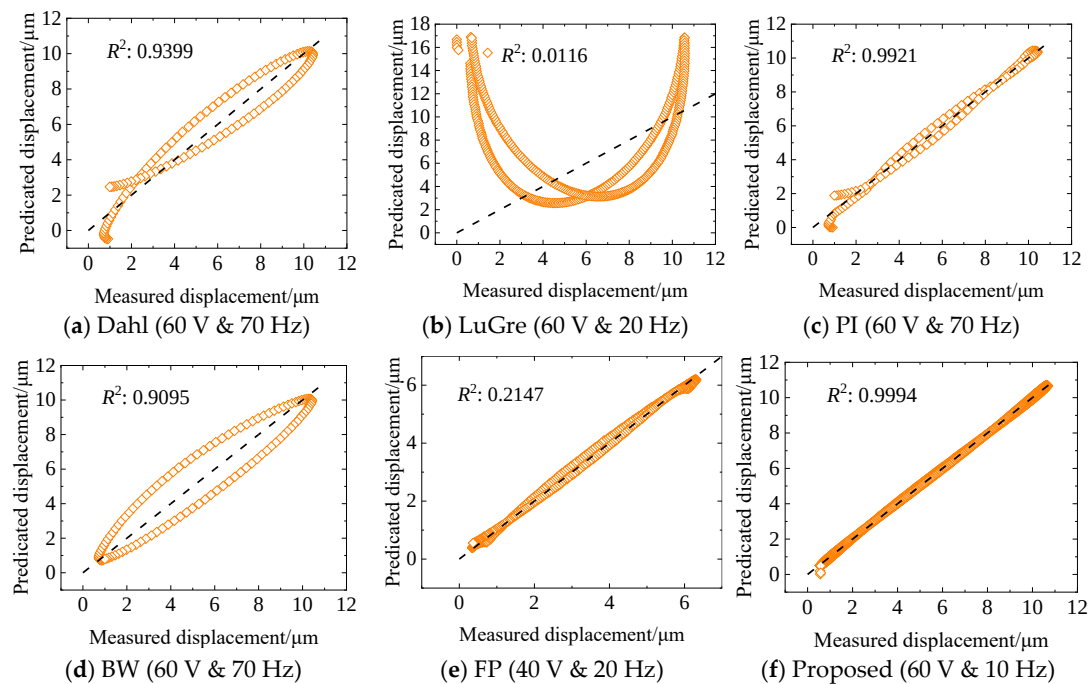


Figure 16. Worst R^2 analysis among the six models.

4.4. Fitting Error Analysis of the Proposed NT Model

After confirming the advantages of the proposed NT model in terms of fitting accuracy and stability, the analysis of fitting error is further conducted, as shown in Figure 17. The relative error between the fitted data and the experimental data is presented, with frequencies ranging from 10 to 80 Hz at driving voltages of 20 V, 40 V, and 60 V. As shown in Figure 17, the hysteresis fitting error generally increases with the increase in driving voltage. Under the driving voltages of 20 V, 40 V, and 60 V, the error is within the ranges of $-0.02 \sim 0.02 \mu\text{m}$, $-0.06 \sim 0.06 \mu\text{m}$, and $-0.13 \sim 0.12 \mu\text{m}$, respectively, indicating that the fitting error of the proposed NT model presents a satisfactory acceptable level. The maximum fitting error occurs under the loading condition of 60 V and 40 Hz, with a maximum fitting error of $0.13 \mu\text{m}$. It can be seen that under the 20 V driving voltage, the maximum fitting error of the proposed NT model is only $0.02 \mu\text{m}$, indicating that the fitting accuracy of the proposed NT model is very good. In summary, the proposed NT model has a small fitting error and a relatively stable distribution and can effectively reflect the nonlinear hysteretic features of the piezoelectric actuator.

