

Article

# Deviation Sequence Neural Network Control for Path Tracking of Autonomous Vehicles

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**Abstract:** Despite its excellent performance in path tracking control, the model predictive control (MPC) is limited by computational complexity in practical applications. The neural network control (NNC) is another attractive solution by learning the historical driving data to approximate optimal control law, but a concern is that the NNC lacks security guarantees when encountering new scenarios that it has never been trained on. Inspired by the prediction process of MPC, the deviation sequence neural network control (DS-NNC) separates the vehicle dynamic model from the approximation process and rebuilds the input of the neural network (NN). Taking full use of the deviation sequence architecture and the real-time vehicle dynamic model, the DS-NNC is expected to enhance the adaptability and the training efficiency of NN. Finally, the effectiveness of the proposed controller is verified through simulations in Matlab/Simulink. The simulation results indicate that the proposed path tracking NN controller possesses adaptability and learning capabilities, enabling it to generate optimal control variables within a shorter computation time and handle variations in vehicle models and driving scenarios.

**Keywords:** path tracking control; neural network control; autonomous vehicles; model predictive control



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## 1. Introduction

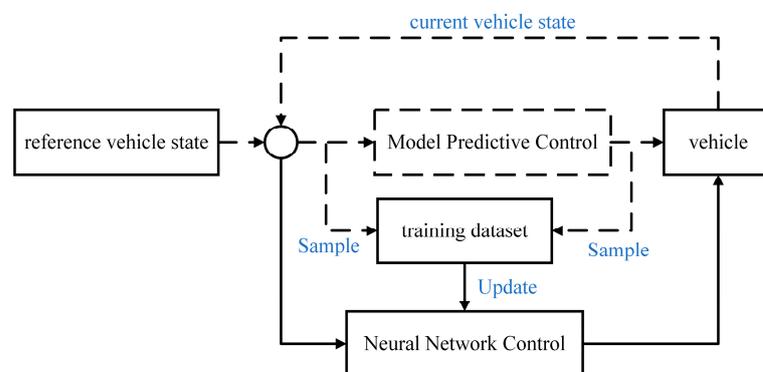
Path planning and tracking control are fundamental and important components of autonomous vehicles (AVs). An investigation in reference [1] highlights that the research topic of path tracking has significantly grown in recent years. For secure and efficient driving, autonomous vehicles need to track precisely the reference trajectory generated by the path planning module. In the past years, to improve the path tracking control performance of AVs, many significant results have been reported with the applications of advanced linear and nonlinear control techniques, such as PID [2,3], LQR [4,5], MPC [6,7] and SMC [8,9]. Model predictive control (MPC) is a highly competitive solution with respect to the other possible control technologies [1]. An advantage is the better tracking performance during high speed and medium-to-high lateral acceleration conditions, compared to the kinematic or geometry-based path tracking methods, such as the pure pursuit [10] and Stanley [11] methods. Furthermore, the capability of managing multi-variable problems and systematically considering constraints on states and control actions make it an ideal choice for multiple-input multiple-output (MIMO) systems, e.g., for AVs with multiple actuators.

Despite its various advantages in path tracking control, MPC is limited by computational cost of the online solution in real-time applications [12,13]. As the system dimension and predictive horizon expand, the resulting exponential growth in computational complexity for solving the optimal control problem (OCP) severely consumes computing resources. The last generations of solvers and control hardware solutions have helped to mitigate the problem. For example, qpOASES speed-up the QP solution based on tracing the solution along a linear homotopy between a QP problem with known solution and the QP problem

to be solved [14]. In addition, the custom solver can also compute control actions faster than the conventional method [15,16].

Considering the requirement of real-time, offline methods is another solution for the path tracking control of AVs. Explicit MPC (EMPC), which determines the control law by designing a piecewise affine function over polyhedral regions, has been implemented in various fields [17–19]. Typically, computing the control variable from the piecewise affine function is faster than solving the QP online. EMPC is verified as the optimal control law for linear time-invariant (LTI) systems [20]. However, the vehicle dynamic model is not time-invariant but varies with changes in longitudinal velocity and sideslip angle. The piecewise affine function cannot cover all conditions and thus, the optimal control solutions may not be obtained with the piecewise affine function designed by the preset condition.

