

Article

Synchronous Control of the Anti-Back-Slip Support System for Hard-Rock TBMs in Large-Inclination Shafts

Linxiao Yao ^{1,2}, Mingzhao Li ^{1,3}, Linjian Shangguan ^{1,3,*}, Bing Li ¹ and Jiahui Wang ¹

¹ School of Mechanical Engineering, North China University of Water Resources and Electric Power, Zhengzhou 450045, China; yaolinxiao@ncwu.edu.cn (L.Y.); limingzhao0326@163.com (M.L.); david19881007@163.com (B.L.); 18438133285@163.com (J.W.)

² School of Mechanical Engineering, Northwestern Polytechnical University, Xi'an 710072, China

³ School of Intelligent Manufacturing, Luoyang Institute of Science and Technology, Luoyang 471023, China

* Correspondence: sgljbh@163.com

Abstract

The underground caverns of pumped-storage power stations generally feature large inclination angles. During the bottom-up oblique excavation by hard-rock Tunnel Boring Machines (TBMs), the Anti-Back-Slip (ABS) support system is the core device ensuring safe operations. Specifically, the synchronization of the multiple hydraulic cylinders within the ABS system is a critical factor determining the stability and safety of the TBM. Therefore, this paper designs a hydraulic control system for the ABS device and proposes an adjacent cross-coupling synergistic control strategy based on adaptive backstepping. This strategy innovatively integrates an adaptive backstepping control law into the adjacent cross-coupling topology to achieve high-precision multi-cylinder control. Utilizing the AMESim-Simulink platform, high-fidelity co-simulations are conducted under both uniform and eccentric load conditions. The results demonstrate that under nominal conditions, the proposed algorithm exhibits asymptotic convergence at the mathematical level. The system maintains robust stability under dynamic excitations. When subjected to sudden asymmetric eccentric loads of 1.0–2.0 times, the system prevents tracking divergence and limits the maximum multi-cylinder synchronization error to within 1.82 mm. This research satisfies the requirements for synchronous control and provides a theoretical and engineering reference for the disturbance-rejection synergy of inclined shaft TBM support systems.

Keywords: inclined shaft TBM; Anti-Back-Slip (ABS) support system; electro-hydraulic servo system; adaptive backstepping; synchronous control



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1. Introduction

As the “stabilizers” and “high-capacity batteries” of modern power grids, pumped-storage power stations play a key role in peak shaving, valley filling, and ensuring the large-scale integration of renewable energy [1–3]. However, the underground cavern groups constructed for pumped-storage power stations often feature complex geological structures and large excavation inclination angles. Traditional hard-rock tunnel boring machines are mostly designed for flat or mildly sloping tunnels. During bottom-up excavation in large-inclination shafts, the equipment is continuously subjected to the component of its own gravity, rendering it highly susceptible to slipping or falling accidents [4–6].

To address the challenges of large-inclination excavation, the Anti-Back-Slip support system has been widely introduced and serves as the core device ensuring the operational safety of inclined shaft TBMs [7]. During the dynamic excavation and regripping process,

the ABS support system must withstand axial eccentric loads and rock reaction forces. Furthermore, the posture and displacement synchronization accuracy of its hydraulic cylinders directly determine the overall safety, excavation efficiency, and support stability of the TBM [8,9]. Directional control valves or servo valves are extensively used for the precise control of hydraulic cylinders. The latest solutions for synchronizing cylinder operations utilize proportional valves coupled with the individual, continuous measurement of cylinder positions. However, inaccuracies in actuator synchronization still occur in practical applications. This is due, among other things, to the capacitance of the system (such as the supply lines) and the unstable operation of the actuator control valves. Furthermore, the unstable operation of the valves and the uneven operation of the actuators may be caused by the excitation of spool vibrations induced by external mechanical vibrations [10]. Once the multiple hydraulic cylinders become desynchronized, it will cause uneven loading on the gripper shoes, physical distortion of the frame, and even the overturning and instability of the entire excavation system [7]. Research on high-precision multi-cylinder synchronous control specifically dedicated to the ABS systems of inclined shaft TBMs remains scarce.

Existing research on the anti-falling and posture control of heavy-duty equipment mostly focuses on mine hoisting systems, coal mine hydraulic supports, or traditional lifting equipment. Giraud and Lonkwic designed mechanical safety catches for vertical shaft hoisting systems [11,12]. However, these purely mechanical structures are primarily passive trigger-locking devices. They cannot meet the real-time posture adjustment requirements during the continuous dynamic excavation process of TBMs. Zhang et al. thoroughly investigated the support control strategies of hydraulic supports in fully mechanized coal mining faces. However, their application scenarios are primarily static support, which does not involve dynamic multi-cylinder synergy while overcoming a significant gravitational component [13,14]. In addition, Zhang, Wang, et al. studied dual-cylinder and multi-cylinder synchronous lifting systems of large cranes, mostly adopting master-slave control or traditional PID algorithms [15,16]. Loren and Koren proposed three typical multi-cylinder synchronous control methods: master-slave synchronization, equal-status synchronization, and cross-coupling synchronization [17,18]. Among them, master-slave control exhibits certain hysteresis, and equal-status control lacks mutual feedback between individual loops. While the cross-coupling strategy provides accuracy and stability for dual-cylinder control, it becomes structurally complex when expanded to four cylinders. This complexity can lead to deteriorated synchronization accuracy and stability. Furthermore, due to the environment inside the TBM inclined shaft, traditional linear control algorithms often exhibit limitations such as insufficient robustness, weak disturbance-rejection stiffness, and a tendency for synchronization errors to diverge. These limitations render them inadequate for the safety requirements of the ABS system.

In summary, traditional anti-falling mechanisms and conventional synchronous control algorithms are no longer sufficient to meet the synergistic requirements of inclined shaft TBMs under complex working conditions. Therefore, taking the ABS hydraulic support system of inclined shaft TBMs as the research object, this paper designs a hydraulic control system for the ABS device and proposes an adjacent cross-coupling synergistic control strategy based on adaptive backstepping. This approach achieves synchronous control without requiring a precise mathematical model of the system. Relying on the AMESim-Simulink platform, co-simulations are conducted under both nominal and eccentric load conditions. The results indicate that under nominal conditions, the proposed algorithm exhibits asymptotic convergence at the mathematical level when tracking four different sets of desired signals. When subjected to sudden asymmetric dynamic eccentric loads of 1.0–2.0 times, the system prevents tracking divergence and limits the maximum multi-cylinder synchronization error to within 1.82 mm. The proposed method enhances

the synchronization accuracy and stability of the ABS support system under complex working conditions.

2. Mechanical Model Construction

2.1. Scheme Design of the ABS Hydraulic System

The overall structure of the inclined shaft TBM is shown in Figure 1a. Specifically, the cutterhead 1 is primarily utilized for rock-breaking operations; the main shaft 2 provides the excavation power; the main support device 3 is responsible for supporting the TBM main machine and bearing the thrust force; the segment erector 4 is tasked with assembling the tunnel lining rings; the dual ABS support devices 5 jointly maintain the stability of the inclined shaft TBM during the dynamic excavation and regripping process to prevent the equipment from sliding; and the trailing gear 6 undertakes tasks such as muck transportation.

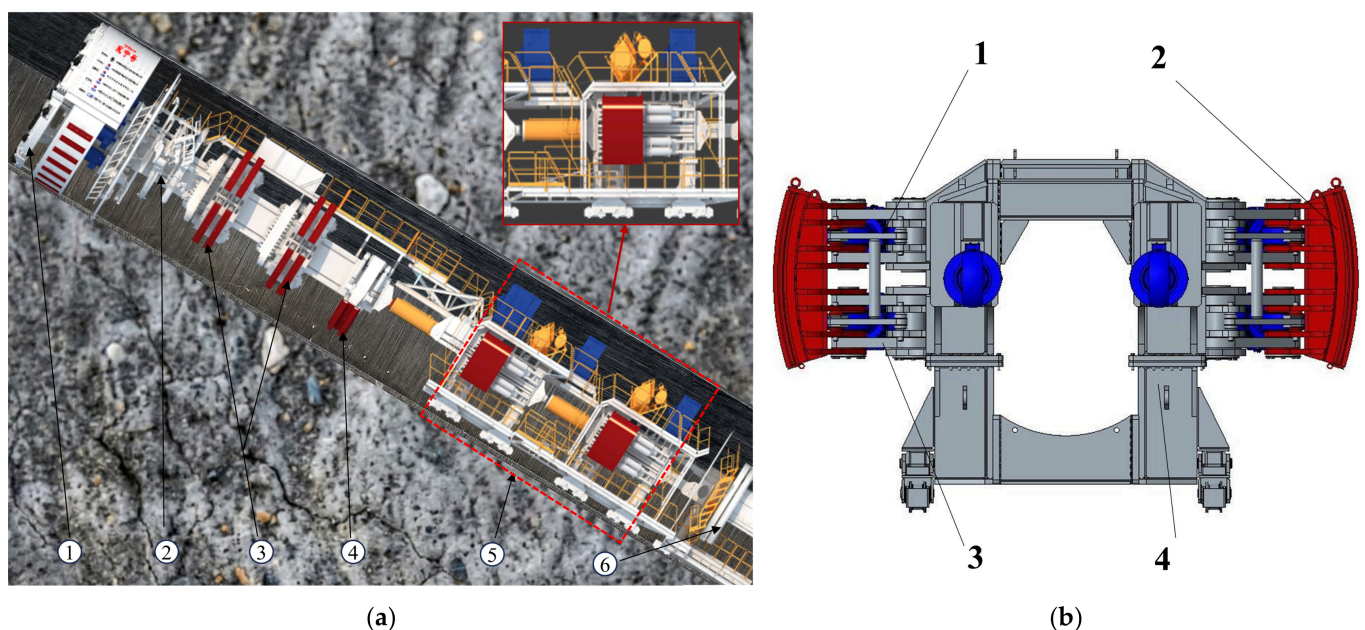


Figure 1. The hard-rock TBM and its Anti-Back-Slip support device: (a) Overall structure of the TBM; (b) Cross-sectional view of the ABS device. Note: (a) 1. Cutterhead. 2. Main shaft. 3. Main support device. 4. Segment erector. 5. Dual ABS support devices. 6. Trailing gear. (b) 1. Hydraulic cylinder. 2. Gripper shoe. 3. Support plate. 4. Main beam.

