

Article

Stiffness Optimization of a Robotic Drilling System for Enhanced Accuracy in Aerospace Assembly

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Abstract: The low stiffness of robots significantly limits their applicability within the aerospace assembly and manufacturing sectors. The majority of existing research focuses on optimizing robot posture; however, the efficacy of these approaches is constrained in situations with minimal posture variation. To address this challenge, this study examines a robotic drilling system designed for use in confined spaces. An in-depth analysis of its stiffness model is conducted, and the system's stiffness limitations are identified using the stiffness ellipsoid evaluation method. Based on the mechanical analysis of the drilling state, a stiffness enhancement method grounded in the local force closure of the end effector is proposed. This method involves locking the end effector's expansion module with the substrate during the drilling process, thereby enabling the axial drilling forces to be jointly borne by the expansion module and the robot's base joints. Consequently, the system's stiffness, particularly in the axial direction, is substantially improved. A series of experiments rigorously validate the effectiveness of the proposed stiffness enhancement method. The experimental results demonstrate that the stiffness-optimized robot reduces axial deformation during drilling by a factor of ten and significantly improves hole quality and exit burr height.

Keywords: robotic drilling system; stiffness enhancement method; axial stiffness; aircraft assembly



Academic Editor: Zhuming Bi

Received: 25 December 2024

Revised: 8 February 2025

Accepted: 9 February 2025

Published: 11 February 2025

Citation: Xu, H.; Xue, J.; Guo, G.; Liu, Y.; Liu, M.; Zhang, D. Stiffness Optimization of a Robotic Drilling System for Enhanced Accuracy in Aerospace Assembly. *Actuators* **2025**, *14*, 86. <https://doi.org/10.3390/act14020086>

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1. Introduction

In the manufacturing sector, particularly within the aerospace industry where high precision is paramount, robots are widely utilized due to their flexibility, cost-effectiveness, and efficient performance in repetitive tasks [1]. Specifically, robotic technologies have demonstrated significant advantages in key machining processes, such as drilling [2–4], boring [5,6], milling [7–9], and grinding [10–12]. For instance, in the assembly of aircraft skins, drilling precision directly affects fatigue life, making the enhancement of machining accuracy a critical objective [13]. However, industrial robots, especially serial robots, are limited in their application to high-precision tasks due to their inherent low stiffness characteristics. Compared to traditional machine tools, these robots are more susceptible to deformation during the drilling process, thereby impacting machining accuracy and quality [14,15].

To mitigate the challenges posed by low stiffness, researchers have proposed various strategies, including stiffness modeling, error compensation, and posture optimization. Chen et al. [16] developed stiffness and cutting force models specifically for robots

and employed an offline compensation method to correct errors caused by deformation. Li et al. [17] established an equivalent stiffness model for a tracked drilling robot and proposed a global position accuracy compensation optimization algorithm based on the elastic deformation of the transmission system's coupling surfaces. This method reduced the positional accuracy error in drilling by 68.67%. Zhang et al. [18] utilized machine learning techniques for error compensation, achieving reductions in positional error and machining error by 48.67% and 28.73%, respectively, compared to traditional methods. Tan et al. [19] introduced an optimization method that accounts for variations in cutting forces under different tool postures by adjusting the tool direction and redundancy angles to minimize tool tip deformation errors caused by these forces. Experimental results demonstrated that this method significantly improved the precision of robotic milling operations compared to unoptimized and existing approaches. However, these methods primarily target dynamic robotic operations, such as milling and grinding. In contrast, for quasi-static operations, like drilling, existing error compensation methods rely mainly on offline compensation for drilling position errors and countersink depth errors, making it difficult to effectively address drilling diameter errors and quality issues caused by insufficient stiffness.

Robot stiffness varies with their posture; therefore, posture optimization has become a critical approach to enhancing machining performance. Tian et al. [20] proposed a stiffness performance indicator to quantify the impact of robot configurations on overall stiffness and developed a posture optimization model based on this indicator, with experimental results validating its effectiveness. Bu et al. [21] established a Cartesian flexibility model for robot stiffness and introduced quantitative evaluation indicators for machining performance, investigating the variation patterns of axial stiffness under different postures. Zargarbashi et al. [22] proposed a kinematic-hydrostatic performance metric called the robot transmission ratio (RTR) to evaluate the effectiveness of attitude optimization of a six-degree-of-freedom tandem industrial robot in machining tasks. The rigidity of the machining system was improved by optimizing the positional attitude of the robot under redundant degrees of freedom. Song et al. [23] constructed an attitude optimization model that comprehensively considers kinematic characteristics and milling stability. The robot posture is optimized by error similarity analysis and an improved gray wolf algorithm, which significantly improves the robot's side milling performance. Cvitanic et al. [24] proposed enhancing the robot's processing stability and stiffness by increasing its redundant degrees of freedom and adjusting its posture, thereby further improving the robot's machining performance and adaptability. The aforementioned studies all assume that the workpiece is fixed and immovable in space. If the workpiece's pose can be adjusted, higher system stiffness can be achieved by simultaneously optimizing the poses of both the robot and the workpiece. Lee et al. [25] proposed a deformation energy model based on energy angles and simultaneously optimized the robot's posture and the workpiece's position through a global optimization approach, with a simulation verifying the effectiveness of the optimization results. Wang et al. [26], focusing on large spacecraft compartments, proposed a synchronous optimization method for the mobile robot base position and compartment angle based on a uniform stiffness domain indicator. Utilizing the sparrow search algorithm to simultaneously optimize the base position and compartment angle, simulations and milling experiments demonstrated that the optimization method effectively improves machining quality. Posture optimization is a widely studied and important means of enhancing stiffness performance. However, for operating in confined spaces, the limited range of posture variations restricts the effectiveness of optimization.

Structural optimization is another crucial approach to improving the stiffness of robots. In recent years, there has been considerable research on soft robots, primarily focusing on compliant mechanisms [27,28] and variable stiffness actuators [29,30]. However,

the majority of studies on soft robotics have concentrated on tasks involving human–robot interaction, medical devices, and dynamic environments, where compliance and variable stiffness are critical. In contrast, applications, such as aerospace assembly require robots with high stiffness. Currently, the technology for variable stiffness in soft robots does not meet the stiffness requirements for high-precision applications. In the area of structural optimization for robots used in manufacturing, Fang et al. [31] proposed a novel open–closed hybrid motion chain industrial robotic arm structure and established the robot’s stiffness model; they verified the stiffness enhancement method through finite element analysis and experiments. Building on this, Sun et al. [32] introduced deformation influence coefficients based on the strain energy method and Castigliano’s theorem to evaluate the deformation characteristics of open–closed hybrid motion chain robotic arms. Denkena et al. [33] proposed a special design and optimization of dual-motor joints; in addition to equipping the robot with traditional harmonic drive gearboxes, they also integrated motors with high-resolution encoders on the load side to improve system stiffness. Guo et al. [34] enhanced stiffness by adding pressure feet at the front end of the robot’s end effector, utilizing these pressure feet to apply pressure to the workpiece during the drilling process. Zheng et al. [35] developed stiffness enhancement models for the robot’s pressure feet in both axial and horizontal directions, analyzing the effect of pressure variations under pressure feet on drilling quality. Experimental results demonstrated that diameter deviation was reduced by 32.91%, and the average exit burr height decreased by 85.3% compared to scenarios without pressure feet. Although adding pressure feet at the front end of the drilling end effector indeed improves system stiffness, in drilling applications involving weakly rigid laminated materials, excessive foot force pressure may lead to elastic deformation of the material and hole misalignment, thereby reducing machining quality.

To address these challenges, this paper proposes a stiffness enhancement method based on the local force closure of the end effector, aimed at improving the robot’s stiffness performance during the drilling process, particularly in the axial direction. This method is also applicable to serial industrial robotic drilling, allowing for stiffness enhancement while preventing the deformation of materials with weak rigidity in laminated structures. The first section of this paper focuses on stiffness modeling of the robotic drilling system and employs simulation or parameter identification techniques to conduct detailed measurements and evaluations of the stiffness of each module within the drilling system, thereby identifying weak points in stiffness. The second section introduces the stiffness optimization method and establishes theoretical models of drilling forces and module deformations during the robot’s drilling process both before and after optimization, providing an in-depth analysis of the optimization effects. Finally, the effectiveness of the proposed stiffness optimization scheme is validated through experimental testing.

2. Structural Design and Stiffness Modeling Analysis of the Robotic Drilling System

2.1. Structural Design of the Robotic Drilling System

The robotic drilling system is designed to perform drilling tasks in confined and complex environments, particularly in aerospace applications. It is well-suited for aircraft manufacturing and maintenance, where conventional drilling equipment faces challenges in accessing narrow spaces, such as engine inlets and nacelles. Figure 1a illustrates a typical application scenario, where the robot is deployed in restricted working areas to carry out high-precision drilling operations.