It is noteworthy that practical AVs involve a large amount of repetitive maneuvers in certain circumstances, such as racing or parking [21]. Therefore, it is possible to take the advantages of system repetition and in turn, improve the tracking performance based on the historical driving data [22], which motivates learning-based control techniques. The general framework of neural network control (NNC) is shown in Figure 1. In this framework, the conventional MPC computes the optimal control law and generates the training dataset. Generally, the training dataset contains system state and corresponding optimal control variables in each sample time. Next, NN is used to fit training data generated by sampling the original controller many times and the result is applied “as is” to replace the controller. The NNC trained on the dataset is expected to simulate the optimal solving process. NNC offers unique advantages, including employing simple mathematical expressions and possessing excellent approximation capacity. More importantly, unlike the exponential increase in computational burden experienced by MPC, the computational demand of NNC grows moderately as the system complexity rises. Compared to EMPC, NNC does not use the dynamic model directly. As a result, it alleviates the adverse effect brought by time-variant parameters.



**Figure 1.** General framework of NNC. The black solid arrows represent the NNC program and the black dotted arrows represent the simulation process of the conventional MPC.

One of the earliest applications of artificial neural networks to the vehicle control problem was the autonomous land vehicle in a neural network (ALVINN) system by Pomerleau in 1989 which was first described in [23]. That neural network has a  $30 \times 32$ -neuron input layer, one 4-neuron hidden layer and a 30-neuron output layer. In the ALVINN system, the neural network is provided with image information from a camera together with the steering commands of the human driver and then generates discrete steering action. The neural networks utilized in early works are significantly smaller when compared to what is feasible with today’s technology [24]. End-to-end learning, where observations are mapped directly to low-level vehicle control interface commands, is a popular approach for plants of different complexity. Mariusz et al. [24] trained a convolutional neural network (CNN) to map raw pixels directly from a single front-facing camera to steering commands. The inputs for the end-to-end learning network are not limited to image information. For longitudinal speed control of a vehicle, the error between the speed output command and the actual

speed is a reasonable input [25]. Ghoniem et al. [26] introduced a suspension controller to generate a valve opening signal. Similarly, the input of NNC is the error between road input and suspension displacement. Direct state information is an effective input, but for complex tasks, such as autonomous driving, the diversity of the data set should be ensured if the aim is to train a generalizable model which can drive in all different environments [27]. Furthermore, a concern is that NNC lacks security guarantees when encountering new scenarios that it has never been trained on. The new scenarios not only represent a different steering angle and velocity, but also represent different vehicle parameters such as sprung mass and yaw inertia moment. Feature engineering, where the training data generated by running the original controller are used to craft artificial features that serve as inputs to the NN approximation, is a promising approach to improve generalization. The training still occurs via pure regression and the crafting of features can include the use of inputs that are not ever presented to the original MPC controller.

For large scale MIMO control problems in the building sector, Drgoňa et al. [28] introduced a versatile framework for mimicry of the behavior of optimization-based controllers. The approach employs deep time delay neural networks (TDNN) and regression trees (RT) to derive the dependency of multiple control inputs on parameters. Karg and Lucia [29] consider as input data the parameters of the mixed-integer quadratic programs (MIQPs) and as output data the first element of the optimal solution. In addition to approaches that directly redesign the inputs of NN, creating addition features is also effective. Lovelett et al. [30] used state feedback to project the system's position in state space onto a latent manifold, and then estimated the optimal control policy. By leveraging such a lowdimensional structure of the control policies, simple functions can be found to approximate the control law using fewer training data points.

Recent studies offer various types of approaches to realize feature engineering. However, predictive sequence is still an MPC factor that is not fully considered. In this paper, the deviation sequence neural network control (DS-NNC) is presented for AVs. In order to enhance the adaptability to a time-variant dynamic model and various scenarios, DS-NNC separates the vehicle dynamic model from the approximation process and rebuilds the input of the neural network. Finally, the effectiveness of the proposed controller is verified through simulations in Matlab/Simulink.