The detailed mechanical configuration of the ABS device for the inclined shaft TBM is illustrated in Figure 1b. It mainly comprises the hydraulic cylinder 1, gripper shoe 2, support plate 3, main beam 4, and other associated components. The working principle of the ABS support system is as follows: the main beam remains relatively stationary in the radial direction, while the gripper shoes perform the actions of extending to grip tightly and retracting under the actuation of the hydraulic cylinders. The gripping force required when the gripper shoes tightly press against the cavern wall is jointly provided by the support plates and the hydraulic cylinders.

According to the working principle of the ABS support system, this paper adopts a multi-cylinder synchronous control circuit configuration in which each hydraulic cylinder is independently controlled by an electro-hydraulic servo valve. By integrating the multi-cylinder synchronous control strategy into the single-cylinder servo control loop, high-precision synchronous control of the entire system is achieved. The designed hydraulic synchronous control system is illustrated in Figure 2. The primary components

comprising this system include dual-motor pump units 3 and 4, a three-position four-way solenoid directional valve 6, a relief valve 5, a pilot-operated check valve 7, a two-position two-way solenoid directional valve 8, and an accumulator 11. Specifically, this design incorporates six hydraulic pumps. Five of these pumps (labeled as 4.2) are dedicated to low-pressure rapid synchronous extension, constituting the core synchronous control circuit investigated in this study. The remaining hydraulic pump (labeled as 4.1) is primarily responsible for high-pressure gripping and charging the accumulator. Furthermore, the accumulator 11 is utilized for pressure holding during power failures and shock absorption during the cylinder gripping process, while the relief valve 5 ensures overload protection. Ultimately, this structural design not only conserves energy but also extends the service life of the equipment.

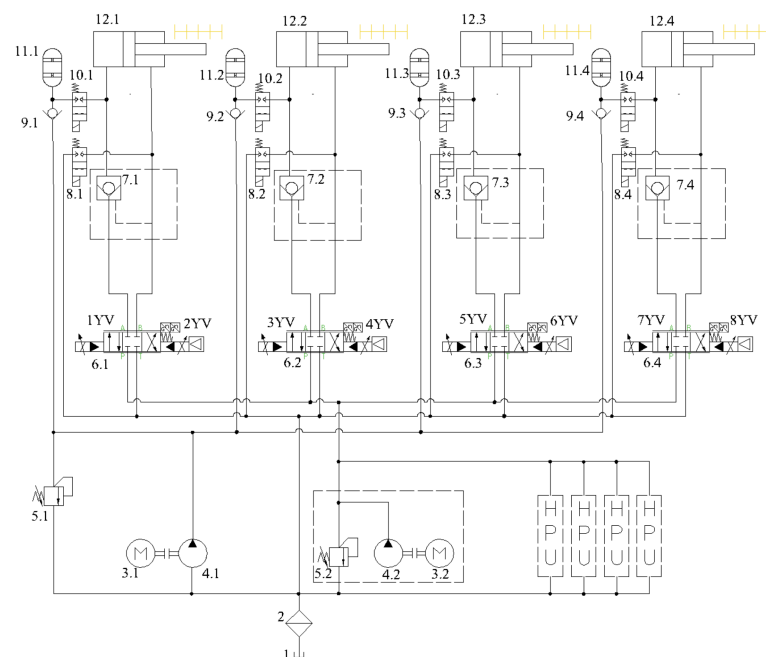


Figure 2. Hydraulic system schematic of the ABS support system. Note: 1. Oil tank; 2. hydraulic return filter; 3. Motor; 4. Hydraulic pump; 5. Relief valve; 6. Three-position four-way solenoid directional valve; 7. Pilot-operated check valve; 8. Two-position two-way solenoid directional valve; 9. Check valve; 10. Two-position two-way solenoid directional valve; 11. Accumulator; 12. Hydraulic cylinder. HPU denotes the Hydraulic Power Unit.

The selection of two-stage proportional directional control valves (6.1–6.4) alongside direct-acting single-stage relief valves (5.1–5.2) is strictly dictated by the distinct fluid power requirements and dynamic objectives of their respective sub-circuits. The primary hydraulic cylinders (12.1–12.4) of the Anti-Back-Slip support system demand an exceptionally large flow capacity combined with high-precision throttling capabilities to sustain the structural load of the TBM during large-inclination shaft excavation. If direct-acting (single-stage) proportional or servo valves were employed to handle such high volumetric flow rates, they would require excessively large, high-power solenoids. This would inevitably result in severe flow-force disturbances, sluggish dynamic responses, and a bulkier footprint. Thus, pilot-operated (two-stage) configurations are selected for components 6.1–6.4 to guarantee high flow rates while maintaining precise. Conversely, the relief valves (5.1 and 5.2) function purely as safety and system-pressure regulation devices designed to mitigate sudden dynamic pressure spikes. For these components, minimizing operational latency is the absolute priority. A single-stage relief valve successfully bypasses the inherent pilot-stage oil propagation delays and pilot-line fluid capacitance, thereby achieving an

instantaneous pressure-override response to safeguard the entire hydraulic circuit from catastrophic pressure spikes.

The control architecture for each hydraulic cylinder unit is illustrated in Figure 3. The underlying control principle is characterized by a closed-loop feedback mechanism: various sensors detect the real-time position, velocity, and acting force of the piston rod, converting these physical parameters into electrical signals for the controller. The controller then processes these signals and outputs a command, which is amplified by a proportional amplifier into a control voltage to actuate the servo-proportional valve. The resulting spool displacement modifies the orifice area, thereby regulating the inflow and outflow of hydraulic fluid according to the flow equations. Consequently, the dynamic motion of the hydraulic cylinder's piston rod is precisely modulated.

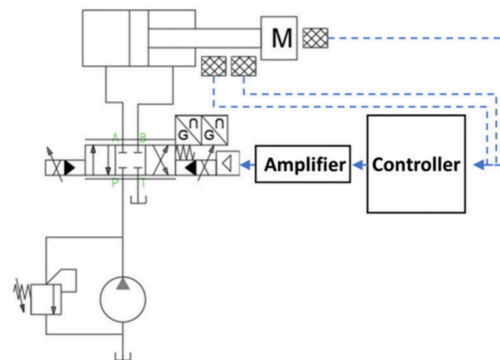


Figure 3. Schematic diagram of the single-cylinder position servo control system.

2.2. Mathematical Modeling of the Hydraulic Control System

In practical electro-hydraulic servo systems, it is challenging to establish a precise mathematical model due to parametric uncertainties and external disturbances. To facilitate controller design while preserving the primary dynamic characteristics of the system, the following reasonable simplifications are made:

1. The servo valve is assumed to be a symmetric four-way valve with a constant orifice area gradient.
2. The bulk modulus of the hydraulic oil and the oil temperature remain constant.
3. The supply pressure is stable, and the return pressure is neglected.
4. External leakage of the hydraulic cylinder is ignored, with only internal leakage considered.

The schematic of the valve-controlled single-cylinder system is shown in Figure 4.

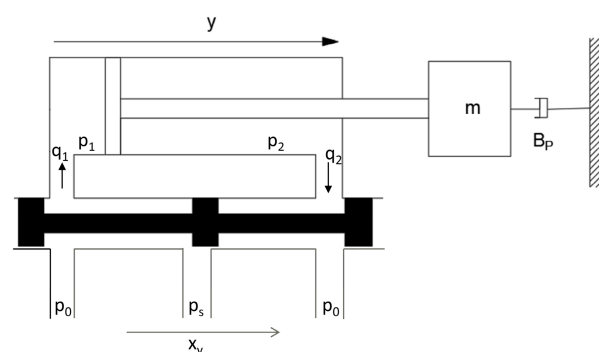


Figure 4. Schematic of the valve-controlled hydraulic cylinder.

