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Asymptotic Motion Control of Motor Servo Systems with Disturbance Compensation and Time-Varying Asymmetric Output Constraints

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Abstract

This paper proposes a novel asymptotic secure tracking controller for motor servo systems. By introducing a novel asymptotic disturbance observer, the system uncertainties can be asymptotically estimated and compensated. Furthermore, by introducing a tracking error-based barrier function, it can achieve that the system output remains within the prescribed time-varying constraint boundaries. Additionally, a nonlinear asymptotic filter is incorporated into the design, which effectively circumvents the inherent "explosion of complexity" issue and achieves asymptotic tracking performance. Finally, the stability of the system is demonstrated through Lyapunov-based theoretical analysis, and the effectiveness of the proposed controller is validated by simulation results.

Keywords: motor servo system; output constraint; asymptotic disturbance observer; disturbance compensation; asymptotic filter; asymptotic tracking

MSC: 68T07; 93B52; 93C10; 93C40; 93C73; 93D20; 93D21

1. Introduction

Motor servo systems are widely used in industrial automation, precision manufacturing, and other high-performance motion control applications. Owing to their advantages in high precision and flexible actuation, they have attracted sustained research interest in recent years. However, the control performance of motor servo systems is often significantly degraded by various uncertainties, which makes the design of high-performance controllers challenging [1–3]. In general, these uncertainties can be categorized into endogenous uncertainties and external disturbances. To compensate for these uncertainties and improve the tracking performance and robustness of servo systems, a variety of advanced control methods have been developed, including adaptive control [4], robust control [5], and intelligent control [6,7]. Adaptive control is effective in dealing with parametric uncertainties by adjusting controller parameters online, while robust control aims to maintain stability and performance in the presence of bounded disturbances and modeling errors. Intelligent control methods, such as neural-network-based and fuzzy-logic-based approaches, further enhance the ability of the controller to approximate unknown nonlinear dynamics. For example, an adaptive robust control strategy was proposed in [8] for DC motor systems to compensate for model uncertainties and external disturbances. In ref. [9], robustness was enhanced through an error-symbol robust integral feedback scheme, which improved disturbance rejection capability and tracking accuracy. These controllers require high feedback

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gains to maintain performance when facing large disturbances. In practical engineering, however, high feedback gains amplify noise, which limits their application.

Disturbance compensation has become an important approach for improving the control performance of servo systems, since it can effectively attenuate the adverse effects of model uncertainties and external disturbances. The central idea of this class of methods is to estimate the lumped disturbance online through an observer and then compensate for it in a feedforward manner. Among the existing approaches, the extended state observer (ESO) proposed by Han has been widely adopted because of its simple structure, limited model dependence, and practical effectiveness in real-time disturbance estimation [10]. In ref. [11], a sliding-mode-observer-based scheme was developed for servo mechanisms to enhance disturbance rejection and state estimation performance. In ref. [12], a switching fuzzy observer was constructed to estimate unmeasured system states under uncertain operating conditions. In parallel, neural-network-based methods have also been introduced to approximate unknown nonlinearities and modeling uncertainties, thereby improving the adaptability of the controller to complex dynamics [13,14]. Although these methods have achieved promising results, most of them can only guarantee bounded estimation errors, which usually implies that the corresponding closed-loop tracking error remains uniformly bounded rather than asymptotically converging to zero. Compared with bounded tracking errors, asymptotic tracking control ensures that the system converges to zero steady-state error [15]. Designing a disturbance observer with accurate estimation capabilities to achieve asymptotic tracking control deserves further exploration.

In practical engineering applications, motor servo systems are often required to satisfy not only prescribed control performance specifications but also stringent safety constraints throughout the entire operation process [16]. Once such constraints are violated, the system may experience performance deterioration, reduced tracking accuracy, and even closed-loop instability. For this reason, constraint handling has become an essential issue in the control design of high-performance servo systems. In the control of nonlinear uncertain systems, barrier functions have been widely applied to address constraint issues [17–19]. Nevertheless, when state constraints are transformed into equivalent tracking-error bounds, additional feasibility conditions are often introduced, which may restrict the admissible initial conditions and complicate controller synthesis [20]. In ref. [21], barrier Lyapunov functions were combined with prescribed performance functions to guarantee bounded tracking errors for nonlinear systems. However, the resulting method did not ensure asymptotic tracking. For motor servo systems that require both safety and high precision, output constraints are of great importance. Therefore, investigating asymptotic secure tracking control for motor servo systems is of great significance.

