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Nonlinear Optimal Control for Spacecraft Rendezvous and Docking Using Symplectic Numerical Method

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Abstract: This paper addresses the autonomous rendezvous and docking between a chaser spacecraft and a target spacecraft. An optimal control method is employed to plan the rendezvous and docking maneuver, considering various constraints, including force, velocity, field of view, and collision avoidance with a diamond-shaped obstacle. The optimal trajectories are derived using a symplectic algorithm, which ensures high accuracy and enhances computational efficiency. These trajectories serve as the reference for the maneuver. A PD-based tracking control method is proposed to enable real-time feedback control. An air-bearing experimental system, encompassing state measurement, data transmission, and processing, is established to conduct ground-based tracking experiments. Furthermore, specialized simulators for the chaser and target spacecraft, equipped with a docking mechanism, are designed. Experimental results validate both the feasibility of the reference trajectories and the effectiveness of the PD tracking control approach.

Keywords: rendezvous and docking; optimal control; superquadratic obstacle model; symplectic numerical method; air-bearing experiment



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1. Introduction

Autonomous Rendezvous and Docking (ARVD) plays a crucial role in emerging space missions such as on-orbit assembly, on-orbit servicing, and deep space exploration [1,2]. As a fundamental operation, the ARVD of two spacecraft has garnered significant attention from both academia and industry. For instance, in the context of ARVD for micro-satellites in on-orbit assembly, Fear et al. proposed a guidance algorithm based on model predictive control [3]. Gao et al. applied time-synchronized control theory to the stable maneuvering of a six-degree-of-freedom service spacecraft, enhancing control performance while reducing energy consumption [4]. Colagrossi et al. developed a comprehensive guidance, navigation, and control framework tailored to ARVD requirements in the Cislunar environment [5]. In terms of engineering applications, NASA's Demonstration for Autonomous Rendezvous Technology (DART) project tested both the software and hardware involved in the mission [6]. Additionally, the International Space Station (ISS), as a typical example of a large spacecraft assembled in orbit, has completed over forty construction flights, providing extensive validation of ARVD technology [7].

To achieve stable and safe relative maneuvers between the Chaser Spacecraft (CS) and the Target Spacecraft (TS), it is crucial to incorporate various mission or path constraints into the guidance control algorithm. Sun explicitly integrated control input saturation into the design of a fault-tolerant controller [8]. Zhao et al. further addressed field-of-view constraints for optical sensors during the final phase of docking [9]. Li et al. introduced

boundary constraints on the maneuvering velocity of the CS to ensure its capability to avoid emergencies during the process [10]. Collision avoidance is another critical consideration. Boyarko et al. imposed a constraint on the minimum distance between the two spacecraft, ensuring it remains above a predefined threshold to prevent collisions [11]. Xu et al. proposed a method to avoid unexpected collisions between the CS and TS by utilizing a circular envelope representation [12]. Beyond direct collision with the TS, Weiss et al. tackled the issue of obstacle avoidance by approximating the obstacle's envelope using an ellipsoid and representing the collision avoidance constraint geometrically with rotating hyperplanes [13]. Similarly, Wei et al. addressed collision avoidance between the TS and space debris by calculating the distance between two circular envelopes [14]. However, in the studies mentioned above, collision avoidance objects were often described using simplified geometric shapes. While this improves computational efficiency, it may limit the planning space for feasible paths. Therefore, methods for contour characterization that strike a balance between complex shape descriptions and computational simplicity merit further attention.

The implementation of ARVD is fundamentally a problem of trajectory planning and control under multiple constraints. Optimal control theory provides an effective approach to solving such problems. For nonlinear control optimization, solution methods can be broadly categorized into direct and indirect methods. The direct method first converts the optimal control problem into a finite-dimensional nonlinear programming problem, which is then solved using established numerical tools. The primary advantage of the direct method lies in its ability to handle any optimization problem within a unified algorithmic framework, making it widely applicable in engineering. For instance, Sanjeev et al. studied the trajectory optimization of low-thrust spacecraft using the Hermite–Legendre–Gauss–Lobatto method. They introduced the use of the Edelbaum trajectory to set bounds, scale the problem, and generate an appropriate initial guess [15]. In a similar vein, Zhong et al. developed a piecewise two-phase optimal control strategy for rapid nanosatellite deorbiting, leveraging a short electrodynamic tether. They utilized a direct collocation method grounded in the Hermite–Simpson approach with discretization schemes that feature coincident nodes. This method effectively reduces the problem size and improves computational efficiency [16].

Unlike the direct method, the indirect method begins by applying variational techniques or Pontryagin's Maximum Principle to derive the first-order necessary conditions for the optimal control problem [17]. These conditions are subsequently discretized and converted into a two-point boundary value problem for solving. The main advantage of the indirect method is that it directly satisfies the first-order necessary conditions of the optimal control problem. However, its drawback is the complex and tedious derivation required for specific optimization problems [18]. Fabien introduced a multiple shooting method to solve boundary-value problems involving both differential and algebraic equations, presenting two examples to demonstrate the effectiveness of the approach [19]. Despite this, most numerical algorithm research tends to focus primarily on numerical approximation accuracy and computational speed, often neglecting the inherent properties of optimal control systems. Symplectic algorithms, derived from appropriate difference schemes or variational principles, maintain an indirect symplectic structure. This structure is highly efficient due to its sparsity, which enhances both solution speed and convergence rate. Consequently, symplectic algorithms not only preserve the intrinsic properties of the optimal control system but also ensure numerical accuracy and computational efficiency through the sparse structure of the associated matrices [20–22]. Peng et al. proposed three different symplectic numerical approaches based on the variational principle to solve the optimal rendezvous control problem. The numerical results demonstrated that, although the three

approaches exhibited different numerical performances, they all converged to the same results as the number of time intervals increased [23,24].

