

Article

Passivity-Based Twisting Sliding Mode Control for Series Elastic Actuators

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Abstract: This paper presents a passivity-based twisting sliding mode control (PBSMC) approach for series elastic actuators (SEAs). To address the time-varying position trajectory tracking control problem in SEAs, a fourth-order dynamic model is developed to accurately characterize the system. The control framework comprises an internal loop and an external loop controller, each designed to ensure precise trajectory tracking. The internal loop controller manages the second derivative of the joint trajectory position error, while the external loop focuses on the error itself. Both controllers are based on the PBSMC methodology to reduce complex nonlinear disturbances and minimize tracking errors. The finite-time convergence of the proposed method is rigorously analyzed. The performance and advantages of the method are evaluated and compared through various simulations.

Keywords: series elastic actuators; twisting sliding mode; internal loop control; external loop control; finite time



Academic Editor: Zhuming Bi

Received: 21 November 2024

Revised: 28 January 2025

Accepted: 29 January 2025

Published: 11 February 2025

Citation: Zhang, H.; Wang, J.; Zhang, L.; Zhang, S.; Zhang, J.; Zhang, Z. Passivity-Based Twisting Sliding Mode Control for Series Elastic Actuators. *Actuators* **2025**, *14*, 87. <https://doi.org/10.3390/act14020087>

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1. Introduction

In recent years, flexible joints have garnered significant attention as a pivotal robotics technology across various domains, including intelligent manufacturing, service robotics, and rehabilitation medicine [1]. Unlike traditional rigid joints, flexible joints, often referred to as series elastic actuators (SEAs), substantially enhance robots' flexibility, adaptability, and safety by incorporating elastic elements or flexible materials. This design reduces collision impact forces during interactions with external environments or humans, enhancing the safety of human–robot collaboration [2]. Additionally, SEAs possess inherent advantages in energy absorption and potential energy recovery, enabling a superior performance in complex tasks and dynamic settings [3]. However, the unique elastic configuration of SEAs introduces complexities in dynamic characteristics, including nonlinearity, time-varying behavior, and vibrations. Consequently, effective control strategies for SEAs must address the challenges of position control accuracy arising from elastic deformation, as well as dynamic uncertainties and vibration suppression.

Current research on control methods for SEAs primarily focuses on the development of torque and position control algorithms to ensure precise tracking. Prominent techniques include PID control, sliding mode control (SMC), adaptive control, backstepping control, model predictive control (MPC), and various intelligent algorithms. While PID control is

widely used in robotic systems and demonstrates commendable tracking performance, the presence of elastic components often leads to high-frequency oscillations and significant delays between input and output, resulting in considerable tracking errors. Furthermore, unknown external disturbances can diminish the robustness and adaptability of the PID approach. Enhancements to PID performance are typically achieved through the integration of additional methods, such as the fractional-order PID, which aims to achieve the precise trajectory tracking of SEAs while minimizing joint vibrations by employing a prescribed performance function [4]. Particle-swarm-optimization-tuned PID controllers have also been implemented to mitigate vibrations in flexible robotic arms with SEAs [5]. To counteract the impact of joint torsional vibrations on system accuracy, fuzzy PID controllers have been proposed, effectively suppressing elastic torsional vibrations in SEA systems and achieving the synchronous optimization of control accuracy and dynamic quality [6]. Fractional-order fuzzy PID controllers for trajectory tracking aim to enhance robustness against model uncertainties, interference, and noise [7]. Additionally, the integration of fuzzy logic and fractional-order techniques into PID error manifolds has been explored to improve the closed-loop system's tracking capability [8].

SMC offers robust performance and the effective handling of nonlinearities and uncertainties [9]. Recent developments include robust finite-time control frameworks utilizing continuous terminal SMC and high-order sliding mode observers to address both matched and mismatched time-varying disturbances, thus ensuring the trajectory tracking of SEAs [10]. Robust output feedback control schemes, which incorporate sliding mode observers to create innovative dynamic terminal sliding surfaces, also address disturbance-affected tracking challenges [11]. Furthermore, adaptive SMC techniques have been employed to counteract friction and torque deviations, facilitating trajectory tracking [12]. Continuous fuzzy non-singular terminal SMC approaches utilize nonlinear finite-time observers to achieve position tracking in multi-joint link systems amidst uncertainties in electrical and mechanical equations [13]. The adaptive fractional-order SMC introduces sliding mode disturbance observers to address composite disturbances, enhancing the trajectory tracking control of uncertain robotic arms [14]. Other strategies, such as adaptive fuzzy compensation-based SMC, improve control the accuracy of torque by identifying nonlinear friction torques and minimizing end-effector tracking errors [15]. Modified proportional–integral–derivative SMC approaches leverage fuzzy logic to optimize control gains, effectively reducing end vibrations in flexible robotic arms [16].

Adaptive control methods are pivotal in dynamically adjusting controller parameters to maintain system stability and performance amid model inaccuracies or external disturbances. One proposed scheme for flexible joint robots, which interact with unknown environments, employs singular perturbation methods to design inner-loop controllers that ensure accurate position tracking and neural networks for compensating for uncertainties in robot dynamics [17]. Robust output feedback controls based on adaptive observers and self-recursive wavelet neural networks have demonstrated excellent position tracking capabilities while maintaining robustness against payload uncertainties and external disturbances [18]. Self-adaptive dynamic surface controllers based on radial basis function (RBF) neural networks facilitate precise position tracking [19], while event-triggered self-adaptive neural tracking control methods reduce the command transmission frequency and actuator response rates under unknown dynamics and input saturation conditions [20]. Moreover, a novel adaptive controller for SEAs, founded on input–output feedback linearization, aims to achieve the accurate tracking of the desired position trajectories [21]. Robust control strategies utilizing adaptive neural networks have been proposed to achieve error compensation and suppress elastic vibrations in response to load variations and other uncertainties encountered in practical engineering scenarios [22]. The development of

stable inversion techniques for robot dynamics using only joint angle measurements also addresses trajectory tracking challenges for highly uncertain systems [23].

