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Adaptive Predefined Time Control for Strict-Feedback Systems with Actuator Quantization

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Abstract: An adaptive predefined-time quantized control issue is considered for strict-feedback systems with actuator quantization. To handle the unknown nonlinearities of a system, the neural networks are first applied to model them. To analyze the predefined-time stability under approximation error, a stability lemma is first introduced. Then, a refreshing predefined-time quantized control strategy is presented. Compared with the existing control studies for actuator quantization, the stability time is not influenced by the initial state and can be set in advance. Furthermore, unlike the available predefined-time control studies, a new parameter adaptive law and virtual controllers are designed. This design not only ensures the predefined-time stability, but overcomes the singularities of system in conventional backstepping control design because of repeating differentiation for virtual controllers.

Keywords: predefined-time control; actuator quantization; backstepping; neural network control



Citation: Zhang, W.; Yu, B. Adaptive Predefined Time Control for Strict-Feedback Systems with Actuator Quantization. *Actuators* **2024**, *13*, 366. <https://doi.org/10.3390/act13090366>

Academic Editors: Guanyu Lai, Fang Wang, Weijun Yang and Hanzhen Xiao

Received: 26 July 2024

Revised: 5 September 2024

Accepted: 11 September 2024

Published: 19 September 2024



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1. Introduction

With the development of the network, quantized control is receiving more and more attention due to the impact of bandwidth limitation in communication. The stabilization problem was firstly considered for discrete quantized linear systems in [1,2]. Then, several quantized feedback control designs followed for nonlinear systems in [3–6]. Given that the control methods in [3–6] were only fit for certain model control systems, two types of adaptive quantized feedback strategies were presented for uncertain systems in [7,8]. One thing that should be pointed out is that an effective stability condition cannot be achieved beforehand, as found in [7,8]. By establishing a nonlinear decomposition of a quantizer, several fuzzy quantized feedback controllers were designed for nonlinear systems to improve the stability conditions in [9–12]. Specifically, the above control strategies are personalized to the infinite-time stability of systems. However, the system performance needs to be accomplished within a very limited time in practical engineering applications.

To improve the stability time, numerous finite-time control results have sprung up over the last few years. A sliding mode controller was established in [13] to ensure the finite-time stability of the rigid robots. On the grounds of a Lyapunov theory, two finite-time control strategies were put forward in [14,15], which prevented a possible chattering phenomenon in [13]. Further intelligent finite-time controllers were designed on the grounds of the approximation of fuzzy logic systems/neural networks in [16–25] for uncertain systems. Despite these achievements, the settling time in [13–25] depends upon the initial conditions of systems. The designer cannot achieve the effective initial conditions in practical applications. Thus, the settling time is impossible to obtain, which limits the application area of the finite-time control. To cope with this issue, a fixed-time stability was considered in [26]. The advantage of fixed-time control is that the stability time has an upper bound, which is not affected by initial conditions. Based on the stability lemma established in [26], several fixed-time control strategies were presented in [27–29]. Furthermore, a fixed-time stability lemma for uncertain nonlinear systems was put forward in Ref. [30]. In

view of the a stability lemma, some interesting adaptive fixed-time control approaches were raised in [30,31]. Although some achievements have been made in [26–31], the boundary of settling time is dependent on the design parameters. What is far more important is that the connection between the convergence time and design parameters is not clear, which leads to the design parameters being difficult to adjust according to actual control requirements.

To further optimize the convergence time, a definition of predefined-time stability was firstly introduced in Refs. [32,33]. The boundary of stability time in [32,33] can be pre-defined according to practical control needs. In view of the definition in [32,33], a predefined-time stability was given consideration for a second-order multi-agent system in [34]. By applying a sliding mode control method, an output feedback strategy is developed for a second-order multi-agent system in [35], which ensures the predefined-time stability of the system. The predefined-time stability was considered for high-order systems with immeasurable states in [36,37]. A predefined-time control approach was put forward in [38] for distributed order systems. Some predefined-time schemes were proposed for strict-feedback systems in [39–43]. A predefined-time stability problem was considered in [44] for stochastic nonlinear system. An adaptive predefined-time control scheme was presented for nonlinear systems with output hysteresis in [45]. Despite recent progress in predefined-time control in [32–45], a predefined-time control is not yet available for quantized nonlinear systems. However, more and more industrial systems require remote control through network, which makes the quantized control inevitable. Therefore, predefined-time control is significant for quantized systems. Motivated by the above statements, a predefined-time control will be intended to handle the tracking problem of nonlinear quantized systems. In this paper, the nonlinear functions of the system are completely unknown, and the neural networks are used to model these nonlinearities. To analyse the predefined-time stability under approximation error, a stability criterion is first introduced. By applying an adaptive neural network method, a brand-new predefined-time quantized control strategy is proposed. This manuscript makes contributions to the following areas:

(1) Compared with the available quantized control research, the settling time is not influenced by the initial states and can be set in advance. To analyse the predefined-time stability under approximation error, this paper establishes a new stability criterion, which gives an effective solution to the predefined-time control under neural networks approximation.

(2) Compared with the existing predefined-time control studies, a novel parameter adaptive law and virtual controllers are designed. This new adaptive parameter design can not only ensure the predefined-time stability but can also reduce control gain. The proposed design for virtual controllers can overcome the singularities of the system in the subsequent backstepping control design caused by repeated differentiations of virtual controllers.

2. Prior Knowledge and Problem Statement

2.1. Prior Knowledge

Consideration is given to the following system:

$$\dot{\omega} = f(\omega, t), \omega(0) = \omega_0, \quad (1)$$

where $\omega \in R^m$ represents a state variable and the continuous function $f : R_+ \times R^m \rightarrow R^m$ meets the condition $f(0, 0) = 0$.

Definition 1 ([32]). *The system (1) is practically predefined-time stable if there exists a constant $\varepsilon > 0$ and a predefined time T_f , such that $\|\omega(t)\| < \varepsilon$ for all $t \geq T_f$.*

Definition 2. *The strictly increasing and continuous function $\beta(r) (r \in [0, \infty))$ is called κ_∞ - and functions if it satisfies $\beta(0) = 0$ and $\lim_{r \rightarrow \infty} \beta(r) = \infty$.*

Lemma 1. For system (1) and a predefined time T_j , if a Lyapunov function $V(\omega(t))$ and a constant $0 < p < 0.5$ is present to meet the following inequality

$$\begin{cases} \beta_1(\|\omega\|) \leq V(\omega(t)) \leq \beta_2(\|\omega\|), \\ \dot{V}(\omega) \leq -\frac{\pi}{pT_j} \left(V^{1+p}(\omega) + V^{1-p}(\omega) \right) + \aleph, \end{cases} \quad (2)$$

where β_1 and β_2 are κ_∞ -functions and $\aleph > 0$ is a constant, then system (1) is practically stable in any given predefined time T_j .

Proof. Define $\Omega_\omega = \left\{ \omega(t) \mid V(\omega(t)) \leq \frac{p\aleph T_j}{\pi} \right\}$ and $\bar{\Omega}_\omega = \left\{ \omega(t) \mid V(\omega(t)) > \frac{p\aleph T_j}{\pi} \right\}$.