Inspired by the aforementioned achievements, this paper proposes a novel asymptotic secure tracking controller for motor servo systems. The system uncertainties are estimated and compensated through an asymptotic disturbance observer, with a focus on asymmetric and time-varying output constraints, while achieving asymptotic tracking. The primary contributions to this work are listed as follows:

(1) Different from most adaptive control and disturbance-rejection control methods, which can only guarantee bounded estimation errors and consequently uniform ultimate boundedness of the tracking error, this paper introduces an asymptotic disturbance observer (ADO) to asymptotically estimate and compensate for the lumped uncertainties, thereby achieving asymptotic tracking performance.
(2) Different from related methods that only consider bounded tracking, this paper directly incorporates time-varying asymmetric output constraints into the control design through a tracking error-based barrier function, so that the system satisfies the prescribed constraint boundaries throughout the entire operation process.

(3) The nonlinear asymptotic filter not only avoids the "explosion of complexity" but also achieves asymptotic tracking performance.

Notably, the present control design is developed under the assumption that the system states are available for feedback. However, the speed signal is often obtained from incremental encoders rather than directly measured. The measurement accuracy may be affected by quantization effects, sampling-time limitations, and implementation constraints, especially in low-speed operating conditions, which may further influence the achievable tracking performance. Relevant studies on adaptive time-base speed estimation, low-cost microcontroller-based acquisition, and FPGA-oriented encoder processing have been reported in [22–24]. These practical measurement issues will be considered in our future work.

Notation: $\hat{\cdot}$ is the estimation of $\cdot$ and $\tilde{\cdot} = \cdot - \hat{\cdot}$ is the estimation error. $\|\cdot\|$ is the Euclidean norm of $\cdot$.

2. Problem Formulation

The state-space equation of a class of motor servo systems can be expressed as [5]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = K_q u + D(x_2, t) \\ y = x_1 \end{cases} \tag{1}$$

where $x_1 \in \mathbb{R}$ and $x_2 \in \mathbb{R}$ are the rotational angular displacement and rotational velocity of the load, respectively; K_q is a parameter related to the electrical gain and the load mass; $D(x_2, t)$ is the modeling uncertainty; u is the system control input.

Control goal: Given the desired trajectory $y_d(t)$, we will design a bounded u such that x_1 asymptotically tracks $y_d(t)$, while ensuring that the tracking error e_1 meets $-F_1(t) < e_1(t) < F_2(t)$ with $F_1(t)$ and $F_2(t)$ representing time-varying constraints.

Assumption 1. *The desired trajectory y_d is second-order differentiable and bounded.*

Assumption 2. *$D(x_2, t)$ and its first-order time derivatives are bounded.*

Remark 1. *Notably, Assumptions 1 and 2 are commonly adopted in servo control problems. In most engineering applications, the desired trajectory is generated by a motion planner or a smoothing mechanism, and is thus bounded and sufficiently differentiable. In addition, although the lumped uncertainty includes modeling errors, load variations, and external disturbances, its magnitude and rate of variation are usually limited by the physical properties of the servo system and the operating environment. Therefore, these assumptions are reasonable for a wide range of practical servo systems.*

Lemma 1 ([25]). *For any variable $\xi \in \mathbb{R}$, we have*

$$0 \leq |\xi| - \xi \tanh(\xi/\Delta) \leq \Delta\chi \tag{2}$$

where $\chi = 0.2785$ and $\Delta > 0$.

3. Controller Design

3.1. Disturbance Observer with Asymptotic Estimation Performance

Based on (1), by redefining the modeling uncertainty term $D(x_2, t)$ in the system as an uncertain state $x_{\delta 2}$, that is, letting $x_{\delta 2} = D(x_2, t)$. According to Assumption 2, the system can be rewritten as:

$$\begin{cases} \dot{x}_2 = K_q u + x_{\delta 2} \\ \dot{x}_{\delta 2} = P_2(t) \end{cases} \tag{3}$$

where $P_2(t)$ denotes the derivative of $x_{\delta 2}$.

Therefore, an ADO can be designed as [26]:

$$\begin{cases} \dot{\hat{x}}_2 = K_q u + \hat{x}_{\delta 2} + r_1 \omega_{o2}(x_2 - \hat{x}_2) \\ \dot{\hat{x}}_{\delta 2} = r_2 \omega_{o2}^2 (x_2 - \hat{x}_2) + \hat{\eta}_2 \mathrm{sgn}(\kappa_2) \end{cases} \tag{4}$$

where ω_{o2} is a tunable positive design parameter; the parameters $r_1 = 2$ and $r_2 = 1$, respectively. The expression for κ_2 will be given later. $\eta_2 = \sup_{t \geq 0} |P_2(t)|$ is an unknown positive constant which can be updated online as [26]:

$$\dot{\hat{\eta}}_2 = \frac{m_2}{\omega_{o2}} \left| \theta_2^{\mathrm{T}} B_o C_o \right| \tag{5}$$

where m_2 represents an adjustable positive gain; $\kappa_2 = \theta_2^{\mathrm{T}} B_o C_o$, in which the matrices B_o and C_o will be described in detail below and $\theta_2 = [\theta_{21}, \theta_{22}]^{\mathrm{T}} = [\tilde{x}_2, \tilde{x}_{\delta 2}/\omega_{o2}]^{\mathrm{T}}$.