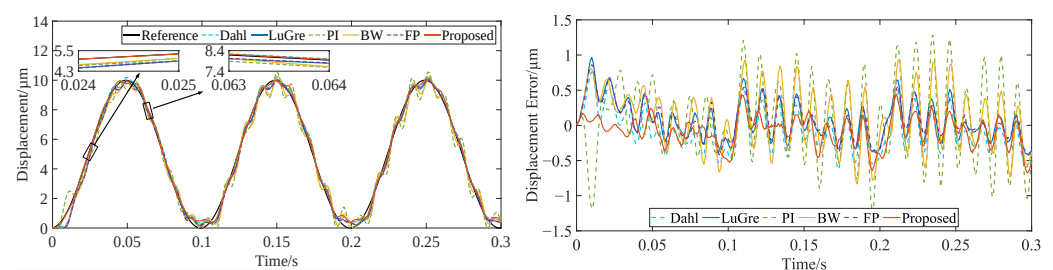


Figure 17. Feedforward compensation performance of different models.

4.5. Performance Comparison

Based on the above analysis, a performance comparison of the six hysteresis models is provided and listed in Table 3. The comparison demonstrates that the proposed NT model has the advantages of simple structure, few parameters, convenient solving, and

high fitting accuracy. It comprehensively outperforms the Dahl, LuGre, PI, BW, and FP models in terms of *RMSE*, *MAPE*, *SMAPE*, and R^2 .

Table 3. Performance comparisons between the proposed model and other models.

Performance	Proposed	Dahl	LuGre	PI	BW	FP
None of nonlinear equation	✓	×	×	✓	×	×
None of intermediate variable	✓	×	×	✓	×	×
Parameters number	4	5	6	4	5	4
Maximum <i>RMSE</i>	0.0812	0.8485	35.6933	0.3219	1.0414	3.6322
Maximum <i>MAPE</i>	0.045	0.2927	7218.6	0.1694	0.2591	0.6044
Maximum <i>SMAPE</i>	0.0456	0.539	0.7143	0.2195	0.239	1.2133
Minimum R^2	0.9994	0.9399	0.0116	0.9921	0.9095	0.2147
Maximum error (μm) (20 V)	0.02	0.03	0.03	0.15	0.05	0.03
Maximum error (μm) (40 V)	0.08	0.24	0.32	0.52	0.94	67.11
Maximum error (μm) (60 V)	0.13	1.47	659.31	0.91	1.73	67.11

Footnote: ✓ stands for conformity; × stands for non-conformity.

5. Trajectory Tracking Control and Positioning Control Performance of the Inverse Model

To systematically evaluate the effectiveness of the proposed NT hysteresis inverse model in the motion control of piezoelectric actuators, feedforward control, sinusoidal trajectory tracking control and step positioning control experiments are carried out. Through comparative analyses with existing hysteresis models and conventional feedback control methods, the performance superiority of the proposed model in hysteresis compensation, dynamic trajectory tracking and fast positioning control is verified.

First, to investigate the feedforward compensation performance of different hysteresis inverse models, the corresponding inverse control voltages are calculated based on the proposed NT, Dahl, LuGre, PI, BW and FP models, respectively, which are applied to the piezoelectric actuator as feedforward inputs. The feedforward control performances under different models are presented in Figure 17 and Table 4. As indicated by the plotted data and tabulated results, the feedforward control based on the proposed NT hysteresis inverse model achieves the minimum trajectory tracking error, with a maximum tracking error $e_{\max}$ of 0.675 μm and a Root Mean Square Error (*RMSE*) of 0.227 μm , which are lower than those obtained from the Dahl, LuGre, PI, BW and FP models. Compared with the PI model, the proposed model reduces $e_{\max}$ and *RMSE* by 47.5% and 59.0%, respectively. In contrast to the BW model, the two indices drop by 27.3% and 46.2%; versus the FP model, the reductions reach 21.7% and 14.3%. In addition, the proposed model also yields smaller maximum error and *RMSE* than the Dahl and LuGre models. The above results demonstrate that the proposed NT model can not only accurately characterize the hysteresis nonlinearity of piezoelectric actuators, but its inverse version can generate more precise feedforward compensation voltage, thereby effectively suppressing the tracking error induced by hysteresis effects.

Table 4. Feedforward control performance comparisons between the proposed model and other models.

