

Article

Load-Current-Compensation-Based Robust DC-Link Voltage Control for Flywheel Energy Storage Systems

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Abstract: DC-link voltage control needs to be achieved for flywheel energy storage systems (FESSs) during discharge. However, load disturbances and model nonlinearity affect the voltage control performance. Therefore, this paper proposes a load-current-compensation-based robust DC-link voltage control method for FESSs. In the proposed method, the model is linearized via load current feedforward compensation and dq-axis current-to-DC-current conversion. The uncertainty of the linear model is analyzed and an H_∞ robust control method is applied to overcome the uncertainty. Furthermore, experiments involving the proposed method are conducted on a 1.2 kWh magnetic suspended FESS prototype. Compared with the general proportional integral control method, the proposed method can increase the voltage response speed by 37.1% and reduce the voltage fluctuations by 29.5%. The effectiveness of the proposed method is verified experimentally.

Keywords: flywheel energy storage system; DC-link voltage control; load current compensation; robust control; disturbance rejection



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1. Introduction

Energy storage has been widely applied in power systems and can effectively alleviate peak power loads, reduce system losses, improve power quality, and enhance system stability [1–3]. As a type of energy storage system, a flywheel energy storage system (FESS) offers advantages such as high efficiency, high power density, fast response, and environmental friendliness and is suitable for applications requiring rapid charging and discharging [4,5]. As a result, FESS shows promising applications across various fields. It attracts attention in aerospace due to its long lifetime and ability to operate in a vacuum [6]. In a renewable energy system, it can compensate for periodic energy fluctuations [7,8]. In power grids, it can enhance efficiency and reliability [9–11]. For transportation systems, it can improve power performance and has application prospects in electric vehicles [12,13] and railways [14]. Additionally, the application of FESS in maritime systems has been investigated [15,16].

During the discharge process of a FESS, control of the DC-link voltage is essential, which can provide a stable DC-link voltage for the power converter at the load port [17,18], or directly supply power for the DC load [19]. However, DC-link voltage control is often affected by load disturbances and system nonlinearities. Traditional proportional-integral-derivative (PID) controllers struggle to achieve satisfactory performance in systems with

nonlinearities, uncertainties, and disturbances. To address the challenges posed by system disturbances and nonlinearities in DC-link voltage control, various control methods have been proposed in recent years. In [20], a nonlinear model predictive control (MPC) method to overcome system uncertainties and disturbances while accounting for the constraints on system states and actuator outputs is proposed. In [21], an adaptive frequency control scheme for FESSs based on MPC, which uses linear-fuzzy control in a frequency regulation controller (FRC) to adapt to frequency changes and the FESS energy, is proposed to suppress frequency fluctuations in microgrids. To achieve wide-speed-range discharge, a robust control method is proposed in [22], in which the impact of speed variation on the DC-link voltage is considered and a speed-adaptive feedback control law to ensure consistent dynamic performance across the entire speed range is designed. In [23], an active disturbance rejection control (ADRC) method is proposed for FESSs. This strategy treats the DC-link voltage and its differential as state variables, with disturbances as expanded states, by applying a nonlinear state feedback control law, which ensures fast and robust voltage control across different states of charge. In [24], a DC-link voltage adaptive controller using first- and second-order ADRC based on permanent magnet synchronous motor (PMSM) analysis is proposed. To ensure rapid dynamic performance over a wide operating range, a fast DC-link voltage control strategy is proposed in [25], in which the inner and outer loop controls of the motor are integrated, the nonlinear and uncertain terms in the power balance equation are treated as a total disturbance, and an extended state observer is employed to compensate for this disturbance. To ensure system stability over a broad speed range, an adaptive control method based on an immersion and invariance (I&I) approach is proposed in [26], in which the inner and outer loops of the motor control system are treated as a whole. In [27], a linear ADRC control method based on RLC filter is proposed, which uses the observer and internal model control to adapt to the speed changes.

The existing papers can overcome disturbances and nonlinearities and achieve satisfactory control performance. However, these methods achieve voltage control by adjusting the dq-axis current, which involves numerous coupled system parameters, and complicates the design of controllers. The current directly related to the DC-link voltage is the current flowing into the DC-link capacitor. If the controller can regulate the capacitor current to control the DC-link voltage, then the control model can be greatly simplified, facilitating controller design.

To overcome disturbances and nonlinearity with a simple controller design, this paper proposes a robust control method for the DC-link voltage based on load current compensation. In this method, the control model is simplified by using load current feedforward compensation and converting dq-axis currents to the DC-link current to obtain the control input for the capacitor current. The simplified control model introduces some uncertainties. To address this, the H_∞ robust control method is employed to overcome these system uncertainties, ensuring stable control of the DC-link voltage.

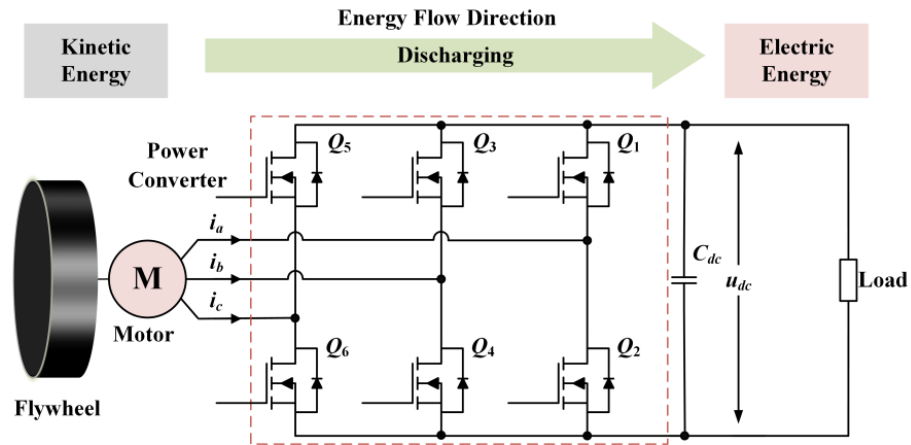
The main contributions and innovations of this paper include the following:

- (1) A H_∞ robust DC-link voltage control method based on load current compensation is proposed.
- (2) Solid experimental research on the robust DC-link voltage control method based on load current compensation is conducted.