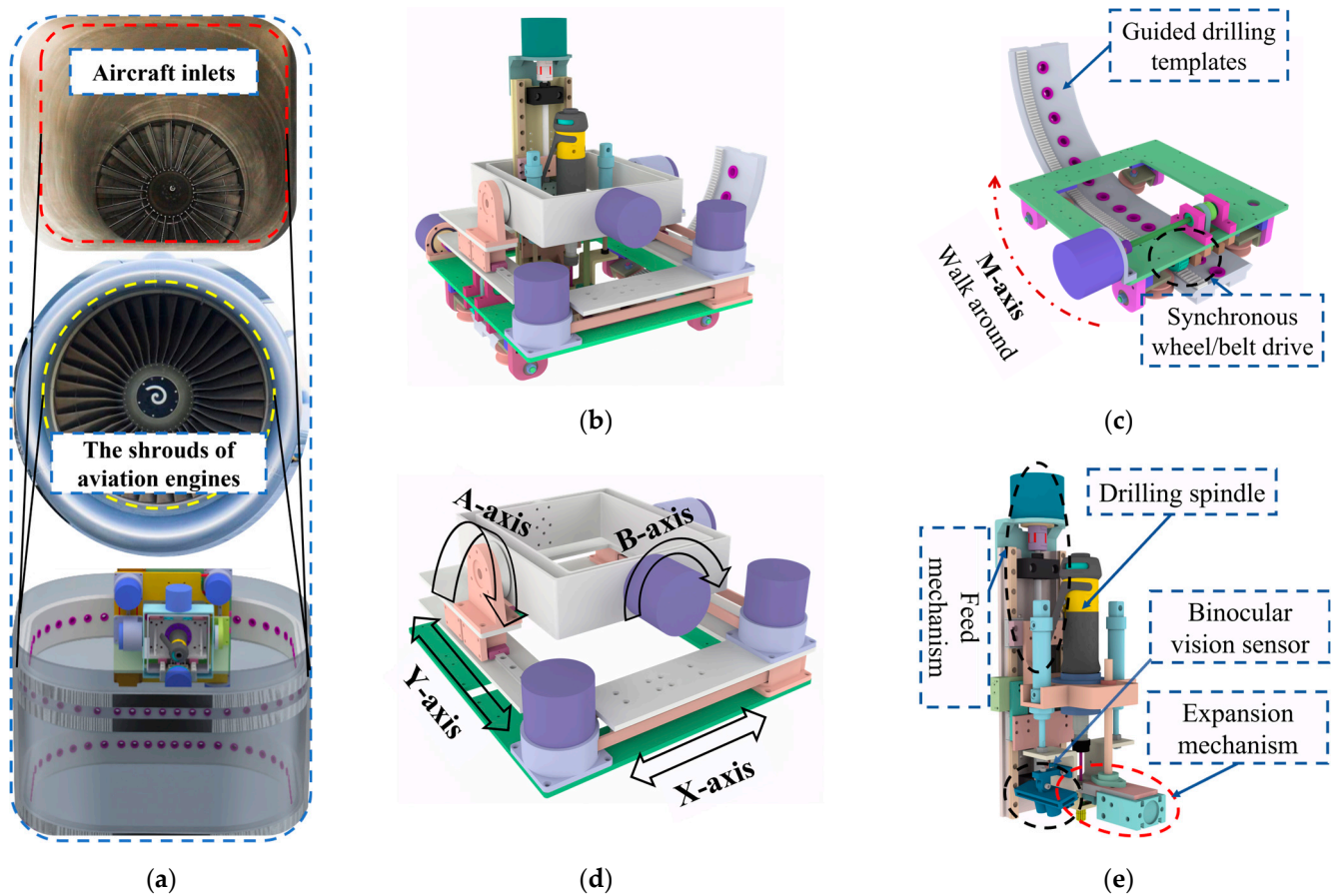


Figure 1. Structure and application of the robotic drilling system: (a) application scenario in a confined space; (b) robotic drilling system; (c) chassis system; (d) four-degree-of-freedom (4-DOF) posture adjustment platform; (e) drilling end effector.

Structurally, the robot consists of three major modules: the chassis system, a four-degree-of-freedom (4-DOF) posture adjustment platform, and the drilling end effector, as illustrated in Figure 1. The key parameters of the robot are detailed in Table 1, which also includes relevant industrial robot parameters for comparison. The chassis system consists of a chassis frame, passive steering wheels, and active drive wheels, as shown in Figure 1b. The passive steering wheels work in conjunction with the guided drilling template to constrain the chassis' degrees of freedom. The active drive wheels are driven by motors and the transmission system, enabling the chassis system module to move along the guided drilling template. The 4-DOF posture adjustment platform comprises X/Y translation joints and Rx/Ry rotation joints, as depicted in Figure 1c. This platform operates as a serial module, configured in the sequence X-Y-Rx-Ry, and maintains a serial connection with the chassis system, allowing the chassis to carry the posture adjustment platform along the guided drilling template during operation. The drilling end effector consists of a Z-axis feed unit, a drilling spindle, an expanding positioning system, and a vision measurement unit, as shown in Figure 1d. The feed unit controls the spindle's feed motion; the vision measurement unit is responsible for measuring the reference holes and target holes on the guided drilling template. Through vision measurement, the robot's pose errors are fed back to the motor drivers, achieving the initial positioning of the drilling apparatus. The expanding positioning system manages the expansion and contraction of the expanding sleeve, working in tandem with the drill sleeve within the guided drilling template to achieve precise positioning of the end effector. The drilling spindle is utilized for skin drilling applications.

Table 1. Parameters of the robotic drilling system and industrial drilling robot.

Category	Size (mm)	Weight (kg)	Working Skin Radius of Curvature (mm)	Drilling Efficiency (holes/min)	Drilling Quality
1 *	294 × 372 × 375	10	200	2	>H9
2 *	N/A	2375	N/A	3~4	H8

1 * Robotic drilling system; 2 * industrial drilling robot [20].

The operational workflow of the robotic drilling system comprises the following steps: first, the robot chassis system moves to the designated work area; second, the four-degree-of-freedom (4-DOF) posture adjustment platform, in conjunction with a binocular vision camera, performs precise positioning; third, the expanding positioning system collaborates with the drill sleeve on the drilling template to achieve accurate posture adjustment; and, finally, the drilling process is executed.

2.2. Stiffness Modeling of the Robotic Drilling System

Robot stiffness refers to its ability to resist deformation when subjected to external forces and is typically categorized into static stiffness and dynamic stiffness. Static stiffness denotes the robot's capacity to withstand deformation under static loads and is usually quantified by the ratio of applied force to the resulting deformation. In contrast, dynamic stiffness reflects the robot's ability to resist vibrations under dynamic loads and is typically assessed using the robot's natural frequencies. Generally, static stiffness and dynamic stiffness are positively correlated, meaning that higher static stiffness corresponds to higher dynamic stiffness. The robotic drilling system examined in this study primarily features a serial configuration, which inherently results in relatively low stiffness—significantly lower than that of CNC machine tools. Furthermore, based on the stiffness data comparison in Appendix A, it is evident that the robot's stiffness is considerably lower than that of large-scale serial industrial robots. In practical machining processes, the quality of drilled holes—such as diameter and roughness—fails to meet the stringent requirements for aircraft skin drilling. Therefore, this section, considering the overall structural characteristics of the robotic drilling system, develops a stiffness model for the robot's drilling system and analyzes the weak points within the stiffness chain.

2.2.1. Stiffness Analysis of the Robot System

The structural configuration of the drilling system for the robotic drilling system studied in this paper is illustrated in Figure 2a. It comprises the chassis, four-degree-of-freedom (4-DOF) posture adjustment joints, expanding positioning system, feed module, drilling spindle, tool, drill template, and workpiece. The multibody topology structure is depicted in Figure 2b. Analysis of the multibody topology diagram reveals that the stiffness chain of the robotic drilling system consists of two parallel stiffness chains, as follows:

1. Workpiece → drill template → chassis → 4-DOF posture adjustment joint → feed module → drilling spindle → tool;
2. Workpiece → drill template → expanding positioning system → feed module → drilling spindle → tool.

To investigate the stiffness characteristics and modeling methods of the robotic drilling system, the following assumptions are made in this study:

1. Rigid bodies: The end effector baseplate and the feed module (comprising the ball screw components) are considered rigid bodies. Under load, any minor deformations of these components are negligible;

2. Elastic deformation: All components of the robot undergo elastic deformation when subjected to external loads.

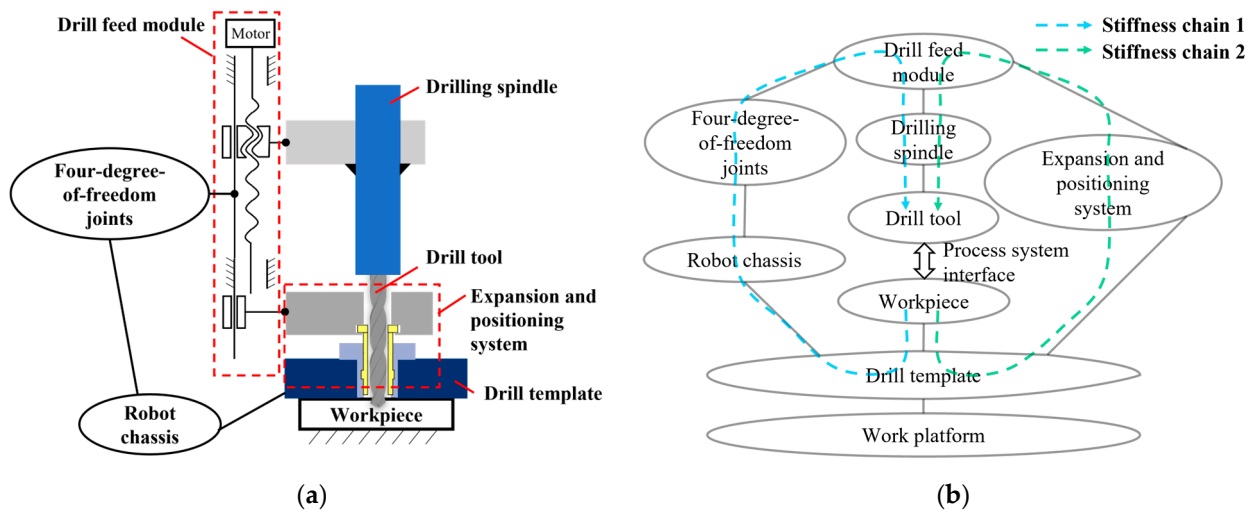


Figure 2. Schematic diagram and stiffness chain of the robotic drilling system: (a) schematic diagram; (b) stiffness chain.

According to classical statics theory, the static equilibrium equation of the robotic drilling system under applied forces is expressed as follows:

$$F = K \cdot \delta, \quad (1)$$

where K is the system's stiffness matrix, describing the structural resistance to deformation, δ is the deformation vector of the system resulting from the applied external forces, and F denotes the external force matrix, encompassing the forces exerted on the end effector during the drilling process.

Based on foundational principles of industrial robot stiffness modeling, the stiffness matrix of the robot in the Cartesian coordinate system is related to the stiffness matrices of its joints and components through the following relationship:

$$K = J^{-T} \cdot K_i \cdot J^{-1}, \quad (2)$$

where J is the Jacobian matrix, which can be derived from the robot's kinematic model, K is the Cartesian stiffness matrix of the robot, and K_i denotes the stiffness matrix of the robot's joints and components. The notation J^{-T} refers to the inverse transpose of the Jacobian matrix, which is commonly used in robot kinematics and stiffness analysis.

When external forces are applied to the robot's end effector, its joints and components undergo deformations across multiple degrees of freedom. For example, consider a cantilever beam; when a load is applied perpendicular to the free end, the beam not only displaces in the direction of the force but also undergoes rotation. This coupled behavior makes it infeasible to directly determine the stiffness matrices of each joint and component within the robot. Consequently, most research focuses on constructing the robot's flexibility matrix based on the relationship between deformation and force and subsequently inverting the flexibility matrix to obtain the stiffness matrix. The system flexibility matrix is formulated as follows:

$$\delta = C \cdot F = K^{-1} \cdot F, \quad (3)$$

$$F = \begin{bmatrix} F_x & F_y & F_z & M_x & M_y & M_z \end{bmatrix}^T, \quad (4)$$

$$\delta = [\delta_x \quad \delta_y \quad \delta_z \quad \theta_x \quad \theta_y \quad \theta_z]^T, \quad (5)$$

$$\begin{bmatrix} \delta_x \\ \delta_y \\ \delta_z \\ \theta_x \\ \theta_y \\ \theta_z \end{bmatrix} = \begin{bmatrix} c_{11} & \cdots & c_{16} \\ \vdots & \ddots & \vdots \\ c_{61} & \cdots & c_{66} \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix} = \begin{bmatrix} k_{11} & \cdots & k_{16} \\ \vdots & \ddots & \vdots \\ k_{61} & \cdots & k_{66} \end{bmatrix}^{-1} \begin{bmatrix} F_x \\ F_y \\ F_z \\ M_x \\ M_y \\ M_z \end{bmatrix} \quad (6)$$

where F represents the external forces applied to the end effector tool, while M represents the corresponding external moments, δ denotes the corresponding displacement of the end effector, including both translational and rotational movements, and C is the system's flexibility matrix, which is the inverse of the stiffness matrix K .