Backstepping control is particularly effective for addressing nonlinear issues associated with SEAs. Robust finite-time command-filtered backstepping strategies reconstruct unmeasurable system states and accommodate total matched and mismatched disturbance observations, achieving finite-time convergence [24]. Robust impedance controllers employing integral sliding mode control and backstepping techniques enhance the impedance control performance and system robustness [25]. Adaptive backstepping controllers based on interval type-2 fuzzy neural networks have been proposed to approximate the unknown nonlinear models of SEAs [26], and H_∞ tracking control strategies using backstepping methods have been designed to manage trajectory tracking amidst external disturbances [27].

MPC for SEAs enables preemptive adjustments to control strategies in response to detected disturbances, effectively mitigating the influence of external disturbances and reducing abrupt changes and vibrations during control processes to ensure accurate trajectory tracking. MPC technologies that incorporate predictions of SEA dynamic behaviors into controller design have been shown to improve position control performance and robustness against external forces [28]. Robust nonlinear predictive controllers facilitate the observation of system uncertainties and errors [29], while nonlinear MPC methodologies achieve effective torque control despite disturbances and uncertainties [30]. Model predictive interactive control enables iterative solutions for motion prediction robot models, accommodating both elastic and rigid contact scenarios [31].

Neural networks and other intelligent algorithms excel in identifying complex patterns within large datasets, making them highly effective for tasks like image and speech recognition. Their versatility allows their application across a broad spectrum of industries, including healthcare, finance, and autonomous driving technologies [32–34]. Intelligent algorithms enhance the real-time adjustment of control strategies through adaptive mechanisms, effectively managing uncertainties and disturbances while improving system robustness and control accuracy. The integration of disturbance-observer-based neural network integral sliding mode control employs disturbance estimation to mitigate system uncertainties, with integral sliding mode techniques further addressing steady-state errors [35]. Sliding mode boundary controllers leveraging adaptive RBF neural networks enhance control robustness against modeling uncertainties and external disturbances [36]. Moreover, control strategies based on artificial neural networks reduce the feedback sensitivity of learning rates to changes in load inertia [37].

Based on the perspectives discussed, this paper aims to develop an angular position trajectory controller for SEAs that relies solely on angular position feedback. The proposed methodology is implemented using a control scheme based on the passivity-based twisting sliding mode control (PBSMC) framework. The key contributions of this paper, in comparison to the existing literature, are as follows:

1. Unlike controllers designed exclusively for position regulation, this paper addresses both position regulation and time-varying trajectory tracking control for SEAs.
2. A dual-loop control scheme with “internal” and “external” loops ensures stable and robust performance. The internal loop controller manages the second derivative of the joint trajectory position error, while the external loop focuses on the error itself.
3. The proposed controller is accompanied by a rigorous stability analysis using a passivity-based approach. Simulations demonstrate the effectiveness and advantages of the method presented in this paper.

The remainder of this paper is structured as follows. Section 2 outlines the general dynamic model of SEAs. Section 3 introduces the passivity-based twisting sliding mode

control and presents a stability analysis of the SEA system. Section 4 presents the simulation results under various conditions. Section 5 concludes the paper.

2. General Dynamic Model of SEA

As shown in Figure 1, the general dynamic model of SEA can be equivalent to a dual-mass spring damper system.

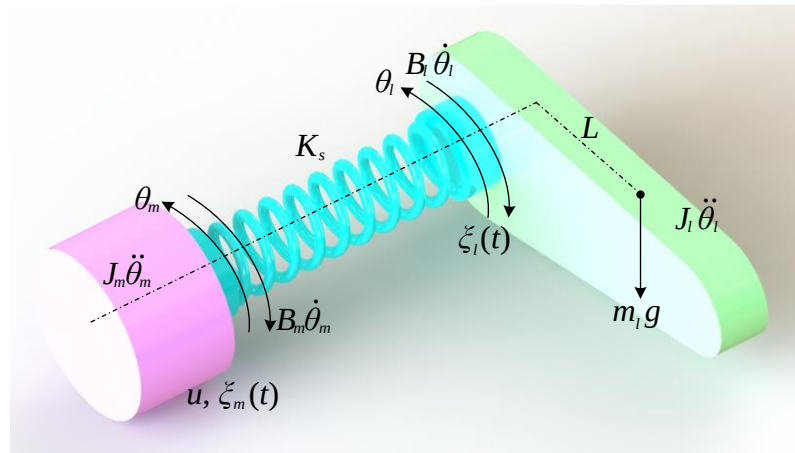


Figure 1. The equivalent schematic diagram of the SEA.