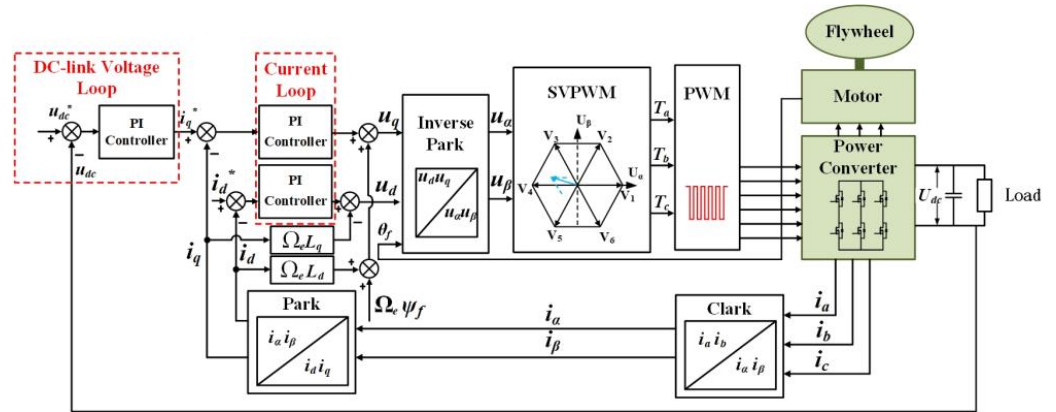
The remainder of this paper is organized as follows. In Section 2, the configuration of the DC-link voltage control system of the FESS is explained, and the control block diagram of the overall system is presented. In Section 3, the robust DC-link voltage control method is proposed, which includes load current compensation, system uncertainty analysis, and design of the H_∞ robust controller via a linear matrix inequality (LMI). Section 4 provides experimental validation. Finally, Section 5 concludes the paper.

2. System Description

For a FESS, a three-phase full-bridge circuit is applied as a power converter of discharge, and the topology is shown in Figure 1a. The field-oriented control (FOC) strategy can be used to achieve DC-link voltage control, with the control strategy block diagram shown in Figure 1b. The DC-link voltage control system is a dual-loop control system consisting of an inner current loop and an outer voltage loop. The inner loop controls the dq-axis currents, whereas the outer loop controls the DC-link voltage. In the FOC strategy for DC-link voltage control, the system operates with a control mode of $i_d = 0$, in which the DC-link voltage stability is maintained by adjusting the q-axis current. The q-axis current is typically negative, as the motor operates in generation mode, converting the mechanical energy stored in the flywheel rotor into electrical energy.



(a)



(b)

Figure 1. System description during discharge of the FESS. (a) The overall topology of the FESS during discharge; (b) Block diagram of DC-link voltage control in the FESS.

The DC-link voltage control model can be derived as follows:

$$\frac{du_{dc}}{dt} = -\frac{3u_q}{2C_{dc}u_{dc}}i_q - \frac{i_l}{C_{dc}} \quad (1)$$

where u_q is the q-axis voltage, i_q is the q-axis current, i_l is the load current, C_{dc} is the DC-link capacitance, and u_{dc} is the DC-link voltage.

From (1), the q-axis voltage u_q is related to the flywheel speed, the q-axis current, and the rate of change in the current. The DC-link voltage control model is a time-varying

nonlinear model. Additionally, the load current i_l can be viewed as a disturbance input to the system, which depends on the type of load connected and its operating state. The use of a conventional PI controller for DC-link voltage control has the following drawbacks: (1) The system is affected by disturbances arising from the load current. The PI controller uses an integrator to track the load current. When the load current dramatically changes, the integrator cannot quickly follow these changes, leading to fluctuations in the DC-link voltage, which is detrimental to system stability. (2) The DC-link voltage control model is a time-varying nonlinear model, and the application of a PI controller for such a model has certain limitations.

Considering the above analysis, the DC-link voltage controller for the FESS needs to have the ability to suppress disturbances arising from the load current and overcome the system nonlinearities.

3. Robust Control Method for the DC-Link Voltage

3.1. Control Strategy Based on Load Current Compensation

According to (1), the DC-link voltage control model is nonlinear. To simplify the model, the control strategy based on load current compensation is proposed. The relationships among the DC-link voltage, DC-link current, capacitor current, and load current are illustrated in Figure 2. The current directly related to the DC-link voltage is the capacitor current i_{ca} , and their relationship can be derived as follows:

$$\frac{du_{dc}}{dt} = \frac{i_{ca}}{C_{dc}} \quad (2)$$

where i_{ca} is the capacitor current, C_{dc} is the DC-link capacitance, and u_{dc} is the DC-link voltage.

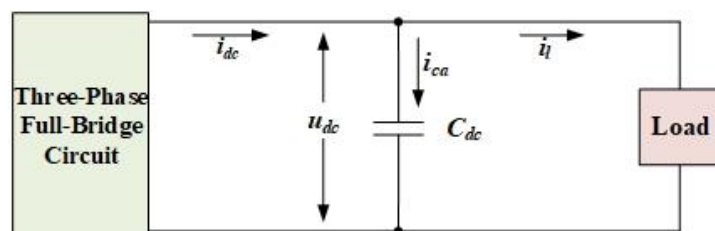


Figure 2. Relationships among the DC-link voltage, DC-link current, and load current.

According to (2), if the voltage controller output is the capacitor current i_{ca} , the DC-link voltage control model can be greatly simplified. The relationships among the capacitor current, DC-link current, and load current can be expressed as:

$$i_{ca} = i_{dc} - i_l \quad (3)$$

The inner loop of the motor remains the dq-axis current loop and the system cannot directly control the capacitor current. Therefore, a conversion relationship between the capacitor current and the dq-axis currents must be established. In the control mode of $i_d = 0$, the relationship between the DC-link current and the motor q-axis current can be derived as follows:

$$i_{dc} = -\frac{\frac{3}{2}u_q i_q + P_{loss}}{u_{dc}} \quad (4)$$

where P_{loss} is the loss of the rectifier.

Compared with the back electromotive force of the motor, the voltage drops across the resistance and inductance are relatively small, so these voltage drops can be neglected. Therefore, the q-axis voltage u_q in (4) can be approximately represented as:

$$u_q \approx \Omega_e \psi_f \quad (5)$$

where Ω_e is the electrical angular speed and Ψ_f is the permanent magnet flux linkage.

By substituting (5) into (4) and neglecting the loss term P_{loss} , the conversion relationship between i_q and i_{dc} can be further derived as follows:

$$i_{dc} = -\frac{3\Omega_e \psi_f}{2u_{dc}} i_q = T_{qdc} i_q \quad (6)$$

where T_{qdc} is the conversion coefficient between the DC-link current and the q-axis current.