To investigate the weak points in the stiffness of the robotic drilling system, this paper divides the robot system into four main components based on the stiffness chain and their corresponding serial-parallel relationships: the tool, spindle, expanding positioning system, and chassis-four-degree-of-freedom (4-DOF) posture adjustment system. Let K_D denote the stiffness of the robot's tool and K_Z denote the stiffness of the robot's spindle. The components comprising the drill template, chassis, and 4-DOF joints are equivalently modeled as an elastic body with stiffness K_1 , while the components comprising the drill template and expanding positioning system are equivalently modeled as an elastic body with stiffness K_2 . Based on stiffness chain analysis, the system constitutes a serial-parallel hybrid system, and the relationship between the system's total stiffness K and the stiffness of its individual components is expressed as follows:

$$K = \frac{1}{\frac{1}{K_D} + \frac{1}{K_Z} + \frac{1}{K_T}}, \quad (7)$$

$$K_T = K_1 + K_2, \quad (8)$$

The relationship between the system's total flexibility C and the flexibility of its individual components is expressed as follows:

$$C = C_D + C_Z + C_T, \quad (9)$$

$$C_T = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}, \quad (10)$$

According to Equation (7), the overall system stiffness is inferior to that of the robot's tool K_D , the robot's spindle K_Z , and the stiffness of the system formed by the parallel arrangement of the chassis and expanding positioning system K_T . To optimize the system stiffness, it is essential to identify and enhance the component with the lowest stiffness among these three. Therefore, theoretical modeling or parameter identification of these components is required.

2.2.2. Stiffness Modeling and Evaluation of Robot System Components

In the robotic drilling system, both the tool and the drilling spindle exhibit relatively regular structures, each characterized by one fixed end and one loaded end. Consequently, they can be equivalently modeled as cantilever beam structures. By referencing the stiffness matrix formula for cantilever beams and considering the corresponding materials of the tool and spindle, simulations conducted using ANSYS Workbench 2023R1 software enable

the determination of the tool's stiffness matrix K_D and the spindle's stiffness matrix K_Z in the Cartesian coordinate system, as follows:

$$K_D = \begin{bmatrix} 4.582 \times 10^6 & 0 & 0 & 0 & 2.599 \times 10^4 & 0 \\ 0 & 4.582 \times 10^6 & 0 & -2.599 \times 10^4 & 0 & 0 \\ 0 & 0 & 2.628 \times 10^7 & 0 & 0 & 0 \\ 0 & -2.599 \times 10^4 & 0 & 2.870 \times 10^2 & 0 & 0 \\ 2.599 \times 10^4 & 0 & 0 & 0 & 2.870 \times 10^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 15.335 \end{bmatrix}$$

$$K_Z = \begin{bmatrix} 5.171 \times 10^7 & 0 & 0 & 0 & 1.340 \times 10^6 & 0 \\ 0 & 5.1712 \times 10^7 & 0 & -1.340 \times 10^6 & 0 & 0 \\ 0 & 0 & 5.280 \times 10^8 & 0 & 0 & 0 \\ 0 & -1.340 \times 10^6 & 0 & 4.594 \times 10^4 & 0 & 0 \\ 1.340 \times 10^6 & 0 & 0 & 0 & 4.594 \times 10^4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.535 \times 10^9 \end{bmatrix}$$

Due to the presence of multiple motor joints within the drill template–chassis–four-degree-of-freedom (4-DOF) posture adjustment system module (hereafter referred to as the chassis module), obtaining accurate stiffness information through simulation is unfeasible. Therefore, referencing stiffness parameter identification methods used in industrial robotics, the stiffness matrix K_1 of the chassis module in the Cartesian coordinate system is determined as follows:

$$K_1 = \begin{bmatrix} 2.087 \times 10^3 & 1.174 \times 10^4 & 6.899 \times 10^3 & 3.86 \times 10^2 & 1.38 \times 10^2 & -70 \\ 1.174 \times 10^4 & 3.91 \times 10^3 & -1.896 \times 10^3 & -70 & 3.86 \times 10^2 & 1.38 \times 10^2 \\ 6.899 \times 10^3 & -1.896 \times 10^3 & 1.353 \times 10^4 & -1.592 \times 10^3 & -3.52 \times 10^2 & -1.24 \times 10^2 \\ 3.86 \times 10^2 & -70 & -1.592 \times 10^3 & 43 & 5 & 52 \\ 1.38 \times 10^2 & 3.86 \times 10^2 & -3.52 \times 10^2 & 5 & 24 & 7 \\ -70 & 1.38 \times 10^2 & -1.24 \times 10^2 & 52 & 7 & 1.15 \times 10^2 \end{bmatrix}$$

The stiffness matrix K_2 of the drill template–drill sleeve–expanding positioning system module (hereafter referred to as the expanding module) in the Cartesian coordinate system is as follows:

$$K_2 = \begin{bmatrix} 2.146 \times 10^5 & 0 & 0 & 0 & -3.407 \times 10^4 & 0 \\ 0 & 1.891 \times 10^5 & 0 & 2.489 \times 10^4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2.489 \times 10^4 & 0 & 6.771 \times 10^3 & 0 & 0 \\ -3.407 \times 10^4 & 0 & 0 & 0 & 1.255 \times 10^4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Therefore, after the chassis module and the expanding module are connected in parallel, the overall stiffness matrix K_T in the Cartesian coordinate system is as follows:

$$K_T = \begin{bmatrix} 2.167 \times 10^5 & 1.174 \times 10^3 & 6.899 \times 10^3 & 3.86 \times 10^2 & -3.393 \times 10^4 & -70 \\ 1.174 \times 10^3 & 1.930 \times 10^5 & -1.896 \times 10^3 & 2.330 \times 10^4 & -3.52 \times 10^2 & -1.24 \times 10^2 \\ 6.899 \times 10^3 & -1.896 \times 10^3 & 1.353 \times 10^4 & 7.06 \times 10^2 & -2.14 \times 10^2 & -5.77 \times 10^2 \\ 3.86 \times 10^2 & 2.33 \times 10^4 & 7.06 \times 10^2 & 6.814 \times 10^3 & 5 & 52 \\ -3.393 \times 10^4 & -3.52 \times 10^2 & -2.14 \times 10^2 & 5 & 1.257 \times 10^4 & 7 \\ -70 & -1.24 \times 10^2 & -5.77 \times 10^2 & 52 & 7 & 1.15 \times 10^2 \end{bmatrix}$$

By combining the stiffness matrices of the tool and spindle, the overall system stiffness matrix K can be obtained according to Equation (6), as follows:

$$K = \begin{bmatrix} 1.179 \times 10^5 & -1.174 \times 10^3 & 5.734 \times 10^3 & 13.602 & 3.855 \times 10^2 & -6.022 \\ -1.171 \times 10^3 & 1.061 \times 10^5 & -5.409 \times 10^3 & -2.341 \times 10^2 & -10.004 & -34.018 \\ 5.744 \times 10^3 & -5.433 \times 10^3 & 1.083 \times 10^4 & 47.671 & 32.284 & -68.656 \\ 14.484 & -3.118 \times 10^2 & 51.579 & 1.156 \times 10^2 & 0.106 & 0.328 \\ 4.725 \times 10^2 & -10.908 & 36.484 & 0.109 & 1.182 \times 10^2 & -0.033 \\ -6.029 & -34.172 & -68.656 & 0.304 & -0.028 & 13.525 \end{bmatrix}$$

In evaluating the stiffness of a robot, the use of stiffness ellipsoids proves to be an effective tool. A stiffness ellipsoid is a mathematical model that describes the distribution of a robot's stiffness under specific working conditions. The lengths of the ellipsoid's axes represent the robot's stiffness in various directions. Shorter axis lengths indicate greater stiffness in that direction, signifying a stronger ability of the robot to resist deformation caused by external forces. Conversely, longer axis lengths denote lower stiffness in those directions, resulting in larger deformations.

To assess the stiffness performance of system, stiffness ellipsoids can be constructed through eigenvalue decomposition of the stiffness matrix. The eigenvalues and eigenvectors of the stiffness matrix satisfy the following relationship:

$$K \cdot v_i = \lambda_i \cdot v_i, i = 1, 2, 3, 4, 5, 6, \quad (11)$$

where λ_i represents the eigenvalues of the stiffness matrix K while v_i represents the corresponding unit eigenvectors for $i = 1, 2, 3, 4, 5, 6$.

The eigenvalues λ_i describe the robot's stiffness in the direction of v_i with larger eigenvalues indicating stronger stiffness in those directions. To construct the stiffness ellipsoid, the reciprocals of the eigenvalues are utilized to represent the semi-axis lengths of the ellipsoid. Specifically, the relationship between the semi-axis length a_i of the stiffness ellipsoid and the eigenvalue λ_i is given by the following:

$$a_i = \frac{1}{\sqrt{\lambda_i}}, i = 1, 2, 3, 4, 5, 6, \quad (12)$$

where a_i represents the semi-axis length of the stiffness ellipsoid in the direction of v_i . This relationship indicates that directions with higher stiffness correspond to shorter semi-axes, while directions with lower stiffness correspond to longer semi-axes.

Since the robot's drilling process is primarily subjected to axial and radial forces, this study focuses solely on evaluating the displacement stiffness ellipsoid of the system. The expression for the displacement stiffness ellipsoid is as follows:

$$\frac{x^2}{a_1^2} + \frac{y^2}{a_2^2} + \frac{z^2}{a_3^2} = 1, \quad (13)$$

Substituting the system stiffness matrix K into Equations (11)–(13) yields the corresponding displacement stiffness ellipsoid, as illustrated in Figure 3.

The stiffness ellipsoid analysis reveals that the robotic drilling system exhibits significantly lower stiffness along the Z-axis compared to the X and Y axes. Based on the comparison of the stiffness matrix values for each module mentioned earlier, it is evident that the stiffness of the chassis module and expanding positioning module is significantly lower than that of the tool and spindle modules. Therefore, optimizing the chassis and expanding the positioning modules is crucial to enhance the overall system stiffness.

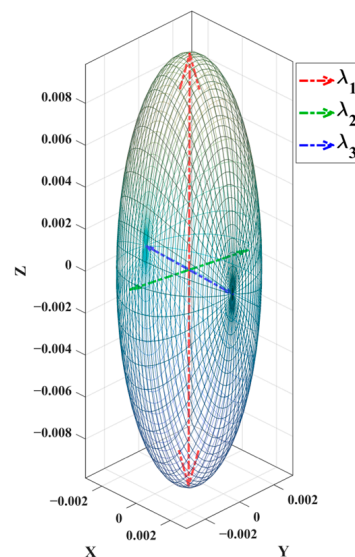


Figure 3. Displacement stiffness ellipsoid of the robotic drilling system.