If $\omega(t) \in \bar{\Omega}_\omega$, we have

$$\begin{aligned} \dot{V}(\omega) &\leq -\frac{\pi}{2pT_j} \left(V^{1+p}(\omega) + V^{1-p}(\omega) \right) + \aleph \\ &\quad - \frac{\pi}{2pT_j} \left(V^{1+p}(\omega) + V^{1-p}(\omega) \right) \\ &\leq -\frac{\pi}{2pT_j} \left(V^{1+p}(\omega) + V^{1-p}(\omega) \right) + \aleph \\ &\quad - \frac{\pi}{pT_j} \sqrt{V^{1+p}(\omega) V^{1-p}(\omega)} \\ &\leq -\frac{\pi}{2pT_j} \left(V^{1+p}(\omega) + V^{1-p}(\omega) \right) - \frac{\pi V(\omega)}{pT_j} + \aleph \\ &\leq -\frac{\pi}{2pT_j} \left(V^{1+p}(\omega) + V^{1-p}(\omega) \right). \end{aligned} \quad (3)$$

Here, (4) represents monotone decreasing in $\bar{\Omega}_\omega$, which means that $\omega(t)$ will get into the set Ω_ω and not leave Ω_ω . Denote T_r as the required time that $\omega(t)$ needs to get into the set Ω_ω , then, according to (4), we can obtain

$$\int_0^{T_r} \frac{\dot{V}(\omega)}{V^{1+p}(\omega) + V^{1-p}(\omega)} dt \leq - \int_0^{T_r} \frac{\pi}{2pT_j} dt. \quad (4)$$

Thus, we have

$$\begin{aligned} T_r &\leq -\frac{2pT_j}{\pi} \int_0^{T_r} \frac{\dot{V}(\omega)}{V^{1+p}(\omega) + V^{1-p}(\omega)} dt \\ &= -\frac{2T_j}{\pi} \int_0^{T_r} \frac{pV^{p-1}(\omega)\dot{V}(\omega)}{V^{2p}(\omega) + 1} dt \\ &= -\frac{2T_j}{\pi} \int_0^{T_r} \frac{dV^p}{V^{2p} + 1} \\ &= -\frac{2T_j}{\pi} \left(\arctan V^p(\omega(T_r)) - \arctan V^p(\omega(0)) \right) \\ &= -\frac{2T_j}{\pi} \left(\arctan V^p(\omega(T_r)) - \arctan V^p(\omega(0)) \right) \\ &\leq \frac{2T_j}{\pi} \arctan V^p(\omega(0)) \\ &\leq \frac{2T_j}{\pi} \frac{\pi}{2} = T_j. \end{aligned} \quad (5)$$

From the above discussion, $\omega(t)$ will get into the set Ω_ω within the predefined time T_j . According to (2), $\|\omega(t)\| \leq \beta_1^{-1}(\frac{p\aleph T_j}{\pi}) = \varepsilon$; therefore, the system is practically stable in the given time T_j . $\square$

Remark 1. Lemma 1 provides an effective stability analysis approach for the following neural network predefined-time control. Compared with the available finite-time control study, the settling time T_j is not influenced by the initial state in this article and may be set beforehand based upon the system requirement.

Lemma 2 ([46]). For $\forall \omega \geq \varrho \geq 0$ and $\sigma > 1$, one has

$$\varrho(\omega - \varrho)^\sigma \leq \frac{\sigma}{\sigma + 1}(\omega^{\sigma+1} - \varrho^{\sigma+1}). \quad (6)$$

Remark 2. Inequality (6) will be applied to cope with the scaling difficulty in (28) and (43).

Lemma 3 ([46]). To design the change rate of the adaptive parameter, the following differential Equation (7) will be employed:

$$\dot{\omega}(t) = -e\omega(t) - \gamma\omega^h(t) + \varphi(t), \quad (7)$$

where $\varphi(t) \geq 0$, constants $e > 0$, $\gamma > 0$, and $h > 1$. In (7), $\omega(t) \geq 0$ may be derived from $\omega(t_0) \geq 0$ for $\forall t \geq t_0$.

Remark 3. From Lemma 3, the positivity of $\omega(t)$ may be guaranteed if the adaptive parameter is designed as form (7). We will apply this property to the following design for the adaptive parameters. The details can be found in (28) and (43).

Lemma 4 ([47]). For $\chi_j \in R_+$, $0 < \varsigma < 1$, $\gamma > 1$, the following relationships can be obtained:

$$\begin{cases} \left(\sum_{j=1}^m \chi_j\right)^\varsigma \leq \sum_{j=1}^m \chi_j^\varsigma, \\ \left(\sum_{j=1}^m \chi_j\right)^\gamma \leq m^{\gamma-1} \left(\sum_{j=1}^m \chi_j^\gamma\right). \end{cases} \quad (8)$$

Lemma 5 ([48]). The following inequality holds for $\mu \in (0, 1)$ and $\iota > 0$:

$$|w|^\mu |\chi|^{1-\theta} \leq \theta \iota |w| + (1-\theta) \iota^{\frac{-\theta}{1-\theta}} |\chi|, \quad (9)$$

where w and χ are variables, $\theta \in (0, 1)$ and $\iota > 0$ are constants.

Lemma 6 ([49]). For a real variable χ and a positive variable w , one has

$$0 \leq |\chi| - \frac{\chi^2}{\sqrt{\chi^2 + w^2}} < w. \quad (10)$$

2.2. Problem Statement

Let us give consideration to a tracking issue of the nonlinear quantized system, which is described as

$$\begin{cases} \dot{x}_s = x_{s+1} + g_s(\bar{x}_s), 1 \leq s \leq m-1, \\ \dot{x}_m = Q(u) + g_m(\bar{x}_m), \\ y = x_1, \end{cases} \quad (11)$$

where $\bar{x}_s = [x_1, \dots, x_s]^T$ stands for state vector; $g_s(\bar{x}_s)$ indicates an unknown nonlinear function; u is a designed controller to fulfill the requirement of the system; and y represents the system output. $Q(u)$ is an input of the system and is the output of the quantizer, which is defined in the following form [7]:

$$Q(u) = \begin{cases} u_i \operatorname{sgn}(u), & \frac{u_i}{1+\kappa} < |u| \leq u_i, \dot{u} < 0, \text{ or} \\ & u_i < |u| \leq \frac{u_i}{1-\kappa}, \dot{u} > 0 \\ u_i(1+\kappa) \operatorname{sgn}(u), & u_i < |u| \leq \frac{u_i}{1-\kappa}, \dot{u} < 0, \text{ or} \\ & \frac{u_i}{1-\kappa} < |u| \leq \frac{u_i(1+\kappa)}{1-\kappa}, \dot{u} > 0 \\ 0, & 0 \leq |u| < \frac{u_{\min}}{1+\kappa}, \dot{u} < 0, \text{ or} \\ & \frac{u_{\min}}{1+\kappa} \leq |u| \leq u_{\min}, \dot{u} > 0 \\ Q(u(t^-)), & \text{other cases,} \end{cases} \quad (12)$$

where $u_i = \rho^{1-i} u_{\min}$ ($u_{\min} > 0$ and $\kappa = \frac{1-\rho}{1+\rho}$, $\rho \in (0, 1)$) represents a measure of quantization density. Then, $Q(u) \in U = \{0, \pm u_i, \pm u_i(1+\kappa), i = 1, 2, \dots\}$.

In this article, a quantized control strategy is going to be presented to make y fall in a small neighborhood of a given signal y_r in a predefined time.

Remark 4. In contrast to the available quantized control studies in [1–12], in this paper, the proposed settling time may be set beforehand, and its boundary is not influenced by the initial states and design parameters. Thus, the presented predefined-time control method is more in line with the needs of the practical application.