To verify the applicability of the guidance algorithm, a ground-based experiment is essential. For spacecraft proximity rendezvous and docking tasks, Wei et al. developed an air-bearing experimental system, which consists of air-bearing spacecraft simulators, a granite platform, and a binocular vision measurement system. This setup was used to validate the proposed analytical rendezvous and docking control algorithm [25]. Xu et al. extended this work by incorporating a collision avoidance scenario with an obstacle simulator, successfully achieving collision-free autonomous rendezvous and docking between a chaser spacecraft simulator and a stationary target spacecraft simulator [26]. Additionally, in ground-based experiments, it is crucial to account for the hardware performance of the simulators and the potential impact of the measurement systems on the results.

This paper focuses on the numerical simulation and ground-based experiment of ARVD between a chaser spacecraft and a target spacecraft, where the entire process is limited to the orbital plane [1,14,25,27–30]. In this study, the symplectic algorithm is employed to solve the optimal control problems of ARVD, ensuring that the characteristics of the original system are preserved, while maintaining high accuracy and improving computational efficiency. Additionally, a superquadratic curve is used to describe the shape of the obstacle, leveraging its ability to represent complex shapes while remaining simple. An analytical expression is applied to avoid numerical differentiation and reduce calculation errors. To address experimental challenges, this paper introduces a ground-based experimental system and proposes an indirect method to meet the requirements of real-time feedback control. The optimal trajectories obtained from the symplectic algorithm serve as reference trajectories, and a PD-based tracking control method is introduced to ensure that the simulator follows these reference trajectories.

This paper is organized as follows. Section 2 presents the dynamic model for the ARVD system, along with the superquadratic curve, the relevant constraints, and the formulation of the optimal control problem. Section 3 introduces the symplectic algorithm for generating optimal trajectories and the PD tracking control method for the ground-based experiment. The numerical simulation is provided in Section 4, followed by the experimental studies in Section 5. Finally, Section 6 concludes the paper.

2. Problem Formulation

2.1. System Dynamics

The ARVD mission between a chaser spacecraft and a target spacecraft is considered. When the target spacecraft is in a circular orbit and the distance between the chaser spacecraft and the target spacecraft is much smaller than the orbital radius, the relative translational motion of the chaser with respect to the target is described by the Clohessy–Wiltshire (CW) equations, as follows [1]:

$$\begin{aligned}\ddot{x}^H - 2n\dot{y}^H - 3n^2x^H &= \frac{F_x^H}{m} \\ \ddot{y}^H + 2n\dot{x}^H &= \frac{F_y^H}{m} \\ \ddot{z}^H + n^2z^H &= \frac{F_z^H}{m}\end{aligned}\quad (1)$$

where x^H , y^H , and z^H are the projection coordinates of the relative position vector in the Hill coordinate system; $F_{(\cdot)}^H$ is the control force; m is the mass of the chaser spacecraft; and n is the orbital rate of the target spacecraft.

The rotational dynamics of a single spacecraft lead to the following Euler's equations [14]:

$$\begin{aligned}\dot{\omega}_{ix} &= \frac{(J_{iy} - J_{iz})\omega_{iy}\omega_{iz} + \tau_{ix}}{J_{ix}} \\ \dot{\omega}_{iy} &= \frac{(J_{iz} - J_{ix})\omega_{iz}\omega_{ix} + \tau_{iy}}{J_{iy}} \\ \dot{\omega}_{iz} &= \frac{(J_{ix} - J_{iy})\omega_{ix}\omega_{iy} + \tau_{iz}}{J_{iz}}\end{aligned}\quad (2)$$

where $\{J_{Cx}, J_{Cy}, J_{Cz}\}$ and $\{J_{Tx}, J_{Ty}, J_{Tz}\}$ denote the principal moments of inertia of the chaser and target spacecraft, respectively.

Additionally, if the mission duration is much shorter than the orbital period, orbital effects are neglected. When the entire process is confined to the orbital plane, the resulting dynamic equations simplify to a double integral form [1,14,28]. As illustrated in Figure 1, the target spacecraft is treated as a stationary object, while the dynamic equations of the chaser spacecraft, with one rotational and two translational degrees of freedom, are given by:

$$\ddot{x} = \frac{F_x}{m}, \quad \ddot{y} = \frac{F_y}{m}, \quad \ddot{\theta} = \frac{T}{I_z} \quad (3)$$

where F_x , F_y , and T are the control forces and torque; m is the mass of the chaser spacecraft; and I_z is the moment of inertia about the vertical axis. The state-space form of the chaser spacecraft model can be written as:

$$\dot{x} = Ax + Bu \quad (4)$$

where $x = [x, y, \theta, \dot{x}, \dot{y}, \dot{\theta}]^T$ denotes the state vector, $u = [F_x, F_y, T]^T$ denotes the control force, and A and B are the corresponding state and control constant matrices, respectively.

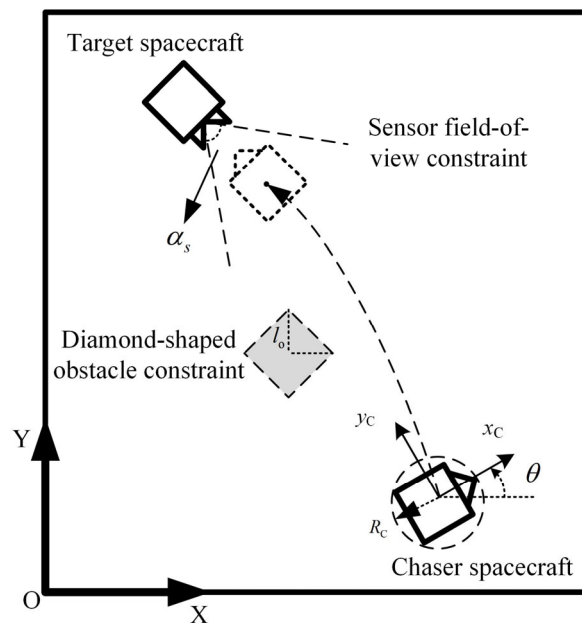


Figure 1. Schematic of the ARVD mission.