The dynamics of the SEA are expressed by

$$\begin{aligned} J_l \ddot{\theta}_l + B_l \dot{\theta}_l + m_l g L \sin(\theta_l) &= K_s(\theta_m - \theta_l) + \zeta_l(t) \\ J_m \ddot{\theta}_m + B_m \dot{\theta}_m + K_s(\theta_m - \theta_l) &= u + \zeta_m(t) \end{aligned} \quad (1)$$

where θ_l , $\dot{\theta}_l$ and $\ddot{\theta}_l$ denote the load angular position, angular velocity, and angular acceleration, respectively; θ_m , $\dot{\theta}_m$ and $\ddot{\theta}_m$ denote the angular position, angular velocity, and angular acceleration of the motor, respectively; J_l is the load inertia, B_l is the load damping coefficient, m_l is the mass, J_m is the motor inertia, B_m is the motor damping coefficient, L is the distance from the axis of rotation to the center of mass of the load, K_s is the spring stiffness, $\zeta_l(t)$ denotes unknown disturbances at the load end, and $\zeta_m(t)$ denotes unknown disturbances at the motor end.

Selecting the state variables $x_1 = \theta_l$, $x_2 = \dot{\theta}_l$, $x_3 = \theta_m$ and $x_4 = \dot{\theta}_m$, the system model (1) is expressed as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -a_{l1}x_1 - a_{l2}x_2 + a_{l1}x_3 - a_{l3}\sin(x_1) + d_l \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= a_{m1}x_1 - a_{m1}x_3 - a_{m2}x_4 + a_{m3}u + d_m \\ d_l &= \frac{1}{J_l}\zeta_l(t) \\ d_m &= \frac{1}{J_m}\zeta_m(t) \end{aligned} \quad (2)$$

where $a_{l1} = \frac{K_s}{J_l}$, $a_{l2} = \frac{B_l}{J_l}$, $a_{l3} = \frac{G_l}{J_l}$, $a_{m1} = \frac{K_s}{J_m}$, $a_{m2} = \frac{B_m}{J_m}$, $a_{m3} = \frac{1}{J_m}$, $d_l = \frac{\zeta_l(t)}{J_l}$ and $d_m = \frac{\zeta_m(t)}{J_m}$.

Assumption 1. The disturbances d_l and d_m are bounded, i.e., $|d_l| \leq \delta_l$ and $|d_m| \leq \delta_m$

3. Passivity-Based Twisting Sliding Mode Control

Before designing the controller, the tracking error is defined as

$$\begin{aligned} e_1 &= x_1 - x_{1d}, & e_2 &= x_2 - \dot{x}_{1d} \\ e_3 &= x_3 - x_{3d}, & e_4 &= x_4 - \dot{x}_{3d} \end{aligned} \quad (3)$$

A sliding mode surface for the load subsystem is designed as $s_1 = e_1$. Using Equations (2) and (3),

$$\ddot{s}_1 = -a_{l1}x_1 - a_{l2}x_2 + a_{l1}x_3 - a_{l3}\sin(x_1) + d_l - \ddot{x}_{1d}, \quad (4)$$

Based on Equation (4), the internal loop controller is designed as

$$x_{3d} = \frac{1}{a_{l1}}(a_{l1}x_1 + a_{l2}x_2 + a_{l3}\sin(x_1) + \ddot{x}_{1d}) - \frac{\alpha_l}{a_{l1}}(\text{sign}(s_1) + 0.5\text{sign}(\dot{s}_1)), \quad (5)$$

where α_l is a positive control gain.

Substituting the internal loop controller (5) into Equation (4) yields

$$\ddot{s}_1 = -\alpha_l[\text{sign}(s_1) + 0.5\text{sign}(\dot{s}_1)] + d_l + a_{l1}s_3, \quad (6)$$

The sliding mode surface for the motor subsystem is designed as $s_3 = e_3$. Using Equations (2) and (3),

$$\ddot{s}_3 = a_{m1}x_1 - a_{m1}x_3 - a_{m2}x_4 + a_{m3}u + d_m - \ddot{x}_{3d}, \quad (7)$$

Based on Equation (7), the external loop controller is designed as

$$u = \frac{1}{a_{m3}}(-a_{m1}x_1 + a_{m1}x_3 + a_{m2}x_4 + \ddot{x}_{3d}) - \frac{\alpha_m}{a_{m3}}(\text{sign}(s_3) + 0.5\text{sign}(\dot{s}_3)), \quad (8)$$

where α_m is a positive control gain.

Remark 1. $\ddot{x}_{3d}$ is used in control law (8). However, x_{3d} is non-differentiable. In practical control applications, the tanh function can be infinitely close to the sign function, so in practical applications, the tanh function can be used instead of sign to make x_{3d} differentiable.

Remark 2. For the control laws (8) with Equation (3), x_1 , x_2 , x_3 and x_4 are required in practical applications. However, x_2 and x_4 often cannot be directly measured and can be obtained using tracking differentiators using x_1 and x_2 .

Substituting the internal loop controller (5) into Equation (4) yields

$$\ddot{s}_3 = -\alpha_m[\text{sign}(s_3) + 0.5\text{sign}(\dot{s}_3)], \quad (9)$$

Theorem 1. Considering the system model (2) under Assumption 1, the controller (8) with the virtual controller (5) guarantees that the tracking errors e_1 , e_2 , e_3 and e_4 can fast converge to 0.

Proof of Theorem 1. The proof can be divided into three steps.