Substituting (6) into (3), the relationship among i_{ca} , i_l , and i_q can be expressed as:

$$i_{ca} = T_{qdc} i_q - i_l \quad (7)$$

The input signal of the inner current loop is the reference signal for the q-axis current i_q^* . To ensure that the output of the DC-link voltage controller is the control input for the capacitor current i_{ca}^* , a conversion relationship between i_{ca}^* and i_q^* must be derived. Based on (7), the conversion relationship between i_{ca}^* and i_q^* can be derived as follows:

$$i_q^* = T_{qdc}^{-1} (i_{ca}^* + i_l^*), \quad (8)$$

where i_l^* is the load current feedforward compensation signal.

The q-axis current loop can be viewed as a first-order inertial element and the relationship between i_q and i_q^* can be expressed as:

$$G_q(s) = \frac{I_q(s)}{I_q^*(s)} = \frac{1}{T_q s + 1} \quad (9)$$

where $I_q(s)$ is the Laplace transform of the q-axis current, $I_q^*(s)$ is the Laplace transform of the reference signal for the q-axis current and T_q is the time constant of the q-axis current loop.

The load current feedforward compensation signal in (8) is designed to overcome the influence of load disturbances and is expressed as:

$$I_l^*(s) = (T_q s + 1) I_l(s) \quad (10)$$

where $I_l^*(s)$ is the Laplace transform of the output of the feedforward compensator and $I_l(s)$ is the Laplace transform of the load current.

Based on (8)–(10), the relationship between the reference capacitor current i_{ca}^* and the capacitor current i_{ca} can be derived as follows:

$$\frac{I_{ca}(s)}{I_{ca}^*(s)} = \frac{1}{T_q s + 1} \quad (11)$$

where $I_{ca}^*(s)$ is the Laplace transform of the capacitor current control input and $I_{ca}(s)$ is the Laplace transform of the capacitor current.

From the above analysis, the control strategy block diagram for the DC-link voltage based on load current compensation can be obtained, as shown in Figure 3.

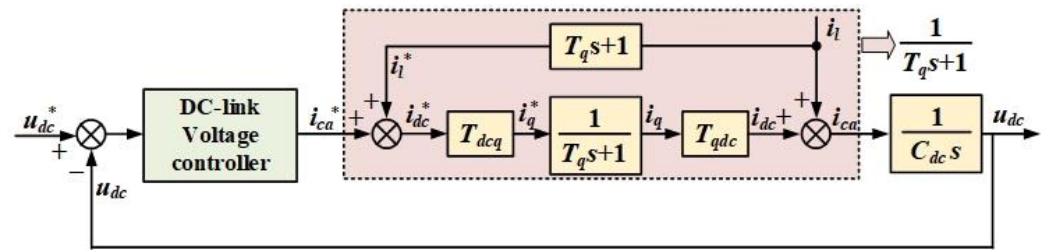


Figure 3. Block diagram of the control strategy based on load current compensation.

Based on Figure 3, the linearized model of the DC-link voltage can be expressed as:

$$G_{pdc}(s) = \frac{U_{dc}(s)}{I_{ca}^*(s)} = \frac{1}{C_{dc}s(T_q s + 1)} \quad (12)$$

The DC-link voltage control strategy based on load current compensation leads to a linearized model of the DC-link voltage, which is related only to the DC-link capacitance and the time constant of the q-axis current loop, with minimal coupling to the parameters of the motor system. The use of this voltage control model facilitates the design of the DC-link voltage controller.

3.2. System Uncertainty Analysis

There is an error between the linearized model of (12) and the actual model and this model error can be regarded as the system uncertainty. In this paper, the uncertainties in the linearized model are analyzed and an uncertainty model is established. The established uncertainty model can be used for subsequent H_∞ robust control studies.

In this paper, the DC-link capacitor current i_{ca} is defined as the system state x_1 and the deviation of the actual DC-link voltage from the reference signal $u_{dc} - u_{ref}$ is defined as the system state x_2 . Based on (12), the state-space representation of the linearized model can be derived as follows:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases} \quad (13)$$

where:

$$A = \begin{bmatrix} -\frac{1}{T_q} & 0 \\ \frac{1}{C_{dc}} & 0 \end{bmatrix}, B = \begin{bmatrix} \frac{1}{T_q} \\ 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

In the linearized model, the derivation of the conversion coefficients i_{dc}^* and i_q^* neglects the losses in the pulse-width modulation (PWM) rectifier and the voltage drops across the resistance and inductance, which introduces uncertainty into the conversion coefficients. The uncertainty in the conversion coefficients i_{dc}^* and i_q^* can be expressed as:

$$I_q(s)^* = I_{dc}(s)^* (T_{dcq} + \Delta T) \quad (14)$$

where ΔT represents the uncertainty coefficient in the conversion of i_{dc}^* and i_q^* .

The compensation amount of the current feedforward compensator is determined based on the load current, which has measurement errors. Therefore, the load current measurement error is also one of the uncertainties in the system. The load current compensation considering measurement errors can be expressed as:

$$I_l^*(s) = (T_q s + 1)(I_l(s) + \Delta I(s)) \quad (15)$$

where $\Delta I(s)$ represents the Laplace transform of the load current measurement error.

The load current compensation error can be expressed as:

$$\Delta I^*(s) = (T_q s + 1) \Delta I(s) \quad (16)$$

where $\Delta I^*(s)$ represents the Laplace transform of the load current compensation error.

According to (13)–(15), the uncertainties in the system can be summarized as input disturbances, as shown in Figure 4. The input disturbance of the system can be expressed as:

$$W(s) = \Delta I^*(s) + \Delta T I_{dc}^*(s) \quad (17)$$

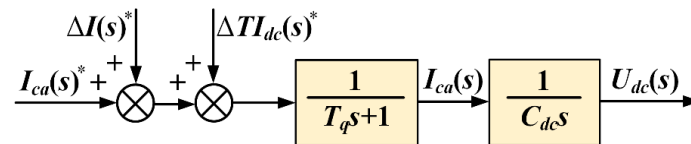


Figure 4. Input disturbance model of the system.

Based on Figure 4, the DC-link voltage uncertainty model can be derived as follows:

$$\begin{cases} \dot{x} = Ax + Bu + Bw \\ y = Cx \\ z = C_1 x + D_1 u + D_2 w \end{cases} \quad (18)$$

where w is the input disturbance, z is the system output evaluation, C_1 is a matrix related to the system state, D_1 is a coefficient related to the control input, and D_2 is a coefficient related to the input disturbance.

The system output z is related to the system state, control input, and disturbance input, which can be used to evaluate the control performance. Let the two elements of the matrix C_1 be C_{11} and C_{12} . C_{11} , C_{12} , D_1 , and D_2 can be considered weight coefficients for the evaluation output. In robust control, the control performance can be improved by adjusting these weight coefficients.