Performance	Proposed	Dahl	LuGre	PI	BW	FP
$e_{\max}$ (μm)	0.675	0.759	0.966	1.285	0.929	0.862
<i>RMSE</i> (μm)	0.227	0.298	0.286	0.554	0.422	0.265

Further, sinusoidal trajectory tracking experiments under various operating conditions are implemented to verify the compensation capability of the proposed NT hysteresis

inverse model in closed-loop trajectory tracking control. Three control strategies are compared in the experiments: pure PID feedback control, feedforward control based on the NT inverse model, and composite control combining NT inverse-model feedforward with PID feedback. To evaluate the adaptability of the proposed control scheme under varying frequencies and complicated trajectories, three reference trajectories are adopted: low-frequency sinusoidal trajectory at 0.5 Hz, high-frequency sinusoidal trajectory at 10 Hz, and variable-amplitude variable-frequency sinusoidal trajectory at frequencies ranging from 0.5 to 10 Hz. The mathematical expression of the sinusoidal reference trajectory is given as

$$d_r(t) = \frac{A_i}{2} \sin(2\pi f_i t - \pi/2) + \frac{A_i}{2} \quad (17)$$

where A_i denotes the sinusoidal amplitude and f_i stands for the trajectory frequency.

Experimental results are summarized in Figure 18 and Table 5.

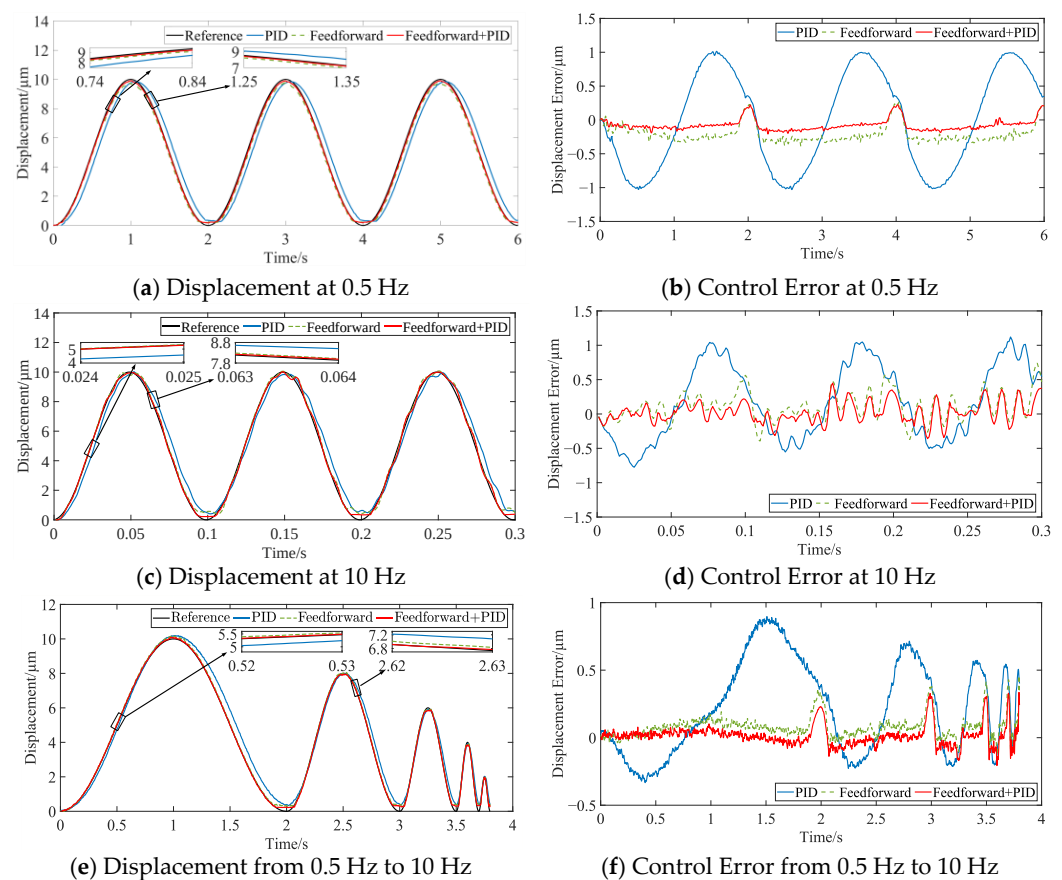


Figure 18. Sinusoidal trajectory tracking under different control methods.

Table 5. Sinusoidal trajectory tracking performance under different control methods.