The physical notations in Figure 4 are defined as follows: x_v is the displacement of the servo valve spool; p_0 and p_s denote the supply and return pressures, respectively; q_1 and q_2

represent the flow rates into the rodless and rod chambers of the hydraulic cylinder; p_1 and p_2 are the pressures in the rodless and rod chambers, respectively; y is the displacement of the piston rod; m is the equivalent load mass; and B_p is the viscous damping coefficient of the load.

2.2.1. Flow Equation of the Electro-Hydraulic Servo Valve

1. Dynamic Equation of the Servo Valve

The simplification of the servo valve's dynamic equation depends on the ratio of its bandwidth to the natural frequency of the actuating mechanism. Under working conditions where the response bandwidth of the servo valve is significantly larger than the natural frequency of the power system, the dynamic equation of the servo valve can be simplified to a first-order form, as shown in Equation (1):

$$x_v = k_i \mu \quad (1)$$

where k_i is the proportional coefficient between the servo valve spool displacement and the control signal; and μ is the control input.

2. Flow Equation of the Spool Valve

The flow rate entering the electro-hydraulic servo valve q is proportional to the input displacement of the spool x_v . Assuming that the flow gains at both ports of the servo valve are equal, denoted as k_q , and the effective bulk moduli of the hydraulic oil at both actuator ports are equal, denoted as β_e , a function is defined as follows:

$$s(u) = \begin{cases} 1, & u \geq 0 \\ 0, & u \leq 0 \end{cases} \quad (2)$$

Based on this assumption, the flow equation of the servo valve q_1 and q_2 is given by:

$$\begin{cases} q_1 = g R_1 \mu \\ q_2 = g R_2 \mu \\ R_1 = s(u) \sqrt{p_s - p_1} + s(-u) \sqrt{p_1 - p_r} \\ R_2 = s(u) \sqrt{p_s - p_2} + s(-u) \sqrt{p_2 - p_r} \end{cases} \quad (3)$$

where $g = \sqrt{2} k_q k_i$, p_r represents the return pressure of the system.

2.2.2. Flow Continuity Equation of the Hydraulic Cylinder

In contemporary hydraulic systems, owing to innovations in sealing technology, the phenomenon of external leakage has been effectively controlled [19]. Consequently, internal leakage has become the primary influencing factor. Therefore, during the system modeling process, the impact of external leakage can be safely neglected, requiring only the consideration of internal leakage factors [20]. Accordingly, the flow continuity equations of the hydraulic cylinder are expressed as Equations (4) and (5):

$$q_1 = A_1 \frac{dx_p}{dt} + C_t p_L + \frac{V_1}{\beta_e} \frac{dp_1}{dt} \quad (4)$$

$$q_2 = A_2 \frac{dx_p}{dt} + C_t p_L + \frac{V_2}{\beta_e} \frac{dp_2}{dt} \quad (5)$$

where

$$\begin{cases} p_L = p_1 - p_2 \\ V_1 = V_{01} + A_1 y \\ V_2 = V_{02} - A_2 y \end{cases} \quad (6)$$

In the above equations, β_e is the effective bulk modulus of the hydraulic oil; C_t is the internal leakage coefficient of the hydraulic cylinder; V_1 and V_2 denote the volumes of the rodless and rod chambers of the hydraulic cylinder, respectively; and V_{01} and V_{02} represent the initial volumes of the rodless and rod chambers, respectively.

To better reflect the state variations of the system, Equations (4) and (5) can be rewritten as:

$$\dot{p}_1 = \frac{\beta_{e1}}{V_1} (-A_1 \dot{y} - C_t p_L + q_1) \quad (7)$$

$$\dot{p}_2 = \frac{\beta_{e2}}{V_2} (A_2 \dot{y} + C_t p_L - q_2) \quad (8)$$

2.2.3. Dynamic Balance Equation of the Hydraulic Cylinder

In the modeling and analysis of hydraulic systems, the piston rod is typically selected as the object of study. The relationship between the output force of the hydraulic cylinder and the load force experienced during normal operation can be derived through a force analysis of the piston rod, yielding the force balance equation shown in Equation (9):

$$p_1 A_1 - p_2 A_2 = m \ddot{y} + B_P \dot{y} + A_f S_f + f(z_1, z_2) \quad (9)$$

where $f(z_1, z_2)$ represents the uncertain nonlinear forces caused by unmodeled friction and other hard-to-model disturbances; A_f is the amplitude of the modeled Coulomb friction; and S_f is a continuous shape function approximating the Coulomb friction.

The above equation describes the nonlinear model of the valve-controlled asymmetric cylinder, reflecting the nonlinear characteristics of the system. The subsequent design of the nonlinear controller for the electro-hydraulic position servo system in this paper is formulated based on this fundamental nonlinear model.

2.2.4. Cylinder State-Space Mathematical Model of the Electro-Hydraulic Position Servo System

As a time-domain modeling tool, the state-space method describes system dynamics through matrix operations. This approach plays a crucial role in controller design and stability analysis. Based on the nonlinear model of the hydraulic cylinder system, the state variables are defined as:

$$\vec{X} = [y, \dot{y}, \dot{p}_1, \dot{p}_2]^T \quad (10)$$

Then we have:

$$\begin{cases} \dot{x}_1 = x_2 \\ m \dot{x}_2 = A_1 x_3 - A_2 x_4 - B_P x_2 - A_f S_f - f(z_1, z_2) \\ \dot{p}_1 = \frac{\beta_e}{V_1} [-A_1 x_2 - C_t (p_1 - p_2) + q_1] \\ \dot{p}_2 = \frac{\beta_e}{V_2} [A_2 x_2 + C_t (p_1 - p_2) - q_2] \end{cases} \quad (11)$$

By defining $x_3 = A_1 p_1 - A_2 p_2$, the kinematic model of the electro-hydraulic servo system can be simplified to:

$$\begin{cases} \dot{x}_1 = x_2 \\ m \dot{x}_2 = x_3 - B_P x_2 - A_f S_f - f(z_1, z_2) \\ \dot{x}_3 = \frac{A_1 \beta_e}{V_1} [-A_1 x_2 - C_t (p_1 - p_2) + q_1] - \frac{A_2 \beta_e}{V_2} [A_2 x_2 + C_t (p_1 - p_2) + q_2] \end{cases} \quad (12)$$

The above equation reduces the motion model of the electro-hydraulic servo system into a set of first-order differential equations containing state variables. Through this methodology, the design of the control system is no longer confined merely to inputs, outputs, and error variables, providing a mathematical tool to enhance system performance.

2.3. Adaptive Backstepping Controller Design

Adaptive backstepping is a prominent method utilized for designing controllers for nonlinear systems, particularly suitable for nonlinear systems possessing a strict-feedback structure [21]. This methodology is fundamentally based on the Lyapunov stability theory for recursive design. By constructing Lyapunov functions step-by-step and sequentially introducing virtual control variables, it achieves stable control of the nonlinear system [22]. This approach is effective not only when the system parameters are completely known, but it also maintains system stability and tracking performance under conditions with multiple unknown parameters or parametric uncertainties by constructing adaptive laws for online parameter estimation.

Based on the state equations of the electro-hydraulic position servo system established previously and Equations (1)–(11), defining $\theta = [\theta_1, \theta_2, \theta_3, \theta_4]^T = [B_p, A_f, d_n, C_t]^T$ simplifies Equation (12) to:

$$\begin{cases} \dot{x}_1 = x_2 \\ m\dot{x}_2 = x_3 - \theta_1 x_2 - \theta_2 S_f - \theta_3 - d \\ \dot{x}_3 = g_3 u - f_c - \theta_4 f_u \end{cases} \quad (13)$$

where

$$\begin{cases} g_3 = \left(\frac{A_1 R_1}{V_1} + \frac{A_2 R_2}{V_2} \right) g \beta_e \\ f_c = \left(\frac{A_1^2}{V_1} + \frac{A_2^2}{V_2} \right) \beta_e x_2 \\ f_u = \left(\frac{A_1}{V_1} + \frac{A_2}{V_2} \right) \end{cases} \quad (14)$$

where d_n is the lumped nominal value of unmodeled dynamics and external disturbances, $d = f(z_1, z_2) - d_n$. It is assumed that the viscous damping coefficient of the piston and load, the elastic stiffness of the load, the unmodeled load force of the system, and the total leakage coefficient of the hydraulic cylinder act as unknown parameters. The control objective of this paper is: under the conditions of parameter uncertainties and external disturbances, to design an adaptive backstepping controller that enables the hydraulic cylinder piston displacement x_1 to accurately track the desired trajectory $x_d(t)$ while ensuring the stability of the closed-loop system.

Step 1:

Define the tracking error variable:

$$e_1 = x_1 - x_{1d} \quad (15)$$

To ensure the error e_1 converges to zero, a positive semi-definite Lyapunov function is defined as:

$$V_1(e_1) = \frac{1}{2} e_1^2 \quad (16)$$

Taking the derivative of the function and substituting the derivatives from the system state Equations (13) and (15) yields:

$$\dot{V}_1(e_1) = e_1 \dot{e}_1 = e_1 (\dot{x}_1 - \dot{x}_{1d}) = e_1 (x_2 - \dot{x}_{1d}) \quad (17)$$

For the derivative to be negative definite, the virtual control law can be designed as $\dot{V}_1(e_1) = -k_1 e_1^2$, thus:

$$x_2 = \dot{x}_{1d} - k_1 e_1 \quad (18)$$

Here x_2 is x_{2d} , that is $x_{2d} = \dot{x}_{1d} - k_1 e_1$.