## 2. Deviation Sequence Neural Network Control

### 2.1. Reformulation of Approximate Function

Considering a mapping from current vehicle state vector  $x_k$  (longitudinal velocity and yaw rate) and reference vehicle state vector  $r_k$  to control variable  $u_k$ , a general structure of NN controller generates the control law with the form below:

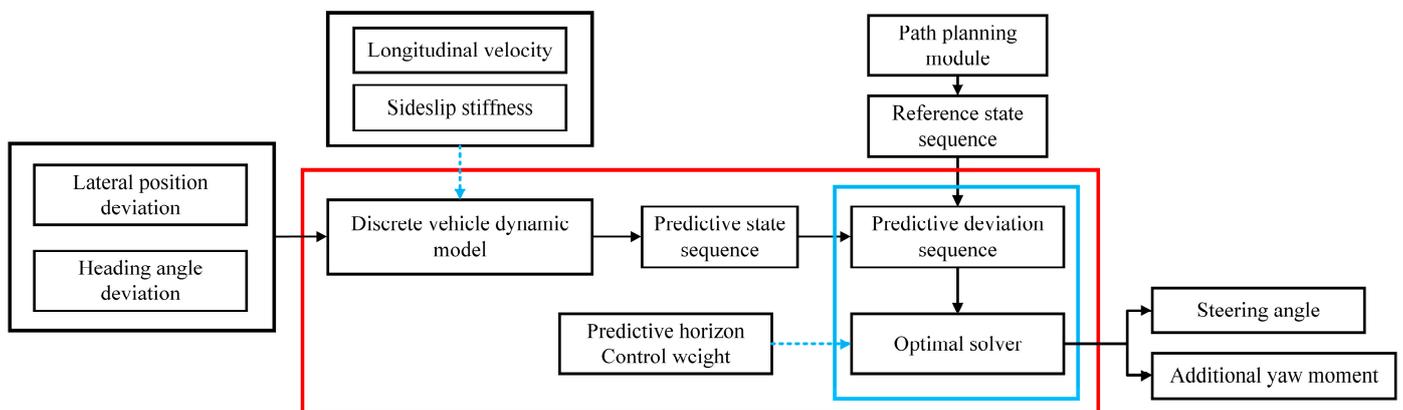
$$u_k = F_1(x_k, r_k). \quad (1)$$

As is shown in Figure 2, this mapping is expected to simulate the optimal solving process where the vehicle controller generates the optimal control variables. Unfortunately, the state variable  $x_k$  is not suitable as an input variable. The reason is that the values of the vehicle state can vary significantly, making it challenging for the function to accommodate all the data generated from the optimal control law in simulations. Another concern is overfitting. Due to the limited availability of abundant training data, the neural network can only perform well in a few specific scenarios. What is worse, this problem becomes more severe when the NN is trained perfectly, but solely for that particular dataset. In order to make it more suitable for a vehicle controller, some improvement is presented as follows.

If one hypothesizes that a multi-layer neural network can approximate  $F_1(x_k, r_k)$ , then it is equivalent to hypothesize that they can asymptotically approximate part of this function, i.e., the following:

$$u_k = F_2(E) = F_2(H(x_k, r_k)) \quad (2)$$

where the variable  $E$  represents the deviation sequence and the second equation is based on the explicit calculation process from the discrete system state-space equation to the deviation sequence. This reformulation is motivated by considering of the known relationship between the current vehicle state vector  $x_k$  and the reference vehicle state vector  $r_k$  to the error sequence vector  $E$ . In other words, this improvement integrates prior knowledge (the established vehicle dynamic model) into the architecture of NN control rather than letting NN approximate the real-time vehicle dynamic model. A similar concept is physics-informed machine learning (PINN) which integrates (noisy) data and mathematical models, and can be trained from additional information obtained by enforcing the physical laws [31,32]. So rather than expect an NN to approximate  $F_1$ , we explicitly let these layers approximate the simplified function  $F_2$ . Although both forms can be asymptotically approximate (as hypothesized), the ease of learning might be different. From another perspective, the relation between control output and error is similar in different scenarios. Remarkably, the premise of this assumption is that the vehicle is stable. Therefore, the deviation form represents more various scenarios compared with the function  $F_1$  of which the input is the current vehicle state and reference vehicle state. It is unlikely that the deviation sequence is optimal, but our reformulation may help to reduce the impact of model change and extend the working range of the NN controller.



**Figure 2.** General control architecture of the conventional MPC. The solid black arrows and the blue dotted arrows represent the controller data and the real-time controller parameters, respectively. The red area represents the optimal mapping approximated by NNC. The blue area represents the optimal mapping approximated by the proposed DS-NNC.