The mission consists of two phases: the obstacle avoidance phase and the docking phase. The shape of the diamond obstacle is represented by a superquadratic curve, while the chaser spacecraft is assumed to have a circular appearance with a specified

radius R_C . The final system configuration of the docking phase is shown in Figure 2. The corresponding states of the chaser spacecraft are given by:

$$\begin{aligned} x_f &= x^T + L_{\text{total}} \cos \gamma, \quad y_f = y^T - L_{\text{total}} \sin \gamma, \quad \theta_f = \pi + \theta^T \\ \dot{x}_f &= \dot{x}^T, \quad \dot{y}_f = \dot{y}^T, \quad \dot{\theta}_f = \dot{\theta}^T \end{aligned} \quad (5)$$

with:

$$L_{\text{total}} = L_{\text{EDM}} + \frac{L_{\text{CS}}}{2} + \frac{L_{\text{TS}}}{2} \quad (6)$$

where $\mathbf{x}_f = [x_f, y_f, \theta_f, \dot{x}_f, \dot{y}_f, \dot{\theta}_f]^T$ denotes the final state of the chaser spacecraft, $\mathbf{x}^T = [x^T, y^T, \theta^T, \dot{x}^T, \dot{y}^T, \dot{\theta}^T]^T$ denotes the position of the target spacecraft, γ denotes the angle between the line connecting the centroids of the two spacecraft and the X -axis, $L_{\text{EDM}} = 0.068$ m is the length of the Electromagnetic Docking Mechanism (EDM), and L_{CS} and L_{TS} are the side lengths of the chaser spacecraft and target spacecraft, respectively. Specifically, L_{CS} and L_{TS} hold the same value, equal to $L_{\text{spacecraft}}$.

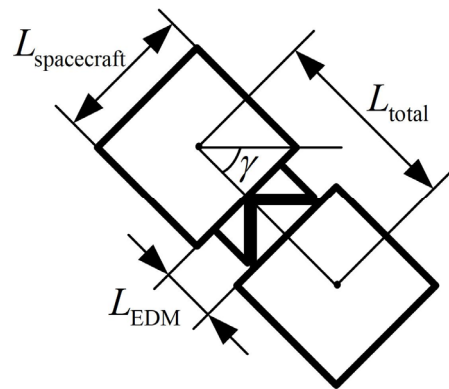


Figure 2. Schematic of the final system configuration in the docking phase.

2.2. Constraints Modeling

Due to the complex shapes of obstacles and the target spacecraft, it is challenging to represent these shapes using a general curve. The superquadratic curve offers a flexible way to describe such complex shapes using a simple formula while also improving calculation accuracy and avoiding the need for numerical differentiation [31–33]. In this subsection, the superquadratic curve is used to model the obstacle, as shown in Figure 1.

The standard expression of the superquadratic curve can be expressed as:

$$\left(\frac{x_{\text{sq}}^2}{a^2} \right)^s + \left(\frac{y_{\text{sq}}^2}{b^2} \right)^s = 1 \quad (7)$$

where x_{sq} and y_{sq} are the independent variables; a , b , and s are positive real numbers.

Figure 3 shows a group of superquadratic curves. The value of s is from 0 to ∞ . As shown in the figure, the shape changes among the following seven conditions: (1) “+” line ($s = 0$); (2) concave asteroid ($0 < s < 0.5$); (3) diamond ($s = 0.5$); (4) thin ellipse ($0.5 < s < 1$); (5) ellipse ($s = 1$); (6) fat ellipse ($s > 1$); (7) rectangle ($s \rightarrow \infty$). Table 1 further introduces some special superquadratic curves.

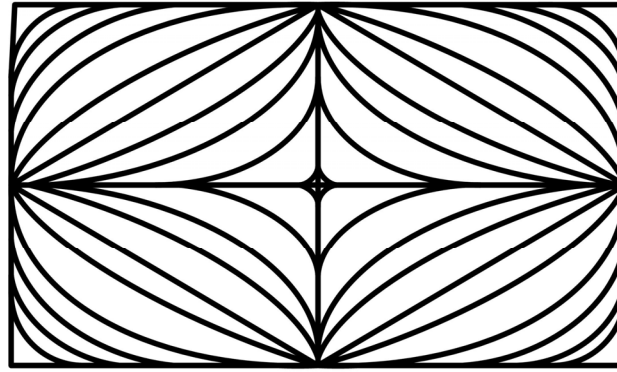


Figure 3. A group of superquadratic curves of $a = 5$, $b = 3$.

Table 1. Some special superquadratic curves.

Curve	Equation	Shape
"+" ($s = 0$)	x -axis and y -axis	
Asteroid ($s = 1/3$ and $a = b$)	$x^{\frac{2}{3}} + y^{\frac{2}{3}} = c^{\frac{2}{3}}$	
Diamond ($s = 0.5$)	$\pm \frac{x}{a} \pm \frac{y}{b} = 1$	
Ellipse ($s = 1$)	$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$	
Rectangle ($s \rightarrow \infty$)	$s = 6$	

To ensure that the chaser spacecraft follows a safe and feasible trajectory until it makes contact with the target spacecraft through the docking mechanism, the following constraints are imposed.

Sensor field-of-view constraint: Sensors are installed on the target spacecraft to detect the relative distance and attitude of the chaser spacecraft. Therefore, the centroid of the chaser spacecraft must remain within the sensor's field of view throughout the entire process. This constraint can be expressed mathematically as:

$$\tan\left(\theta^T - \frac{\alpha_s}{2}\right) \leq \frac{y - y_s}{x - x_s} \leq \tan\left(\theta^T + \frac{\alpha_s}{2}\right) \quad (8)$$

where (x_s, y_s) denotes the origin of the sensor.