This yields:

$$\dot{V}_1(e_1) = e_1 \dot{e}_1 = -k_1 e_1^2 + e_1 e_2 \quad (19)$$

Step 2:

Define the secondary error variable:

$$e_2 = x_2 - x_{2d} \quad (20)$$

To drive the e_2 to zero, a Lyapunov function must be formulated that includes both e_1 and e_2 . Only when both converge to zero can the system be considered stable:

$$V_2(e_1, e_2) = \frac{1}{2} k_1^2 e_1^2 + \frac{1}{2} m e_2^2 \quad (21)$$

$$\dot{V}_2(e_1, e_2) = k_1^2 e_1 \dot{e}_1 + m e_2 \dot{e}_2 \quad (22)$$

Substituting the relevant equations, and introducing the parameter estimates $\hat{\theta}$ in this section ensures the system can achieve dynamic adjustment.

$$e_3 = x_3 - x_{3d}, \tilde{\theta} = \hat{\theta} - \theta \quad (23)$$

Then we obtain:

$$\begin{aligned} \dot{V}_2(e_1, e_2) = & -k_1^3 e_1^2 + e_2(e_3 + x_{3d} - \hat{\theta}_1 x_2 - \hat{\theta}_2 S_f - \hat{\theta}_3) \\ & - (-\hat{\theta}_1 x_2 - \hat{\theta}_2 S_f - \hat{\theta}_3) - m \dot{x}_{2d} - d + k_1^2 e_1 \end{aligned} \quad (24)$$

To satisfy the negative-definite condition for the first-order derivative of Equation (24), it must satisfy:

$$x_{3d} = \hat{\theta}_1 x_2 + \hat{\theta}_2 S_f + \hat{\theta}_3 + m \dot{x}_{2d} + d - k_2 e_2 \quad (25)$$

Thus:

$$\dot{V}_2(e_1, e_2) = -k_1^3 e_1^2 + e_2 e_3 + k_1^2 e_1 e_2 - k_2 e_2^2 - e_2(-\hat{\theta}_1 x_2 - \hat{\theta}_2 S_f - \hat{\theta}_3) \quad (26)$$

Step 3:

Similarly, to ensure e_3 to zero, a Lyapunov function is designed that encompasses e_1 , e_2 and e_3 to guarantee holistic system stability.

$$V_3(e_1, e_2, e_3) = V_2(e_1, e_2) + \frac{1}{2} e_3^2 \quad (27)$$

Based on the derived equations, we obtain:

$$\dot{V}_3(e_1, e_2, e_3) = -k_1^3 e_1^2 + e_2 e_3 + k_1^2 e_1 e_2 - k_2 e_2^2 - e_2(-\hat{\theta}_1 x_2 - \hat{\theta}_2 S_f - \hat{\theta}_3) \quad (28)$$

By setting the appropriate conditions $g_3 u - f_c - \hat{\theta}_4 f_u - \dot{x}_{3d} = -k_3 e_3$, the final control law and the virtual variable are calculated as:

$$u = \frac{1}{g_3} (\dot{x}_{3d} + f_c + \hat{\theta}_4 f_u + \dot{x}_{3d} - k_3 e_3) \quad (29)$$

$$\dot{V}_3(e_1, e_2, e_3) = -k_1^3 e_1^2 + e_2 e_3 + k_1^2 e_1 e_2 - k_2 e_2^2 - e_2(-\hat{\theta}_1 x_2 - \hat{\theta}_2 S_f - \hat{\theta}_3) - k_3 e_3^2 + e_3 \hat{\theta}_4 f_u \quad (30)$$

Step 4:

Define:

$$V_4(e_1, e_2, e_3, \tilde{\theta}) = V_3(e_1, e_2, e_3) + \frac{1}{2} \tilde{\theta}^T \Gamma^{-1} \tilde{\theta} \quad (31)$$

Then:

$$\dot{V}_4(e_1, e_2, e_3, \tilde{\theta}) = -k_1^3 e_1^2 + e_2 e_3 + k_1^2 e_1 e_2 - k_2 e_2^2 - k_3 e_3^2 + \tilde{\theta}^T \left[\Gamma^{-1} \dot{\tilde{\theta}} - \varphi_2 e_2 - \varphi_3 e_3 \right] \quad (32)$$

where:

$$\begin{cases} \varphi_2^T = [-x_2, -S_f, -1, 0] \\ \varphi_3^T = [0, 0, 0, -f_u] \end{cases} \quad (33)$$

Furthermore, to ensure:

$$\begin{cases} \dot{V}_4(e_1, e_2, e_3, \tilde{\theta}) \leq 0 \\ \Gamma^{-1} \dot{\tilde{\theta}} - \varphi_2 e_2 - \varphi_3 e_3 = 0 \end{cases} \quad (34)$$

The adaptive parameter estimation law is derived as:

$$\dot{\tilde{\theta}} = \Gamma(\varphi_2 e_2 + \varphi_3 e_3) \quad (35)$$

According to the Lyapunov stability theory, all state variables tend towards stability at this stage, and the hydraulic cylinder displacement error converges to a stable value. Therefore, it is evident that the system error is in an asymptotically stable state, showing that the original nonlinear system possesses asymptotic stability.

3. Simulation Analysis

To verify the effectiveness of the designed adaptive backstepping controller, simulation validations are conducted within the MATLAB/Simulink R2018a environment. Given that the primary focus of this study is the synchronous control performance of the ABS support system, the system model established in Simcenter AMESim is appropriately simplified by neglecting secondary factors that are weakly correlated with the synchronous control characteristics. By retaining the critical mechanisms that dictate the system's dynamic response and synchronization error, the computational efficiency and analytical targetedness of the simulation are improved, provided the model continues to accurately reflect the primary dynamic characteristics. Based on the hydraulic operational principles of the electro-hydraulic position servo system, the corresponding electro-hydraulic servo control model is established in the AMESim 2020.1 software.

Adjacent cross-coupling synchronization builds upon equal-status synchronization by comparing the displacement difference between adjacent cylinders. After processing by the synchronous controller, these differential signals are fed back to individual loops, realizing high-precision control of each branch [23,24]. Adjacent cross-coupling effectively realizes the distributed synchronous control of the hydraulic system through a local error mutual-feedback mechanism among the actuators [16]. Its principle is illustrated in Figure 5, where each controller not only corrects the error of its own actuator but also outputs compensation signals to adjacent branches. This synergistic suppression of synchronization errors improves the synchronous performance of multiple actuators. Based on the operational principle of this strategy, the adaptive backstepping controller is se-

lected as the single-cylinder displacement controller, while a PID controller serves as the synchronous compensator. The overall control strategy is depicted in Figure 5.

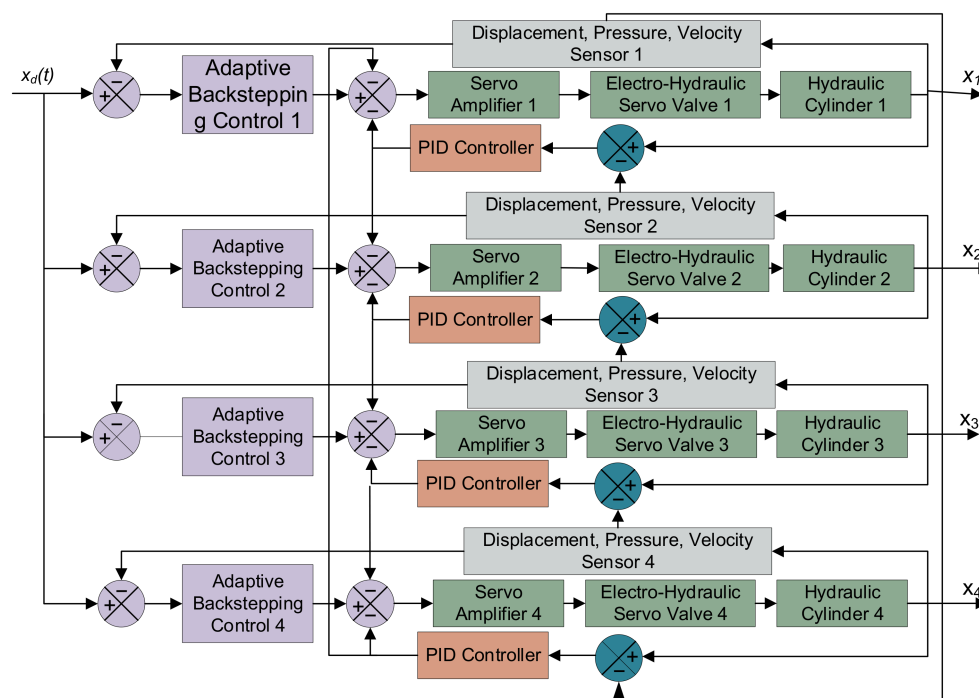


Figure 5. Schematic diagram of the adjacent cross-coupling synchronous control strategy.