## 2.2. Implementation

The precondition to train the NN controller is the dataset. In this paper, the training data are sampled from a complete model predictive control process. First, the vehicle dynamic model is described, followed by the deviation model and the path tracking control problem formulation. In addition, some details about the improvement of the proposed NN controller are illustrated.

For simplicity, the single-track model is used to describe vehicle dynamics. This form is the most commonly used when designing the vehicle lateral controller because it contains the necessary elements to describe the lateral motion. Figure 3. shows the schematic of this model. In this model, the vehicle dynamic can be modeled in the non-linear form of the equations of motion. Because of the complexity of non-linear equations, the design of an MPC controller is difficult. In addition, the sideslip stiffness of a turning vehicle changes greatly and the tire parameters depend heavily on the road surface and environmental conditions [33]. To estimate the system state accurately, the nonlinear equations derived from basic principles of dynamics are linearized based on two assumptions, i.e., unchanged vehicle velocity and linear tire sideslip characteristics. The linear mathematic expression of the vehicle single-track model is shown as (3):

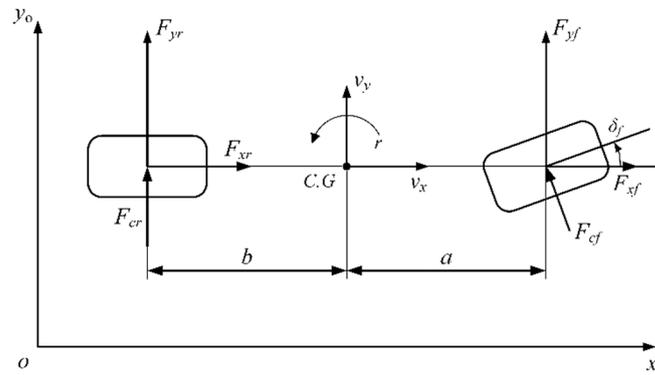
$$\begin{aligned}\dot{v}_y &= \frac{-(c_f + c_r)}{mv_x} v_y + \left[ \frac{(bc_r - ac_f)}{mv_x} - v_x \right] r + \frac{c_f}{m} \delta_f \\ \dot{r} &= \frac{bc_r - ac_f}{I_z v_x} v_y + \frac{-(a^2 c_f + b^2 c_r)}{I_z v_x} r + \frac{ac_f}{I_z} \delta_f + \frac{1}{I_z} \Delta M_z\end{aligned}\quad (3)$$

Here  $c_r$  is the cornering stiffness of the rear axle,  $c_f$  is the cornering stiffness of the front axle,  $v_x$  and  $v_y$  are the vehicle longitudinal and lateral velocity respectively,  $m$  is the vehicle mass,  $r$  is the yaw rate,  $a$  is the distance from the front axis to the center of gravity,  $b$  is the distance from the rear axis to the center of gravity,  $I_z$  is the vehicle yaw inertia. The control variables  $\delta_f$  and  $\Delta M_z$  represent the front wheel steering angle and the additional yaw moment generated by the differential moment of the wheel, respectively.

As mentioned earlier, the deviation form represents various scenarios. In the path tracking scenario, the deviation model can better describe the changes in vehicle state. As shown in the Figure 4, considering a preset reference trajectory, the deviation variables are built as follows:

$$\begin{aligned}\dot{e}_{cg} &= v_y + v_x e_\theta \\ \ddot{e}_{cg} &= \dot{v}_y + v_x \dot{e}_\theta \\ \dot{e}_\theta &= r - r(s) \\ \ddot{e}_\theta &= \dot{r}\end{aligned}\quad (4)$$

where  $e_{cg}$  represents the distance between the vehicle centroid and the closest point in the reference path,  $e_\theta$  represents the error between the heading angle of the vehicle direction and the heading angle of the closest point,  $r(s)$  is the reference yaw rate.