Step 1: The dynamic system (6) is re-expressed as

$$\begin{aligned} \dot{s}_1 &= s_2 \\ \dot{s}_2 &= -\alpha_l\text{sign}(s_1) - 0.5\alpha_l\text{sign}(\dot{s}_1) + d_l + a_{l1}s_3 \end{aligned} \quad (10)$$

Defining $z_l = [|s_1|^{1/2} \text{sign}(s_1), s_2]^T$, the Lyapunov function for Equation (10) is considered as

$$V_1(s_1, s_2) = |s_1| z_l^T A_l z_l + \frac{1}{4} s_2^4, \quad (11)$$

where $A_l = \begin{bmatrix} \alpha_l^2 & \gamma_l/2 \\ \gamma_l/2 & \alpha_l \end{bmatrix}$, $\gamma_l \leq 2\alpha_l^{3/2}$.

Defining $\eta_l = [|s_1|, |s_2|^2]^T$, and taking into account $\lambda_{\min}(A_l) \|z_l\|^2 \leq z_l^T A_l z_l \leq \lambda_{\max}(A_l) \|z_l\|^2$ and Young's inequality, it follows that

$$\begin{aligned} V_1(s_1, s_2) &\leq \lambda_{\max}(A_l) (|s_1|^2 + |s_1||s_2|^2) + \frac{1}{4} s_2^4 \\ &\leq \frac{3}{2} \lambda_{\max}(A_l) s_1^2 + \frac{1}{2} \left(\lambda_{\max}(A_l) + \frac{1}{2} \right) s_2^4, \\ &\leq \eta_l^T P' \eta_l \end{aligned} \quad (12)$$

where $P' = \begin{bmatrix} \frac{3}{2} \lambda_{\max}(A_l) & 0 \\ 0 & \frac{\lambda_{\max}(A_l)}{2} + \frac{1}{4} \end{bmatrix}$.

The matrix P' is a positive definite diagonal matrix. Taking into account $\lambda_{\min}(P') \|\eta_l\|^2 \leq \eta_l^T P' \eta_l \leq \lambda_{\max}(P') \|\eta_l\|^2$, then

$$V_1(s_1, s_2) \leq \lambda_{\max}(P') \|z_l\|^2 \leq \lambda_{\max}(P') (|s_1|^{1/2} + |s_2|)^4, \quad (13)$$

Equation (12) can be rewritten as

$$V_1(s_1, s_2) \geq \lambda_{\min}(A_l) (|s_1|^2 + |s_1||s_2|^2) + \frac{1}{4} s_2^4 \geq \lambda_{\min}(A_l) |s_1|^2 + \frac{1}{4} s_2^4, \quad (14)$$

Equation (14) shows that $V_1(s_1, s_2)$ is a positive definite matrix. Using Equation (12), differentiating Equation (13) yields

$$\begin{aligned} \dot{V}_1(s_1, s_2) &= \left(2\alpha_l^2 s_1 + \frac{3}{2} \gamma_l |s_1|^{1/2} s_2 + \alpha_l \text{sign}(s_1) s_2^2 \right) \dot{s}_1 + \left(\gamma_l |s_1|^{3/2} \text{sign}(s_1) + 2\alpha_l |s_1| s_2 + s_2^3 \right) \dot{s}_2 \\ &= -\gamma_l \left(\alpha_l - d_l \text{sign}(s_1) + \frac{1}{2} \alpha_l \text{sign}(s_1 s_2) \right) |s_1|^{3/2} - \alpha_l (\alpha_l - 2d_l \text{sign}(s_2)) |s_1| |s_2| + \frac{3}{2} \gamma_l |s_1|^{1/2} s_2^2 \\ &\quad - \left(\frac{1}{2} \alpha_l - d_l \text{sign}(s_2) \right) |s_2|^3 + a_{l1} \left(\gamma_l |s_1|^{3/2} \text{sign}(s_1) + 2\alpha_l |s_1| s_2 + s_2^3 \right) s_3 \\ &= -\gamma_l \left(\alpha_l - d_l \text{sign}(s_1) + \frac{1}{2} \alpha_l \text{sign}(s_1 s_2) \right) |s_1|^{3/2} + a_{l1} \left(\gamma_l |s_1|^{3/2} \text{sign}(s_1) + 2\alpha_l |s_1| s_2 + s_2^3 \right) s_3 \\ &\quad - |s_2| \left[\alpha_l (\alpha_l - 2d_l \text{sign}(s_2)) |s_1| - \frac{3}{2} \gamma_l |s_1|^{1/2} |s_2| + \left(\frac{1}{2} \alpha_l - d_l \text{sign}(s_2) \right) s_2^2 \right] \end{aligned} \quad (15)$$

Assuming that $\alpha_l > 2\delta_l$ and since $|d_l| \leq \delta_l$, Equation (16) implies

$$\begin{aligned} \dot{V}_1(s_1, s_2) &\leq -|s_2| \left[\alpha_l (\alpha_l - 2\delta_l) |s_1| - \frac{3}{2} \gamma_l |s_1|^{1/2} |s_2| + \left(\frac{1}{2} \alpha_l - \delta_l \right) s_2^2 \right] - \gamma_l \left(\frac{1}{2} \alpha_l - \delta_l \right) |s_1|^{3/2} \\ &\quad + a_{l1} \left(\gamma_l |s_1|^{3/2} + 2\alpha_l |s_1| |s_2| + |s_2|^3 \right) s_3 \\ &= -\gamma_l \left(\frac{1}{2} \alpha_l - \delta_l \right) |s_1|^{3/2} - |s_2| B^T P_1 B + a_{l1} \left(\gamma_l |s_1|^{3/2} + 2\alpha_l |s_1| |s_2| + |s_2|^3 \right) s_3 \end{aligned} \quad (16)$$

where $B = [|s_1|^{1/2}, |s_2|]^T$, and $P_1 = \begin{bmatrix} 2\alpha_l \left(\frac{1}{2} \alpha_l - \delta_l \right) & -\frac{3}{4} \gamma_l \\ -\frac{3}{4} \gamma_l & \frac{1}{2} \alpha_l - \delta_l \end{bmatrix}$.