3.3. H_∞ Robust Control Based on a Linear Matrix Inequality (LMI)

As derived in (18), the linearized DC-link voltage model contains uncertainties. Therefore, a suitable control method is needed to overcome these uncertainties. The H_∞ robust control method is an effective approach for handling system uncertainties and has been widely applied. In designing an H_∞ robust controller, the objective is to obtain an optimal or suboptimal solution for the H_∞ norm of the transfer function from the disturbance input to the evaluation output. The suboptimal H_∞ norm problem can often be transformed into an LMI solution [28] and solving the LMI yields a state feedback matrix. Therefore, this paper proposes an H_∞ robust control method for the DC-link voltage based on an LMI to address the uncertainties in the DC-link voltage model.

The uncertain model in (18) can be rewritten in the form of a transfer function matrix as follows:

$$\begin{bmatrix} Z(s) \\ Y(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} W(s) \\ U(s) \end{bmatrix} \quad (19)$$

where $W(s)$ is the Laplace transform of the input disturbance signal and $Z(s)$ is the Laplace transform of the system evaluation output.

The goal of H_∞ robust control is to design a state feedback controller that minimizes the H_∞ norm of the transfer function from the disturbance input to the system evaluation output. The block diagram of the H_∞ robust control system is shown in Figure 5, where

$K(s)$ is the transfer function of the state feedback controller. From Figure 5, the relationships between $Z(s)$, $Y(s)$, and $W(s)$ can be derived as follows:

$$Z(s) = G_{11}(s)W(s) + G_{12}(s)K(s)Y(s) \quad (20)$$

$$Y(s) = G_{21}(s)W(s) + G_{22}(s)K(s)Y(s) \quad (21)$$

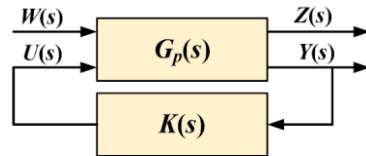


Figure 5. H_∞ robust control system block diagram.

Substituting (20) into (21), the transfer function from $W(s)$ to $Z(s)$ can be derived as follows:

$$G_{wz}(s) = \frac{Z(s)}{W(s)} = \left(G_{11}(s) + G_{12}(s)K(s)(sI - G_{22}(s)K(s))^{-1}G_{21}(s) \right) \quad (22)$$

The design objective of H_∞ robust control is to solve for $K(s)$ such that the H_∞ norm of $G_{wz}(s)$ is minimized. However, in practical applications, obtaining the optimal solution for the H_∞ norm is difficult. Typically, a suboptimal solution for the H_∞ norm of $G_{wz}(s)$ is sought, as shown in (23):

$$\|G_{wz}(s)\|_\infty < r \quad (23)$$

where r is a positive constant.

A sufficient and necessary condition for a suboptimal solution for the H_∞ norm of $G_{wz}(s)$ is that positive definite matrices $\mathbf{L}$ and $\mathbf{M}$ exist such that the LMI in (24) is satisfied:

$$\begin{bmatrix} \mathbf{A}\mathbf{L} + \mathbf{B}\mathbf{M} + (\mathbf{A}\mathbf{L} + \mathbf{B}\mathbf{M})^T & \mathbf{B} & (\mathbf{C}_1\mathbf{L} + \mathbf{D}_2\mathbf{M})^T \\ \mathbf{B}^T & -1 & \mathbf{D}_1 \\ \mathbf{C}_1\mathbf{L} + \mathbf{D}_2\mathbf{M} & \mathbf{D}_1 & -r^2 \end{bmatrix} < 0 \quad (24)$$

If the LMI is satisfied, then the state feedback matrix for the H_∞ robust controller is given as:

$$\mathbf{K} = \mathbf{L}\mathbf{M}^{-1} \quad (25)$$

The state feedback matrix $\mathbf{K}$ in (25) ensures the stability of the closed-loop system, i.e., the real part of the eigenvalues of $(\mathbf{A} - \mathbf{B}\mathbf{K})$ is less than zero. The LMI can be solved via elimination or variable substitution methods and the LMI toolbox integrated with MATLAB can be used to solve for the matrices $\mathbf{L}$ and $\mathbf{M}$.

According to (24), the state feedback matrix $\mathbf{K}$ is related to the weight coefficients C_{11} , C_{12} , D_1 , and D_2 , and thus, the selection of these weight coefficients affects the performance of the robust controller. The influence of the weight coefficients on the control performance can be summarized as follows:

- (1) Increases in the weight coefficients C_{11} and C_{12} increase the feedback matrix $\mathbf{K}$, accelerating the convergence of the system state and reducing the evaluation output. An increase in C_{11} accelerates the current convergence but decreases the speed of voltage regulation. An increase in C_{12} accelerates the control error convergence, enhancing the voltage response speed but potentially causing overshoot.

- (2) An increase in D_1 increases the impact of the control input on the evaluation output. To reduce the H_∞ norm of $G_{wz}(s)$, the feedback matrix $\mathbf{K}$ is decreased, resulting in a slower DC-link voltage response.
- (3) An increase in D_2 increases the impact of the input disturbance on the evaluation output. To reduce the H_∞ norm of $G_{wz}(s)$, the feedback matrix $\mathbf{K}$ is decreased, reducing the effect of disturbances on the evaluation output. However, an excessively large D_2 may slow the system response speed.

To minimize the impact of system uncertainties, the H_∞ norm of $G_{wz}(s)$ needs to be as small as possible. However, solving the LMI yields only a suboptimal solution. To achieve the smallest possible H_∞ norm of $G_{wz}(s)$, multiple iterations of solving the LMI can be performed to reduce r toward the minimum value r_{min} of the H_∞ norm. This approach results in the H_∞ robust controller design flowchart based on the LMI shown in Figure 6. The design process involves three main steps:

- (1) The model parameters and weight coefficients are initialized and the LMI based on these initial values is constructed.
- (2) The LMI is iteratively solved, reducing the value of r in each iteration. The iteration is stopped when reducing r no longer yields a solution for the LMI.
- (3) The state feedback matrix $\mathbf{K}$ is obtained based on the LMI solution.