Collision avoidance: Any collision with obstacles or target spacecraft must be avoided. Based on Equation (7) and Table 1, by setting $a = b = l_o$, the concerned diamond-shaped obstacle constraint can be expressed as:

$$|x - x_o| + |y - y_o| \geq l_o + R_C \quad (9)$$

where (x_o, y_o) denotes the position of the center of the obstacle.

Thruster force Constraints: Considering the spacecraft design, the control force is subject to finite upper and lower bounds:

$$|\mathbf{u}| \leq \mathbf{u}_{\max} \quad (10)$$

where $\mathbf{u}_{\max} = [F_{x\max}, F_{y\max}, T_{\max}]^T$ denotes the bound of the control force.

Velocity and angular velocity constraints: To ensure the spacecraft's controllability in the event of an emergency, both the velocity and angular velocity are constrained by finite upper and lower bounds:

$$|v| \leq v_{\max} \quad (11)$$

where $v = [\dot{x}, \dot{y}, \dot{\theta}]^T$ and $v_{\max} = [\dot{x}_{\max}, \dot{y}_{\max}, \dot{\theta}_{\max}]^T$ denote the bound of the velocity and angular velocity.

2.3. Optimal Control Problem Formulation

The performance index function is defined as:

$$J = \frac{1}{2} \int_{t_0}^{t_f} [(x - x_f)^T P (x - x_f) + (u - u_f)^T R (u - u_f)] dt \quad (12)$$

where t_0 and t_f denote the initial and final times, respectively, and x_f can be obtained from Equations (5) and (6). $u_f = [F_{xf}, F_{yf}, T_f]^T$ represents the final control variable, which is given as $u_f = [0, 0, 0]^T$. The matrices P and R represent the performance index function weights associated with the final states and control variables, respectively.

The optimal control problem for the ARVD maneuver can be formulated as follows: find the control histories u and state histories x that minimize the performance index J , while simultaneously transferring the system described by Equation (4) from an initial state $x_0 = [x_0, y_0, \theta_0, \dot{x}_0, \dot{y}_0, \dot{\theta}_0]^T$ to a final state $x_f = [x_f, y_f, \theta_f, \dot{x}_f, \dot{y}_f, \dot{\theta}_f]^T$ that satisfies Equations (5) and (6), and respecting the path constraints outlined in Equations (8)–(11).

3. Trajectory Generation and Tracking

In this section, the symplectic algorithm is used to obtain the solutions to the Nonlinear Optimal Control (NOC) problem [34,35], and based on the derived optimal trajectories, a PD-based tracking control method is then proposed.

3.1. Theory of Symplectic Algorithm

The performance index of the NOC problem is defined as:

$$J = \int_{t_0}^{t_f} L(x, u, t) dt \quad (13)$$

where the detailed formulation of $L(x, u, t)$ is given by Equation (12). The dynamics equations, boundary conditions, and state-control constraints are as follows:

$$\begin{aligned} \dot{x} &= f(x, u, t), \quad x(t_0) = x_0, \quad x(t_f) = x_f \\ h(x, u, t) &\leq 0 \end{aligned} \quad (14)$$

where $f(x, u, t)$ is system dynamics shown in Equation (4); $h(x, u, t)$ denotes the inequality constraints presented by Equations (8)–(11).

To reduce computation and complexity, a quasi-linearization method is adopted in this work to linearize the nonlinear optimal control problem defined in Equations (13) and (14). By extending the performance index, dynamic equations, and state-control constraints, the iterative process of the linearized optimal control problem is obtained as follows.

The performance index function can be written as:

$$J^{(k+1)} = \int_{t_0}^{t_f} L^{(k+1)} dt \quad (15)$$

where:

$$\begin{aligned} L^{(k+1)} &= \bar{L}^{(k)} + (\mathbf{x}^{(k+1)} - \mathbf{x}^{(k)})^T \mathbf{L}_x^{(k)} + (\mathbf{u}^{(k+1)} - \mathbf{u}^{(k)})^T \mathbf{L}_u^{(k)} \\ &+ \frac{1}{2} (\mathbf{x}^{(k+1)} - \mathbf{x}^{(k)})^T \mathbf{L}_{xx}^{(k)} (\mathbf{x}^{(k+1)} - \mathbf{x}^{(k)}) \\ &+ (\mathbf{x}^{(k+1)} - \mathbf{x}^{(k)})^T \mathbf{L}_{xu}^{(k)} (\mathbf{u}^{(k+1)} - \mathbf{u}^{(k)}) + \frac{1}{2} (\mathbf{u}^{(k+1)} - \mathbf{u}^{(k)})^T \mathbf{L}_{uu}^{(k)} (\mathbf{u}^{(k+1)} - \mathbf{u}^{(k)}) \end{aligned} \quad (16)$$

with

$$\bar{L}^{(k)} = L(\mathbf{x}^{(k)}, \mathbf{u}^{(k)}, t) \quad (17)$$

$$\mathbf{L}_x^{(k)} = \left. \frac{\partial \bar{L}(\mathbf{x}, \mathbf{u}, t)}{\partial \mathbf{x}} \right|_{\mathbf{x}^{(k)}, \mathbf{u}^{(k)}}, \quad \mathbf{L}_u^{(k)} = \left. \frac{\partial \bar{L}(\mathbf{x}, \mathbf{u}, t)}{\partial \mathbf{u}} \right|_{\mathbf{x}^{(k)}, \mathbf{u}^{(k)}} \quad (18)$$

$$\mathbf{L}_{xx}^{(k)} = \left. \frac{\partial^2 \bar{L}(\mathbf{x}, \mathbf{u}, t)}{\partial \mathbf{x}^2} \right|_{\mathbf{x}^{(k)}, \mathbf{u}^{(k)}}, \quad \mathbf{L}_{xu}^{(k)} = \left. \frac{\partial^2 \bar{L}(\mathbf{x}, \mathbf{u}, t)}{\partial \mathbf{x} \partial \mathbf{u}} \right|_{\mathbf{x}^{(k)}, \mathbf{u}^{(k)}}, \quad \mathbf{L}_{uu}^{(k)} = \left. \frac{\partial^2 \bar{L}(\mathbf{x}, \mathbf{u}, t)}{\partial \mathbf{u}^2} \right|_{\mathbf{x}^{(k)}, \mathbf{u}^{(k)}} \quad (19)$$