Lemma 7 ([9]). $Q(u)$ can be further expressed as

$$Q(u) = \Gamma(u)u + K(t), \quad (13)$$

where $\Gamma(u)$ and $K(t)$ meet

$$1 - \kappa \leq \Gamma(u) \leq 1 + \kappa, |K(t)| \leq u_{\min}. \quad (14)$$

2.3. RBF Neural Networks

Considering that functions $g_s(\bar{x}_s)$ are unknown, they will be approximated by radial basis function neural networks (RBF NNs):

$$g(\varsigma) = \xi^T \Phi(\varsigma), \quad (15)$$

where $\varsigma \in \Omega_\varsigma \subset R^q$ is an input vector, $\xi = [\xi_1, \dots, \xi_l]^T \in R^l$ with $l > 1$ being a weight vector, and $\Phi(\varsigma) = [\phi_1(\varsigma), \phi_2(\varsigma), \dots, \phi_l(\varsigma)]^T$ is a basis function vector with $\phi_k(\varsigma)$ being the Gaussian function in the following form:

$$\phi_k(\varsigma) = \exp\left[-\frac{(\varsigma - \iota_k)^T(\varsigma - \iota_k)}{\omega_k^2}\right], k = 1, 2, \dots, l, \quad (16)$$

where $\iota_k = [\iota_{k1}, \iota_{k2}, \dots, \iota_{kq}]^T$ denotes the center of the receptive field and ω_k denotes the width of the basis function $\phi_k(\varsigma)$.

In view of [49], for any continuous function $g(\varsigma)$ defined on a compact set $\Xi \subset R^q$, $\forall \varepsilon > 0$, there exists an RBF NN (15) to make the following equality hold:

$$g(\varsigma) = \xi^{*T} \Phi(\varsigma) + \epsilon(\varsigma), \forall \varsigma \in \Xi \subset R^q,$$

where ξ^* denotes an ideal weight vector in the following form:

$$\xi^* := \arg \min_{\xi \in R^l} \left\{ \sup_{\varsigma \in \Xi} |g(\varsigma) - \xi^T \Phi(\varsigma)| \right\},$$

and $\epsilon(\varsigma)$ denotes an approximation error which meets $|\epsilon(\varsigma)| \leq \varepsilon$.

3. Predefined-Time Neural Networks Control Design

This part proposes a predefined-time neural network control strategy for (11) on the grounds of backstepping steps. At step $s - th$ ($1 \leq s \leq m$), a neural network $\zeta_s^T \Phi_s(X_s)$ is going to be applied to approximate an unknown nonlinear function. In the controller design process, $\eta_s = \|\zeta_s\|^2$ needs to be defined, where $\|\zeta_s\|$ represents a two-norm of a neural network weighted vector ζ_s . It is clear that η_s is an unknown positive constant. Considering that η_s is unknown, $\hat{\eta}_s$ is used to estimate η_s . Define $\tilde{\eta}_s = \eta_s - \hat{\eta}_s$. Define $\bar{y}_r^{(s)} = [y_r, y_r^{(1)}, \dots, y_r^{(s)}]^T$. Let us suppose y_r and its derivative $y_r^{(s)}$ are continuous and bounded.

Here is a coordinate conversion:

$$\begin{aligned}\chi_1 &= y - y_r, \\ \chi_s &= x_s - v_{s-1}, s = 2, \dots, m,\end{aligned}\quad (17)$$

where v_s is an intermediate controller, which is going to be constructed in the ensuing control design.

Step 1: Choose the following Lyapunov function:

$$V_1 = \frac{\chi_1^2}{2} + \frac{\tilde{\eta}_1^2}{2\lambda_1}, \quad (18)$$

where $\lambda_1 > 0$ denotes a design parameter.

Operating the derivative of V_1 along (11), one has

$$\dot{V}_1 = \chi_1 (g_1(x_1) + v_1 + \chi_2 - \dot{y}_r) - \frac{1}{\lambda_1} \tilde{\eta}_1 \dot{\hat{\eta}}_1. \quad (19)$$

Taking advantage of Young's inequality, we have

$$\chi_1 \chi_2 \leq \frac{1}{2} (\chi_1^2 + \chi_2^2). \quad (20)$$

Denote $\bar{g}_1 = g_1(x_1) - \dot{y}_r + \chi_1$. Considering that $\bar{g}_1$ is unknown, by using the approximation ability of neural networks, for $\forall \varepsilon_1 > 0$, we have

$$\bar{g}_1 = \zeta_1^T \Phi_1(X_1) + \epsilon_1(X_1), |\epsilon_1(X_1)| \leq \varepsilon_1, \quad (21)$$

where the definitions of ζ_1^T and $\Phi_1(X_1)$ can be found in (15) with $X_1 = [x_1, y_r, \dot{y}_r]^T$. Applying Young's inequality, one has

$$\begin{aligned}\chi_1 \bar{g}_1 &= \chi_1 \frac{\zeta_1^T}{\|\zeta_1\|} \|\zeta_1\| \Phi_1 + \chi_1 \epsilon_1 \\ &\leq \frac{\eta_1}{2a_1^2} \chi_1^2 \Phi_1^T \Phi_1 + \frac{a_1^2}{2} + \frac{\chi_1^2}{2} + \frac{\varepsilon_1^2}{2},\end{aligned}\quad (22)$$

where $a_1 > 0$ is a constant.

Let us set up the first virtual controller:

$$v_1 = -\frac{\chi_1 \zeta_1^2}{\sqrt{\chi_1^2 \zeta_1^2 + d_1^2}}, \quad (23)$$

where ζ_1 is represented as

$$\zeta_1 = \begin{cases} \Gamma_1 + \frac{\tau_1}{2^{1+p}} \chi_1^{1+2p} + \frac{\tau_2}{2^{1-p}} \chi_1^{1-2p}, & |\chi_1| \geq \varepsilon, \\ \Gamma_1, & |\chi_1| \leq \varepsilon. \end{cases} \quad (24)$$

in which $\Gamma_1 = \frac{\chi_1}{2a_1^2} \hat{\eta}_1 \Phi_1^T(X_1) \Phi_1(X_1)$, $\tau_1 = \frac{r\pi}{pT_j}$ with $r = (2m)^p$, $\tau_2 = \frac{\pi}{pT_j}$, d_1 is a design constant, the definition of p and T_j can be found in Lemma 1, and $\varepsilon > 0$ is a predefined accuracy.

Remark 5. To guarantee the predefined-time stability of the system, the term $\frac{\tau_2}{2^{1-p}} \chi_1^{1-2p}$ needs to be included in the virtual controller v_1 . However, since $0 < 1 - 2p < 1$, this will cause singularity during differentiation for v_1 at $\chi_1 = 0$ in the following stability analysis: To avoid the singularity, Equation (24) is designed. From (24), when the tracking error χ_1 tends to be zero, i.e., $|\chi_1| \leq \varepsilon$, the terms $\frac{\tau_1}{2^{1+p}} \chi_1^{1+2p}$ and $\frac{\tau_2}{2^{1-p}} \chi_1^{1-2p}$ fail to work, which prevents the singularity occurrence.