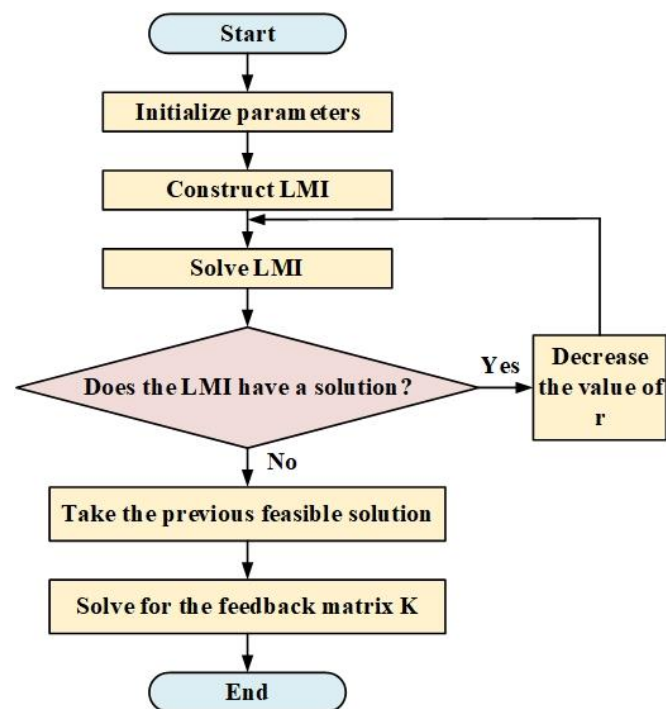


Figure 6. H_∞ robust controller design flowchart based on the LMI.

The block diagram of the proposed control method is presented in Figure 7. Through H_∞ control, load current compensation, and the i_{dc} to i_q conversion current, the control output can be obtained to realize the DC-Link voltage control.

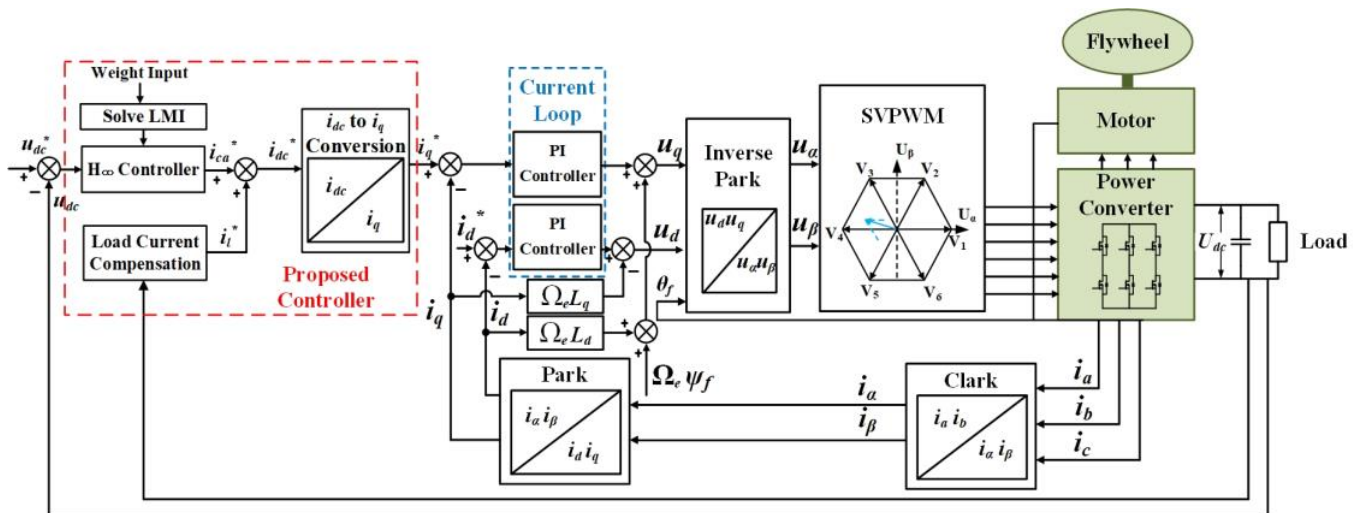


Figure 7. Block diagram of the load-current-compensation-based robust DC-link voltage control.

4. Experimental Results

In this paper, experimental research on the robust DC-link voltage control method based on load current compensation is conducted. The experimental platform for the FESS is shown in Figure 8. The experimental platform mainly includes a magnetic suspended FESS prototype, a magnetic suspended controller, a motor controller, a contactor, a power supply, an oscilloscope, a power analyzer, a spectrum analyzer, and a vacuum pump. The parameter specifications of the FESS prototype are listed in Table 1. The PW3390 power analyzer developed by HIOKI in Nagano, Japan is used to obtain the voltage response curve and the DVL125 voltage sensor developed by LEM in Geneva, Switzerland is used to measure voltage in the control system with an accuracy of 0.5%.

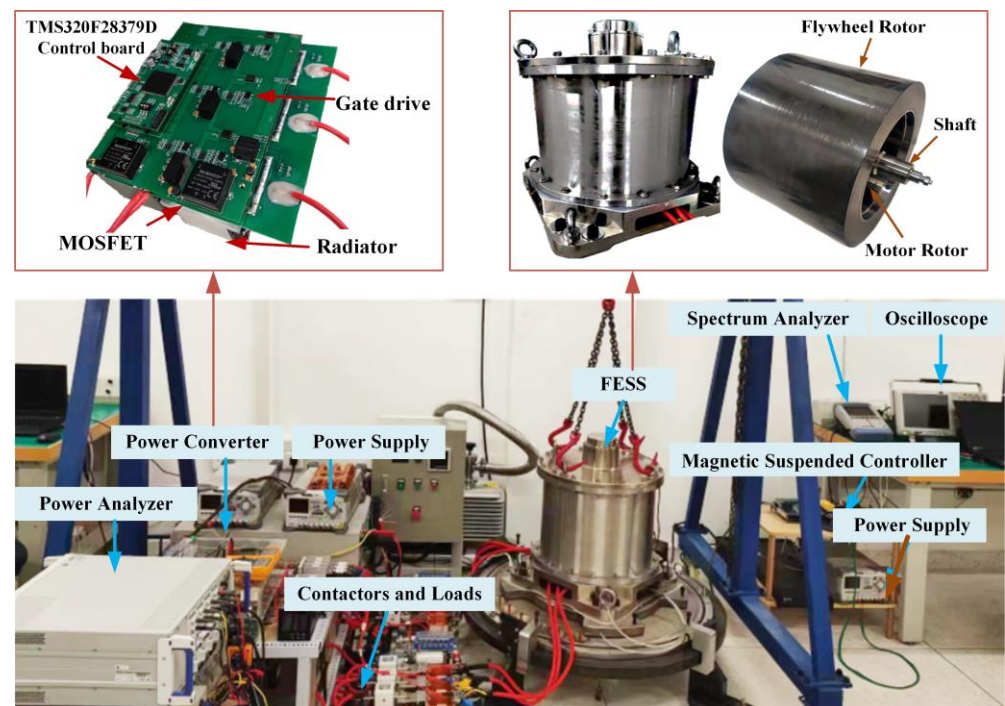


Figure 8. FESS experimental platform.