Based on (3) and (4), the estimation error dynamics of ADO can be expressed as:

$$\begin{cases} \dot{\tilde{x}}_2 = \tilde{x}_{\delta 2} - r_1 \omega_{o2}(x_2 - \hat{x}_2) \\ \dot{\tilde{x}}_{\delta 2} = P_2(t) - r_2 \omega_{o2}^2 (x_2 - \hat{x}_2) - \hat{\eta}_2 \mathrm{sgn}(\kappa_2) \end{cases}. \tag{6}$$

Moreover, (6) can be restructured as:

$$\begin{cases} \dot{\theta}_{21} = -r_1 \omega_{o2} \theta_{21} + \omega_{o2} \theta_{22} \\ \dot{\theta}_{22} = -r_2 \omega_{o2} \theta_{21} + \dfrac{1}{\omega_{o2}} [P_2(t) - \hat{\eta}_2 \mathrm{sgn}(\kappa_2)] \end{cases} \tag{7}$$

which is equivalent to

$$\dot{\theta}_2 = \omega_{o2} A_o \theta_2 + \frac{1}{\omega_{o2}} C_o [P_2(t) - \hat{\eta}_2 \mathrm{sgn}(\kappa_2)] \tag{8}$$

where

$$A_o = \begin{bmatrix} -r_1 & 1 \\ -r_2 & 0 \end{bmatrix}, C_o = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{9}$$

Since A_o is Hurwitz, the matrix B_o satisfies $A_o^{\mathrm{T}} B_o + B_o A_o = -I$ [27]. Accordingly, B_o can be computed as:

$$B_o = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{3}{2} \end{bmatrix}. \tag{10}$$

Remark 2. *θ_2 denotes a normalized error-state vector formed by the observation errors. It is introduced to rewrite the observer error dynamics into a standard matrix form, thereby facilitating the stability analysis. κ_2 denotes a scalar projection of the observation errors. It is introduced to isolate the cross term associated with the unknown disturbance variation rate in the Lyapunov derivative, so that this term can be handled in a tractable scalar form.*

Remark 3. *It is worth noting that the $\mathrm{sgn}(\kappa_2)$ needs to be calculated online when implementing ADO. However, since κ_2 contains the unmeasurable signal θ_{22}, it cannot be directly obtained from*

the expression of $\theta_2^{\mathrm{T}} B_o C_o$. *In order to determine* $\mathrm{sgn}(\kappa_2)$, *we need to introduce a set of integral-type functions* $\phi_2(t)$ *as follows:*

$$\phi_2(t) = \int_0^t \kappa_2(v)dv. \tag{11}$$

Because of $\kappa_2 = \theta_2^{\mathrm{T}} B_o C_o = -\frac{1}{2}\widetilde{x}_2 + \frac{3}{2}\frac{\widetilde{x}_{\delta 2}}{\omega_{o2}}$ and from (6), with $r_1 = 2$ taken in the paper, thus

$$\kappa_2 = \theta_2^{\mathrm{T}} B_o C_o = -\frac{1}{2}\widetilde{x}_2 + \frac{3}{2}\frac{\widetilde{x}_{\delta 2}}{\omega_{o2}} = \frac{5}{2}\widetilde{x}_2 + \frac{3}{2}\frac{\dot{\widetilde{x}}_2}{\omega_{o2}}. \tag{12}$$

Based on (12), (11) can be written as:

$$\phi_2(t) = \frac{5}{2}\int_0^t \widetilde{x}_2 dv + \frac{3}{2}\frac{1}{\omega_{o2}}\widetilde{x}_2. \tag{13}$$

The calculated value of $\mathrm{sgn}(\kappa_2)$ can be obtained from the following expression [24,25]:

$$\mathrm{sgn}(\kappa_2) = \mathrm{sgn}[\phi_2(t) - \phi_2(t - \tau_s)] \tag{14}$$

where τ_s represents the time delay, and when calculating $\mathrm{sgn}(\kappa_2)$, the system sampling time can be set to τ_s [28,29]:

$$\kappa_2(t) = \lim_{\tau_s \to 0}[\phi_2(t) - \phi_2(t - \tau_s)]/\tau_s. \tag{15}$$

Based on (15), we can determine whether $\phi_2(t)$ is decreasing or increasing, thereby achieving ADO.