The adaptive law design of $\hat{\eta}_1$ is defined in the following form:

$$\begin{aligned} \dot{\hat{\eta}}_1 &= \frac{\lambda_1 \chi_1^2}{2a_1^2} \Phi_1^T(X_1) \Phi_1(X_1) - \frac{\tau_1}{\lambda_1^p} \hat{\eta}_1^{1+2p} - \tau_2 \hat{\eta}_1, \\ \hat{\eta}_1(t_0) &\geq 0. \end{aligned} \quad (25)$$

Remark 6. In the existing predefined-time control study, $\dot{\hat{\eta}}_1 = \frac{\lambda_1}{2a_1^2} \chi_1^2 \Phi_1^T(X_1) \Phi_1(X_1) - \tau \hat{\eta}_1$ is often chosen, which can not meet the predefined-time stability condition in Lemma 1. Different from the above design form, a new design for $\hat{\eta}_1$ (25) is designed. Since the nonlinear term $-\frac{\tau_1}{\lambda_1^p} \hat{\eta}_1^{1+2p}$ is contained within (25), the predefined-time performance can be guaranteed.

Bringing (20)–(25) into (19), we have

$$\begin{aligned} \dot{V}_1 &\leq \frac{\hat{\eta}_1}{2a_1^2} \chi_1^2 \Phi_1^T \Phi_1 - \frac{\chi_1^2 \zeta_1^2}{\sqrt{\chi_1^2 \zeta_1^2 + d_1^2}} + \frac{\tau_1}{\lambda_1^p} \hat{\eta}_1 \hat{\eta}_1^{1+2p} + \frac{\tau_2}{\lambda_1} \hat{\eta}_1 \hat{\eta}_1 \\ &\quad + \frac{\chi_2^2}{2} + \sum_{k=1}^m \frac{a_k^2 + \varepsilon_k^2}{2}. \end{aligned} \quad (26)$$

By applying Lemma 6, one has

$$-\frac{\chi_1^2 \zeta_1^2}{\sqrt{\chi_1^2 \zeta_1^2 + d_1^2}} \leq d_1 - |\chi_1 \zeta_1| \leq d_1 - \chi_1 \zeta_1. \quad (27)$$

By applying the definition of $\tilde{\eta}_1$ and Lemmas 2 and 3, we have

$$\hat{\eta}_1 \hat{\eta}_1^{1+2p} = \tilde{\eta}_1 (\eta_1 - \tilde{\eta}_1)^{1+2p} \leq \frac{1+2p}{2+2p} (\eta_1^{2+2p} - \tilde{\eta}_1^{2+2p}). \quad (28)$$

$$\tilde{\eta}_1 \hat{\eta}_1 = -\tilde{\eta}_1^2 + \tilde{\eta}_1 \eta_1 \leq -\tilde{\eta}_1^2 + \frac{\tilde{\eta}_1^2 + \eta_1^2}{2} = -\frac{\tilde{\eta}_1^2}{2} + \frac{\eta_1^2}{2}. \quad (29)$$

If $|\chi_1| \geq \varepsilon$, substituting (24) and (27)–(29) into (26) gives

$$\begin{aligned} \dot{V}_1 &\leq -\tau_1 \left(\frac{\chi_1^2}{2} \right)^{1+p} - \tau_2 \left(\frac{\chi_1^2}{2} \right)^{1-p} - 2^p \tau_1 \left(\frac{\tilde{\eta}_1^2}{2\lambda_1} \right)^{1+p} - \frac{\tau_2 \tilde{\eta}_1^2}{2\lambda_1} \\ &\quad + \frac{\chi_2^2}{2} + \frac{a_1^2 + \varepsilon_1^2}{2} + d_1 + \frac{\tau_1}{\lambda_1^p} \eta_1^{2+2p} + \frac{\tau_2 \eta_1^2}{2\lambda_1}. \end{aligned} \quad (30)$$

If $|\chi_1| < \varepsilon$, the tracking objective has been achieved. Furthermore,

$$\dot{V}_1 \leq -2^p \tau_1 \left(\frac{\tilde{\eta}_1^2}{2\lambda_1} \right)^{1+p} - \frac{\tau_2 \tilde{\eta}_1^2}{2\lambda_1} + \frac{\tau_1}{\lambda_1^p} \eta_1^{2+2p} + \frac{\chi_2^2}{2} + \frac{\tau_2 \eta_1^2}{2\lambda_1}. \quad (31)$$

By comparing (30) and (31), (30) still holds in the case of $|\chi_1| < \varepsilon$.

Step s – th ($2 \leq s \leq m - 1$). Let us suppose that the virtual controllers have been constructed such that $V_{s-1} = \sum_{\ell=1}^{s-1} \frac{z_\ell^2}{2} + \sum_{\ell=1}^{s-1} \frac{\tilde{\eta}_\ell^2}{2\lambda_\ell}$ satisfies the following inequality:

$$\begin{aligned} \dot{V}_{s-1} \leq & -\tau_1 \sum_{\ell=1}^{s-1} \left(\frac{\chi_\ell^2}{2}\right)^{1+p} - \tau_2 \sum_{\ell=1}^{s-1} \left(\frac{\chi_\ell^2}{2}\right)^{1-p} - \sum_{\ell=1}^{s-1} \frac{\tau_2 \tilde{\eta}_\ell^2}{2\lambda_\ell} \\ & - 2^p \tau_1 \sum_{\ell=1}^{s-1} \left(\frac{\tilde{\eta}_\ell^2}{2\lambda_\ell}\right)^{1+p} + \sum_{\ell=1}^{s-1} \frac{a_\ell^2 + \varepsilon_\ell^2}{2} + \sum_{\ell=1}^{s-1} \frac{\tau_2 \eta_\ell^2}{2\lambda_\ell} \\ & + \sum_{\ell=1}^{s-1} \frac{\tau_1}{\lambda_\ell^p} \eta_\ell^{2+2p} + \frac{\chi_s^2}{2} + \sum_{\ell=1}^{s-1} d_\ell. \end{aligned} \quad (32)$$

where $\lambda_\ell, d_\ell, a_\ell$ are positive design constants.

Choose the following Lyapunov function:

$$V_s = V_{s-1} + \frac{\chi_s^2}{2} + \frac{\tilde{\eta}_s^2}{2\lambda_s}, \quad (33)$$

where λ_s is a positive design constant.

Differentiating V_s along (11), one obtains

$$\dot{V}_s = \dot{V}_{s-1} + \chi_s \left(g_s(\bar{x}_s) + v_s - \dot{v}_{s-1} + \chi_{s+1} \right) - \frac{\tilde{\eta}_s \dot{\tilde{\eta}}_s}{\lambda_s} \quad (34)$$

where the definition of $\dot{v}_{s-1}$ is

$$\begin{aligned} \dot{v}_{s-1} = & \sum_{\ell=1}^{s-1} \frac{\partial v_{s-1}}{\partial x_\ell} \left[g_\ell(\bar{x}_\ell) + x_{\ell+1} \right] + \sum_{\ell=1}^{s-1} \frac{\partial v_{s-1}}{\partial \hat{\eta}_\ell} \hat{\eta}_\ell \\ & + \sum_{\ell=0}^{s-1} \frac{\partial v_{s-1}}{\partial y_d^{(\ell)}} y_d^{(\ell+1)}. \end{aligned} \quad (35)$$