In the AMESim simulation model, the physical plant is constructed utilizing components that include hydraulic cylinders, displacement sensors, pressure transducers, velocity sensors, hydraulic pumps, relief valves, proportional amplifiers, and servo valves. The principal parameters of these main components are detailed in Table 1.

Table 1. Main parameters of the co-simulation model.

Component	Main Setting Values
Hydraulic fluid	Density: 850 kg/m ³ , Bulk modulus: 1700 MPa
Hydraulic power supply	Maximum pressure: 32 MPa
Electric motor	Rated power: 220 kW, Speed: 991 r/min
Three-position four-way valve	Maximum flow rate: 450 L/min, Natural frequency: 80 Hz, Damping ratio: 0.8
Hydraulic cylinder	Bore diameter: 550 mm, Piston rod diameter: 300 mm, Stroke: 0.5 m, Mass: 100 kg
Proportional amplifier	Amplification gain: 30
Hydraulic pump	Displacement: 400 mL/r, Rated pressure: 32 MPa, Rated speed: 1000 r/min

The adaptive backstepping controller is programmed utilizing the S-function in Simulink. The adaptive parameters are configured as follows: $\theta_0 = [1000, 10, -400 \sin 0.015\pi + 600, 9 \times 10^{-12}]^T$, $\theta_{\min} = [800, 10, 200, 8 \times 10^{-12}]^T$, $\theta_{\max} = [1100, 20, 1000, 11 \times 10^{-12}]^T$, $\Gamma = [100, 1, 1, 1 \times 10^{-26}]^T$. Taking $S_f = 2 \arctan(1000x_2)/\pi$, $g = 4 \times 10^{-8} \text{ m}^4 / (\text{s} \cdot \text{V} \cdot \sqrt{\text{N}})$ for the system modeling, a sinusoidal signal is applied as the desired tracking reference in Simulink. It should be noted that the expression $-400 \sin 0.015\pi + 600$ in the initial parameter vector θ_0 is a constant value (approximately 581.16), representing the exact initial state at $t = 0$ with an initial phase shift of 0.015π . Through the empirical trial-and-error method, the controller gains of the adaptive backstepping algorithm, specifically k_1 , k_2 and k_3 , are tuned to 13,000, 900, and 150, respectively. Concurrently, the proportional k_p , integral k_i , and derivative k_d gains of the synchronous PID controller are set to 1000, 10, and 10, respectively.

In the practical engineering application of the ABS system on the “Luoning” TBM, the ambient temperature in the inclined shaft is approximately 30 °C, and the hydraulic system utilizes water cooling to maintain the fluid temperature at around 50 °C. This thermal management restricts severe variations in fluid viscosity and effective bulk modulus. To explicitly address the robustness of the system under relaxed modeling assumptions in practical heavy-duty engineering, it should be noted that additional unmodeled dynamics—including but not limited to variable fluid bulk modulus, temperature-induced variations, supply pressure ripples, and inherent valve dead-zone—are mathematically absorbed into the aforementioned lumped uncertainty term d_n . To continuously compensate for these complex dynamic variations without requiring a precise mathematical model, the parameter vector $\hat{\theta}$ is adapted online according to the proposed estimation law $\dot{\hat{\theta}}$. To prevent numerical parameter drift and ensure physical feasibility, this adaptation is strictly confined within the defined lower bound $\theta_{\min}$ and upper bound $\theta_{\max}$ through a smooth projection mapping algorithm. Furthermore, the adaptation rate is dynamically governed by the adaptation gain matrix Γ . While larger gains in Γ can accelerate parameter convergence and theoretically enhance steady-state tracking precision, they can simultaneously induce severe transient peaking, high-frequency control chattering, and potential valve saturation. Therefore, the defined parameters $\theta_{\min}$, $\theta_{\max}$, and Γ are carefully tuned to strike an optimal balance between rapid error convergence and the suppression of transient peaking.

It should be noted that the actual controller gains were determined through an empirical trial-and-error tuning process. In practical engineering applications, a balanced gain tuning guideline must be followed: the gains should be initially set to conservative lower values to prevent high-frequency noise amplification and actuator saturation. Subsequently, the gains should be incrementally increased while monitoring the dynamic response. This iterative heuristic process ensures an optimal trade-off between steady-state tracking precision and the physical constraints of the electro-hydraulic proportional valves.

3.1. Multi-Cylinder Trajectory Tracking Simulation

To comprehensively evaluate the performance of the proposed hydraulic cylinder synchronous control method, multiple desired trajectories are formulated as tracking test signals. These include a pure sinusoidal desired signal, as well as superpositions of the desired signal with sine, cosine, and white noise components, which are utilized to test the actual operational states and tracking errors of the hydraulic cylinders.

During the operational process, the piston rod of the hydraulic cylinder in the ABS support system is continuously subjected to the gravitational force of the gripper shoe. Furthermore, under the specific working conditions of the steeply inclined shaft, the magnitude of this gravitational component is inversely proportional to the extension length of the piston rod. Consequently, in the AMESim software, the sinusoidal load force applied to each hydraulic cylinder is defined as $y = -400 \sin(0.03\pi t) + 600$ N, and the initial displacement of the hydraulic cylinder is set to 0.3 m. The integrated model is subsequently imported into the Simulink environment for co-simulation analysis.

Owing to the highly effective control strategy, the actual trajectories macroscopically coincide with the desired trajectories almost perfectly. The tracking errors of the four hydraulic cylinders exhibit similar transient and steady-state behaviors, resulting in basically the same tracking trajectory for each cylinder. To prevent severe curve overlapping and to ensure visual clarity, this paper exclusively presents the tracking error plot of Cylinder 1 with respect to the desired signal. The synchronization accuracy of the system is further demonstrated through the inter-cylinder synchronization errors of Cylinders 2, 3,

and 4 relative to Cylinder 1. The original desired tracking signal defined in Simulink is $y = 0.15 \sin(0.03\pi t)$ m. The corresponding simulation results are illustrated in Figure 6.

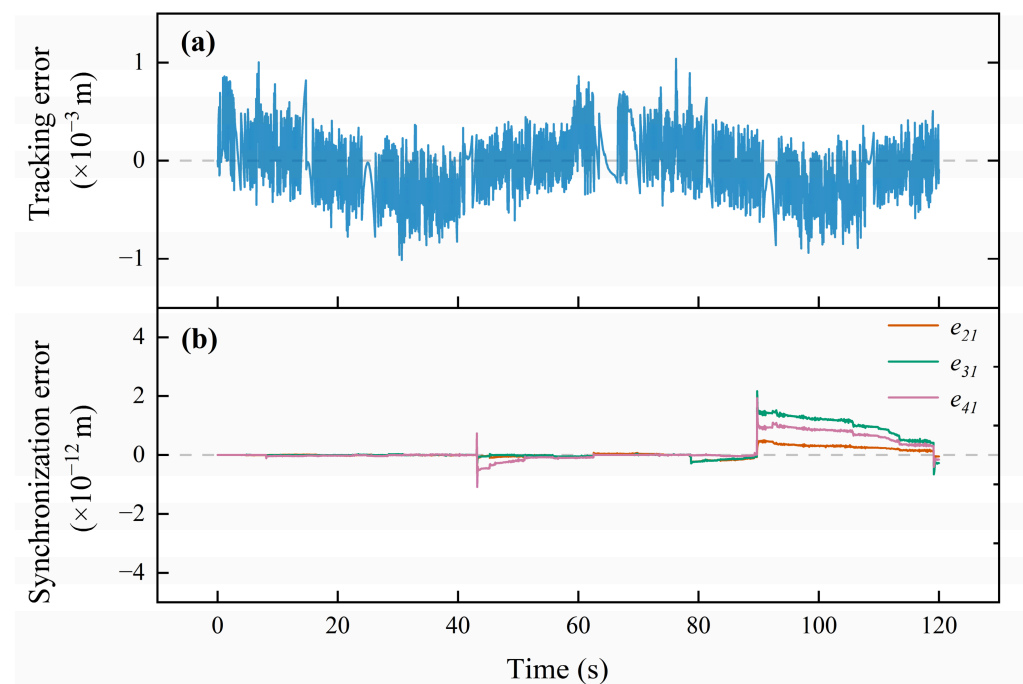


Figure 6. Simulation results tracking the sinusoidal signal: (a) Trajectory tracking error of the piston rod; (b) Inter-cylinder synchronization error.

An analysis of Figure 6 reveals that the designed adaptive adjacent cross-coupling control strategy provides stable steady-state control performance. Regarding trajectory tracking, as illustrated in Figure 6a, the maximum position tracking error of the piston rod is kept within 1.03 mm, representing merely 0.68% of the given signal's amplitude. The high-frequency micro-fluctuations observed in the error curve reflect the real-time, high-frequency dynamic compensation process executed by the adaptive backstepping controller to counteract the system's inherent nonlinearities.

In terms of multi-cylinder synergistic synchronization, as shown in Figure 6b, the maximum inter-cylinder synchronization error between Cylinders 2, 3, and 4 relative to Cylinder 1 is merely 2.2×10^{-12} m. This picometer-level magnitude reflects the theoretical convergence limit of the proposed control algorithm within the double-precision numerical environment, rather than a macroscopic physical deviation. To address potential concerns regarding numerical stability and distinguish control performance from round-off errors, sensitivity checks were conducted across different solver configurations, as detailed in Table 2.