Remark 4. *According to [28], by integrating both sides of* $\dot{\hat{\eta}}_2$ *in Equation (5), we can still obtain the real-time updated value* $\hat{\eta}_2$, *as follows:*

$$\hat{\eta}_2 = \frac{5}{2}\frac{m_2}{\omega_{o2}}\int_0^t \widetilde{x}_2 \mathrm{sgn}(\kappa_2)dv + \frac{3}{2}\frac{m_2}{\omega_{o2}^2}\widetilde{x}_2 \mathrm{sgn}(\kappa_2) \tag{16}$$

3.2. Controller Design

Before designing the controller, define the tracking errors e_1 and z_2.

$$\begin{aligned} e_1 &= x_1 - y_d \\ z_2 &= x_2 - \alpha_{1c} \end{aligned} \tag{17}$$

where α_{1c} is the filtered value of the virtual control law α_1.

To obtain the filtered value of α_1, we construct a nonlinear asymptotic filter [26]:

$$\begin{aligned} w_{c1}\dot{\alpha}_{1c} &= -\alpha_{1c} + \alpha_1 - \varepsilon_{1s}, \alpha_{1c}(0) = \alpha_1(0) \\ \varepsilon_{1s} &= -\frac{w_{c1}\varepsilon_1 \hat{N}_1^2}{\varepsilon_1 \hat{N}_1 \tanh\left[\varepsilon_1 q_1^{-1}(t)\right] + q_1(t)} \end{aligned} \tag{18}$$

where $w_{c1} > 0$ is the adjustable positive parameter; $\varepsilon_1 = \alpha_1 - \alpha_{1c}$ is the filtering error; ε_{1s} denotes the auxiliary compensation term in the nonlinear asymptotic filter; $q_1(t) > 0$ and satisfies $\int_0^t q_1(v)dv \le \Xi_1 < \infty, \forall t \ge 0$, Ξ_1 is a positive constant [30]; additionally, N_1 denotes the online adjustable upper bound of the first-order derivative of α_1, and its update law is given by:

$$\dot{\hat{N}}_1 = \beta_1 |\varepsilon_1| \tag{19}$$

where β_1 denotes the non-negative gain.

Step 1:

Inspired by [31,32], the tracking error e_1 is transformed into the barrier function z_1 as

$$z_1 = \frac{F_1 F_2 e_1}{(F_1 + e_1)(F_2 - e_1)}, -F_1(0) < e_1(0) < F_2(0). \tag{20}$$

Computing the time derivative of z_1 yields:

$$\dot{z}_1 = \mu \dot{e}_1 + \Delta_1 \dot{F}_1 + \Delta_2 \dot{F}_2 \tag{21}$$

where

$$\mu = \frac{F_1 F_2 (F_1 F_2 + e_1^2)}{(F_1 + e_1)^2 (F_2 - e_1)^2}, \Delta_1 = \frac{F_2 e_1^2}{(F_1 + e_1)^2 (F_2 - e_1)}, \Delta_2 = -\frac{F_1 e_1^2}{(F_1 + e_1)(F_2 - e_1)^2}. \tag{22}$$

Based on (1), (17), and (21), we have

$$\dot{z}_1 = \mu (z_2 + \alpha_1 - \varepsilon_1 - \dot{y}_d) + \Delta_1 \dot{F}_1 + \Delta_2 \dot{F}_2. \tag{23}$$

Based on (23), the virtual control signal α_1 is designed as:

$$\alpha_1 = \frac{1}{\mu} \left(-k_1 z_1 - \Delta_1 \dot{F}_1 - \Delta_2 \dot{F}_2 \right) + \dot{y}_d \tag{24}$$

where k_1 is the positive feedback gain.

Substituting (24) into (23), one has

$$\dot{z}_1 = -k_1 z_1 + \mu z_2 - \mu \varepsilon_1. \tag{25}$$

Step 2:

Considering (1) and (17), and differentiating z_2, we obtain

$$\dot{z}_2 = K_q u + D(x_2, t) - \dot{\alpha}_{1c}. \tag{26}$$

To facilitate the Lyapunov-based backstepping design, and according to the error dynamics obtained in Step 1, the variable z_2 is expected to follow the stable form

$$\dot{z}_2 = -k_2 z_2 - \mu z_1 + \widetilde{x}_{\delta 2}. \tag{27}$$

where $\widetilde{x}_{\delta 2} = D(x_2, t) - \hat{x}_{\delta 2}$ denotes the disturbance estimation error. Here, the term $-k_2 z_2$ is introduced to enforce the convergence of z_2, while the coupling term $-\mu z_1$ is used to cancel the cross term generated in the Lyapunov derivative. By substituting $\widetilde{x}_{\delta 2}$ into the desired error dynamics and comparing it with (26), we have

$$K_q u + D(x_2, t) - \dot{\alpha}_{1c} = -k_2 z_2 - \mu z_1 + \widetilde{x}_{\delta 2}. \tag{28}$$