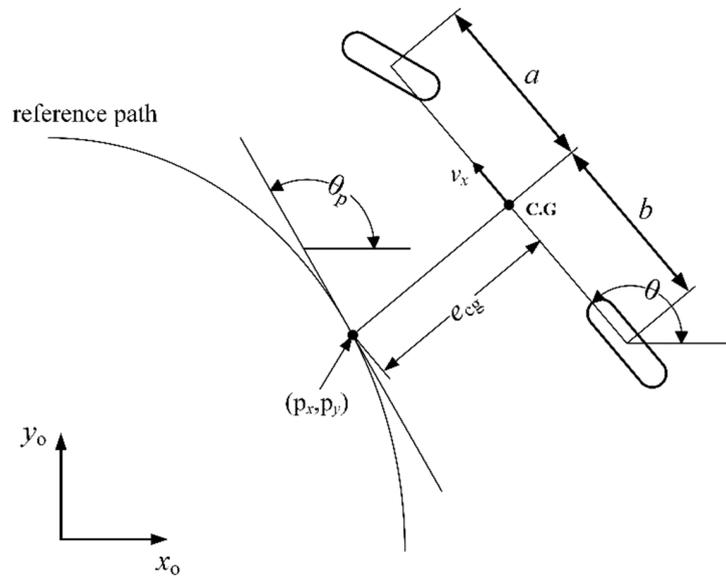


**Figure 3.** Vehicle lateral and yaw dynamic model (single-track model).

By combining Equations (3) and (4), the deviation tracking control model can be described as follows:

$$\begin{aligned}\ddot{e}_{cg} &= \frac{-(c_f + c_r)}{mv_x} \dot{e}_{cg} + \frac{(c_f + c_r)}{m} e_\theta + \frac{(bc_r - ac_f)}{mv_x} \dot{e}_\theta \\ &+ \left[ \frac{(bc_r - ac_f)}{mv_x} - v_x \right] r(s) + \frac{c_f}{m} \delta\end{aligned}\quad (5)$$

$$\begin{aligned}\ddot{e}_\theta &= \frac{bc_r - ac_f}{I_z v_x} \dot{e}_{cg} + \frac{bc_r - ac_f}{I_z} e_\theta \\ &+ \frac{-(a^2 c_f + b^2 c_r)}{I_z v_x} (\dot{e}_\theta + r(s)) + \frac{ac_f}{I_z} \delta\end{aligned}\quad (6)$$



**Figure 4.** The schematic of deviation model based on the single-track model and a reference tracking path.

To facilitate the controller design and analysis, we reformulate Equations (5) and (6) with the state variables into a linear state-space equation:

$$\dot{x} = Ax + Bu + Cr(s) \tag{7}$$

where the state variable vector is modelled as

$$x = [e_{cg} \quad \dot{e}_{cg} \quad e_{\Psi} \quad \dot{e}_{\Psi}]^T \tag{8}$$

and the control variable vector is modelled as follows:

$$u = [\delta_f \quad \Delta M_z]^T \tag{9}$$

The system matrix and input matrix are given by the following:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\frac{c_f + c_r}{mv_x} & \frac{c_f + c_r}{m} & \frac{l_r c_r - l_f c_f}{mv_x} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{l_r c_r - l_f c_f}{I_z v_x} & \frac{l_r c_r - l_f c_f}{I_z} & -\frac{l_f^2 c_f + l_r^2 c_r}{I_z v_x} \end{bmatrix} \tag{10}$$

$$B = \begin{bmatrix} 0 & 0 \\ \frac{c_f}{m} & 0 \\ 0 & 0 \\ \frac{l_f c_f}{I_z} & \frac{1}{I_z} \end{bmatrix} \tag{11}$$

The matrix C comes from the definition of the heading angle error  $e_{\theta}$  as shown in (4) and is given by the following:

$$C = \begin{bmatrix} 0 \\ \frac{l_r c_r - l_f c_f}{m v_x} - v_x \\ 0 \\ \frac{l_f^2 c_f + l_r^2 c_r}{I_z v_x} \end{bmatrix} \quad (12)$$

To predict the future state, the discrete form of the Equation (7) is as follows:

$$x(k+1) = A_k x(k) + B_k u(k) + C_k r(s) \quad (13)$$

where  $A_k$  and  $B_k$  represent the discrete system matrix and control matrix respectively. At each time step  $k$ , the predictive system state sequence vector

$$X_k^p = \begin{bmatrix} x(k+1|k)^T & x(k+2|k)^T & \cdots & x(k+p|k)^T \end{bmatrix} \quad (14)$$

can be derived by the p-step recursive calculation of the discrete system transition Equation (13). The calculation process is shown as follows:

$$\begin{aligned} x(k+1|k) &= A_k x(k) + B_k u(k|k) + C_k r(s) \\ x(k+2|k) &= A_k^2 x(k) + A_k B_k u(k|k) + B_k u(k+1|k) \\ &\quad + A_k C_k r(s) + C_k r(s) \\ &\vdots \\ x(k+p|k) &= A_k^p x(k) + \sum_{i=0}^{p-1} A_k^{p-1-i} B_k u(k+i|k) \\ &\quad + \sum_{i=0}^{p-1} A_k^{p-1-i} C_k r(s) \end{aligned} \quad (15)$$