Table 2. Sensitivity check of the maximum synchronization error (e_{21}) under nominal conditions across different solver configurations.

Solver Type	Step Size/Max Step (s)	Relative Tolerance	Absolute Tolerance	Max Sync Error e_{21} (m)	Convergence Status
ode45	10^{-3}	10^{-5}	10^{-6}	2.17×10^{-12}	original setting
ode45	10^{-5}	10^{-5}	10^{-6}	2.18×10^{-12}	stable convergence
ode15s	10^{-4}	10^{-6}	10^{-7}	2.32×10^{-12}	stable convergence
ode23t	10^{-4}	10^{-7}	10^{-8}	2.15×10^{-12}	stable convergence
ode4	Fixed-step 10^{-4}	N/A	N/A	2.59×10^{-12}	stable convergence

As shown in Table 2, the synchronization error is maintained across different solvers and under stricter tolerance settings. This confirms that the 10^{-12} m precision is the mathematical consequence of the adaptive control law's asymptotic convergence, rather than a numerical artifact or ill-conditioned round-off error. Furthermore, the transient steps in the synchronization error curve indicate that the adaptive parameter learning law can guide the error to reconverge when subjected to internal dynamic excitations.

A sine signal $y = 0.01 \sin(0.3\pi t)$ is superimposed onto the original desired signal to simulate the motion requirements under multi-source excitations or complex working conditions in practical engineering, aiming to verify the control system's comprehensive tracking capability for different frequency components. The simulation results are shown in Figure 7.

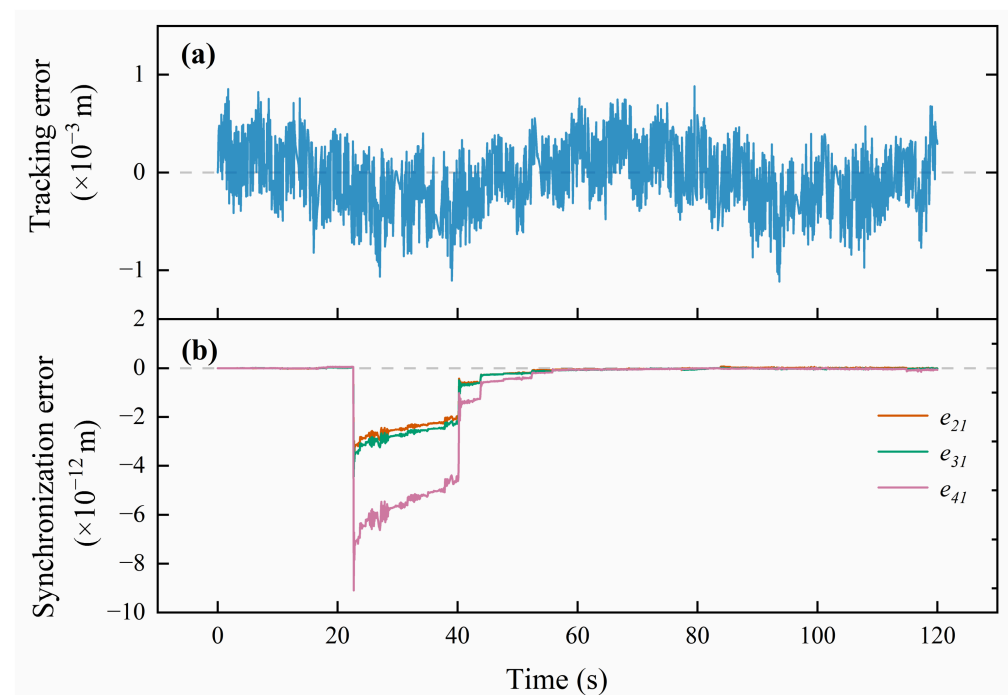


Figure 7. Simulation results tracking the composite sinusoidal signal: (a) Trajectory tracking error of the piston rod; (b) Inter-cylinder synchronization error.

Regarding trajectory tracking, as depicted in Figure 7a, despite the complex composite frequency excitation, the maximum position tracking error of Cylinder 1 remains strictly constrained to 0.88 mm. Figure 7b unveils the microscopic response mechanism of the adaptive cross-coupling strategy when dealing with internal dynamic excitations. Throughout the entire operational cycle, the maximum inter-cylinder synchronization error is 9.1×10^{-12} m. At $t = 22$ s, the synchronization error exhibits a distinct transient plunge, followed by a step-like rapid convergence at $t = 40$ s. This phenomenon does not indicate system instability; rather, it occurs because the frequency superposition of the composite sinusoidal signal generates internal dynamic excitations and local acceleration extremums at specific moments. In the instant when such microscopic excitations induce displacement deviations, the adaptive parameter learning law performs high-frequency iterations to rapidly modify the control parameters. Simultaneously, the adjacent cross-coupling strategy intervenes, ensuring that all slave cylinders maintain highly consistent tracking even under extreme microscopic disturbances. This transient process fully demonstrates the parameter self-learning capability and the strong multi-cylinder coupling constraint capacity of the controller.

A cosine signal $y = 0.05 \cos(0.02\pi t)$ is superimposed onto the original desired signal, causing the desired trajectory to exhibit more complex continuous multi-frequency variations. Consequently, this imposes higher requirements on the dynamic response bandwidth and disturbance-rejection capability of the control system. The simulation results are presented in Figure 8.

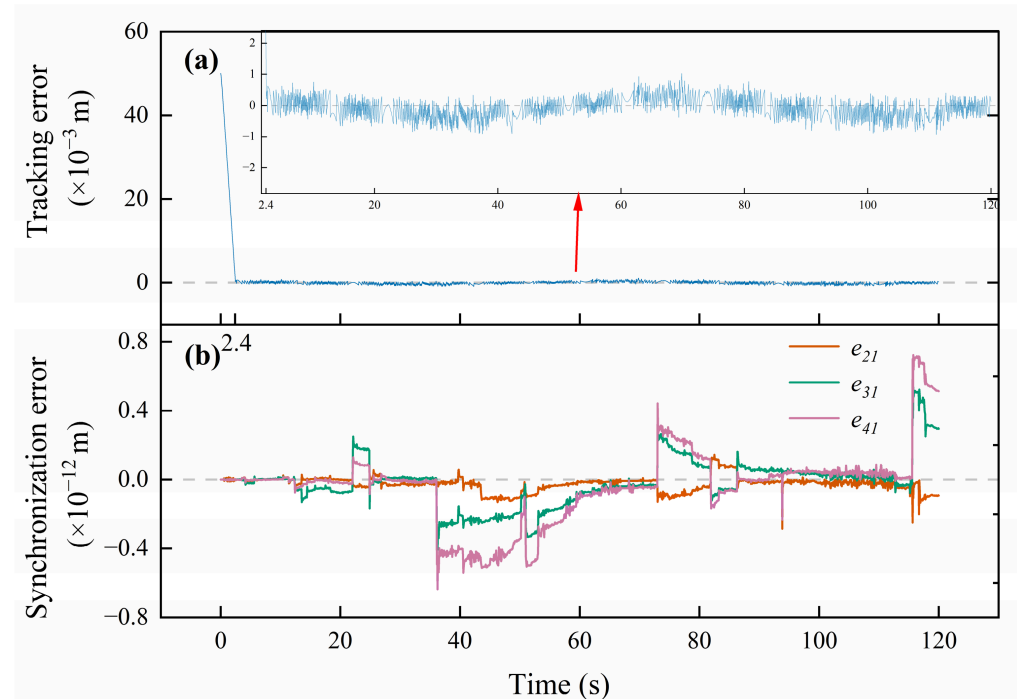


Figure 8. Simulation results tracking the composite cosine signal: (a) Trajectory tracking error of the piston rod; (b) Inter-cylinder synchronization error.

As shown in Figure 8a, the tracking error stabilizes near the zero-scale line after an initial transient duration of 2.4 s. To explicitly illustrate the steady-state tracking precision, the inset in Figure 8a locally magnifies the post-transient dynamic response. This specific magnification clearly demonstrates that the tracking error of Cylinder 1 consistently fluctuates around the zero-scale line, with its maximum value being only 1.02 mm despite the complex frequency reference signals. Regarding multi-cylinder synergistic synchronization, the results in Figure 8b demonstrate that even under continuous dynamic alternating excitations of composite harmonics, the maximum inter-cylinder synchronization error of the adjacent cross-coupling system is merely 0.72×10^{-12} m. The minute steps and dynamic fine-tuning processes exhibited in the error curve indicate that the adaptive parameter learning law possesses acute sensitivity when facing complex variable-frequency commands, achieving the synergistic consistency of the multi-cylinder system at a microscopic scale.

White noise with a power of 1×10^{-6} is superimposed onto the original desired signal. This condition aims to simulate the random disturbances introduced by sensor measurement errors, external perturbations, and system uncertainties in actual engineering scenarios. The simulation results are shown in Figure 9.