Based on Young's inequality, we obtain

$$\chi_s \chi_{s+1} \leq \frac{1}{2} (\chi_s^2 + \chi_{s+1}^2). \quad (36)$$

Now, define the function $\bar{g}_s = g_s(\bar{x}_s) - \dot{v}_{s-1} + \frac{3\chi_s}{2}$, $s = 2, \dots, m - 1$. Considering that $\bar{g}_s$ is unknown, we apply neural networks to approximate $\bar{g}_s$. For $\forall \varepsilon_s > 0$, we have

$$\bar{g}_s = \tilde{\zeta}_s^T \Phi_s(X_s) + \epsilon_s(X_s), |\epsilon_s(X_s)| \leq \varepsilon_s, \quad (37)$$

where the definitions of $\tilde{\zeta}_s^T$ and $\Phi_s(X_s)$ can be found in (15) and (6) with

$$X_s = (x_1, \dots, x_s, y_r, \dots, y_r^{(s)}, \hat{\eta}_1, \dots, \hat{\eta}_s)^T.$$

Applying Young's inequality, one has

$$\begin{aligned} \chi_s \bar{g}_s = & \chi_s \frac{\tilde{\zeta}_s^T}{\|\tilde{\zeta}_s\|} \|\tilde{\zeta}_s\| \Phi_s + \chi_s \epsilon_s \\ \leq & \frac{\eta_s}{2a_s^2} \chi_s^2 \Phi_s^T \Phi_s + \frac{a_s^2}{2} + \frac{\chi_s^2}{2} + \frac{\varepsilon_s^2}{2}, \end{aligned} \quad (38)$$

where $a_s > 0$ is a constant. Design the s th virtual controller as

$$v_s = - \frac{\chi_s \tilde{\zeta}_s^2}{\sqrt{\chi_s^2 \tilde{\zeta}_s^2 + d_s^2}}, \quad (39)$$

where $d_s > 0$ is a constant and ζ_s is represented as

$$\zeta_s = \begin{cases} \Gamma_s + \frac{\tau_1}{2^{1+p}} \chi_s^{1+2p} + \frac{\tau_2}{2^{1-p}} \chi_s^{1-2p}, & |\chi_s| \geq \varepsilon, \\ \Gamma_s, & |\chi_s| \leq \varepsilon. \end{cases} \quad (40)$$

in which $\Gamma_s = \frac{\chi_s}{2a_s^2} \hat{\eta}_s \Phi_s^T(X_s) \Phi_s(X_s)$.

Design the rate of change for $\hat{\eta}_s$ as

$$\begin{aligned} \dot{\hat{\eta}}_s &= \frac{\lambda_s \chi_s^2}{2a_s^2} \Phi_s^T(X_s) \Phi_s(X_s) - \frac{\tau_1}{\lambda_s^p} \hat{\eta}_s^{1+2p} - \tau_2 \hat{\eta}_s, \\ \hat{\eta}_s(t_0) &\geq 0. \end{aligned} \quad (41)$$

Similar to the discussions in (27)–(29), we have

$$-\frac{\chi_s^2 \zeta_s^2}{\sqrt{\chi_s^2 \zeta_s^2 + d_s^2}} \leq d_s - |\chi_s \zeta_s| \leq d_s - \chi_s \zeta_s, \quad (42)$$

$$\tilde{\eta}_s \hat{\eta}_s^{1+2p} \leq \frac{1+2p}{2+2p} (\eta_s^{2+2p} - \tilde{\eta}_s^{2+2p}), \quad (43)$$

$$\tilde{\eta}_s \hat{\eta}_s \leq -\frac{\tilde{\eta}_s^2}{2} + \frac{\eta_s^2}{2}. \quad (44)$$

If $|\chi_s| \geq \varepsilon$, substituting (33) and (36)–(44) into (34) gives

$$\begin{aligned} \dot{V}_s &\leq -\tau_1 \sum_{\ell=1}^s \left(\frac{\chi_\ell^2}{2} \right)^{1+p} - \tau_2 \sum_{\ell=1}^s \left(\frac{\chi_\ell^2}{2} \right)^{1-p} - \sum_{\ell=1}^s \frac{\tau_2 \tilde{\eta}_\ell^2}{2\lambda_\ell} \\ &\quad - 2^p \tau_1 \sum_{\ell=1}^s \left(\frac{\tilde{\eta}_\ell^2}{2\lambda_\ell} \right)^{1+p} + \sum_{\ell=1}^s \frac{a_\ell^2 + \varepsilon_\ell^2}{2} + \sum_{\ell=1}^s d_\ell \\ &\quad + \sum_{\ell=1}^s \frac{\tau_1}{\lambda_\ell^p} \eta_\ell^{2+2p} + \sum_{\ell=1}^s \frac{\tau_2 \eta_\ell^2}{2\lambda_\ell} + \frac{\chi_{s+1}^2}{2}. \end{aligned} \quad (45)$$

If $|\chi_s| < \varepsilon$, the tracking objective has been achieved and (46) still holds.

Step m. Consider the following Lyapunov function:

$$V_m = V_{m-1} + \frac{\chi_m^2}{2} + \frac{\tilde{\eta}_m^2}{2\lambda_m}, \quad (46)$$

where λ_m is a positive design constant.

Differentiating V_m along (11), one has

$$\dot{V}_m = \dot{V}_{m-1} + \chi_m \left(g_m(\bar{x}_m) + Q(u) - \dot{v}_{m-1} \right) - \frac{\tilde{\eta}_m \dot{\eta}_m}{\lambda_m} \quad (47)$$

where $\dot{v}_{m-1}$ is defined in (35).

The following inequality can easily be obtained:

$$\begin{aligned} \chi_m Q(u) &= \chi_m \Gamma(u) u + \chi_m K(t) \\ &\leq \chi_m \Gamma(u) u + \frac{1}{2} \chi_m^2 + \frac{1}{2} u_{\min}^2. \end{aligned} \quad (48)$$

Now, define the function $\bar{g}_m = g_s(\bar{x}_m) - \dot{v}_{m-1} + \frac{3\chi_m}{2}, s = 2, \dots, m-1$. Applying the approximation ability of neural networks, for $\forall \varepsilon_m > 0$, we have

$$\bar{g}_m = \zeta_m^T \Phi_m(X_m) + \epsilon_m(X_m), |\epsilon_m(X_m)| \leq \varepsilon_m, \quad (49)$$

where the definitions of ζ_m^T and $\Phi_m(X_m)$ can be found in (15) and (6) with $X_m = (x_1, \dots, x_m, y_r, \dots, y_r^{(m)}, \hat{\eta}_1, \dots, \hat{\eta}_m)^T$.

Similar to the discussion in (38), we have

$$\chi_m \bar{g}_m \leq \frac{\eta_m}{2a_m^2} \chi_m^2 \Phi_m^T \Phi_m + \frac{a_m^2}{2} + \frac{\chi_m^2}{2} + \frac{\varepsilon_m^2}{2}, \quad (50)$$

where $a_m > 0$ is a constant.