If the conditions $\alpha_l > 2\delta_l$ and $0 < \gamma_l < \frac{4\sqrt{2}}{3} \sqrt{\alpha_l} \left(\frac{1}{2} \alpha_l - \delta_l \right)$ hold, then the matrix P_1 is positive definite. Since the matrix P_1 is positive definite, the following inequality holds

$$\lambda_{\min}(P_1) (|s_1| + s_2^2) \leq B^T P_1 B \leq \lambda_{\max}(P_1) (|s_1| + s_2^2), \quad (17)$$

Using Equation (17), Equation (16) is reorganized as

$$\begin{aligned}
 \dot{V}_1(s_1, s_2) &\leq -\gamma_l \left(\frac{1}{2} \alpha_l - \delta_l \right) |s_1|^{3/2} - |s_2| \lambda_{\min}(P_1) (|s_1| + s_2^2) + a_{l1} \left(\gamma_l |s_1|^{3/2} + 2\alpha |s_1| |s_2| + |s_2|^3 \right) s_3 \\
 &\leq -\gamma_l \left(\frac{1}{2} \alpha_l - \delta_l \right) |s_1|^{3/2} - |s_2|^3 \lambda_{\min}(P_1) + I(s_1, s_2) s_3 \\
 &\leq -K_l \left(|s_1|^{3/2} + |s_2|^3 \right) + I(s_1, s_2) s_3 \\
 &\leq -\frac{K_l}{2^{2/3}} \left(|s_1|^{1/2} + |s_2| \right)^3 + I(s_1, s_2) s_3 \\
 &\leq -\frac{K_l}{2^{2/3} \lambda_{\max}^{3/4}(P_1)} V_1^{3/4}(s_1, s_2) + I(s_1, s_2) s_3
 \end{aligned} \tag{18}$$

where $K_l = \min \left\{ \lambda_{\min}(P_1), \gamma_l \left(\frac{1}{2} \alpha_l - \delta_l \right) \right\}$.

Step 2: The dynamic system (9) is re-expressed as

$$\begin{aligned}
 \dot{s}_3 &= s_4 \\
 \dot{s}_4 &= -\alpha_m \text{sign}(s_3) - 0.5 \alpha_m \text{sign}(\dot{s}_3) + d_m
 \end{aligned} \tag{19}$$

Defining $z_m = \left[|s_3|^{1/2} \text{sign}(s_3), s_4 \right]^T$, and considering the Lyapunov function for Equation (19)

$$V_2(s_3, s_4) = |s_3| z_m^T A_m z_m + \frac{1}{4} s_4^4, \tag{20}$$

where $A_m = \begin{bmatrix} \alpha_m^2 & \gamma_m/2 \\ \gamma_m/2 & \alpha_m \end{bmatrix}$, $\gamma_m \leq 2\alpha_m^{3/2}$.

Referring to the procedure in Step 1, if the conditions $\alpha_m > 2\delta_m$ and $0 < \gamma_m < \frac{4\sqrt{2}}{3} \sqrt{\alpha_m} \left(\frac{1}{2} \alpha_m - \delta_m \right)$ hold, then the matrix P_2 is positive definite and it follows that

$$\dot{V}_2(s_3, s_4) \leq -\frac{K_m}{2^{2/3} \lambda_{\max}^{3/4}(P_2)} V_2^{3/4}(s_3, s_4), \tag{21}$$

where $K_m = \min \left\{ \lambda_{\min}(P_2), \gamma_m \left(\frac{1}{2} \alpha_m - \delta_m \right) \right\}$, $P_2 = \begin{bmatrix} 2\alpha_m \left(\frac{1}{2} \alpha_m - \delta_m \right) & -\frac{3}{4} \gamma_m \\ -\frac{3}{4} \gamma_m & \frac{1}{2} \alpha_m - \delta_m \end{bmatrix}$.

Step 3: Equation (18) can be rewritten as

$$\underbrace{I(s_1, s_2)}_{\text{input}} \underbrace{s_3}_{\text{output}} \geq \dot{V}_1(s_1, s_2) + \frac{K_l}{2^{2/3} \lambda_{\max}^{3/4}(P_1)} V_1^{3/4}(s_1, s_2), \tag{22}$$

Equation (22) shows that the relationship between $I(s_1, s_2)$ and s_3 is strictly output passive. Therefore, it is bounded-input bounded-output (BIBO) stable. The Lyapunov function $V_2(s_3, s_4)$ is positively definite, and Equation (21) shows that the dynamic system (19) is finite time stable, i.e., s_3 and s_4 will converge to 0 within a finite time. Consequently, s_1 and s_2 will converge to zero. $\square$

4. Simulations

To evaluate the performance of the proposed technique, a series of simulations were conducted. The system and controller parameters used in the simulations are provided in Table 1. To demonstrate the advantages of the developed control method, comparisons were made with PID and backstepping control. Considering the output torque limitations of real motors, the torque in this study is constrained to ± 50 Nm.

Table 1. The system parameters of SEA (Case 1).