Table 1. Main parameter specification of FESS prototype.

Symbol	Description	Value
E_m	Stored energy	1.2 kWh
M_r	Flywheel rotor weight	58.0 kg
S_m	Prototype size	$\Phi 500 \times 590$ mm
R_s	Motor stator resistance	56 m Ω
L_d	d-axis inductance	32 μ H
L_q	q-axis inductance	32 μ H
ψ_f	Permanent magnet flux	0.062 Wb
P_0	Pole pair number	3
C_{dc}	DC-link voltage	600 μ F
P_{max}	Power converter maximum power	80 kW
f_{sw}	Switching frequency	40 kHz
e_r	Voltage measurement accuracy	0.5%

The weight coefficients C_{11} , C_{12} , D_1 , and D_2 of the H_∞ robust controller affect the DC-link voltage control performance. Therefore, the effects of these weight coefficients on DC-link voltage control are experimentally analyzed and the experimental results are used to determine the weight coefficients for H_∞ robust control. H_∞ robust control with the determined weight coefficients is subsequently applied to several typical load conditions, including constant resistance load, load step change, motor load, and energy feedback scenarios. Under different conditions, the proposed H_∞ robust control method is compared with the traditional PI control method. The parameter of the PI control is turned by small deviation linearization and frequency correction method. Furthermore, a comparison of the existing ADRC method is performed.

4.1. Parameter Influence Experiment for H_∞ Robust Control

This section discusses the influence of the H_∞ robust control weight coefficients on the DC-link voltage control performance. The experimental results showing the effects of these weight coefficients on the control performance are illustrated in Figure 9. Figure 9a shows the DC-link voltage step response curves for different C_{11} coefficients. An increase in C_{11} reduces system overshoot by increasing system damping. However, a larger C_{11} can slow the voltage response. Figure 9b shows the DC-link voltage step response curves for different C_{12} coefficients. An increase in C_{12} improves the voltage response speed, but an excessively large C_{12} can cause significant overshoot of the DC-link voltage. The H_∞ robust controller requires careful selection of appropriate values for C_{11} and C_{12} to ensure that the DC-link voltage responds quickly without causing excessive overshoot. Figure 9c shows the DC-link voltage step response curves for different D_1 coefficients. A larger D_1 reduces the control input, thereby minimizing system overshoot. However, an excessively large D_1 slows the system response. Figure 9d shows the DC-link voltage step response curves for different D_2 coefficients. A larger D_2 helps reduce the impact of system disturbances. However, an excessively large D_2 restricts the control input, which in turn affects the voltage response speed.

The experimental results demonstrate that the theoretical analysis of the influence of the weight coefficients on the control performance aligns with the experimental findings. Table 2 shows the weight coefficients of the H_∞ robust controller determined through experiments. The H_∞ robust controller in this paper uses these coefficients.

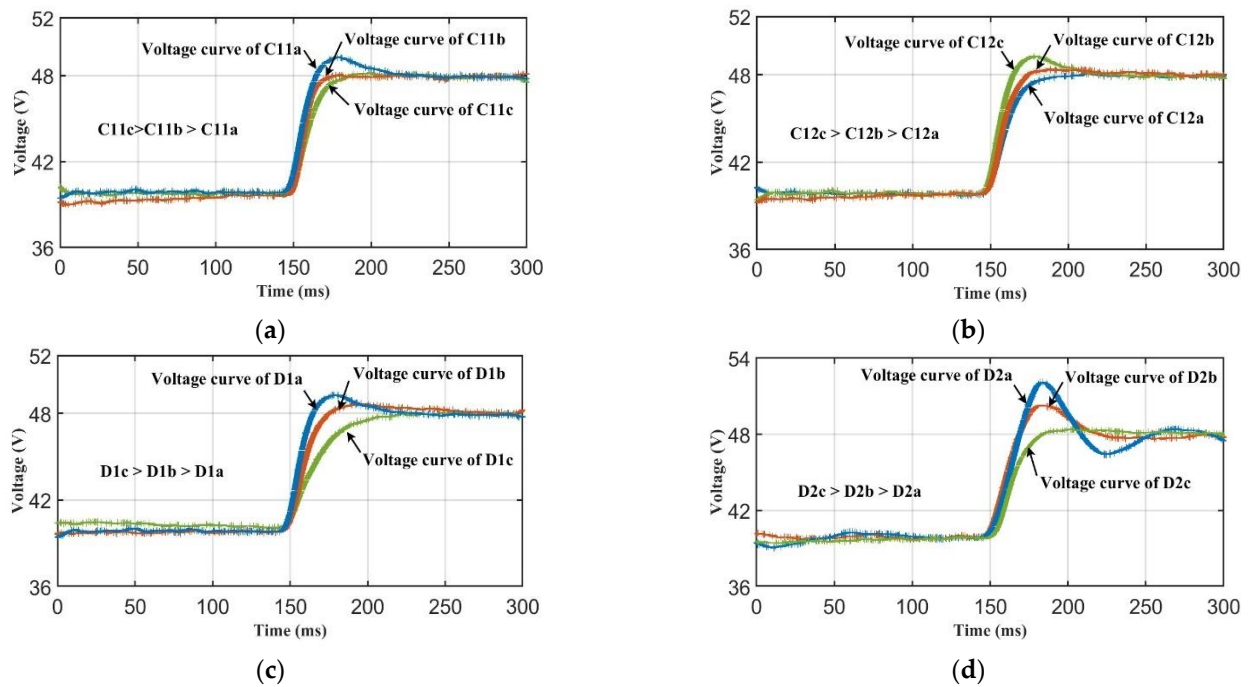


Figure 9. Impact of different weight coefficients on the control performance: (a) experimental results for different C_{11} coefficients; (b) experimental results for different C_{12} coefficients; (c) experimental results for different D_1 coefficients; (d) experimental results for different D_2 coefficients.

Table 2. Weight coefficients for H_∞ robust control of the DC-link voltage.

H_∞ Robust Control Parameter	Value
C_{11}	0.67
C_{12}	0.32
D_1	0.04
D_2	0.32

4.2. Experiments on H_∞ Robust Control Under Different Load Conditions

In this paper, the H_∞ robust control method based on load current compensation is applied to DC-link voltage control under various load conditions and compared with the PI control and ADRC methods. The experimental comparison is conducted under four common load conditions: constant resistance load, load step change, motor load, and energy feedback conditions.