The chassis module comprises the robot chassis and the four-degree-of-freedom (4-DOF) posture adjustment platform, operating in a multi-joint serial configuration akin to serial industrial robots. Research on the stiffness of serial industrial robots has identified that weak stiffness conditions are primarily caused by joint flexibility, where elastic deformations at the motors and reducers account for the majority of the robot's overall deformation. To increase the stiffness of this module, it is necessary to enhance the rigidity of each joint motor. A common approach is to add electromagnetic brakes at the posture adjustment joints. However, the robotic drilling system features numerous joints and a compact structure; installing electromagnetic brakes on every joint would significantly increase the overall structural dimensions and weight of the robot, thereby impairing its ability to navigate confined spaces. Therefore, this study focuses on improving the stiffness of the robot's expanding positioning module to enhance the robot's overall stiffness.

3. Stiffness Optimization of the Expanding Module

3.1. Stiffness Analysis and Optimization Methods of the Expanding Module

Figure 4a presents the structural diagram of the expanding positioning system, which exhibits a relatively complex architecture. The system primarily consists of an expanding sleeve, a conical sleeve, mounting brackets for both sleeves, push-pull cylinders, push-pull rods, pins, and sliders. During the drilling operation, the expanding sleeve at one end of the expanding positioning system extends and engages with the drill sleeve on the drill template, thereby constraining five degrees of freedom: translations along the X, Y, and Z axes, and rotations about the RX and RY axes. Conversely, the slider at the opposite end is connected to the end effector baseplate, restricting five degrees of freedom: translations along the X and Y axes, and rotations about the RX, RY, and RZ axes, as shown in Figure 4b.

Figure 5 depicts the deformation behavior of the end effector during the drilling process when subjected to external forces, resulting in both displacement and rotation. Specifically, the displacement of the end effector baseplate along the X and Y axes, as well as rotation about the RX and RY axes, is jointly managed by the expanding positioning system and the chassis module. However, displacement and rotation along the Z-axis cannot be constrained by the expanding module and must be solely managed by the chassis module. Notably, the Z-axis direction experiences the greatest force during the drilling process. The inability of the expanding module to restrict movement in the Z direction leads to significantly reduced axial stiffness during machining, which negatively impacts

drilling quality under the influence of cutting forces. This area constitutes the weakest link in the system's stiffness. Therefore, this study focuses on optimizing stiffness at this critical juncture by enhancing the expanding module's ability to restrict displacement in the Z direction, thereby increasing the system's overall axial stiffness.

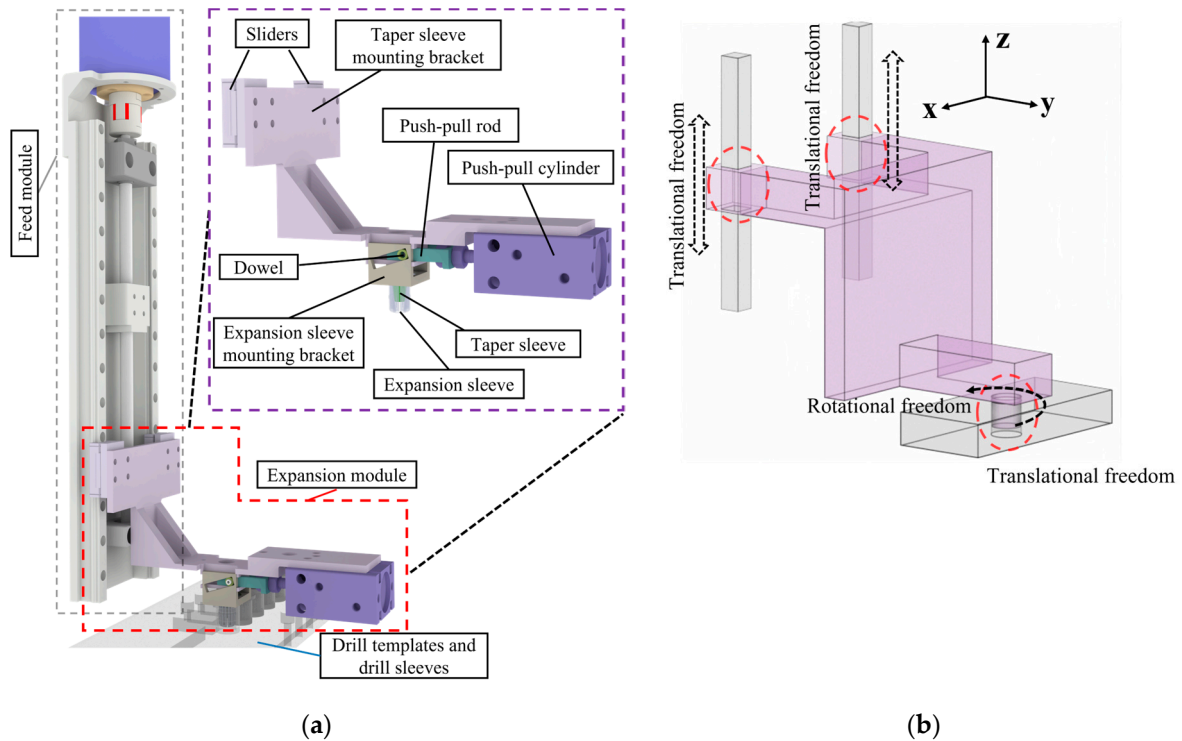


Figure 4. Structural diagram and degrees of freedom analysis of the expanding positioning system: (a) schematic diagram; (b) degrees of freedom analysis.

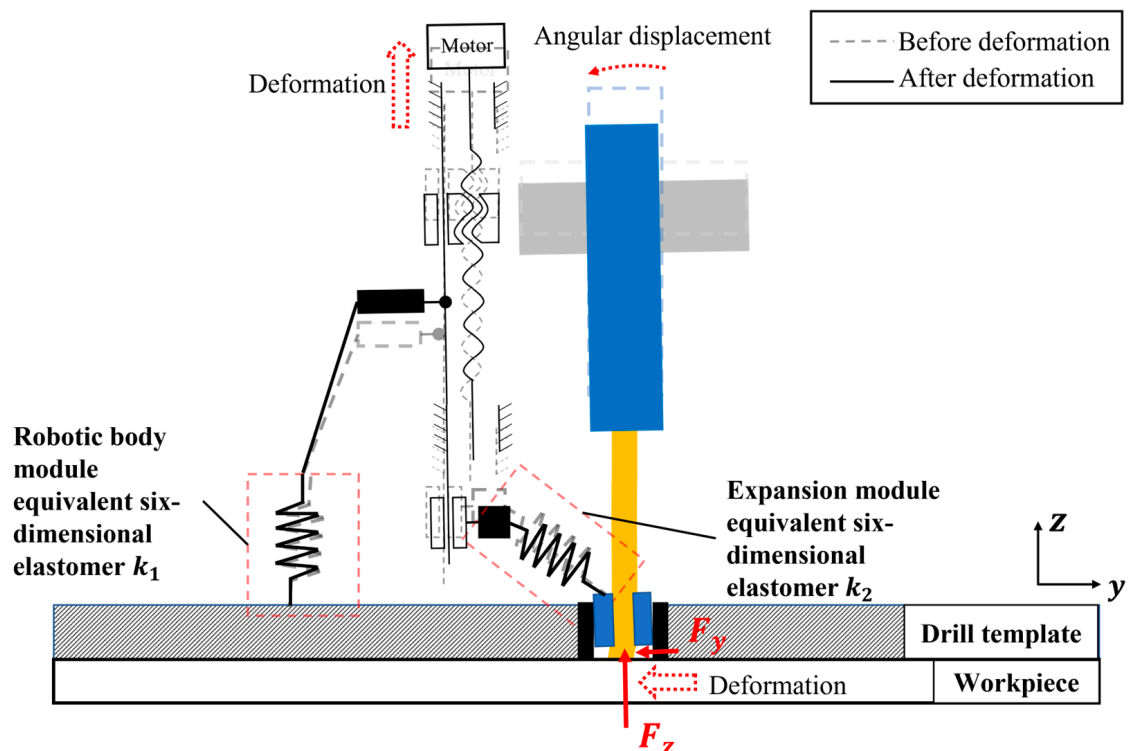


Figure 5. Deformation behavior of the end effector during the drilling process.

The optimization method proposed in this study employs a stiffness enhancement approach based on a localized closed-loop force flow at the end effector. This is achieved by integrating a guide rail clamping device into the existing expanding module, as depicted in Figure 6. Structurally, one end of the clamping module is mounted on the guide rail of the end effector baseplate, while the opposite end is connected to the slider of the expanding module via a connector. During the drilling process, the locking of the guide rail is controlled by an electromagnetic valve, allowing the locking force F_5 to be adjusted by varying the air pressure. These variations in locking force result in corresponding changes in stiffness along the Z-axis.

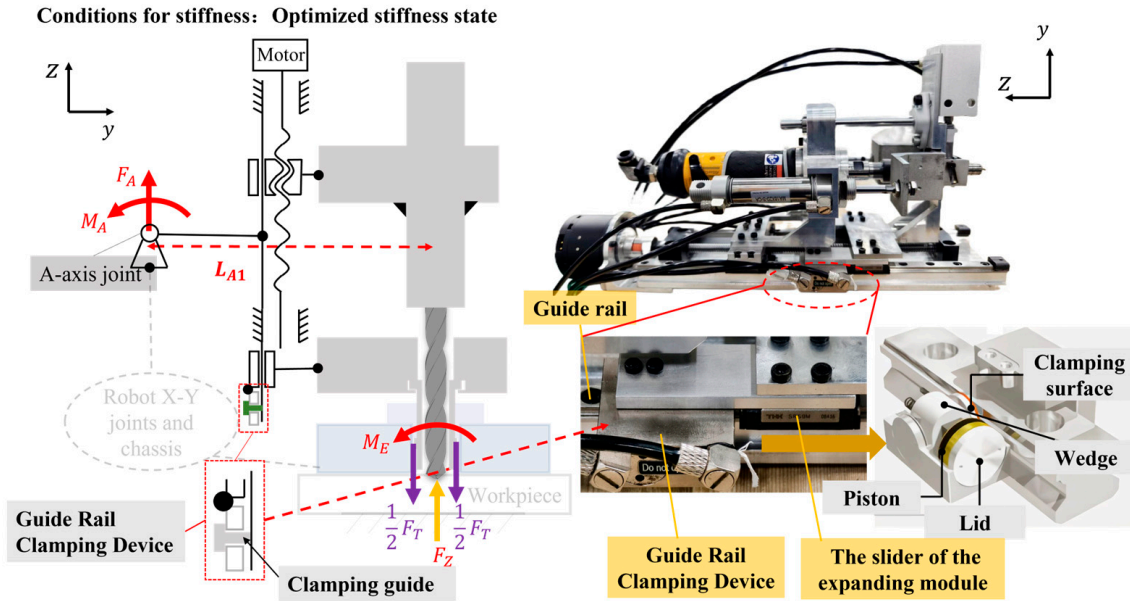


Figure 6. Mechanical analysis of drilling forces in the robot's optimized stiffness state and end effector structure.