The state-control constraints are:

$$\mathbf{C}^{(k)} \mathbf{x}^{(k+1)} + \mathbf{D}^{(k)} \mathbf{u}^{(k+1)} + \mathbf{v}^{(k)} \leq \mathbf{0} \quad (20)$$

where:

$$\begin{aligned} \mathbf{C}^{(k)} &= \left. \frac{\partial h(\mathbf{x}, \mathbf{u}, t)}{\partial \mathbf{x}} \right|_{\mathbf{x}^{(k)}, \mathbf{u}^{(k)}}, \quad \mathbf{D}^{(k)} = \left. \frac{\partial h(\mathbf{x}, \mathbf{u}, t)}{\partial \mathbf{u}} \right|_{\mathbf{x}^{(k)}, \mathbf{u}^{(k)}} \\ \mathbf{v}^{(k)} &= h(\mathbf{x}^{(k)}, \mathbf{u}^{(k)}, t) - \mathbf{C}^{(k)} \mathbf{x}^{(k)} - \mathbf{D}^{(k)} \mathbf{u}^{(k)} \end{aligned} \quad (21)$$

The variables $\mathbf{x}^{(k+1)}$ and $\mathbf{u}^{(k+1)}$ in the above equations represent the state and control vectors for the current $(k+1)$ th iteration, while $\mathbf{x}^{(k)}$ and $\mathbf{u}^{(k)}$ are the state and control vectors from the previous k th iteration, which serve as reference values for the current computation. As a result, the NOC problem is transformed into a series of Linear Quadratic Optimal Control (LQOC) problems, and the symplectic approach is applied to solve each LQOC problem.

To transform the inequality constraints defined in Equation (20) into equality constraints, the parameter variables $\boldsymbol{\alpha}^{(k+1)}$ are introduced as:

$$\mathbf{C}^{(k)} \mathbf{x}^{(k+1)} + \mathbf{D}^{(k)} \mathbf{u}^{(k+1)} + \mathbf{v}^{(k)} + \boldsymbol{\alpha}^{(k+1)} = \mathbf{0}, \quad \boldsymbol{\alpha}^{(k+1)} \geq \mathbf{0} \quad (22)$$

By introducing the cosatate variables $\boldsymbol{\lambda}^{(k+1)}$ and parameter variables $\boldsymbol{\beta}^{(k+1)}$, the Hamilton function can be defined as follows:

$$\begin{aligned} H^{(k+1)} &= L(\mathbf{x}^{(k+1)}, \mathbf{u}^{(k+1)}, t) + (\boldsymbol{\lambda}^{(k+1)})^T f(\mathbf{x}^{(k+1)}, \mathbf{u}^{(k+1)}, t) \\ &+ (\boldsymbol{\beta}^{(k+1)})^T (h(\mathbf{x}^{(k+1)}, \mathbf{u}^{(k+1)}, t) + \boldsymbol{\alpha}^{(k+1)}) \end{aligned} \quad (23)$$

The variations of the augmented performance index function are computed, and the first-order necessary conditions are subsequently derived.

$$\frac{\partial H^{(k+1)}}{\partial \mathbf{u}^{(k+1)}} = \mathbf{0} \quad (24)$$

$$\dot{\mathbf{x}}^{(k+1)} = \frac{\partial H^{(k+1)}}{\partial \boldsymbol{\lambda}^{(k+1)}} \quad (25)$$

$$\dot{\boldsymbol{\lambda}}^{(k+1)} = -\frac{\partial H^{(k+1)}}{\partial \mathbf{x}^{(k+1)}} \quad (26)$$

$$\frac{\partial H^{(k+1)}}{\partial \beta^{(k+1)}} = 0 \quad (27)$$

It is important to note that the following complementary conditions must be satisfied, according to Pontryagin's minimum principle [34,35].

$$\alpha^{(k+1)} \geq 0, \beta^{(k+1)} \geq 0, (\alpha^{(k+1)})^T \beta^{(k+1)} = 0 \quad (28)$$

The optimal solutions can be obtained by solving Equations (24)–(28) with the initial conditions and the final conditions specified in Equations (5) and (6).

3.2. PD Tracking Control

To track the position and velocity, the formulation of the PD control can be described as follows

$$u(t) = -K_p e(t) - K_d \frac{de(t)}{dt} \quad (29)$$

where $e(t)$ and $\frac{de(t)}{dt}$ are the error of position and velocity; K_p and K_d are the proportional and derivative coefficients, respectively.

According to Equation (29), the control law for translational motion can be expressed as

$$u_i(t) = -K_p(x_i - x_d^i) - K_d(\dot{x}_i - \dot{x}_d^i) \quad i = 1, 2, 3 \dots N' \quad (30)$$

where N' is the number of points, K_p and K_d are the control coefficients, and $x_d^i = [x_d^i, y_d^i]^T$ and $\dot{x}_d^i = [\dot{x}_d^i, \dot{y}_d^i]^T$ denote the desired position and velocity at the i th point. $x_i = [x_i, y_i]^T$ and $\dot{x}_i = [\dot{x}_i, \dot{y}_i]^T$ represent the tracking trajectory and velocity of the translational motion from the $(i - 1)$ th point to the i th point, while $u_i(t) = [F_{xi}, F_{yi}]^T$ is the required thrust from the $(i - 1)$ point to the i th point.