Therefore, the actual control law is designed as:

$$u = \frac{1}{K_q} (-k_2 z_2 - \mu z_1 + \dot{\alpha}_{1c} - \hat{x}_{\delta 2}) \tag{29}$$

where k_2 is the positive feedback gain.

Substituting (29) into (26), one has

$$\dot{z}_2 = -k_2 z_2 - \mu z_1 + \tilde{x}_{\delta 2}. \tag{30}$$

3.3. Main Results

Theorem 1. *For (1), by employing the designed ADO and the control laws (24) and (29), and through appropriate selection of design parameters, the controller ensures that all signals in the closed-loop system remain bounded and achieves asymptotic tracking performance. Furthermore, the tracking error e_1 is subject to the time-varying asymmetric constraints. Moreover, the system output can keep within the required constraints.*

Proof. See Appendix A. □

Remark 5. *Notably, the control gain parameters in this paper are mainly used to regulate the convergence speed of the system error and the dynamic response performance; the observer bandwidth parameter mainly affects the estimation speed and observation accuracy of the uncertain terms; and the filter-related parameters are used to balance the filtering effect and the system response. As the values of the design parameters increase, their estimation performance can be further improved. In practical applications, the controller parameters can be selected by using the trial-and-error method. Additionally, we will focus on reinforcement learning [33,34], optimal control [35,36], fault diagnosis [37,38] and so on to further enhance the control performance of motor servo systems.*

4. Simulation Validation

For (1), we set $K_q = 375$ and $D(x_2, t) = -187.5x_2 + 30\sin(t)$. The superiority of the proposed controller is verified by comparing the following three controllers using the desired trajectory $y_d = 0.35\sin(\pi t)\left[1 - \exp\left(-0.5t^2\right)\right]$ rad.

(1) **C1:** This is the controller proposed in this paper, and its detailed design procedure can be found in Section 3. Its parameters are set to: $k_1 = 100$, $k_2 = 30$, $m_2 = 200000$, $\omega_{o2} = 150$, $w_{c1} = 0.0005$, $q_1(t) = 1/\left(t^2 + 0.1\right)$, $\beta_1 = 50$. The time-varying constraint functions are chosen as: $F_1 = 0.02 + 0.005\cos(\pi t)$, $F_2 = 0.02 - 0.005\sin(\pi t)$.

(2) **C2:** This is a secure tracking controller based on an extended state observer, and the observer is designed as follows [10].

$$\begin{cases} \dot{\hat{x}}_2 = K_q u + \hat{x}_{\delta 2} + 2\omega_{o2}(x_2 - \hat{x}_2) \\ \dot{\hat{x}}_{\delta 2} = \omega_{o2}^2(x_2 - \hat{x}_2) \end{cases}. \tag{31}$$

(3) **C3:** This controller is identical to C1, but it does not include compensation for $x_{\delta 2}$. The controller is given as follows.

$$\begin{aligned} \alpha_1 &= \frac{1}{\mu}\left(-k_1 z_1 - \Delta_1 \dot{F}_1 - \Delta_2 \dot{F}_2\right) + \dot{y}_d \\ u &= \frac{1}{K_q}\left(-k_2 z_2 - \mu z_1 + \dot{\alpha}_{1c}\right) \end{aligned}. \tag{32}$$

For fair comparison, all parameters of C2 and C3 are chosen the same as those of C1.

Figure 1 illustrates the tracking performance, where the output state x_1 accurately follows the desired trajectory y_d. Figure 2 shows the tracking error comparison of the three controllers. It can be clearly observed that the C1 controller outperforms the others. By comparing the tracking errors of C1 and C2, the effectiveness of ADO is validated, while a comparison between the tracking errors of C1 and C3 demonstrates the effective

compensation capability of $x_{\delta 2}$. Figures 3 and 4 depict the estimation performance of x_2 and $x_{\delta 2}$, respectively, demonstrating that the estimated values closely approximate the true values. The tracking error and time-varying constraint functions are shown in Figure 5, from which it can be observed that the tracking error remains within the prescribed time-varying constraint bounds. In addition, as shown in Figure 6, the control input is continuous and bounded.