Figure 6 illustrates the force analysis of the robot during the drilling process in the optimized stiffness state. Analyzing the forces acting on joint A of the chassis module yields the following force equilibrium equations:

$$\begin{cases} F_Z - F_T - F_A = 0 \\ (F_Z - F_T) \cdot L_{A1} + M_A + M_E = 0' \end{cases} \quad (14)$$

where F_T is the supporting force exerted by the drill sleeve on the expanding module.

Prior to structural optimization, the thrust F_Q from the auxiliary feed cylinder became an internal force within the end effector and the expanding module due to the presence of the guide rail clamping device, thereby having no impact on the system.

The displacement equilibrium equations are as follows:

$$\begin{cases} \delta'_Z = \frac{F_A}{k_{1z}} = \frac{F_Z - F_T}{k_{1z}} \\ \theta'_x = \frac{(F_Z - F_T) \cdot L_{A1}}{k_{1w} + k_{2w}} \end{cases} \quad (15)$$

where δ'_Z is the displacement in the Z direction of the end effector in the optimized stiffness state, while θ'_x is the rotation angle about the X-axis of the end effector in the optimized stiffness state.

Figure 7 illustrates the force analysis of the robot during the drilling process prior to stiffness optimization. Analyzing the forces acting on joint A of the chassis module yields the following force equilibrium equations:

$$\begin{cases} F_Z - F_A - F_Q = 0 \\ (F_Z - F_Q) \cdot L_{A1} + M_A + M_E = 0 \end{cases} \quad (16)$$

where F_Q is the auxiliary thrust generated by the auxiliary feed cylinder on the end effector, F_Z is the axial drilling force exerted on the workpiece, F_A is the supporting force produced by joint A of the chassis module, L_{A1} is the perpendicular distance from the spindle axis to joint A of the chassis module, M_A is the supporting torque generated by joint A of the chassis module, and M_E is the supporting torque produced by the expanding module.

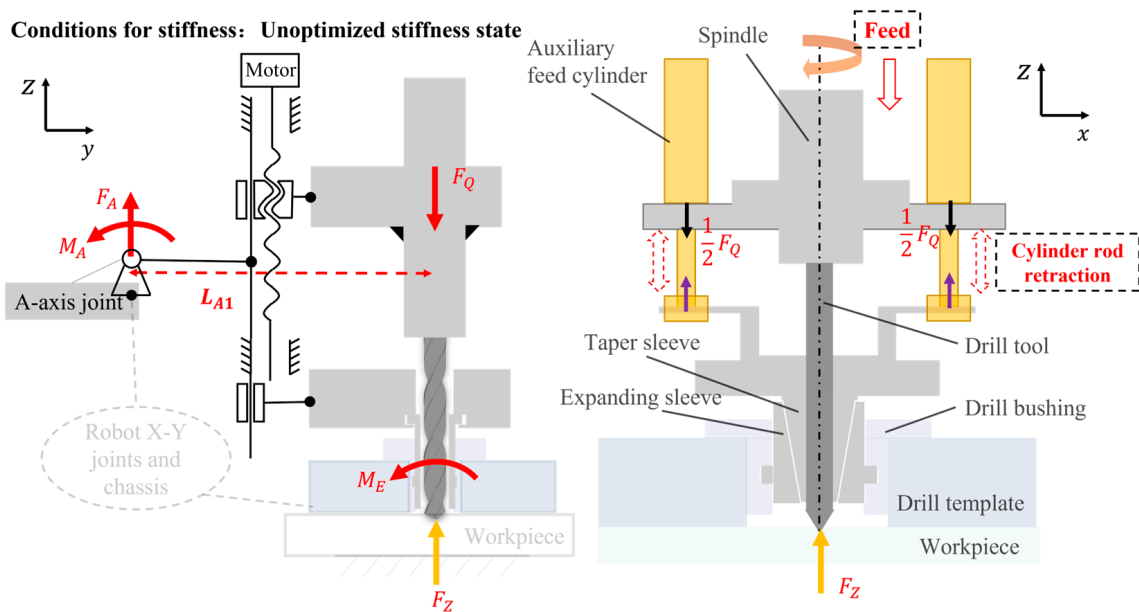


Figure 7. Mechanical analysis of drilling forces in the robot's unoptimized stiffness state.

The displacement equilibrium equations are as follows:

$$\begin{cases} \delta_Z = \frac{F_A}{k_{1z}} = \frac{F_Z - F_Q}{k_{1z}} \\ \theta_x = \frac{(F_Z - F_Q) \cdot L_{A1}}{k_{1w} + k_{2w}} \end{cases} \quad (17)$$

where δ_Z is the displacement in the Z direction of the end effector during the unoptimized state, k_{1z} is the stiffness of the chassis module in the Z-axis direction, θ_x is the rotation angle about the X-axis of the end effector during the unoptimized state, k_{1w} is the stiffness of the chassis module resisting rotation about the A-axis, and k_{2w} is the stiffness of the expanding module resisting rotation about the A-axis.

3.2. Comparison of Unoptimized and Optimized Stiffness State

3.2.1. Comparison of End Effector Z-Axis Displacement

According to Equation (17), the Z-axis deformation before optimization is primarily determined by the axial drilling force F_Z , the auxiliary thrust F_Q from the auxiliary feed cylinder, and the chassis module's Z-axis stiffness k_{1z} . Similarly, Equation (15) indicates that the Z-axis deformation after optimization is mainly influenced by the axial drilling force F_Z , the supporting force F_T provided by the drill sleeve, and the chassis module's Z-axis stiffness k_{1z} . Since the chassis module was not optimized, k_{1z} remains unchanged before

and after optimization, and the axial drilling force F_Z also remains constant. Therefore, only the auxiliary thrust F_Q from the auxiliary feed cylinder and the supporting force F_T from the drill sleeve need to be investigated.

The auxiliary thrust F_Q from the auxiliary feed cylinder is determined by air pressure and remains constant during the drilling process; it cannot be varied. Additionally, during the drilling process, the axial drilling force F_Z fluctuates and is related to various factors, such as feed rate, drilling material, and hole diameter. The auxiliary thrust F_Q from the auxiliary feed cylinder cannot approximate the axial drilling force F_Z . Therefore, $0 < F_A < F_Z$. If the auxiliary thrust F_Q exceeds twice the axial drilling force F_Z , the auxiliary thrust F_Q exerts a detrimental effect compared to having no auxiliary thrust, causing more severe deformation in the Z-axis direction of the end effector.

The magnitude of the supporting force F_T provided by the drill sleeve is determined by the Z-axis stiffness k_{2z} of the expanding module. The relationship between the supporting force F_T , the displacement δ'_Z , and the supporting force F_A can be expressed as follows:

$$\delta'_Z = \frac{F_T}{k_{2z}} = \frac{F_A}{k_{1z}}, \quad (18)$$

If $k_{2z} \gg k_{1z}$, then $0 \approx F_A \ll F_Z$. This indicates that after optimization, the Z-axis deformation of the end effector is significantly reduced compared to the unoptimized state, leading to an enhancement in the system's Z-axis stiffness. Subsequent experiments will be conducted to validate these findings.

3.2.2. Comparison of Rotation Angle About the X-Axis

Equations (15) and (17) describe the relationship between the end effector's rotation angle about the X-axis and stiffness under both unoptimized and optimized stiffness conditions. The addition of the guide rail clamping device increases the Z-axis stiffness of the expanding module but does not enhance its resistance to rotation about the X-axis. Consequently, only the auxiliary feed cylinder thrust F_Q and the supporting force provided by the drill sleeve F_T need to be considered in the analysis.

Based on the preceding analysis, the optimized effect can only be achieved when the auxiliary feed cylinder's thrust F_Q is approximately equal to the drilling force F_Z . However, since F_Q is a fixed value and F_Z fluctuates due to various factors, such as feed rate, material properties, and hole diameter, this condition cannot always be satisfied. As a result, the post-optimization rotation of the end effector about the X-axis θ'_x is ultimately smaller than the pre-optimization rotation θ_x . Relevant experiments will be conducted to validate these findings.

4. Experimental Results and Discussion

4.1. Validation of the Mechanism for System Stiffness Optimization

To validate the proposed theoretical analysis of stiffness optimization, an experimental system was established, as shown in Figure 8. The evaluation was conducted by analyzing the Z-axis deformation of the end effector during the robot's machining process and the torque output of the chassis A-axis joint. The guide rail clamping device of the robotic drilling system is controlled by a pneumatic circuit, including an electromagnetic valve and a pressure regulator. The locking force generated by the clamp module is determined by the input air pressure, which can be adjusted via the pressure regulator to vary the locking force.

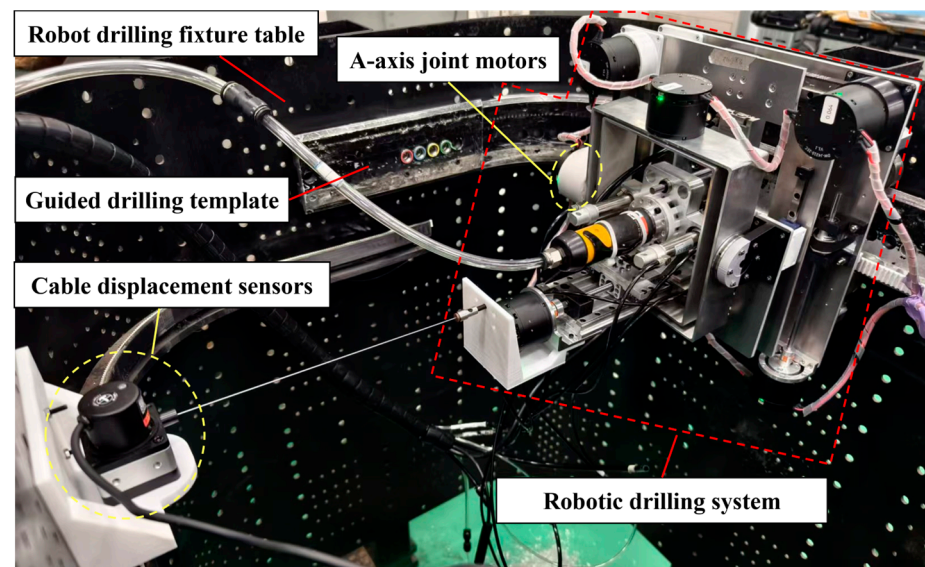


Figure 8. Experimental validation of stiffness optimization for robot drilling.