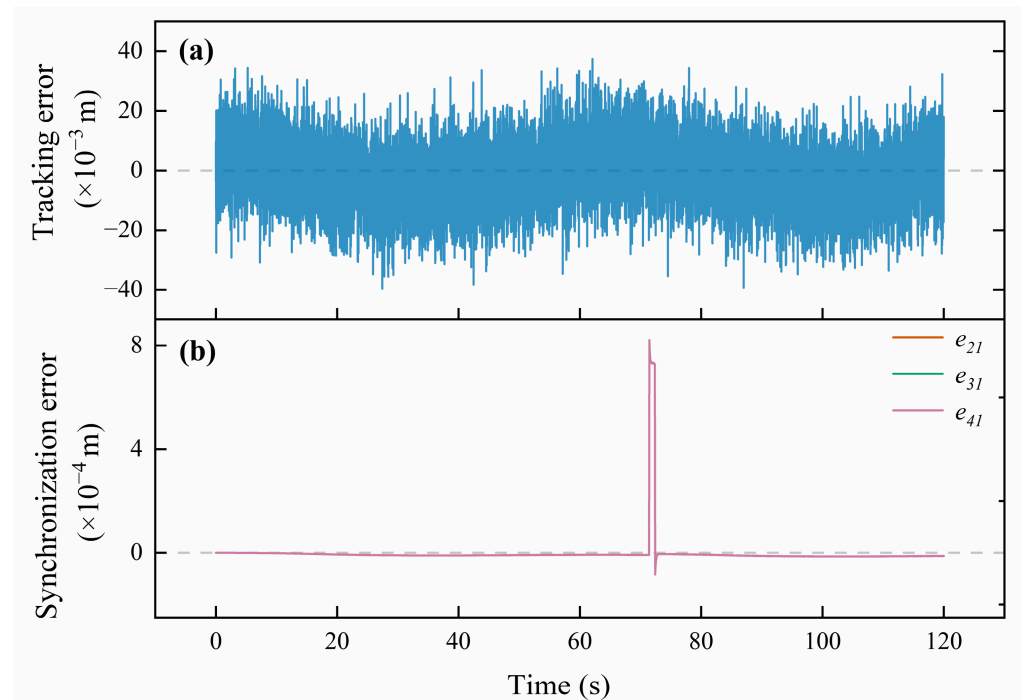


Figure 9. Simulation results tracking the desired signal superimposed with white noise: (a) Trajectory tracking error of the piston rod; (b) Inter-cylinder synchronization error.

Regarding trajectory tracking, as depicted in Figure 9a, under the influence of continuous system dynamics and external environments, the tracking error of Cylinder 1 exhibits a high-frequency broadband fluctuation. This continuous dynamic adjustment band authentically reflects the high-intensity, disturbance-rejection working state of the controller in a complex physical environment. As for multi-cylinder synchronization, shown in Figure 9b, a transient sudden change with a peak value of 0.82 mm occurs in the cylinder synchronization error at $t = 72$ s. This represents an internal state mutation characteristic of nonlinear systems subjected to continuous broadband white noise excitation. When such unpredictable internal chaotic excitations occur, conventional controllers are highly prone to triggering system divergence and instability. However, the adaptive adjacent cross-coupling mechanism of this system responds within a short time, reallocating the control laws to converge the error back to a microscopic steady state approaching zero within 1 s. This process verifies the robustness of the control strategy in maintaining the mechanical synchronization of the platform under disturbance conditions.

3.2. Performance Analysis Against Eccentric Load Interference

The preceding sections have detailed the response characteristics when tracking various desired signals under the adaptive backstepping control. Although the load variations during the extension of the hydraulic rod were considered, these were strictly limited to the ideal working conditions of the gripper shoes. To comprehensively verify the anti-interference performance against eccentric loading, a specific asymmetric load condition is introduced. The external disturbance force on Cylinders 1 and 3 remains unchanged, while a sinusoidal dynamic disturbance with a maximum amplitude of 150% ($1.5\times$) of the original load is suddenly injected into Cylinders 2 and 4 at $t = 20$ s. This setup is designed to simulate the actual operational scenarios where the hydraulic cylinders of the ABS support system are subjected to differential external dynamic disturbances. The tracking results of each hydraulic cylinder against the original desired signal are illustrated in Figure 10.

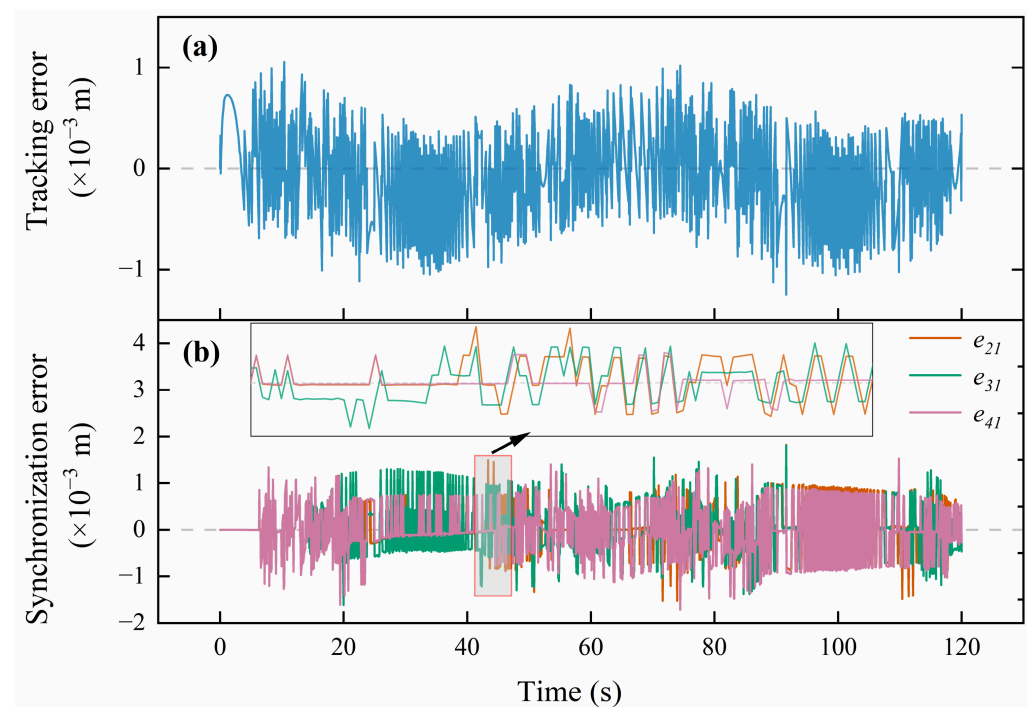


Figure 10. Simulation results under dynamic eccentric load interference: (a) Trajectory tracking error of the piston rod; (b) Inter-cylinder synchronization error.

Regarding trajectory tracking, as illustrated in Figure 10a, even under the dynamic eccentric load excitation, the maximum position tracking error of Cylinder 1 is merely 1.24 mm. The continuous error oscillation band depicts the dynamic compensation processes executed by the adaptive backstepping control law. This continuous compensation rejects the external sinusoidal disturbances and ensures macroscopic trajectory tracking accuracy. In terms of multi-cylinder synchronization and synergy, as shown in Figure 10b, following the injection of the strong disturbance at $t = 20$ s, the disparity in the physical loads borne by the cylinders inevitably induces mechanical and hydraulic transient deformations, causing the synchronization error to jump to the millimeter scale. The synchronization error curves (e_{21} , e_{31} , and e_{41}) visually overlap on a macroscopic scale due to the high convergence precision of the controller. To demonstrate their dynamic differences, a locally magnified inset is added in Figure 10b. This zoom-in view clearly captures the distinct transient adjustment processes and minute differential details of each cylinder. Ultimately, under the strong constraints of the adjacent cross-coupling strategy and the extreme rapid adjustment of the adaptive parameters, the maximum inter-cylinder synchronization error is strictly contained at 1.82 mm. This explicitly proves the controller's synergistic capacity against eccentric loads, strong robustness, and high engineering application value.

To further investigate the robustness boundary and the extreme limit of anti-eccentric loading of the proposed control strategy, gradient tests were conducted within the range of $1.0\times$ to $2.0\times$ asymmetric dynamic disturbance loads. The resulting maximum error envelope trends are illustrated in Figure 11.