Frequency	Performance	PID	Feedforward	Feedforward + PID
0.5 Hz	$e_{\max}$ (μm)	1.029	0.394	0.270
	RMSE (μm)	0.718	0.248	0.113
10 Hz	$e_{\max}$ (μm)	1.121	0.741	0.659
	RMSE (μm)	0.567	0.242	0.183
0.5 to 10 Hz	$e_{\max}$ (μm)	0.897	0.461	0.336
	RMSE (μm)	0.414	0.114	0.077

For the low-frequency sinusoidal trajectory with an amplitude of 10 μm at 0.5 Hz, large tracking errors under standalone PID control were observed, with a maximum tracking error e_{max} of 1.029 μm and an $RMSE$ of 0.718 μm , accounting for 10.29% and 7.18% of the full stroke, respectively. After introducing feedforward compensation using the proposed NT hysteresis inverse model, the e_{max} drops to 0.394 μm and $RMSE$ decreases to 0.248 μm , corresponding to 3.94% and 2.48% of the total travel stroke. When the composite feedforward-plus-PID control is applied, the tracking precision is remarkably improved, with an e_{max} of 0.270 μm and $RMSE$ of 0.113 μm , which occupy merely 2.70% and 1.13% of the full stroke. Compared with conventional standalone PID control, the feedforward-only strategy reduces the e_{max} and $RMSE$ by 61.7% and 65.5%. If the composite feedforward and PID control is adopted, error reductions can be achieved to 73.7% and 84.3%. These results demonstrate that the proposed NT hysteresis inverse model can effectively compensate hysteresis nonlinearity during low-frequency motion, and the cooperation with feedback regulation can further boost trajectory tracking accuracy.

To evaluate the dynamic tracking performance of the proposed control method under high-frequency excitation, high-frequency sinusoidal trajectory tracking experiments with an amplitude of 10 μm at 10 Hz are carried out. The experimental results are presented in Figure 18c,d, and the corresponding 10 Hz data is shown in Table 5. Under the 10 Hz high-frequency sinusoidal excitation, standalone PID control yields a maximum tracking error e_{max} of 1.121 μm and an $RMSE$ of 0.567 μm . After adopting the NT inverse-model feedforward control, these two indices drop to 0.741 μm and 0.242 μm , representing reductions of 33.9% and 57.3% compared with pure PID control. When the composite feedforward-plus-PID control is further implemented, the e_{max} and $RMSE$ are reduced to 0.659 μm and 0.183 μm , decreasing by 41.2% and 67.7% relative to PID alone. Although high-frequency input aggravates the adverse effects of dynamic lag and modeling errors on tracking accuracy, the proposed NT inverse model still considerably suppresses tracking errors, verifying its favorable compensation capability for high-speed motion control. Moreover, the composite control further reduces tracking errors, which reveals excellent complementarity between feedforward compensation and feedback regulation.

To further verify the robustness of the developed control method under complicated operating conditions, variable-amplitude and variable-frequency sinusoidal tracking experiments are performed. The reference trajectory amplitude A_i is sequentially set to 10, 8, 6, 4 and 2 μm with corresponding frequencies of 0.5, 1, 2, 5 and 10 Hz. The experimental outcomes are plotted in Figure 18e,f and listed in the relevant 0.5~10 Hz columns of Table 5. For the variable-amplitude variable-frequency trajectory, pure PID control results in an e_{max} of 0.897 μm and $RMSE$ of 0.414 μm . Under NT inverse-model feedforward control, the e_{max} and $RMSE$ decline to 0.461 and 0.114 μm , which are reduced by 48.6% and 72.5% in contrast to PID control. The composite feedforward-PID control achieves further error reduction, with the e_{max} and $RMSE$ reaching 0.336 and 0.077 μm , corresponding to error reductions of 62.5% and 81.4% against the PID strategy. These results demonstrate that the proposed NT hysteresis inverse model maintains effective feedforward compensation even when the amplitude and frequency of reference trajectories vary, which confirms the satisfactory dynamic adaptability and control robustness of the proposed method.

In all, it can be concluded that the feedforward control of the NT inverse model can significantly decrease the tracking errors of pure PID control under low-frequency, high-frequency, and variable-amplitude variable-frequency trajectories. Further, the feedforward-PID composite control achieves the minimum e_{max} and $RMSE$ across all tested working conditions. It is concluded that the proposed NT hysteresis inverse model can not only improve open-loop hysteresis compensation for piezoelectric actuators but also further

elevate the trajectory tracking precision of the system under complex dynamic motions when combined with feedback control.