4.1.1. Torque of the A-Axis Joint

During the drilling process, the torque of the robot's joints can be estimated using motor current measurements [36]. This method was applied in this study to evaluate the torque of the A-axis joint. The experiment involved the robot executing drilling operations according to a predefined program. Variations in the clamping states and locking forces of the guide rail clamping device resulted in changes to the system's stiffness, which were reflected in notable differences in the torque output of the A-axis joint. Four different clamping states of the guide rail clamping device were tested, as detailed in Table 2. For each state, five drilling operations were performed, and the corresponding torque curves of the A-axis joint were recorded. The torque output curves of the A-axis joint during the drilling process are presented in Figure 9.

Table 2. Stiffness state of the robot.

Stiffness State	Clamping Device Status	Clamping Device Pressure (MPa)
1	Off	-
2	On	0.2
3	On	0.4
4	On	0.7

Figure 9 presents the A-axis torque output curves during the drilling process under different system stiffness states. It is evident that, without stiffness optimization, the maximum A-axis joint torque reaches $2.159 \text{ N} \cdot \text{m}$, which is substantially higher than the torque value of $0.072 \text{ N} \cdot \text{m}$ achieved after stiffness optimization (under 0.7 MPa conditions). Furthermore, in the unoptimized stiffness state, the torque continuously increases during the drilling process, particularly during the initial tool entry phase, where the torque escalation rate surpasses that of the stable drilling stage.

This phenomenon arises because at the onset of drilling, as the cutting tool progressively engages with the workpiece material, the cutting force increases. However, due to the poor axial stiffness of the unoptimized system, axial deformation occurs under the cutting force, and the deformation increases as the cutting force grows (as shown in Figure 10). According to the mechanical analysis described previously, without stiffness optimization, the energy associated with axial deformation and the transmitted axial cutting force is transferred to the A-axis joint, causing a gradual increase in the joint torque.

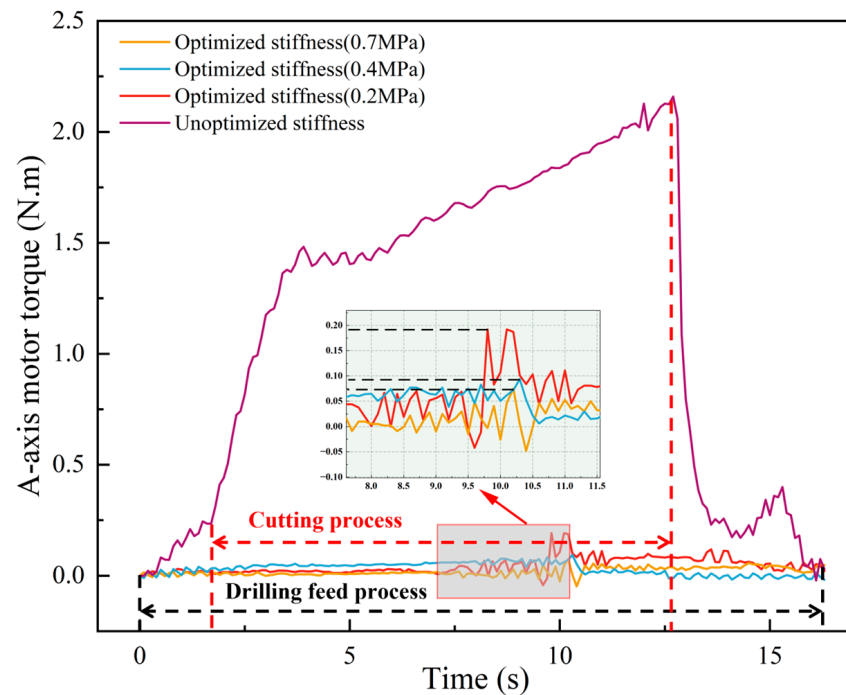


Figure 9. Variation in A-axis joint torque during the robot drilling process.

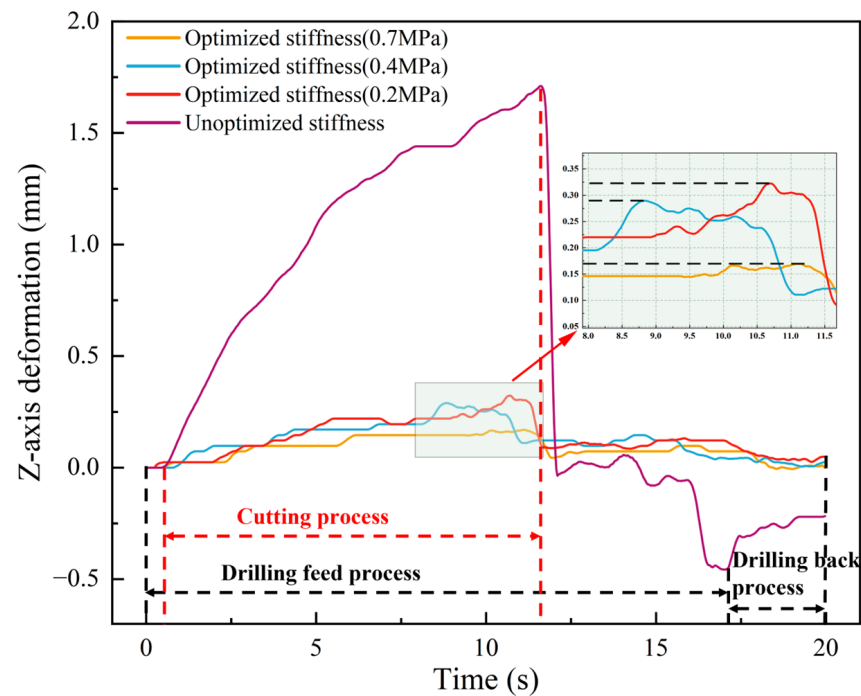


Figure 10. Z-Axis deformation of the end effector during the robot drilling process.

In contrast, when stiffness optimization is implemented, the A-axis joint torque remains relatively stable during the drilling process, showing no significant fluctuations. The magnified view indicates that torque fluctuations occur near the drilling exit position. The peak fluctuation values follow the order of 0.2 MPa > 0.4 MPa > 0.7 MPa, corresponding to the axial stiffness levels of the system. Although the system's axial stiffness is enhanced, it still exhibits relatively low stiffness compared to a conventional machine tool. Consequently, minor axial deformations occur during drilling, and when the tool exits the material, the release of stored deformation energy leads to these A-axis torque fluctuations.

In summary, stiffness optimization markedly reduces the forces and torques acting on the chassis module during drilling, thereby increasing the overall system stiffness. Quantitatively, based on the A-axis joint torque curves, the torque value at the A-axis joint after stiffness optimization is reduced by a factor of 30 compared to the unoptimized state.

4.1.2. Deformation of the Z-Axis

During the drilling process, the Z-axis displacement of the robot's end effector is measured using a draw-wire displacement sensor, as illustrated in Figure 8. The sensor is fixed to the drilling fixture, remaining stationary while the robot operates. Its cable end is attached to a mounting plate on the end effector. The experimental procedure follows the one described in Section 4.1.1. The Z-axis displacement curve of the end effector during the drilling process is presented in Figure 10.

As illustrated in Figure 10, the Z-axis displacement progressively increases throughout the drilling process until completion under both unoptimized and optimized stiffness conditions. However, without stiffness optimization, the Z-axis deformation reaches approximately 1.7 mm, notably exceeding the 0.17 mm recorded after stiffness optimization at 0.7 MPa. Additionally, deformation varies at different pressure levels. As shown in the magnified view of Figure 10, the peak deformation values follow the order of 0.2 MPa > 0.4 MPa > 0.7 MPa, indicating that as pressure increases, deformation gradually decreases. This trend directly correlates with the system's axial stiffness improvement, as higher clamping pressure enhances structural rigidity, reducing displacement and deformation. These results confirm that implementing the guide rail clamping device effectively enhances the system's Z-axis stiffness, thereby reducing the end effector's Z-axis displacement and minimizing overall robotic deformation during drilling. Consequently, the deformation after stiffness optimization is reduced by a factor of ten compared to the unoptimized state.

4.2. Verification of the Stiffness Improvement Effect

This section evaluates the proposed stiffness optimization approach for enhancing the robot's machining performance by conducting drilling experiments. The experimental parameters and tool types were selected based on commonly used industrial robot drilling conditions [37]. Table 3 lists the drilling parameters. The workpiece material used was an Al-7075 aluminum alloy plate measuring 120 mm × 80 mm × 6 mm. The system's stiffness was adjusted by varying the air pressure of the clamping device. Under each of the 4 stiffness conditions (as indicated in Table 2), the robot drilled 18 holes into the workpiece. The robot drilling setup is shown in Figure 8.

Table 3. Process parameters related to drilling.

Tool Diameter (mm)	Tool Material	Air Drill Speed (r/min)	Feed Rate (mm/s)	Pressure (MPa)
3.5	Cemented carbide	3500	0.8	0.7

Figure 11a presents the aluminum alloy workpieces processed under four distinct robot stiffness conditions. A total of 72 holes were drilled (4 workpieces, each with 18 holes) using the same drill bit. The resulting hole diameter errors were measured with a coordinate measuring machine (Aberlink Axiom Too 15097, Aberlink, Eastcombe, Gloucestershire, UK), as depicted in Figure 11b. Furthermore, a digital microscope (AM7915MZT, Dino-Lite, New Taipei City, Taiwan) was employed to examine the exit burrs on the workpieces, as illustrated in Figure 11c.

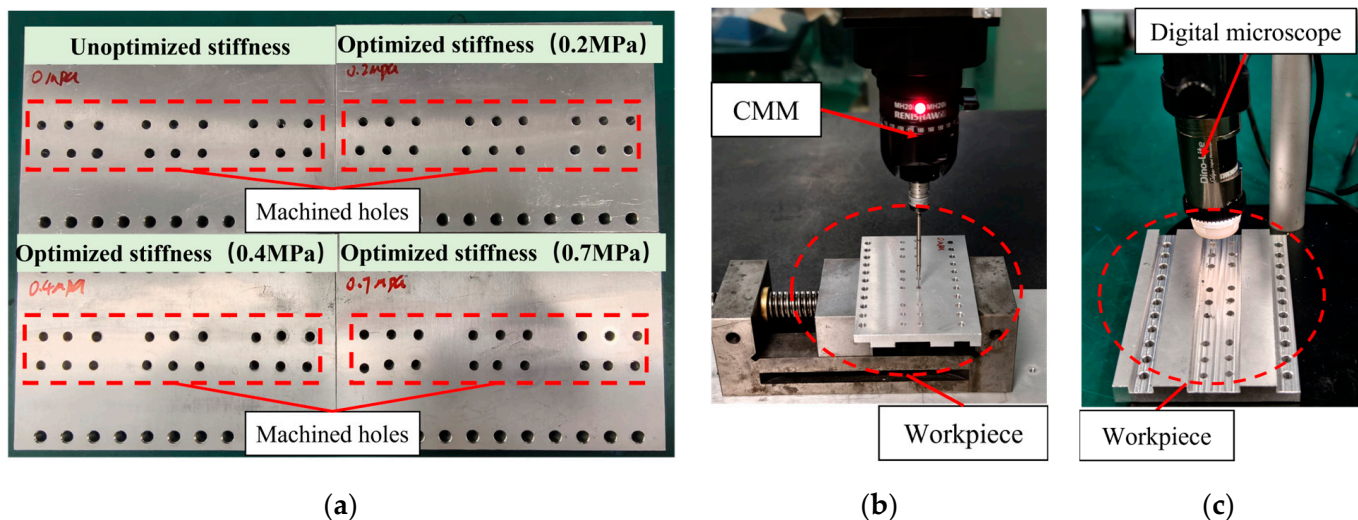


Figure 11. Workpiece and measurement: (a) drilled workpiece; (b) diameter measurement of hole; (c) exit burr observation of holes.