Parameters	Value	Parameters	Value
J_{mn}	1.6 kg·m ²	G_{ln}	8.2 Nm
B_{mn}	0 Nm/(rad·s ^{−1})	m_l	1.7 kg
J_{ln}	0.54 kg·m ²	L	0.5 m
B_{ln}	0 Nm/(rad·s ^{−1})	α_l	3000
K_{sn}	1126 Nm/rad	α_m	2000

The desired trajectory x_r of the SEA (Case 1) is defined in Equation (23) and Figure 2a.

$$\begin{aligned}\ddot{x}_r &= 10 \sin(\pi t) - 5x_r - 2\dot{x}_r \\ x_r(0) &= -0.2, \dot{x}_r(0) = 0\end{aligned}\quad (23)$$

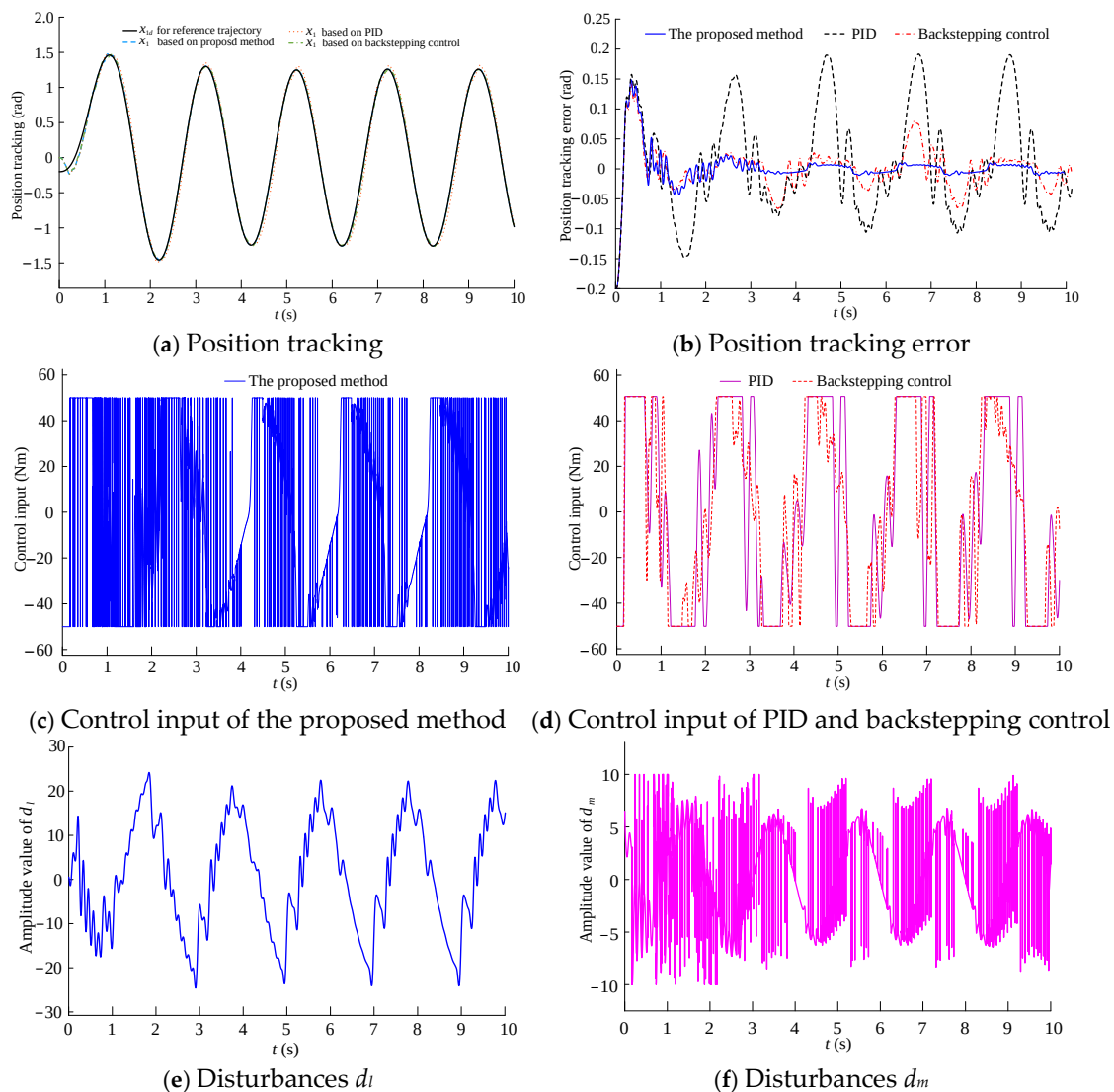


Figure 2. Comparison of simulation results between PID, backstepping method, and the proposed method without external interference.

Figure 2 illustrates the performance of the proposed method in tracking the desired trajectory x_r . Figure 2a shows the position tracking curves for the proposed method, PID control, and backstepping control, all demonstrating a strong tracking performance.

Figure 2b highlights the high tracking accuracy and minimal errors achieved by the proposed method, with tracking errors ranging between -0.05 rad and 0.05 rad. In contrast, the errors for PID and backstepping control are significantly larger, with the maximum PID error exceeding 0.2 rad. Figure 2c,d present the control inputs, where the control input curve shows some fluctuations due to the signum functions used in Equations (5) and (9). Despite these fluctuations, the design enables the SEA to achieve excellent tracking performance. The control input curves for PID and backstepping control exhibit less variation but result in larger tracking errors compared to the proposed method. Figure 2e,f depict the changes in disturbances d_l and d_m .