When supplying power to a DC load, the DC-link voltage is usually set through a step input. In addition, the system response time and overshoot can be obtained through the step response curve. Therefore, the step response experiment is carried out. The DC-link voltage step response under a constant resistance load is shown in Figure 10. Both H_∞ robust control and ADRC involve feedforward compensation, which improves the system response speed. Thus, H_∞ robust control and ADRC achieve faster response times than PI control, with response times of 13.7 ms, 14.2 ms, and 21.8 ms. Although H_∞ robust control shows an overshoot of 0.8 V, this is within an acceptable range. The overshoot for PI control is 1.1 V. Compared with that of PI control, the response speed of H_∞ robust control is improved by 37.1%. The control performance of H_∞ robust control is comparable to that of ADRC, but the H_∞ robust controller design requires fewer motor parameters, making it simpler than the ADRC method.

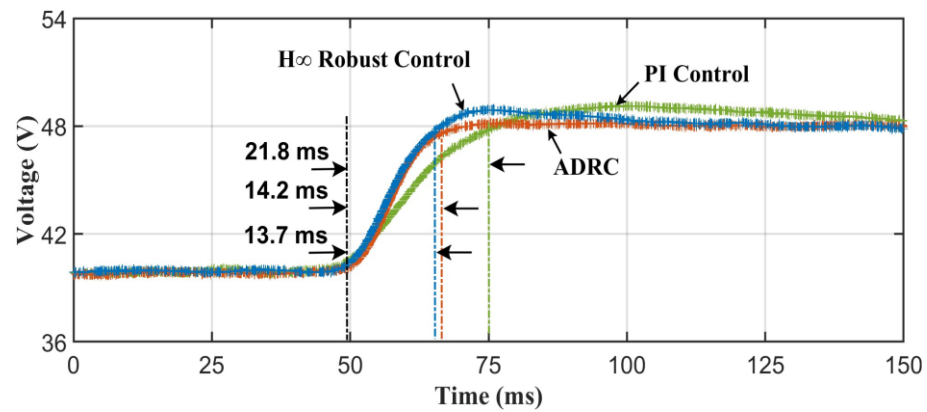


Figure 10. DC-link voltage step response curves under a constant resistance load.

When the DC-link load is turned on or off, this can be viewed as a load change. In this study, the performance of DC-link voltage control during such changes is investigated. The resistance load on the DC link is switched from $12\ \Omega$ to $6\ \Omega$ and the DC-link voltage curves for different control methods are shown in Figure 11. Both H_∞ robust control and ADRC can suppress load disturbances, whereas PI control cannot. Therefore, H_∞ robust control and ADRC result in lower voltage fluctuations and shorter voltage recovery times. The voltage fluctuations for H_∞ robust control, ADRC, and PI control are 5.13 V, 4.92 V, and 7.28 V, respectively, with recovery times of 13.25 ms, 14.62 ms, and 26.98 ms. Compared with PI control, H_∞ robust control reduces the voltage fluctuations by 29.5% and improves the recovery speed by 51.3%. The control performances of H_∞ robust control and ADRC are comparable.

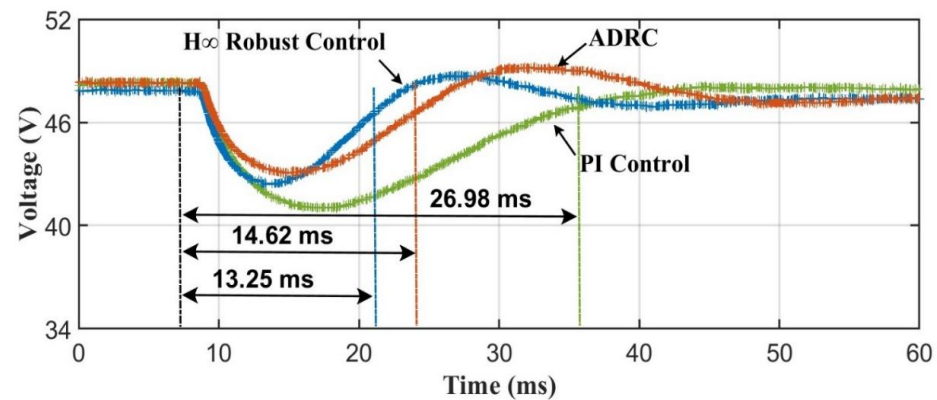


Figure 11. DC-link voltage curves during load step changes.

Motor loads are common in power systems and the current and back electromotive force varies from startup to steady-state operation, making it a time-varying nonlinear load. Figure 12 shows the DC-link voltage curves during motor startup for different control methods. After motor startup, the current gradually increases, causing a slight voltage drop. The voltage fluctuations for H_∞ robust control, ADRC, and PI control are 0.98 V, 1.03 V, and 1.36 V, respectively. Under motor load conditions, H_∞ robust control can quickly track the changes in the motor current, preventing large voltage drops and outperforming PI control. The experimental result illustrates that the proposed method can obtain a satisfactory control performance that can be achieved under time-varying nonlinear load motor loads.

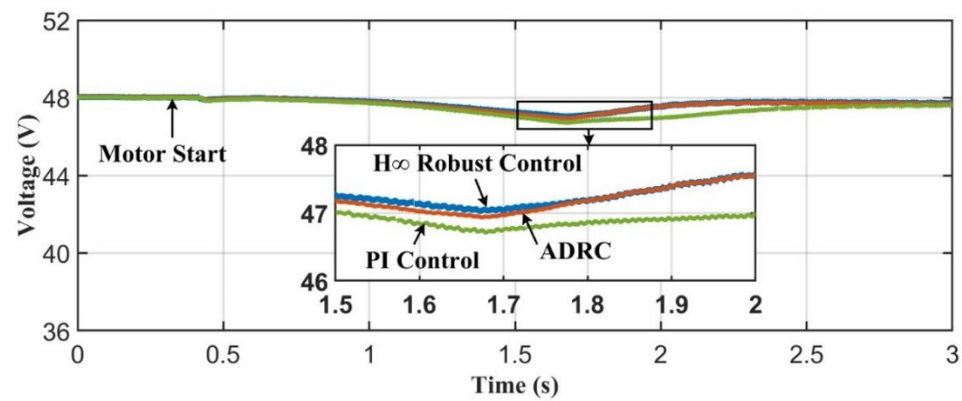


Figure 12. DC-link voltage curves under a motor load.

In some scenarios, the load feeds energy back to the DC link. For example, during rapid motor braking, the mechanical energy of the motor is converted into electrical energy and fed back to the DC link. The FESS must quickly absorb the returned energy to prevent excessive DC-link voltage fluctuations, which could affect system stability. The voltage curves during energy feedback for different control methods are shown in Figure 13. The voltage fluctuations for H_∞ robust control, ADRC, and PI control are 1.25 V, 1.32 V, and 5.22 V, respectively. Compared with PI control, H_∞ robust control allows the FESS to absorb the feedback energy more quickly, reducing the voltage fluctuations by 76.1%. The voltage fluctuations for H_∞ robust control and ADRC are comparable.