4.2.1. Diameter of Hole

As shown in Figure 12, the hole diameter measurement results indicate that when the clamp operates at 0.7 MPa, the average hole diameter is 3.535 mm; at 0.4 MPa, the average hole diameter is 3.540 mm; at 0.2 MPa, the average hole diameter is 3.543 mm; and with the clamp disabled, the average hole diameter is 3.588 mm. The data indicate that hole diameter accuracy is higher under optimized stiffness conditions than in the un-optimized state. Furthermore, as clamping pressure increases, hole accuracy improves, and the hole diameter variation range becomes narrower. Strengthening the axial stiffness improves both the hole diameter accuracy and its stability, primarily for the following reasons:

1. As a robotic drilling system with inherently low stiffness, the system exhibits poor centering capability at the hole entrance. When the cutting tool contacts the material, the resulting cutting force induces axial deformation of the system (as shown in Section 4.1.2, where Z-axis deformation is observed at the initial drilling stage). This deformation reduces the actual tool feed and material removal and may even cause the tool to slip in place at the entrance. Such conditions increase lateral tool deflection and enlarge the hole diameter. Greater axial stiffness reduces the severity of this phenomenon.
2. Weak axial stiffness leads to smaller actual material removal than the programmed feed amount, exacerbating work hardening and increasing the axial cutting force. Previous studies [38] indicate that the radial drilling force is approximately 30% of the axial drilling force. Since stiffness optimization only enhances axial stiffness, the increased axial cutting force still affects the radial direction, potentially enlarging the hole diameter.
3. Inadequate axial stiffness compromises the stability of tool–workpiece contact, making instantaneous load changes more likely and introducing severe vibrations during machining. These vibrations reduce drilling stability and further increase the range of hole diameter values.

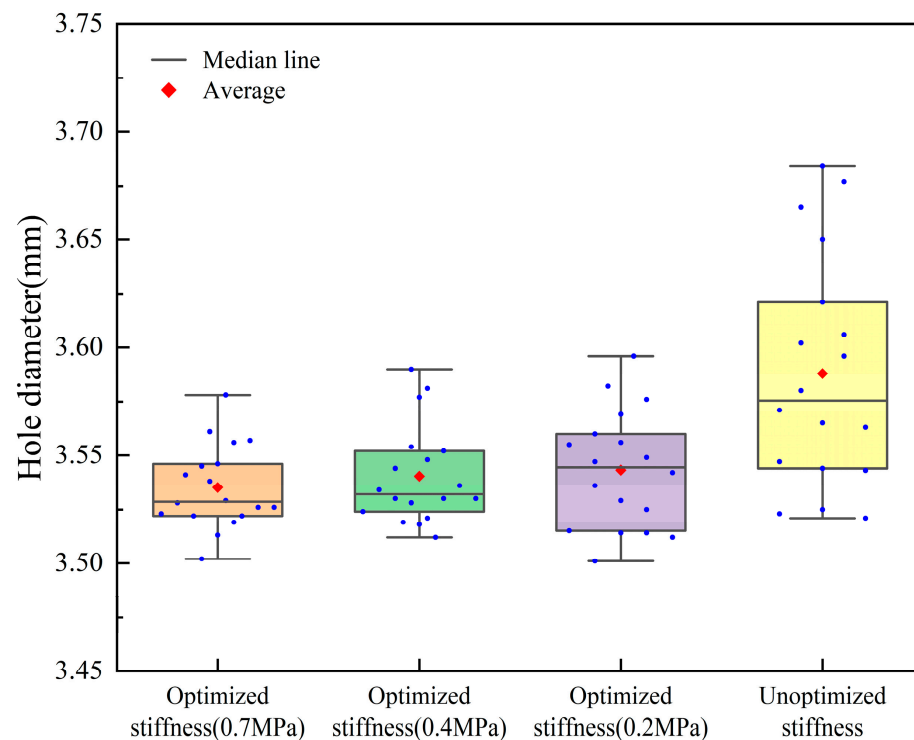


Figure 12. Hole diameter of robot drilled holes with different stiffness conditions.

4.2.2. Exit Burr

Figure 13 shows the exit quality of the workpieces produced by the robot under different stiffness conditions. Under the same cutting parameters (feed rate and spindle speed), the exit quality achieved by the stiffness-optimized robot is noticeably superior to that of the unoptimized robot. Burr formation was generally consistent across different pressure levels; however, at 0.2 MPa, a small number of holes exhibited visible exit burrs. In contrast, at 0.4 MPa and 0.7 MPa, exit burrs were significantly reduced or nearly eliminated. This trend aligns with the increased axial stiffness observed at higher clamping pressures, reinforcing the conclusion that greater rigidity improves exit hole quality. This improvement can be explained from two perspectives:

1. **Axial stiffness and deformation energy:** When the robot's axial stiffness is weak, its axial deformation gradually increases as the spindle feeds into the workpiece (as illustrated in Figure 10). This leads to the accumulation of deformation energy within the robot's structure. Upon reaching the bottom of the material, due to the softness of the aluminum alloy, the accumulated energy causes the tool to burst through the material rather than cutting it cleanly. As a result, the hole exit is not formed by the rotating drill bit slicing through the material, but rather by the tool pushing through it, leaving significant burrs at the hole exit.
2. **Cutting force distribution and stability:** Under weak axial stiffness conditions, the transmission of cutting forces becomes uneven, causing intermittent contact between the cutting edges and the workpiece surface. Such instantaneous fluctuations and uneven distributions of cutting forces not only increase the complexity of the cutting process but may also result in localized overload. This localized overload further promotes burr formation at the exit.

In summary, by enhancing the robot's axial stiffness, both the stability of the cutting process and the quality of the hole exit are significantly improved.

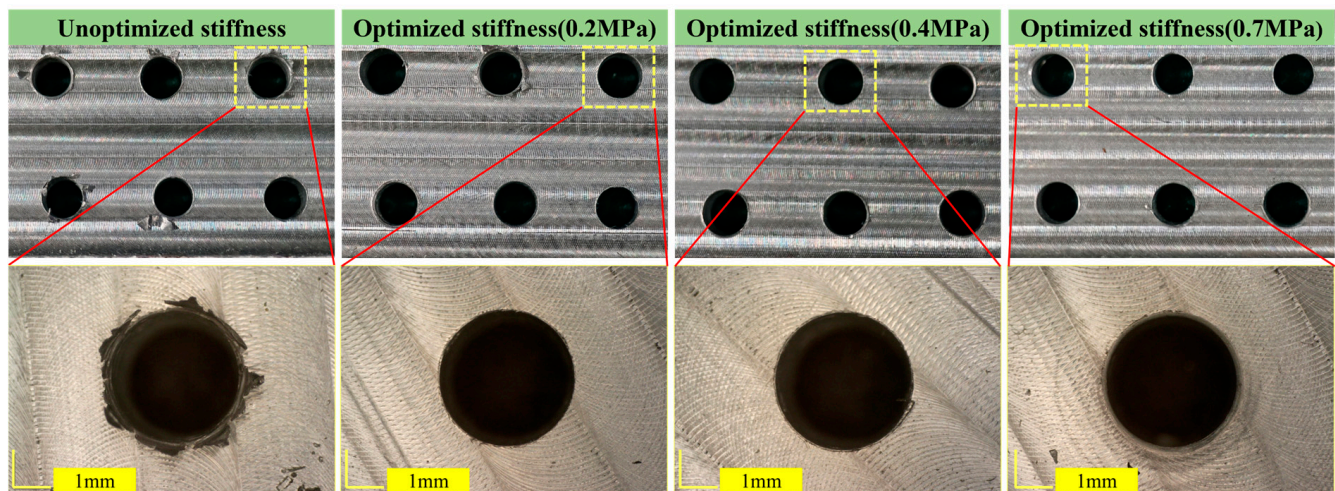


Figure 13. Robot drilling exit burrs under different stiffness conditions.

5. Conclusions

This study focused on the stiffness characteristics of a robotic drilling system and proposed a stiffness enhancement method based on local force closure at the end effector. The following conclusions can be drawn from the theoretical analysis and experimental verification:

1. Considering the structural characteristics of the robotic drilling system, a system stiffness model was established, and a stiffness ellipsoid evaluation method was employed. This approach enabled the clear identification of the weakest links in the robot's stiffness.
2. A stiffness enhancement method based on local force closure at the end effector was introduced. By analyzing the force equilibrium conditions before and after stiffness optimization during the robot's drilling process, the mechanical analysis results indicate that stiffness optimization significantly improves the robot's stiffness performance.
3. By adjusting the air pressure of the guide rail clamping device, the system's axial stiffness can be effectively altered. Experimental results show that, with stiffness optimization, the axial deformation during drilling is reduced by a factor of ten compared to the unoptimized state, and the torque acting on the A-axis joint is reduced by a factor of thirty. Moreover, with improved axial stiffness, both the hole diameter accuracy and the exit burr quality are markedly enhanced.

The proposed stiffness enhancement method provides effective reference and guidance for stiffness optimization in serial-type robots.

Author Contributions: Conceptualization, D.Z. and H.X.; methodology, H.X. and D.Z.; software, J.X. and M.L.; validation, J.X., G.G. and Y.L.; formal analysis, H.X.; investigation, H.X. and G.G.; resources, D.Z.; data curation, H.X.; writing—original draft preparation, H.X.; writing—review and editing, H.X.; visualization, H.X. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by National Natural Science Foundation of China (No. 91960203).

Data Availability Statement: The original contributions presented in the study are included in the article; further inquiries can be directed to the corresponding authors upon reasonable request.

Conflicts of Interest: The authors declare no conflicts of interest.