To verify the performance of the presented method against different interferences, a significant external disturbance is introduced into x_2 of Equation (2) and expressed as follows:

$$d_E = 10 \sin(2\pi t) \quad (24)$$

Figure 3a shows the position tracking curves for the different methods used to track the desired trajectory x_r . Figure 3b demonstrates that the proposed method achieves high tracking accuracy with minimal errors. The corresponding tracking errors range between -0.07 rad and 0.06 rad. The performance of the proposed method is slightly better than the backstepping control method and significantly better than the PID method. The absolute values of the steady-state errors for the PID and backstepping control methods exceed 0.2 rad and 0.08 rad, respectively. As seen in Figure 3c,d, the control input curve based on the proposed method fluctuates significantly, while the control input curves for the other methods show less variation. Figure 3e,f illustrate the changes in disturbances d_l and d_m during trajectory tracking. The disturbance d_l exhibits larger amplitude changes compared to Figure 2e due to the introduction of external interference d_E , while d_m remains similar to that shown in Figure 2f.

Considering both Figures 2 and 3, the method presented in this paper demonstrates an excellent tracking performance for SEAs under various external interference conditions. Compared to PID and backstepping control methods, the proposed approach offers higher tracking accuracy, smaller tracking errors, stronger robustness, and faster response capabilities, making it more suitable for controlling flexible joints and flexible robotic arms.

To validate the effectiveness of the controller across different SEA systems, the simulation parameters were established, as detailed in Table 2. The desired trajectory x_r of the SEA (Case 2) is defined in Equation (25).

$$\begin{aligned} \ddot{x}_r &= 8 \sin(2\pi t) - 5x_r - 2\dot{x}_r \\ x_r(0) &= -0.2, \dot{x}_r(0) = 0 \end{aligned} \quad (25)$$

Table 2. The system parameters of SEA (Case 2).

Parameters	Value	Parameters	Value
J_{mn}	$2.1 \text{ kg}\cdot\text{m}^2$	G_{ln}	7.2 Nm
B_{mn}	$0.2 \text{ Nm}/(\text{rad}\cdot\text{s}^{-1})$	m_l	1.7 kg
J_{ln}	$0.40 \text{ kg}\cdot\text{m}^2$	L	0.5 m
B_{ln}	$0.2 \text{ Nm}/(\text{rad}\cdot\text{s}^{-1})$	α_l	3000
K_{sn}	$1000 \text{ Nm}/\text{rad}$	α_m	2000

As can be inferred from Figure 4, the control methodology proposed in this paper exhibits a high degree of generality, rendering it suitable for application across various SEA systems. In other words, compared to PID and backstepping control approaches, the

proposed control method demonstrates superior tracking accuracy, minimal tracking error, enhanced robustness, and faster response characteristics across different parameter sets. These attributes make it particularly well suited to controlling flexible joints and robotic arms with elastic elements.