Similarly, the control law for rotational motion can be given as:

$$T_i(t) = -K'_p(\theta_i - \theta_d^i) - K'_d(\dot{\theta}_i - \dot{\theta}_d^i) \quad i = 1, 2, 3 \dots N' \quad (31)$$

where K'_p and K'_d are the control coefficients; θ_d^i and $\dot{\theta}_d^i$ denote the desired attitude and angular velocity at the i th point. θ_i and $\dot{\theta}_i$ represent the tracking trajectory and angular velocity of the rotational motion from the $(i - 1)$ th point to i th point, while $T_i(t)$ is the required torque from the $(i - 1)$ th point to i th point.

4. Numerical Results

The results of the numerical simulation are presented in this section. Table 2 lists the physical parameters of the chaser spacecraft and the target spacecraft. The initial states of the chaser spacecraft and the position of the target spacecraft are summarized in Table 3. Table 4 provides the parameters of the constraints.

Table 2. The physical parameters of the chaser and target spacecraft.

Parameter	Value
$L_{\text{spacecraft}}$	200 mm
R_C	0.15 m
m_C	9 kg
I_C	0.05 kg · m ²

Table 3. The initial states of the chaser spacecraft and the position of target spacecraft.

Parameter	Value
x_0, y_0, θ_0	0.8 m, 0.2 m, $\frac{\pi}{2}$
$\dot{x}_0, \dot{y}_0, \dot{\theta}_0$	0, 0, 0
$x^T, y^T, \theta^T, \dot{x}^T, \dot{y}^T, \dot{\theta}^T$	0.3 m, 1.5 m, $-\frac{\pi}{4}$

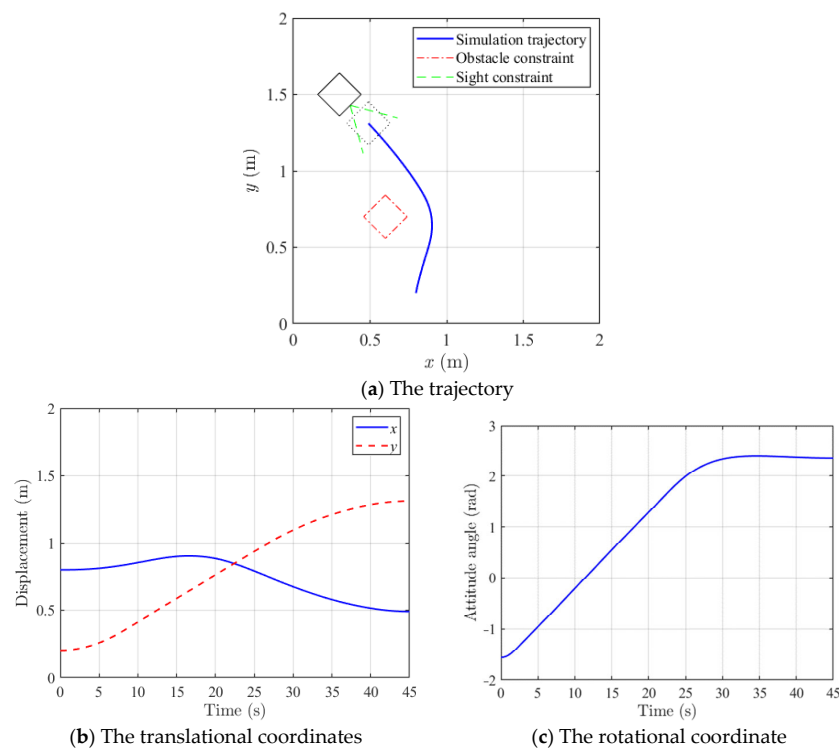
Table 4. The parameters of constraints.

Parameter	Value
Origin of FOV	$(0.3 + \frac{\sqrt{2}}{20}, 1.5 - \frac{\sqrt{2}}{20})$ m
α_s	60°
Roadblock location	(0.6, 0.7) m
a, b, s	$a = b = 0.15, s = 0.5$
$F_{x\max}, F_{y\max}$	0.03 N
$T_{\max}$	0.001 N · m
$\dot{x}_{\max}, \dot{y}_{\max}$	0.035 m/s
$\dot{\theta}_{\max}$	0.15 rad/s

The matrices for the performance index are chosen as:

$$\mathbf{P} = \text{diag}(10, 10, 0.01, 600, 600, 0.02), \mathbf{R} = \text{diag}(400, 400, 800) \quad (32)$$

The simulation is conducted in the Matlab Simulink (R2022b) environment, with a simulation duration of 45 s. Figures 4–6 present the results obtained using the symplectic algorithm. Figure 4 shows the translational and rotational trajectories of the chaser spacecraft. From these figures, the following conclusions can be drawn: (1) at all times, the centroid of the chaser spacecraft remains within the sensor's field of view; (2) the chaser successfully avoids collision while passing by the obstacle. Moreover, the constraints defined in Equations (10) and (11) are satisfied, as shown in Figures 5 and 6.

**Figure 4.** The pose trajectory of the chaser spacecraft obtained by the symplectic algorithm.

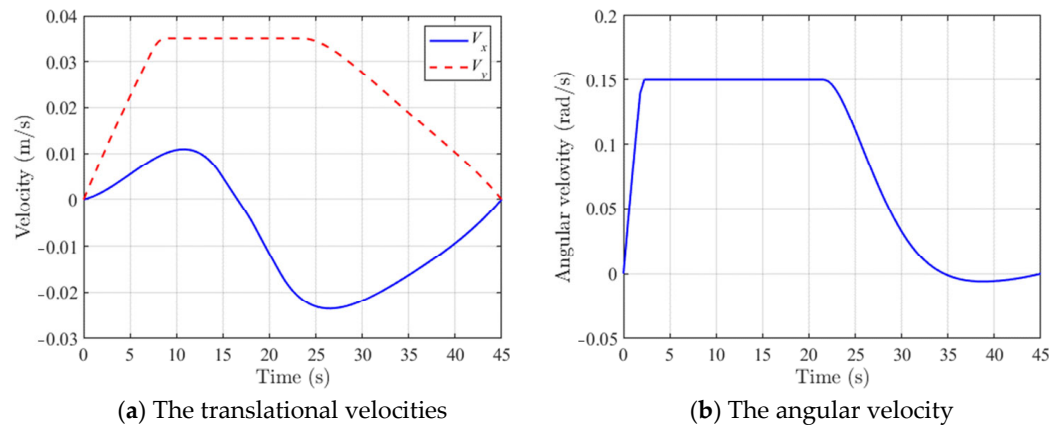


Figure 5. Time history of the translational and rotational velocities obtained by the symplectic algorithm.