Appendix A

The stiffness matrix data for the industrial robot are as follows [20]:

$$K = \begin{bmatrix} 5.86 \times 10^9 & & & & & \\ & 9.85 \times 10^9 & & & & \\ & & 1.11 \times 10^{10} & & & \\ & & & 5.32 \times 10^8 & & \\ & & & & 5.52 \times 10^8 & \\ & & & & & 8.82 \times 10^8 \end{bmatrix}$$

The stiffness matrix data for the robotic drilling system are as follows:

$$K = \begin{bmatrix} 1.179 \times 10^5 & & & & & \\ & 1.061 \times 10^5 & & & & \\ & & 1.083 \times 10^4 & & & \\ & & & 1.156 \times 10^2 & & \\ & & & & 1.182 \times 10^2 & \\ & & & & & 13.525 \end{bmatrix}$$

References

- Verl, A.; Valente, A.; Melkote, S.; Brecher, C.; Ozturk, E.; Tunc, L.T. Robots in Machining. *CIRP Ann.* **2019**, *68*, 799–822. [\[CrossRef\]](#)
- Li, B.; Tian, W.; Zhang, C.; Hua, F.; Cui, G.; Li, Y. Positioning Error Compensation of an Industrial Robot Using Neural Networks and Experimental Study. *Chin. J. Aeronaut.* **2022**, *35*, 346–360. [\[CrossRef\]](#)
- Lee, J.; Hong, T.; Seo, C.-H.; Jeon, Y.H.; Lee, M.G.; Kim, H.-Y. Implicit Force and Position Control to Improve Drilling Quality in CFRP Flexible Robotic Machining. *J. Manuf. Process.* **2021**, *68*, 1123–1133. [\[CrossRef\]](#)
- Farhadi, A.; Lee, S.K.H.; Hinchy, E.P.; O'Dowd, N.P.; McCarthy, C.T. The Development of a Digital Twin Framework for an Industrial Robotic Drilling Process. *Sensors* **2022**, *22*, 7232. [\[CrossRef\]](#) [\[PubMed\]](#)
- Wang, G.; Dong, H.; Guo, Y.; Ke, Y. Chatter Mechanism and Stability Analysis of Robotic Boring. *Int. J. Adv. Manuf. Technol.* **2017**, *91*, 411–421. [\[CrossRef\]](#)
- Guo, Y.; Dong, H.; Wang, G.; Ke, Y. A Robotic Boring System for Intersection Holes in Aircraft Assembly. *Ind. Robot. Int. J. Robot. Res. Appl.* **2018**, *45*, 328–336. [\[CrossRef\]](#)
- Xin, S.; Tang, X.; Wu, J.; Peng, F.; Yan, R.; Yang, W. Investigation of the Low-Frequency Chatter in Robotic Milling. *Int. J. Mach. Tools Manuf.* **2023**, *190*, 104048. [\[CrossRef\]](#)
- Zhu, Z.; Tang, X.; Chen, C.; Peng, F.; Yan, R.; Zhou, L.; Li, Z.; Wu, J. High Precision and Efficiency Robotic Milling of Complex Parts: Challenges, Approaches and Trends. *Chin. J. Aeronaut.* **2022**, *35*, 22–46. [\[CrossRef\]](#)
- Sun, L.; Liao, W.; Zheng, K.; Tian, W.; Liu, J.; Feng, J. Stability Analysis of Robotic Longitudinal-Torsional Composite Ultrasonic Milling. *Chin. J. Aeronaut.* **2022**, *35*, 249–264. [\[CrossRef\]](#)
- Pan, R.; Zhao, W.; Wang, Z.; Ji, S.; Gao, X.; Chen, D.; Fan, J. Research on an Evaluation Model for the Working Stiffness of a Robot-Assisted Bonnet Polishing System. *J. Manuf. Process.* **2021**, *65*, 134–143. [\[CrossRef\]](#)
- Gao, Z.; Lan, X.; Bian, Y. Structural Dimension Optimization of Robotic Belt Grinding System for Grinding Workpieces with Complex Shaped Surfaces Based on Dexterity Grinding Space. *Chin. J. Aeronaut.* **2011**, *24*, 346–354. [\[CrossRef\]](#)
- Huang, H.; Gong, Z.M.; Chen, X.Q.; Zhou, L. Robotic Grinding and Polishing for Turbine-Vane Overhaul. *J. Mater. Process. Technol.* **2002**, *127*, 140–145. [\[CrossRef\]](#)
- Bi, Y.; Li, Y.; Jiang, Y.; Li, J.; Wang, W.; Gao, M.; Li, S. An Industrial Robot Based Drilling System for Aircraft Structures. *Appl. Mech. Mater.* **2013**, *433*, 151–157. [\[CrossRef\]](#)
- Wu, K.; Li, J.; Zhao, H.; Zhong, Y. Review of Industrial Robot Stiffness Identification and Modelling. *Appl. Sci.* **2022**, *12*, 8719. [\[CrossRef\]](#)
- Celikag, H.; Sims, N.D.; Ozturk, E. Cartesian Stiffness Optimization for Serial Arm Robots. *Procedia CIRP* **2018**, *77*, 566–569. [\[CrossRef\]](#)
- Chen, J.; Xie, F.; Liu, X.-J. Stiffness Parameter Identification and Cutting-Force-Induced Error Compensation of an Adsorption Machining Robot. *IEEE/ASME Trans. Mechatron.* **2024**, *29*, 2756–2767. [\[CrossRef\]](#)
- Li, Z.; Tian, W.; Hu, J.; Wang, M.; Li, B.; Liao, W. Deformation Error Compensation by Stiffness Model of Mechanical Joint on a Flexible Track Drilling Robot for Aircraft Assembly. *Int. J. Adv. Manuf. Technol.* **2023**, *127*, 5863–5878. [\[CrossRef\]](#)

18. Zhang, T.; Peng, F.; Tang, X.; Yan, R.; Deng, R.; Zhao, S. A Sparse Knowledge Embedded Configuration Optimization Method for Robotic Machining System toward Improving Machining Quality. *Robot. Comput.-Integr. Manuf.* **2024**, *90*, 102818. [\[CrossRef\]](#)
19. Tan, S.; Yang, J.; Wu, C.; Ding, H. Processing Accuracy Improvement of Robotic Ball-End Milling by Simultaneously Optimizing Tool Orientation and Robotic Redundancy. *Robot. Comput.-Integr. Manuf.* **2025**, *93*, 102904. [\[CrossRef\]](#)
20. Bu, Y.; Liao, W.; Tian, W.; Zhang, J.; Zhang, L. Stiffness Analysis and Optimization in Robotic Drilling Application. *Precis. Eng.* **2017**, *49*, 388–400. [\[CrossRef\]](#)
21. Bu, Y.; Liao, W.; Tian, W.; Zhang, L.; Dawei, L. Modeling and Experimental Investigation of Cartesian Compliance Characterization for Drilling Robot. *Int. J. Adv. Manuf. Technol.* **2017**, *91*, 3253–3264. [\[CrossRef\]](#)
22. Zargarbashi, S.H.H.; Khan, W.; Angeles, J. Posture Optimization in Robot-Assisted Machining Operations. *Mech. Mach. Theory* **2012**, *51*, 74–86. [\[CrossRef\]](#)
23. Song, C.; Liu, Z.; Wang, X.; Qiu, T.; Liang, Z.; Shen, W.; Gao, Y.; Ma, S. Research on Posture Optimization and Accuracy Compensation Technology in Robotic Side Milling. *Meas. Sci. Technol.* **2024**, *35*, 125004. [\[CrossRef\]](#)
24. Cvitanic, T.; Nguyen, V.; Melkote, S.N. Pose Optimization in Robotic Machining Using Static and Dynamic Stiffness Models. *Robot. Comput.-Integr. Manuf.* **2020**, *66*, 101992. [\[CrossRef\]](#)
25. Lee, J.H.; Kim, S.H.; Min, B.-K. Posture Optimization in Robotic Drilling Using a Deformation Energy Model. *Robot. Comput.-Integr. Manuf.* **2022**, *78*, 102395. [\[CrossRef\]](#)
26. Wang, Z.; Gao, D.; Deng, K.; Lu, Y.; Ma, S.; Zhao, J. Robot Base Position and Spacecraft Cabin Angle Optimization via Homogeneous Stiffness Domain Index with Nonlinear Stiffness Characteristics. *Robot. Comput.-Integr. Manuf.* **2024**, *90*, 102793. [\[CrossRef\]](#)
27. Hammond, M.; Cichella, V.; Lamuta, C. Bioinspired Soft Robotics: State of the Art, Challenges, and Future Directions. *Curr. Robot. Rep.* **2023**, *4*, 65–80. [\[CrossRef\]](#)
28. Niiyama, R. Soft Actuation and Compliant Mechanisms in Humanoid Robots. *Curr. Robot. Rep.* **2022**, *3*, 111–117. [\[CrossRef\]](#)
29. Kang, J.; Lee, S.; Park, Y.-L. Soft Bending Actuator With Fiber-Jamming Variable Stiffness and Fiber-Optic Proprioception. *IEEE Robot. Autom. Lett.* **2023**, *8*, 7344–7351. [\[CrossRef\]](#)
30. Jin, H.; Luo, M.; Lu, S.; He, Q.; Lin, Y. Design and Analysis of a Novel Variable Stiffness Joint for Robot. *Actuators* **2022**, *12*, 10. [\[CrossRef\]](#)
31. Sun, L.; Liang, F.; Fang, L. Comparative Study of Stiffness Modeling Methods for A Novel Industrial Robotic Arm with Hybrid Open-and Closed-Loop Kinematic Chains. In Proceedings of the 2018 IEEE International Conference on Mechatronics and Automation (ICMA), IEEE, Changchun, China, 5–8 August 2018; pp. 1765–1770. [\[CrossRef\]](#)
32. Sun, L.; Wang, B.; Huang, S.; Li, J.; Fang, L. Improving Machining Accuracy for a Robotic Arm with Hybrid Kinematic Chains Based on Deformation Characteristics. *Int. J. Adv. Manuf. Technol.* **2022**, *121*, 3539–3550. [\[CrossRef\]](#)
33. Denkena, B.; Bergmann, B.; Lepper, T. Design and Optimization of a Machining Robot. *Procedia Manuf.* **2017**, *14*, 89–96. [\[CrossRef\]](#)
34. Guo, Y.; Dong, H.; Wang, G.; Ke, Y. Vibration Analysis and Suppression in Robotic Boring Process. *Int. J. Mach. Tools Manuf.* **2016**, *101*, 102–110. [\[CrossRef\]](#)
35. Zheng, X.; Zhang, G.; Zhou, C.; Guo, G. Hole Diameter Deviation Control Method for Robotic Hole-Making System Based on Presser Foot Action. *Int. J. Precis. Eng. Manuf.* **2024**, *25*, 319–333. [\[CrossRef\]](#)
36. Yen, S.-H.; Tang, P.-C.; Lin, Y.-C.; Lin, C.-Y. Development of a Virtual Force Sensor for a Low-Cost Collaborative Robot and Applications to Safety Control. *Sensors* **2019**, *19*, 2603. [\[CrossRef\]](#)
37. Zhang, L.; Gao, W.; Lu, D.; Zeng, D.; Lei, P.; Yu, J.; Tang, M. Robotic Simple and Fast Drilling System for Automated Aircraft Assembly. *Int. J. Adv. Manuf. Technol.* **2022**, *122*, 411–426. [\[CrossRef\]](#)
38. Pirtini, M.; Lazoglu, I. Forces and Hole Quality in Drilling. *Int. J. Mach. Tools Manuf.* **2005**, *45*, 1271–1281. [\[CrossRef\]](#)

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