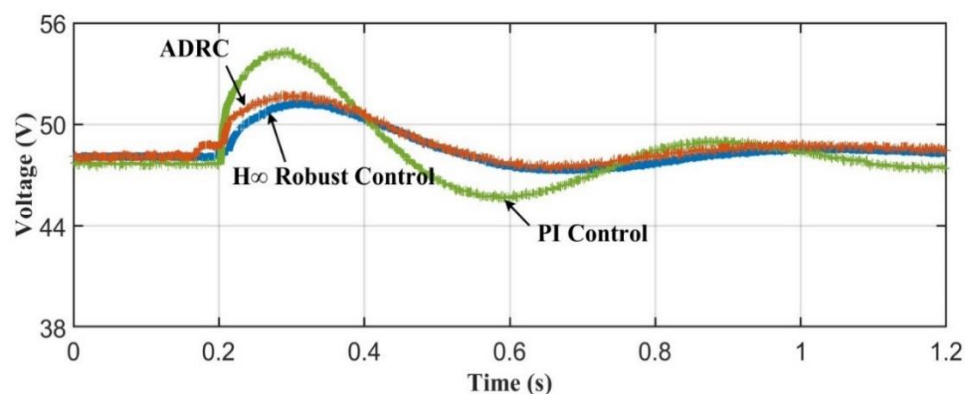


Figure 13. Voltage curves in energy feedback mode.

Table 3 summarizes the control performance of the different methods under various load conditions. The experimental results show that under constant resistance load, load step change, motor load, and energy feedback conditions, the proposed H_∞ robust control method outperforms the PI control method. The performance of the proposed H_∞ robust control method is comparable to that of the ADRC method, but the H_∞ robust control method requires fewer motor system parameters, making its design simpler than that of the ADRC method. Furthermore, the execution time of the control methods in MCU is compared. The control methods are applied to the TMS302F28379D MCU that developed by Texas Instruments in Texas, USA. The main frequency of MCU is 200 MHz and the code runs in RAM. Compared with the PI control method, although the execution time of the proposed robust control method is increased by 4.6 μ s, the control performance is improved.

Table 3. Performance comparison of the control methods.

Control Method	Response Time for a Fixed Resistor Load	Recovery Time for Load Step Changes	Voltage Fluctuations for a Motor Load	Voltage Fluctuations Under Energy Feedback	Code Execution Time in MCU
Proposed	13.7 ms	13.25 ms	0.98 V	1.25 V	9.8 μ s
PI Control	21.8ms	14.62 ms	1.36 V	5.22 V	5.2 μ s
ADRC	14.2 ms	26.98 ms	1.03 V	1.32 V	10.6 μ s

5. Conclusions

This paper proposes a robust control method for the DC-link voltage based on load current compensation for a FESS, which is validated through experiments. The main conclusions are as follows:

- (1) The experimental results align with the theoretical analysis of the influence of weight coefficients on the control performance and appropriate controller parameters can be determined through experiments based on the relationship between weight coefficients and the control performance.
- (2) Under constant resistance load, load step change, motor load, and energy feedback conditions, the proposed robust control method outperforms PI control, with a 37.1% faster response speed and 29.5% lower voltage fluctuations than PI control.
- (3) The performance of the proposed robust control method is comparable to that of ADRC, but the robust control method requires fewer motor system parameters, making its design process simpler than that of the ADRC method. The experiments validate the effectiveness of the robust control method based on load current compensation.

6. Future Works

In this study, a robust control method for the DC-link voltage of FESS is proposed and achieves performance improvement compared to the PI control method. However, there are several aspects of the proposed method that require further research.

- (1) The weight coefficient selection of H_∞ robust control has an important impact on the performance of the control system. In this paper, the weight coefficients are obtained through experimental adjustment. However, it is difficult to obtain the optimal weight coefficient combination in this way. In the future, intelligent algorithms, such as genetic algorithm and particle swarm optimization, can be further used to search for the optimal weight coefficients combination and improve the performance of the system.
- (2) During the long-term operation of the FESS, the model parameters will change with operating conditions. However, the model parameters of the H_∞ robust control method in this paper were measured offline. In future work, it is necessary to consider online model parameter identification of the system to adjust control parameters in real time and improve system stability.

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Abbreviations

FESS	flywheel energy storage system
PID	proportional-integral-derivative
MPC	model predictive control
FRC	frequency regulation controller
ADRC	active disturbance rejection control
PMSM	permanent magnet synchronous motor
LMI	linear matrix inequality
FOC	field-oriented control
PI	proportional-integral
PWM	pulse-width modulation

Key Variables

i_a	the phase A current
i_b	the phase B current
i_c	the phase C current
i_α	the α -axis current
i_β	the β -axis current
u_α	the α -axis voltage
u_β	the β -axis voltage
u_{dc}	the DC-link voltage
u_{dc}^*	the reference signal for the DC-link voltage
u_q	the q-axis voltage
i_q	the q-axis current
i_q^*	the reference signal for the q-axis current
C_{dc}	the DC-link capacitance
i_{ca}	the DC-link capacitor current
i_{ca}^*	the capacitor current control input
i_l	the load current
i_l^*	the load current feedforward compensation signal
i_{dc}	the DC-link current
i_{dc}^*	the reference signal for the DC-link current
T_{qdc}	the conversion coefficient between the DC-link current and the q-axis current
T_{dq}	the conversion coefficient between the q-axis current and the DC-link current
T_q	the time constant of the q-axis current loop
ΔT	the uncertainty coefficient in the conversion of i_{dc}^* and i_q^*
$\Delta I(s)$	the Laplace transform of the load current measurement error
$\Delta I^*(s)$	the Laplace transform of the load current compensation error
A	the state matrix of the system
B	the input matrix of the system
C	the output matrix of the system
w	the input disturbance
z	the system output evaluation
C_1	the weight matrix related to the system state
C_{11}, C_{12}	the elements of the matrix C_1
D_1	the weight coefficient related to the control input
D_2	the weight coefficient related to the input disturbance
$Z(s)$	the Laplace transform of the system evaluation output
$Y(s)$	the Laplace transform of the system output
$W(s)$	the Laplace transform of the input disturbance
$U(s)$	the Laplace transform of the system input
$K(s)$	the transfer function of the state feedback controller
$G_{wz}(s)$	the transfer function from $W(s)$ to $Z(s)$
r	a positive constant
L, M	two positive definite matrices that satisfy the LMI.
K	the state feedback matrix K

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