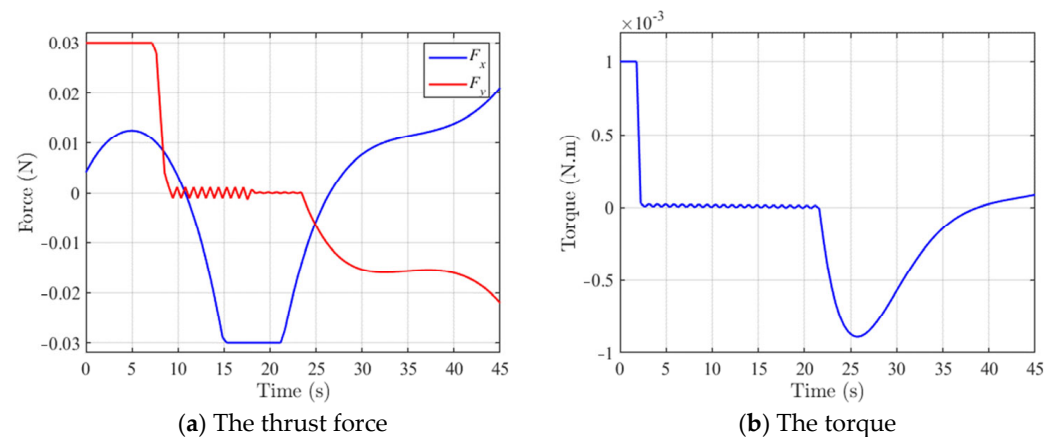


Figure 6. Time histories of the thrust force and torque generated by the symplectic algorithm.

5. Experimental Studies

5.1. Experimental System

As shown in Figure 7, the experimental system primarily consists of a granite platform, a binocular camera, an Image Processing Workstation (IPW), and two spacecraft simulators that float above the granite platform using air bearings. The binocular camera, mounted on the ceiling, captures the positions of fluorescent markers in real time. The acquired image data is then transmitted to the IPW, where the digital image is processed to obtain the three-dimensional coordinates. This information is forwarded to the Industrial PC (IPC) via a ZigBee module. The IPC calculates the control law in real time, converting the continuous control law into a series of fully equivalent switching signals using the pulse width pulse frequency modulation method [36,37]. These switching signals control the motion of the chaser spacecraft simulator through the solenoid valves. Notably, the electromagnet on the Electromagnetic Docking Mechanism (EDM) is also controlled by the IPC. When the distance between the two spacecraft simulators drops below a predefined threshold, a turn-on signal is sent to the relay, activating the electromagnet and generating suction. Figure 8 presents the workflow of the experimental system. For more detailed information about the customized spacecraft simulator, one can refer to [14,25].

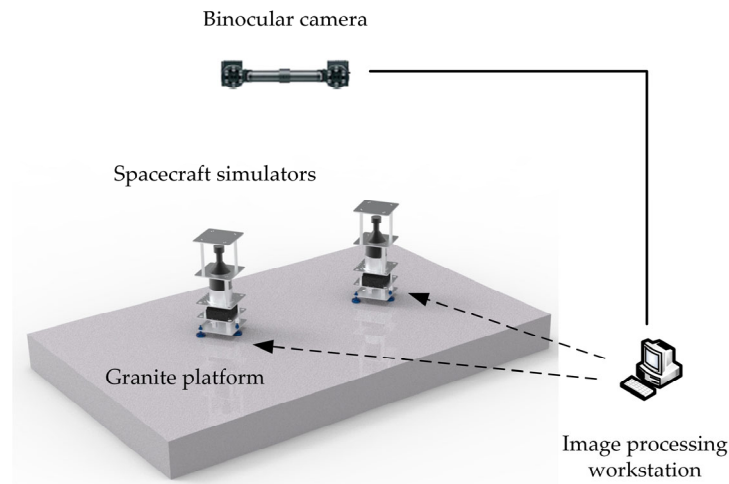


Figure 7. Schematic of the ground-based experimental system.

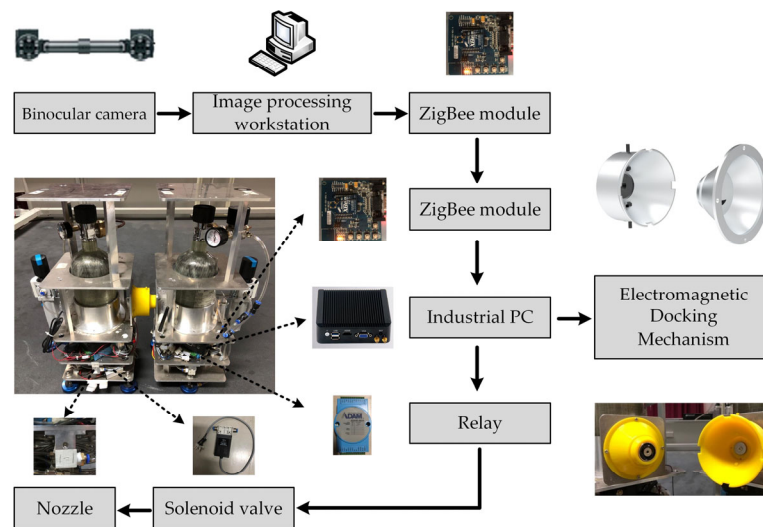


Figure 8. Schematic of the workflow of the experimental system.