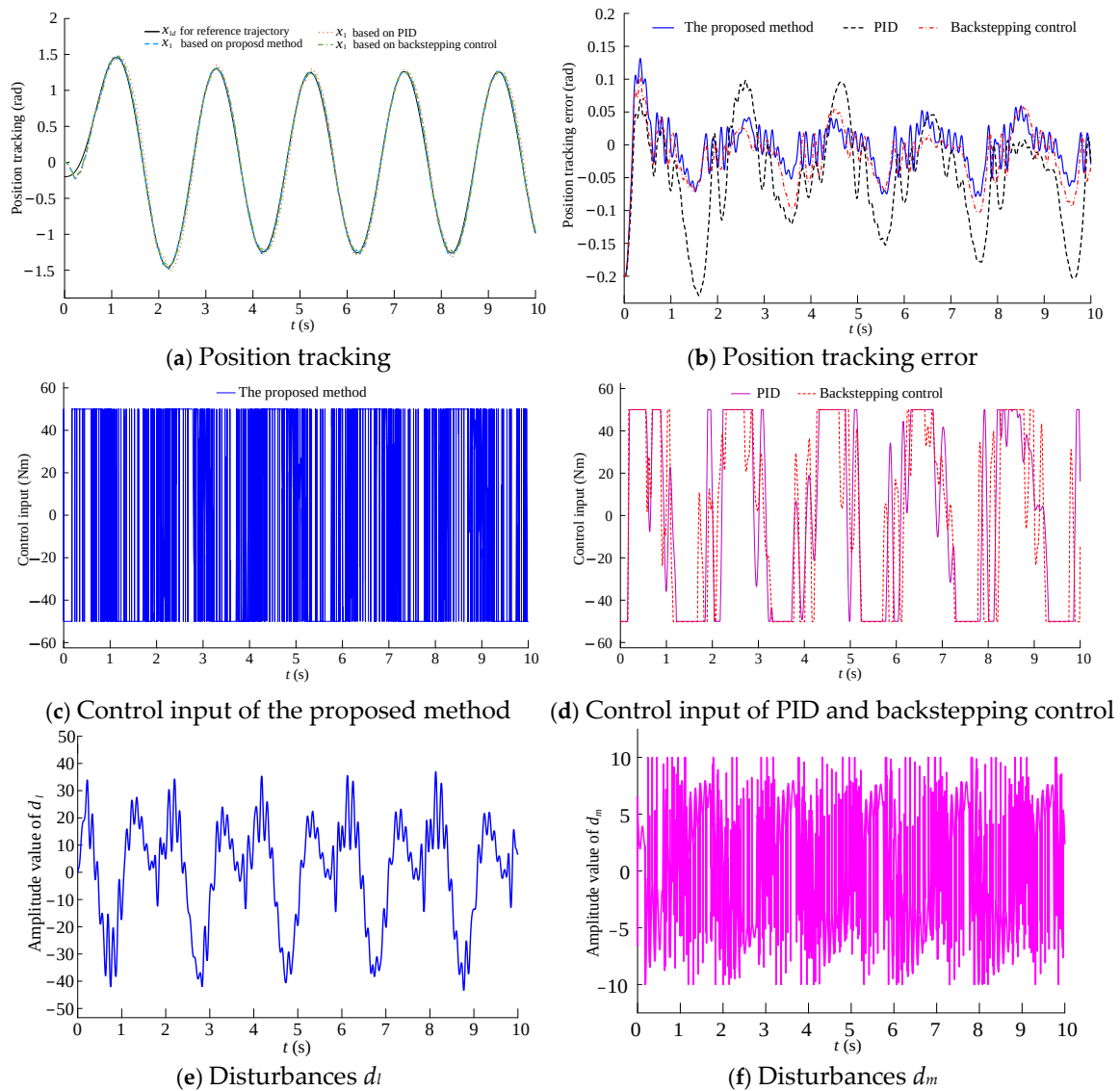


Figure 3. Comparison of simulation results between PID, backstepping method, and the proposed method with external interference.

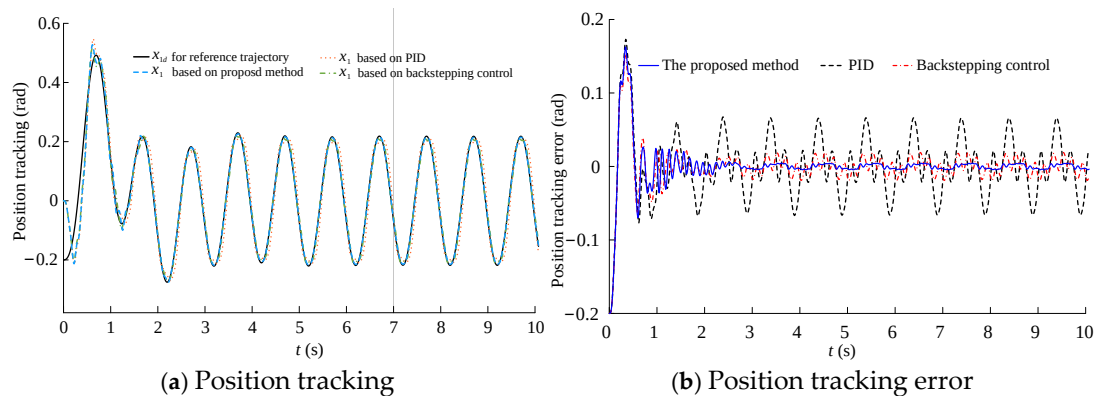


Figure 4. Simulation results for SEA (Case 2).

5. Conclusions

This paper proposes a PBSMC method for SEAs to achieve position trajectory tracking. A fourth-order dynamic model for the SEA is defined, and a control framework comprising an internal loop and an external loop controller is designed based on the PBSMC. The finite-time convergence of the proposed method is rigorously analyzed. The simulation results demonstrate that this method exhibits superior tracking performance and reduced errors compared to PID and backstepping control methods. Furthermore, the proposed approach maintains an excellent trajectory tracking performance, strong robustness, and rapid response capabilities, even in the presence of significant external disturbances. The PBSMC presented in this paper is well suited for flexible joint control. In future work, the proposed PBSMC will be experimentally validated on a dedicated testing platform. This platform will be designed to replicate the dynamic characteristics of flexible joints under various operating conditions, enabling the comprehensive evaluation of the algorithm's robustness, adaptability, and performance. Moreover, the algorithm will be applied to multi-joint flexible robotic arms, emphasizing its ability to achieve precise trajectory tracking and force control in complex, dynamic environments.

Author Contributions: Conceptualization, H.Z., J.W., L.Z., S.Z., J.Z. and Z.Z.; methodology, H.Z., J.W., L.Z., S.Z., J.Z. and Z.Z.; software, H.Z., J.W., L.Z., S.Z., J.Z. and Z.Z.; validation, H.Z. and J.W.; formal analysis, H.Z., J.W., L.Z., S.Z., J.Z. and Z.Z.; investigation, H.Z., J.W., L.Z., S.Z., J.Z. and Z.Z.; resources, H.Z.; data curation, H.Z.; writing—original draft preparation, H.Z.; writing—review and editing, H.Z.; visualization, H.Z.; supervision, H.Z., J.W. and S.Z.; project administration, H.Z., J.W. and S.Z.; funding acquisition, H.Z., J.W. and S.Z. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by the Foundation of the National Natural Science Foundation of China under Grant 52405017, the Foundation of the Key Research and Development Program of Henan Province under Grant 242102220118, the Foundation of the Anhui Province Key Laboratory of Machine Vision Detection and Perception under Grant KLMVI-2023-HIT-18, the Foundation of the Key Research Projects of Higher Education Institutions in Henan Province under Grants 25A410001 and 25A413003, the Foundation of the Open Research Project of the Key Laboratory of Grain Information Processing and Control under Grant KFJJ-2021-110, and the Foundation of the Science Foundation of Henan University of Technology under Grants 2021BS060 and 2023BS109.

Data Availability Statement: Data are contained within the article.

Conflicts of Interest: The authors declare no conflicts of interest.

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