Let us set up the following controller:

$$u = \frac{-\chi_m \zeta_m^2}{(1 - \kappa) \sqrt{\chi_m^2 \zeta_m^2 + d_m^2}}, \quad (51)$$

where $d_m > 0$ is a constant and ζ_m is represented as

$$\zeta_m = \Gamma_m + \frac{\tau_1}{2^{1+p}} \chi_m^{1+2p} + \frac{\tau_2}{2^{1-p}} \chi_m^{1-2p}, |\chi_m| \geq \varepsilon, \quad (52)$$

in which $\Gamma_m = \frac{\chi_m}{2a_m^2} \hat{\eta}_m \Phi_m^T(X_m) \Phi_m(X_m)$. Based on Lemma 6 and (51), we have

$$\chi_m \Gamma(u) u \leq \frac{-\chi_m \zeta_m^2}{\sqrt{\chi_m^2 \zeta_m^2 + d_m^2}} \leq d_m - \chi_m \zeta_m. \quad (53)$$

Design the rate of change for $\hat{\eta}_m$ as

$$\begin{aligned} \dot{\hat{\eta}}_m &= \frac{\lambda_m \chi_m^2}{2a_m^2} \Phi_m^T(X_m) \Phi_m(X_m) - \frac{\tau_1}{\lambda_m^p} \hat{\eta}_m^{1+2p} - \tau_2 \hat{\eta}_m, \\ \hat{\eta}_m(t_0) &\geq 0. \end{aligned} \quad (54)$$

Similar to (43) and (44), one has

$$\tilde{\eta}_m \hat{\eta}_m^{1+2p} \leq \frac{1+2p}{2+2p} (\eta_m^{2+2p} - \tilde{\eta}_m^{2+2p}), \quad (55)$$

$$\tilde{\eta}_m \hat{\eta}_m \leq -\frac{\tilde{\eta}_m^2}{2} + \frac{\eta_m^2}{2}. \quad (56)$$

Substituting (46), (48)–(56) into (47) gives

$$\begin{aligned} \dot{V}_m &\leq -\tau_1 \sum_{\ell=1}^m \left(\frac{\chi_\ell^2}{2} \right)^{1+p} - \tau_2 \sum_{\ell=1}^m \left(\frac{\chi_\ell^2}{2} \right)^{1-p} - \sum_{\ell=1}^m \frac{\tau_2 \tilde{\eta}_\ell^2}{2\lambda_\ell} \\ &\quad - 2^p \tau_1 \sum_{\ell=1}^m \left(\frac{\tilde{\eta}_\ell^2}{2\lambda_\ell} \right)^{1+p} + \sum_{\ell=1}^m \frac{a_\ell^2 + \varepsilon_\ell^2}{2} + \sum_{\ell=1}^m d_\ell \\ &\quad + \sum_{\ell=1}^m \frac{\tau_1}{\lambda_\ell^p} \eta_\ell^{2+2p} + \sum_{\ell=1}^m \frac{\tau_2 \eta_\ell^2}{2\lambda_\ell}. \end{aligned} \quad (57)$$

We apply Lemma 5 to the term $\left(\sum_{\ell=1}^m \frac{\tilde{\eta}_\ell^2}{2\lambda_\ell}\right)^{1-p}$ with $w = \sum_{\ell=1}^m \frac{\tilde{\eta}_\ell^2}{2\lambda_\ell}$, $\chi = 1$, $\theta = 1 - p$, and $\iota = \frac{1}{1-p}$, then the following inequality can be obtained:

$$\left(\sum_{\ell=1}^m \frac{\tilde{\eta}_\ell^2}{2\lambda_\ell}\right)^{1-p} \leq \sum_{\ell=1}^m \frac{\tilde{\eta}_\ell^2}{2\lambda_\ell} + p(1-p)^{\frac{1-p}{p}}. \quad (58)$$

We substitute (58) into (58) and apply Lemma 4 to (58) to obtain

$$\begin{aligned} \dot{V}_m &\leq -\frac{\tau_1}{m^p} \left(\sum_{\ell=1}^m \frac{\chi_\ell^2}{2}\right)^{1+p} - \frac{2^p \tau_1}{m^p} \left(\sum_{\ell=1}^m \frac{\tilde{\eta}_\ell^2}{2\lambda_\ell}\right)^{1+p} \\ &\quad - \tau_2 \left(\sum_{\ell=1}^m \frac{\chi_\ell^2}{2}\right)^{1-p} - \tau_2 \left(\sum_{\ell=1}^m \frac{\tilde{\eta}_\ell^2}{2\lambda_\ell}\right)^{1-p} + \aleph \\ &\leq -\frac{\tau_1}{(2m)^p} \left(\sum_{\ell=1}^m \left(\frac{\chi_\ell^2}{2} + \frac{\tilde{\eta}_\ell^2}{2\lambda_\ell}\right)\right)^{1+p} + \aleph \\ &\quad - \tau_2 \left(\sum_{\ell=1}^m \left(\frac{\chi_\ell^2}{2} + \frac{\tilde{\eta}_\ell^2}{2\lambda_\ell}\right)\right)^{1-p}, \end{aligned} \quad (59)$$

where

$$\begin{aligned} \aleph &= \sum_{\ell=1}^m \frac{a_\ell^2 + \varepsilon_\ell^2}{2} + \sum_{\ell=1}^m \frac{\tau_1}{\lambda_\ell^p} \eta_\ell^{2+2p} + \sum_{\ell=1}^m \frac{\tau_2 \eta_\ell^2}{2\lambda_\ell} \\ &\quad + \tau_2 p(1-p)^{\frac{1-p}{p}} + \sum_{\ell=1}^m d_\ell. \end{aligned}$$

Here, (60) can be represented as

$$\dot{V}_m \leq -\frac{\pi}{pT_j} \left(V_m^{1+p} + V_m^{1-p}\right) + \aleph. \quad (60)$$

In light of Lemma 1, we can understand that the system is practically stable within the predefined time T_j and $V_m \leq \frac{p\aleph T_j}{\pi}$. Furthermore, the tracking error satisfies $|\chi_1(t)| \leq \sqrt{\frac{p\aleph T_j}{\pi}}$ for $\forall t \geq T_j$.

Now, the main result can be concluded in the following Theorem:

Theorem 1. For nonlinear system (11), if the controller (51), the intermediate controllers (39), and parameter adaptive law (41) are applied, the closed system is practically predefined-time stable and the tracking error satisfies $|y - y_r| \leq \sqrt{\frac{2p\aleph T_j}{\pi}}$ within the predefined time T_j .

4. Simulation Example

In this part, an electromechanical system [50] is given to illustrate the validity of the presented control scheme (see Figure 1). Its dynamics can be expressed as

$$\begin{aligned} M\ddot{q} + B\dot{q} + N \sin(q) &= I, \\ L\dot{I} &= v_0 - RI - K_B \dot{q}, \end{aligned} \quad (61)$$

where $M = (J/K_\tau) + (mL_0^2/3K_\tau) + (M_0L_0^2/K_\tau) + (2M_0R_0^2/5K_\tau)$, $N = (mL_0G/2K_\tau) + (M_0G/K_\tau)$ and $B = (B_0/K_\tau)$, J denotes a rotor inertia, m represents a link mass, M_0 denotes a load mass, L_0 denotes a link length, R_0 denotes the radius of the load, G represents a gravity coefficient, B_0 denotes a coefficient of viscous friction at the joint, $q(t)$ represents an angular motor position (and hence the position of the load), $I(t)$ denotes a motor armature current, K_τ is an electromechanical conversion coefficient of armature current to torque, L

denotes an armature inductance, R denotes an armature resistance, K_B denotes a back-emf coefficient, and v_0 denotes a voltage input.

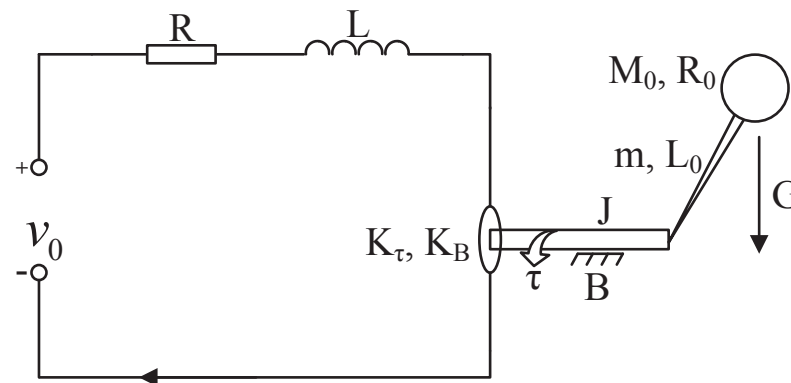


Figure 1. Structure diagram of electromechanical system.