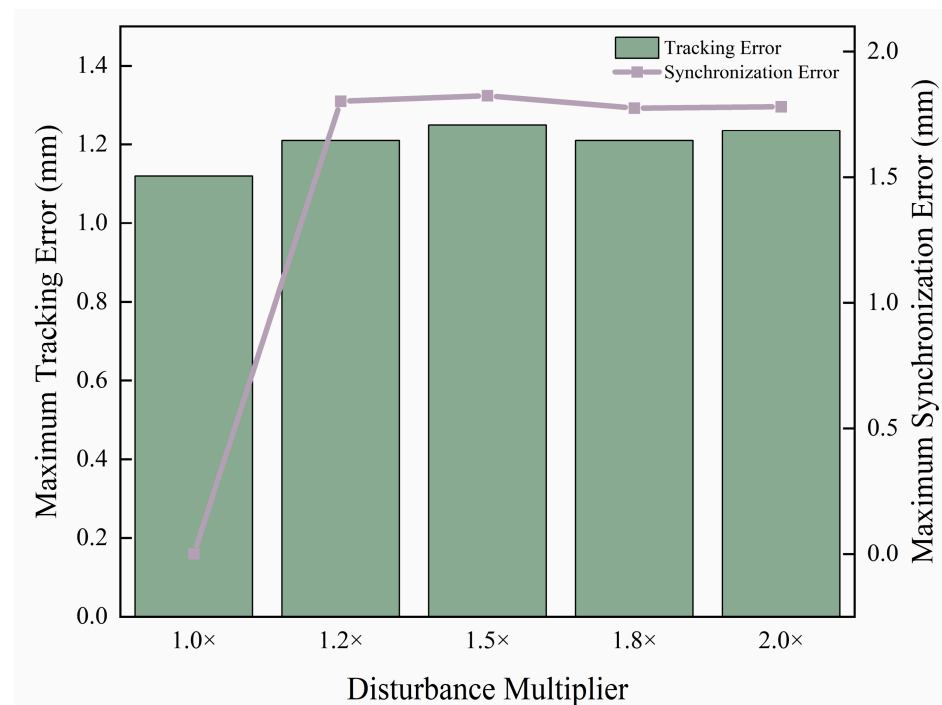


Figure 11. Maximum error envelope trends under varying degrees of eccentric load interference.

An analysis of Figure 11 reveals that the system possesses formidable resilience against extreme asymmetric eccentric loads. On the one hand, the maximum tracking error, represented by the green bar chart, is maintained within a narrow frequency band of 1.12–1.25 mm across the entire load spectrum, highlighting the macroscopic disturbance-rejection stiffness of the adaptive control law. On the other hand, the maximum synchronization error trajectory, denoted by the purple line, indicates that after an initial logical physical transient jump induced by the onset of asymmetric eccentric loading ($1.2\times$), the synchronization error does not exhibit linear or exponential divergence as the load continues to aggressively scale up to $2.0\times$. Instead, it is firmly bounded near a horizontal threshold of approximately 1.8 mm. This proves that the proposed cross-coupling mechanism can effectively suppress the divergent instability of multiple cylinders under limit working conditions, maintaining overall synergistic integrity.

3.3. Quantitative Comparative Analysis with Baseline Controllers

To quantitatively evaluate the performance of the proposed control strategy, a comparative analysis was conducted against two established baseline controllers: a decentralized Proportional-Integral-Derivative (PID) controller with feedforward (FF) compensation, and a classical full cross-coupling controller. The evaluation was performed under identical multi-cylinder plant dynamics and subjected to three distinct working conditions: nominal conditions, measurement noise injection, and a 1.2 times asymmetric dynamic eccentric load. The primary performance metrics—including maximum overshoot, settling time, tracking Root Mean Square (RMS) error, maximum synchronization error, synchronization RMS error, and qualitative convergence status—are summarized in Table 3.

As summarized in Table 3, the proposed method demonstrates quantitative improvements over the baseline controllers across all tested conditions. Under nominal conditions, the proposed method achieves a synchronization error of 5.88×10^{-10} mm and a tracking RMS error of 0.33 mm, outperforming both the decentralized PID and classical cross-coupling controllers. Under measurement noise, the baseline controllers experience high-frequency chattering, whereas the proposed method maintains smooth

convergence with a maximum synchronization error constrained to 0.82 mm. Under the 1.2 times eccentric load, the PID and full cross-coupling controllers exhibit severe synchronization degradation—reaching maximum errors of 37.65 mm and 13.28 mm, respectively—alongside actuator saturation. Conversely, the proposed adaptive strategy restricts the maximum synchronization error to 1.82 mm and the maximum overshoot to 3.25 mm, ensuring stable and smooth operation without saturation.

Table 3. Quantitative performance comparison under various working conditions.

Control Strategy	Working Condition	Max Overshoot (mm)	Settling Time (s)	Tracking RMS (mm)	Max Sync Error (mm)	Sync RMS Error (mm)	Convergence Status
Decentralized PID + FF	Nominal	4.28	1.8	1.64	3.47	1.83	Normal
	Measurement Noise	6.25	6.5	4.78	8.76	5.24	High-freq chattering
	1.2× Eccentric Load	23.71	N/A	25.83	37.65	16.36	Severe saturation
Classical Full Cross-Coupling	Nominal	5.36	3.2	2.43	0.85	0.63	Normal
	Measurement Noise	8.64	7.2	5.32	1.74	1.54	High-freq chattering
	1.2× Eccentric Load	19.73	8.5	17.02	13.28	8.29	Intermittent saturation
Proposed Method	Nominal	1.12	0.08	0.33	5.88×10^{-10}	1.58×10^{-10}	Smooth
	Measurement Noise	1.64	0.17	0.68	0.82	0.02	Smooth
	1.2× Eccentric Load	3.25	1.04	1.23	1.82	0.67	Smooth

4. Conclusions

This paper addresses the anti-falling and synchronous control issues during the operation of hard-rock tunnel boring machines in large-inclination shafts. The Anti-Back-Slip support system of pumped-storage power stations is taken as the research object. Driven by its operational characteristics and engineering requirements, a multi-cylinder electro-hydraulic servo synchronous control system is designed. A nonlinear state-space mathematical model of the electro-hydraulic position servo system is constructed, and an adaptive backstepping controller is introduced into the adjacent cross-coupling strategy. Through the co-simulation analysis using the AMESim-Simulink platform, the main conclusions are drawn as follows:

- The system exhibits steady-state synchronization under nominal conditions. When tracking sinusoidal and multi-frequency composite desired trajectories, the adjacent cross-coupling mechanism limits the inter-cylinder synchronization error. Sensitivity analyses across multiple solvers verify the asymptotic convergence of the nonlinear control law.
- The control strategy demonstrates robust disturbance-rejection stiffness. Continuous broadband white noise was introduced to simulate sensor noise and external random perturbations. Under these conditions, the controller completes dynamic compensation within 1 s following internal system excitations. This enables the error to reconverge rapidly, maintaining stable system operation.
- The system maintains synergistic capability against asymmetric eccentric loads. When subjected to suddenly applied asymmetric dynamic eccentric loads of 1.0 to 2.0 times, the multi-cylinder synchronization error avoids divergence, with the maximum error restricted to 1.82 mm, preventing multi-cylinder instability.
- Comparative analyses indicate that the proposed strategy outperforms decentralized PID with feedforward compensation and classical full cross-coupling controllers. It reduces tracking and synchronization RMS errors and eliminates the actuator saturation observed in baseline controllers under severe eccentric loads.

In summary, the proposed control strategy addresses the limitations of traditional linear control algorithms in heavy-duty and eccentric load environments, achieving coordinated control among multiple actuators. The research findings satisfy the requirements

for synchronous control of the anti-falling system in inclined shaft TBMs and provide a theoretical and engineering reference for the disturbance-rejection coordination of inclined shaft TBM support systems.

However, this study has certain limitations. The 10^{-12} m synchronization precision demonstrated in the simulations represents a theoretical mathematical bound within a double-precision environment. While the designed ABS system has been applied in a 5020 m excavation project, achieving actual macroscopic control precision at the millimeter level, practical performance is fundamentally bounded by physical hardware constraints. Furthermore, a dedicated independent physical test rig or Hardware-in-the-Loop (HiL) setup was not constructed in this study to isolate and evaluate the proposed algorithm. Therefore, future research will focus on establishing HiL testing environments and scaled physical test rigs to evaluate the engineering performance of the control law under actual hardware limitations. Additionally, subsequent studies will incorporate comparative analyses with other advanced nonlinear control strategies, such as Sliding-Mode Control and Robust Model Predictive Control, to systematically evaluate control energy consumption and stability margins.

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Nomenclature

The following abbreviations and symbols are used in this manuscript:

TBM	tunnel boring machine
ABS	directory of open access journals
p_0	supply pressures
p_s	return pressures
q_1	flow rates into the rodless chambers of the hydraulic cylinder
q_2	flow rates into the rod chambers of the hydraulic cylinder
p_1	pressures in the rodless chambers
p_2	pressures in the rod chambers
m	equivalent load mass
B_p	viscous damping coefficient of the load
x_v	input displacement of the spool
k_i	proportional coefficient between the servo valve spool displacement
μ	control input
q	flow rate entering the electro-hydraulic servo valve
k_q	flow gains at both ports of the servo valve
β_e	effective bulk moduli of the hydraulic oil at both actuator ports
p_r	return pressure of the system
C_t	internal leakage coefficient of the hydraulic cylinder
V_1	volumes of the rodless chambers of the hydraulic cylinder
V_2	the volumes of the rod chambers of the hydraulic cylinder

V_{01}	initial volumes of the rodless chambers
V_{02}	initial volumes of the rod chambers
$f(z_1, z_2)$	uncertain nonlinear forces
A_f	amplitude of the modeled Coulomb friction
S_f	continuous shape function approximating the Coulomb friction.
d_n	lumped nominal value of unmodeled dynamics and external disturbances
d	$f(z_1, z_2) - d_n$

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