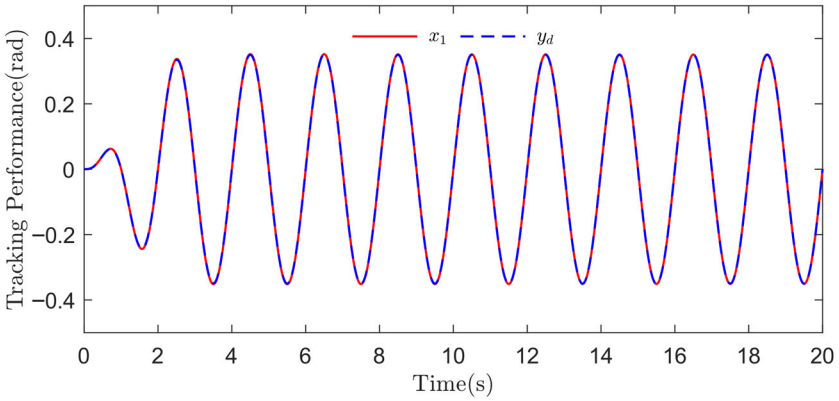


Figure 1. Tracking performance of C1.

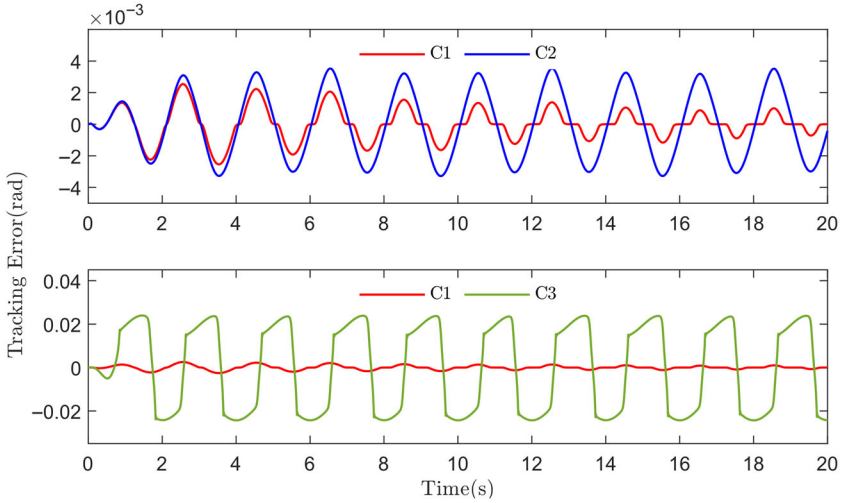


Figure 2. Tracking errors of C1, C2, and C3.

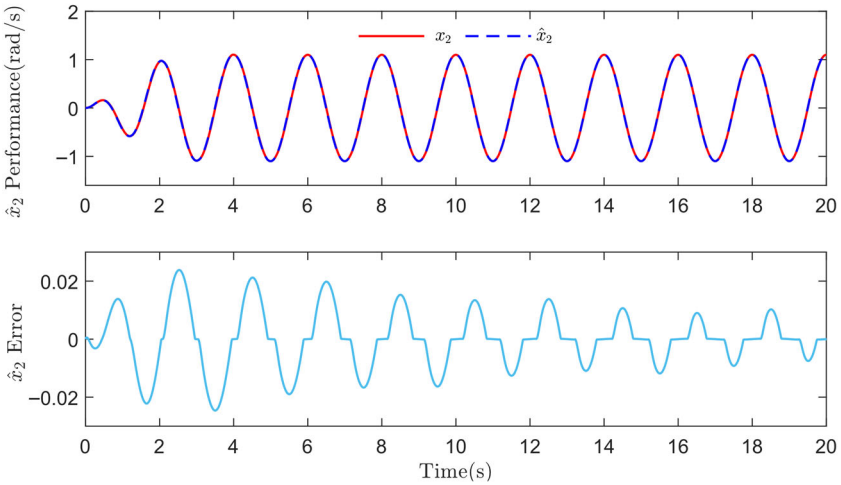


Figure 3. Estimation performance of x_2.