5.2. Experimental Results

Table 5 presents the values of the control gains designed for the PD tracking controller. Figure 9 shows the translational and rotational tracking trajectories of the chaser spacecraft simulator in the ground experiment. The release location of the tracking simulator is (0.82 m, 0.19 m, 88°). As can be seen from the figure, the tracking trajectories almost completely coincide with the reference trajectories, except for the initial phase of the experiment. Figure 10 further illustrates the velocities time history of the chaser spacecraft simulator during the experiment. Based on these experimental results, it can be concluded that the PD tracking controller designed in Section 3.2 is effective.

Table 5. The values of the control gains in the PD tracking controller.

Parameter	Value
K_p	diag(20, 20)
K_d	diag(50, 50)
K'_p	0.1
K'_d	0.3

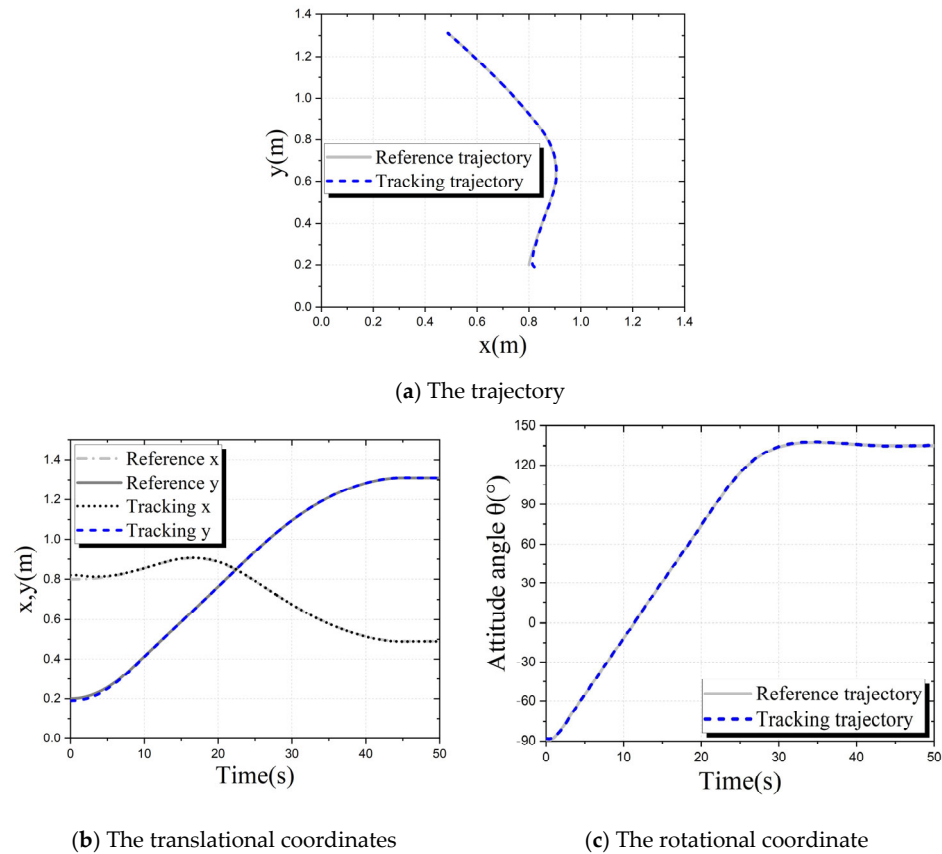


Figure 9. The pose tracking trajectory of the chaser spacecraft in the experiment.

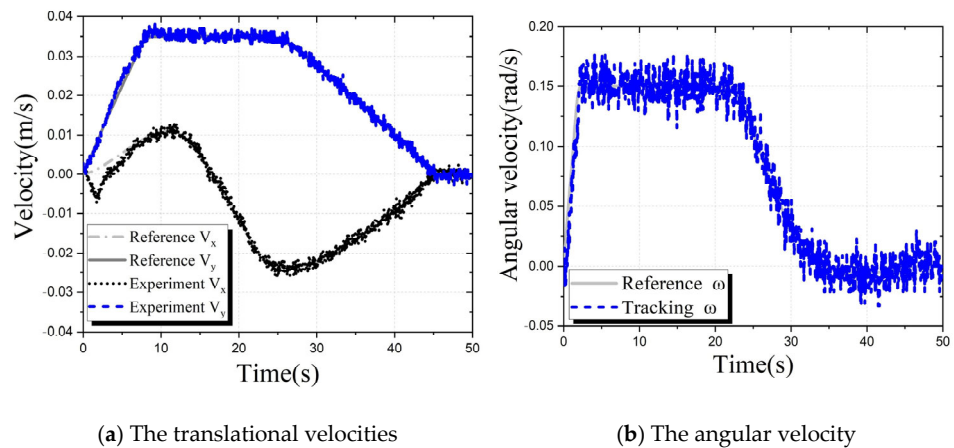


Figure 10. Time history of the translational and rotational tracking velocities in the experiment.

6. Conclusions

In this paper, the symplectic algorithm was employed to execute the ARVD maneuver between a chaser spacecraft and a target spacecraft. The NOC problem was transformed into a series of LQOC problems through quasi-linearization iterations, which could then be solved using symplectic discretization. Additionally, a diamond-shaped obstacle constraint, represented by a superquadratic curve, and a sensor field-of-view constraint were integrated into the optimal control problem formulation. The simulation results demonstrated that the optimal control law derived from the symplectic algorithm effectively guides the chaser spacecraft. Since the guidance algorithm cannot provide real-time control inputs due to hardware limitations, this paper also presents a ground-based experimental system and introduces an indirect method using PD control to track the reference trajectory. The experi-

mental results validate the applicability of the reference trajectories and the effectiveness of the PD tracking control.

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