Considering that the quantization is inevitable in remote control, we think about the quantized input in (58), where $Q(u)$ is defined in (12). By performing the coordinate transformation $x_1 = q$, $x_2 = \dot{q}$, $x_3 = \frac{I}{M}$ and $u(t) = \frac{v_0}{ML}$, the electromechanical system can be further expressed as

$$\begin{aligned}\dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_3 - \frac{N}{M} \sin(x_1) - \frac{B}{M} x_2, \\ \dot{x}_3 &= Q(u) - \frac{K_B}{ML} x_2 - \frac{R}{ML} x_3, \\ y &= x_1.\end{aligned}$$

In the control process, each input variable selects six nodes for $\Phi_1^T \xi_1$, $\Phi_2^T \xi_2$, and $\Phi_3^T \xi_3$ in their compact sets. The width of the basis functions is selected as 2. Thus, 6^3 nodes are contained in $\xi_1^T \Phi_1$, with centers distributed evenly in $[-2, 2]^3$; 6^7 nodes are included in $\Phi_2^T \xi_2$, with centers distributed evenly in $[-2, 2]^7$; and 6^{10} nodes are included in $\Phi_3^T \xi_3$, with centers distributed evenly in $[-2, 2]^{10}$.

In the following, we apply Theorem 1 to make y follow $y_r = \sin(t)$ in a given time $T_f = 20$ s. The block diagram of the control system structure is shown in Figure 2.

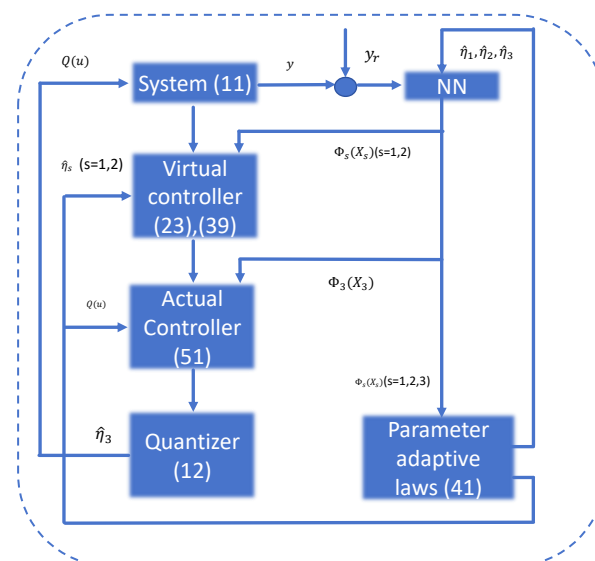


Figure 2. The block diagram of the proposed scheme.

The simulation is conducted using MATLAB, with a sampling time of 0.01 s. The parameters of the electromechanical system are listed in Table 1. We choose the design parameters as $p = 0.07$, $a_1 = 1$, $a_2 = 2$, $a_3 = 2$, $d_1 = 0.001$, $d_2 = 0.01$, $d_3 = 0.01$, $\lambda_1 = 1$, $\lambda_2 = 0.07$, $\lambda_3 = 0.01$, $m = 3$, $\rho = 0.96$, and $u_{min} = 0.99$, and set the initial conditions as $[x_1(0), x_2(0), x_3(0), \hat{\eta}_1(0), \hat{\eta}_2(0), \hat{\eta}_3(0)]^T = [0.1, 0.1, 0.1, 4, 2, 3]^T$. Comparing this with the adaptive predefined-time control scheme in [45], the actuator quantization $Q(u)$ is taken into the control design, which is more applicable to the remote control system. To validate the effectiveness of the proposed scheme, a tracking performance comparison between the paper and [45] is given in Figure 3 under the same initial conditions and parameters, where the output hysteresis is neglected and actuator quantization is considered in [45]. From Figure 3, it is evident that the proposed method in our work achieves higher convergence accuracy under actuator quantization. Figures 4–6 exhibit the curves of state variables x_1 (angular position), x_2 (angular velocity), and x_3 (angular acceleration), respectively. From Figure 7, the adaptive parameters $\hat{\eta}_k$ are bounded. Figure 8 depicts the control signal u and the quantized signal $Q(u)$ (angular jerk). As observed in Figures 3–8, the developed scheme can achieve the control goal. Furthermore, we conducted a statistical analysis of u and $Q(u)$, as summarized in Table 2. The results indicate that the mean value of $Q(u)$ is slightly lower than that of u , and the variance of $Q(u)$ is nearly identical to that of u . The lower mean value of $Q(u)$ compared to u reflects a slight reduction in the control signal's amplitude after quantization.

Table 1. Parameters of the electromechanical system.

Gravity coefficient	$G = 9.8 \text{ N/kg}$
Rotor inertia	$J = 1.625 \times 10^{-3} \text{ kg} \cdot \text{m}^2$
Link mass	$m = 5.06 \times 10^{-1} \text{ kg}$
Viscous friction coefficient at joint	$B_0 = 16.25 \times 10^{-3} \text{ N} \cdot \text{m} \cdot \text{s/rad}$
Armature resistance	$R = 5 \Omega$
Armature inductance	$L = 6 \text{ H}$
Radius of the load	$R_0 = 0.023 \text{ m}$
Load mass	$M_0 = 0.434 \text{ kg}$
Link length	$L_0 = 0.305 \text{ m}$
Back-emf coefficient	$K_B = 0.9 \text{ N} \cdot \text{m/A}$
Coefficient of the armature current to torque conversion	$K_T = 0.9 \text{ N} \cdot \text{m/A}$

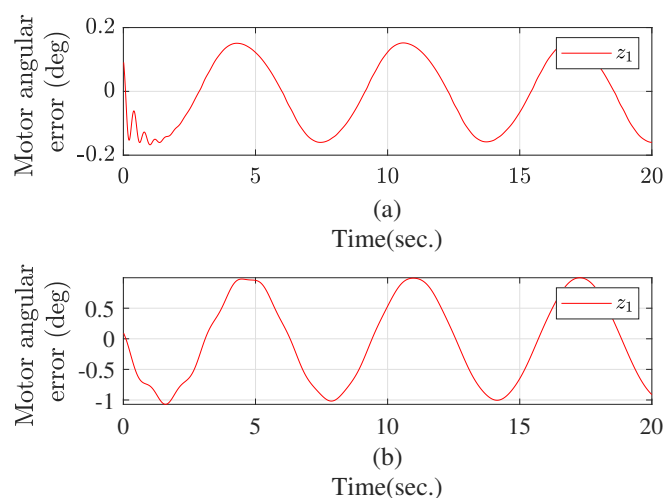


Figure 3. The trajectory of the tracking error under this paper (a) and [45] (b).