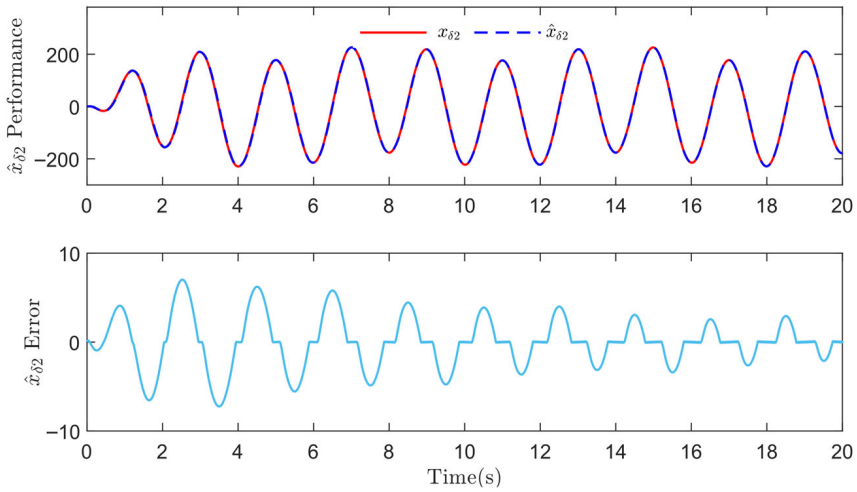


Figure 4. Estimation performance of $x_{\delta 2}$.

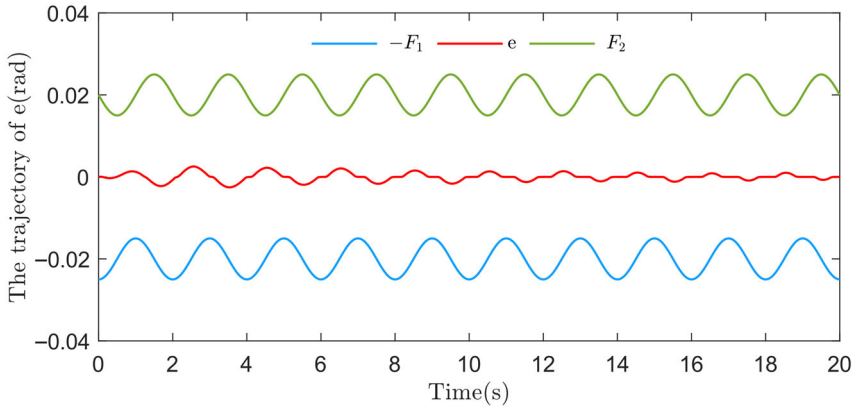


Figure 5. The trajectory of e under constraints.

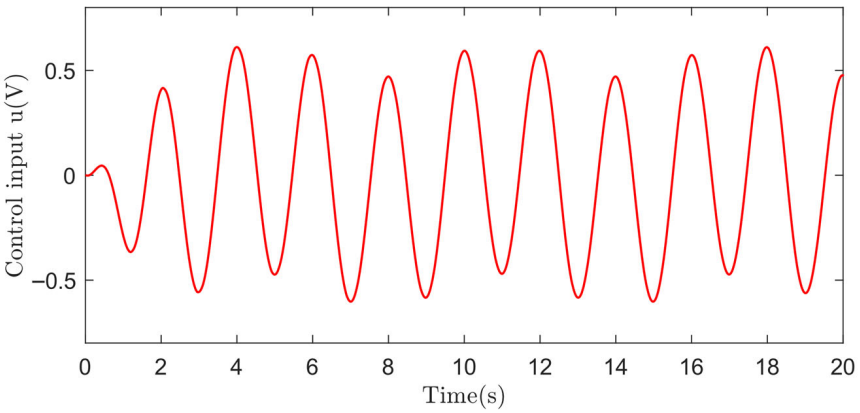


Figure 6. The control input of C1.

5. Conclusions

This paper investigates the asymptotic secure tracking control of uncertain motor servo systems under time-varying asymmetric output constraints. To handle modeling uncertainties and external disturbances, an asymptotic disturbance observer was designed to estimate and compensate for lumped uncertainties online. A tracking-error-based barrier function was introduced to ensure that the tracking error remained within the prescribed constraint boundaries throughout the entire process. In addition, a nonlinear asymptotic filter was adopted to avoid the "explosion of complexity" in backstepping design while

preserving asymptotic tracking performance. Lyapunov analysis proved that all closed-loop signals are bounded and that asymptotic tracking is achieved. Simulation results showed that the proposed controller provides accurate tracking, satisfies the prescribed constraints, and yields smaller tracking errors and better performance indices than the benchmark controllers. Future work will consider prescribed-time and event-triggered control.