To further analyze the response performance of the proposed NT hysteresis inverse model under dynamic working conditions, a step signal with an amplitude of 10 μm was selected as the target displacement. The dynamic response characteristics of conventional PID control and the NT hysteresis inverse-model feedforward + PID compound control were compared, and the results can be observed in Figure 19.

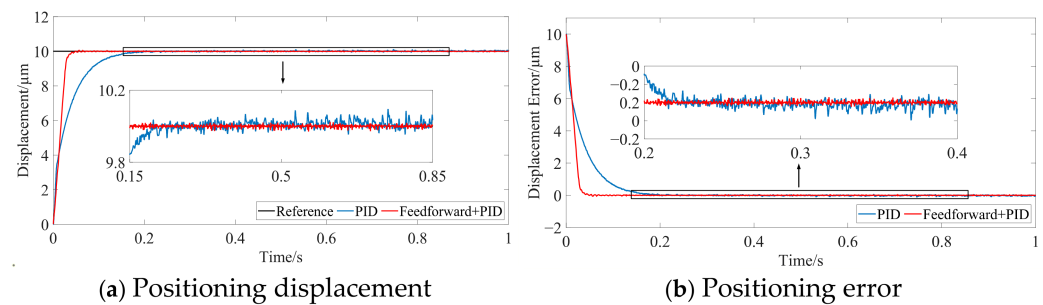


Figure 19. Dynamic responses under different control methods.

As depicted in Figure 19a, upon adopting the compound control strategy, the displacement response can approach the target displacement faster and more accurately. Compared with PID control, the proposed method achieves both faster response speed and higher tracking accuracy, indicating that the established hysteresis inverse model can rapidly predict the required control voltage according to the target displacement. In addition, Figure 19b further shows that the compound control method maintains a lower positioning error during the entire response process, demonstrating that feedforward compensation based on the hysteresis inverse model can effectively suppress the nonlinear error caused by hysteresis.

Table 6 summarizes the key dynamic performance index under the two control strategies, including rise time T_r , settling time T_s , maximum tracking error $e_{\max}$, and Root Mean Square Error $RMSE$. Here, T_r is defined as the time required for the system response to reach 90% of the steady-state value, and T_s denotes the time required for the system response to enter and stabilize within a $\pm 5\%$ error band. The experimental results show that the T_r , T_s , $e_{\max}$ and $RMSE$ under the compound control strategy are 0.027 s, 0.076 s, 0.042 μm and 0.009 μm , respectively, which are reduced by 68.0%, 66.8%, 56.3% and 64.8% compared with PID control. Therefore, the proposed NT hysteresis inverse-model feedforward + PID feedback compound control method exhibits significant advantages in improving response speed and positioning accuracy, showing good application potential in high-precision motion control.

Table 6. Dynamic response performance under different control methods.

Performance	PID	Feedforward + PID
T_r (s)	0.084	0.027
T_s (s)	0.229	0.076
$e_{\max}$ (μm)	0.095	0.042
$RMSE$ (μm)	0.026	0.009

6. Conclusions

Based on the modeling idea of nonlinear transformation, a novel NT model is developed to describe the nonlinear hysteresis response of piezoelectric actuators. The model is simple in structure, has fewer unknown parameters, and does not include highly non-

linear differential equations in its expression. Moreover, it does not require determining intermediate functions, making it easier to solve.

The performance of common hysteresis models and the proposed NT model is comprehensively evaluated, using evaluation indicators such as *RMSE*, *MAPE*, *SMAPE*, and R^2 . The following conclusions can be drawn: (1) the proposed NT model can effectively describe the voltage-displacement response of piezoelectric actuators, achieving high precision in calculation and small predictive deviation; (2) the proposed NT model outperforms the Dahl, LuGre, PI, BW, and FP models with regard to the *RMSE*, *MAPE*, *SMAPE*, and R^2 ; the maximum fitting errors of the proposed NT model under driving voltages of 20 V, 40 V, and 60 V are 0.02 μm , 0.08 μm , and 0.13 μm , respectively.

Hysteresis compensation is further performed. Through combining feedforward compensation with PID feedback, the system performance is significantly improved. The maximum error e_{max} and *RMSE* are reduced to 0.270 μm and 0.113 μm , respectively, accounting for only 2.70% and 1.13% of the total stroke.

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