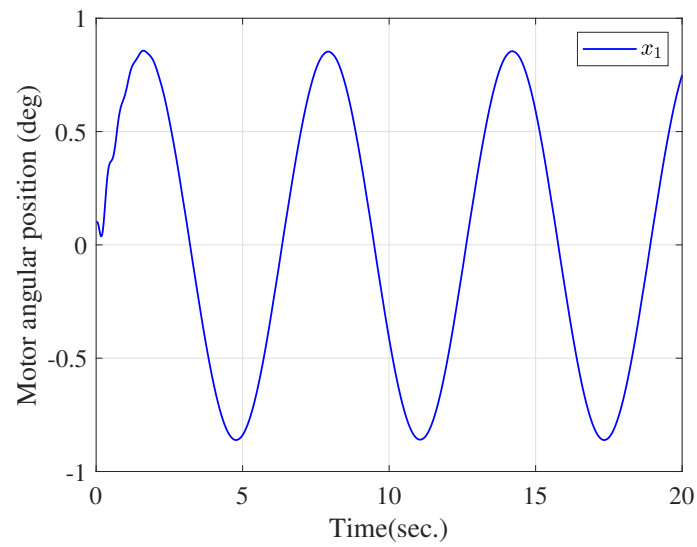


Figure 4. The trajectory of x_1 .

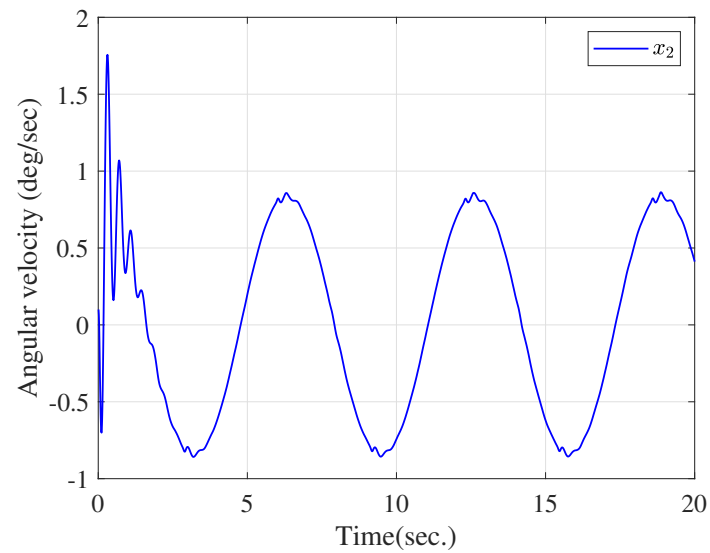


Figure 5. The trajectory of x_2 .

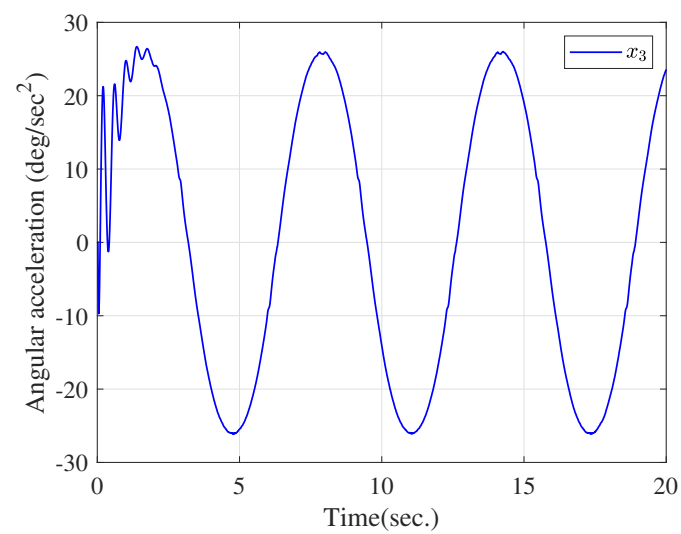


Figure 6. The trajectory of x_3 .

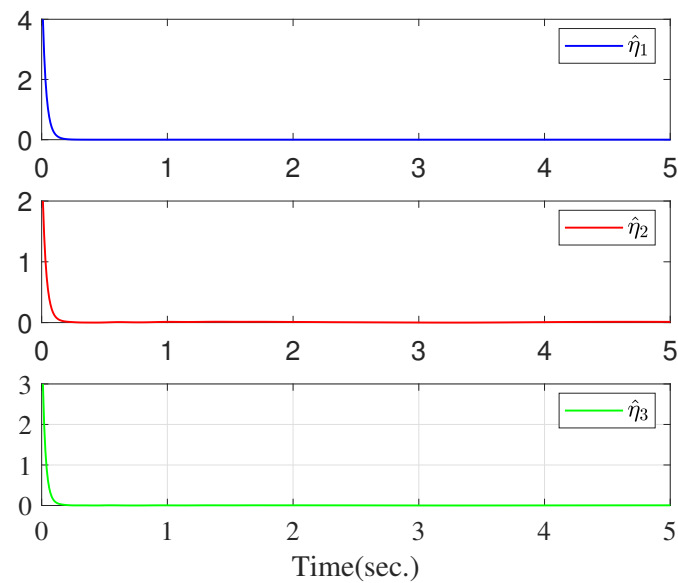


Figure 7. The trajectories of $\hat{\eta}_1$, $\hat{\eta}_2$, and $\hat{\eta}_3$.

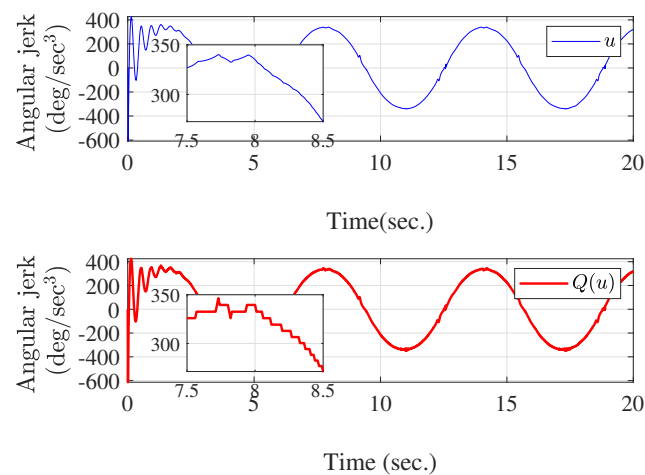


Figure 8. The trajectories of u and $Q(u)$.

Table 2. Performance evaluation of control input u and quantized input $Q(u)$.

Metric	Control Input u	Quantized Input $Q(u)$
Mean value	11.0639	10.9726
Variance	61,306.3621	61,306.3007

5. Conclusions

A predefined-time control strategy is presented for an unknown nonlinear system with actuator quantization. The neural networks are applied to approximate the unknown nonlinearity of the system. To facilitate the stability analysis in the presence of approximation error, the predefined-time stability criterion is first introduced. This criterion gives an effective means of the predefined-time control based on the neural network's framework. A new quantized control design is then presented. This design not only ensures the predefined-time stability, but the singular problem is overcome in the backstepping control design because of repeatedly differentiating for virtual controllers. Compared with the existing quantized control studies, the settling time is not influenced by the initial state and can be set based upon the system requirement. Finally, the feasibility of the main result is tested using an example.

Author Contributions: Conceptualization, W.Z. and B.Y.; methodology, W.Z.; software, W.Z.; validation, W.Z. and B.Y.; formal analysis, W.Z.; investigation, W.Z.; resources, B.Y.; writing—original draft preparation, W.Z.; writing—review and editing, W.Z.; visualization, W.Z.; supervision, B.Y.; project administration, B.Y.; funding acquisition, W.Z. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: No new data were created or analyzed in this study. Data sharing is not applicable to this article.

Conflicts of Interest: The authors declare no conflicts of interest.

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