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Appendix A

Proof of Theorem 1. Construct the Lyapunov function:

$$V = \frac{1}{2}z_1^2 + \frac{1}{2}z_2^2 + \frac{1}{2}\theta_2{}^T B_o \theta_2 + \frac{1}{2}\varepsilon_1^2 + \frac{1}{2\beta_1}\widetilde{N}_1^2 + \frac{1}{2m_2}\widetilde{\eta}_2^2. \tag{A1}$$

Taking the derivative of (A1) and substituting (5), (7), (16), (17), (23) and (26) into it, one has

$$\begin{aligned}\dot{V} \le & -k_1 z_1^2 - k_2 z_2^2 - \mu z_1 \varepsilon_1 + z_2 \widetilde{x}_{\delta 2} - \frac{1}{w_{c1}}\varepsilon_1^2 + \hat{N}_1|\varepsilon_1| - \frac{\varepsilon_1^2 \hat{N}_1^2}{\varepsilon_1 \hat{N}_1 \tanh\left[\varepsilon_1 q_1^{-1}(t)\right] + q_1(t)} \\ & -\frac{\widetilde{\eta}_2}{\omega_{o2}}\left|\theta_2^{\mathrm{T}} B_o C_o\right| - \omega_{o2}\|\theta_2\|^2 + \frac{1}{\omega_{o2}}\theta_2^{\mathrm{T}} B_o C_o [P_2(t) - \hat{\eta}_2 \mathrm{sgn}(\kappa_2)]\end{aligned}. \tag{A2}$$

Based on Lemma 1, we have

$$\dot{V} \le -\varpi_1 z_1^2 - \varpi_2 z_2^2 - \varpi_3 \varepsilon_1^2 - \varpi_4 \theta_{21}^2 - \varpi_5 \theta_{22}^2 + q_1(t). \tag{A3}$$

where ϖ_1, ϖ_2, ϖ_3, ϖ_4, and ϖ_5 are positive constants.

By integrating both sides of (A3) for any time $t > 0$, one has

$$\begin{aligned}V(t) \le & V(0) - \varpi_1 \textstyle\int_0^t z_1^2(\zeta)d\zeta - \varpi_2 \int_0^t z_2^2(\zeta)d\zeta - \varpi_3 \int_0^t \varepsilon_1^2(\zeta)d\zeta \\ & -\varpi_4 \textstyle\int_0^t \theta_{21}^2(\zeta)d\zeta - \varpi_5 \int_0^t \theta_{22}^2(\zeta)d\zeta + \int_0^t q_1(\zeta)d\zeta\end{aligned}. \tag{A4}$$

We further obtain

$$V(t) \le V(0) + \bar{q}_1. \tag{A5}$$

where $\bar{q}_1$ is a bounded-integrable design function, and $\bar{q}_1 = \sup_{t\ge 0}\int_0^t q_1(\zeta)d\zeta < \infty$.

Based on (A5), it can be concluded that z_1, z_2, θ_{21}, θ_{22}, ε_1, $\widetilde{N}_1$, and $\widetilde{\eta}_2 \in L_\infty$. Since $\eta_2 \ge 0$ represents an unknown but bounded constant, we can deduce $\hat{\eta}_2 \in L_\infty$. From $\theta_2 \in L_\infty$, it follows that $\widetilde{x}_{\delta 2}$. Since $x_{\delta 2}$ is bounded, we can obtain $\hat{x}_{\delta 2} \in L_\infty$. Therefore, due to the boundedness of the reference command, all system signals are bounded.

According to Lemma 1 and (18), one has

$$|\varepsilon_{1s}| \leq \frac{\varepsilon_1 \hat{N}_1^2}{\varepsilon_1 \hat{N}_1 \tanh\left[\varepsilon_1 q_1^{-1}(t)\right] + q_1(t)} \leq \hat{N}_1^2. \tag{A6}$$

Because N_1 is bounded and $\widetilde{N}_1 \in L_\infty$, we have $\hat{N}_1 \in L_\infty$. Therefore, based on (A6), it follows that $\varepsilon_{1s} \in L_\infty$. From the expression in (24), we can obtain that $\alpha_1 \in L_\infty$. Because $z_1 \in L_\infty$, $z_2 \in L_\infty$, $\dot{y}_d \in L_\infty$, $\varepsilon_{1s} \in L_\infty$, and $\hat{x}_{\delta 2} \in L_\infty$, we can achieve α_{1c} and $\dot{\alpha}_{1c} \in L_\infty$. Further, we can obtain that $u \in L_\infty$.

Moreover, it follows from (8), (18), (25), and (30) that $\dot{z}_1$, $\dot{z}_2$, $\dot{\varepsilon}_1$, and $\dot{\theta}_2 \in L_\infty$. By Barbalat's lemma, it follows that $\lim_{t\to 0} z_1 = 0$, $\lim_{t\to 0} \varepsilon_1 = 0$, and $\lim_{t\to 0} \theta_2 = 0$. By reusing the barrier transformation, we can obtain $\lim_{t\to 0} e_1 = 0$, thereby completing the proof of Theorem 1. □

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