For simplification, Equation (15) is integrated into the matrix form (16):

$$X_k^p = \Psi x_k + \Theta_1 U_k + \Theta_2 r(s) \quad (16)$$

The matrix  $\Psi$  is given by

$$\Psi = [A_k^1 \quad A_k^2 \quad \cdots \quad A_k^p] \quad (17)$$

The matrix  $\Theta_1$  is given by

$$\Theta_1 = \begin{bmatrix} A_k^{1-1} B_k & \cdots & \mathbf{0} & \mathbf{0} \\ A_k^{2-1} B_k & A_k^{2-2} B_k & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ A_k^{p-1} B_k & A_k^{p-2} B_k & \cdots & A_k^{p-p} B_k \end{bmatrix} \quad (18)$$

The matrix  $\Theta_2$  is given by

$$\Theta_2 = \begin{bmatrix} A_k^{1-1} C_k & \cdots & \mathbf{0} & \mathbf{0} \\ A_k^{2-1} C_k & A_k^{2-2} C_k & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ A_k^{p-1} C_k & A_k^{p-2} C_k & \cdots & A_k^{p-p} C_k \end{bmatrix} \quad (19)$$

Considering a reference vehicle state sequence

$$R_k^p = \begin{bmatrix} r(k+1)^T & r(k+2)^T & \cdots & r(k+p)^T \end{bmatrix} \quad (20)$$

in the next  $p$  time step, the predictive deviation sequence

$$X_k^p - R_k^p = \Psi x_k + \Theta U_k + \Theta_2 r(s) - R_k^p \quad (21)$$

is obtained. Here  $r$  is the vector which consists of the reference sideslip angle and the reference yaw rate. In path tracking, the reference is set to zero to minimize the tracking deviation. Remarkably, the reference sequence vector must be given before calculating the control law and is generally generated by route planning.

In the MPC process, the OCP is solved at each time instant to output the control variable sequence vector  $U_k$ :

$$\min J(U_k) = (X_k^p - R_k^p)^T Q (X_k^p - R_k^p) + U_k^T W U_k \quad (22)$$

subject to the system state transition function (13) and the limits of the actuators

$$\begin{aligned} (\delta_f)_{\min} < \delta_f < (\delta_f)_{\max} \\ (T)_{\min} < T < (T)_{\max} \end{aligned} \quad (23)$$

where  $T$  represents the equivalent torque applied to the tires. In the optimization problem (23),

$$U_k = \begin{bmatrix} u(k|k)^T & u(k+1|k)^T & \cdots & u(k+p-1|k)^T \end{bmatrix} \quad (24)$$

is the control output sequence vector which can be the combination of tracking error, control effort, energy cost, or other factors. The metrics  $Q$  and  $W$  are used to weigh the deviation from the state error and the value of the control vector, respectively, when the importance of state variables is different. Several solution methods exist for the optimization problem (22), including interior point methods, active set methods, gradient projection methods, and dual methods. Each method has its advantages and application range, with the choice of method often depending on the specific problem characteristics, size, and solving requirements. Among these methods, interior point methods (IPMs) provide advantages in constraint handling, global convergence, scalability, flexibility, and parameter tuning in MPC. These advantages make IPMs an effective solution method widely applied in MPC.

In Equation (21), the control variables sequence vector is a variable before the solving process. Because the variable is not allowed in the input of NN, the control variables sequence vector  $U_k$  should be separated from (21), despite it being the basic element of the optimal process. The predictive deviation sequences

$$E = \Psi x_k + \Theta_2 r(s) - R_k^p \quad (25)$$

represents theoretically the predictive state sequence when the vehicle is running without control. This form does not contain any variable and could be calculated by the deviation model (13) and current vehicle state  $x_k$ . Finally, Equation (2) is reformulated as follows:

$$u_k = F_2(E) = F_2(\Psi x_k + \Theta_2 r(s) - R_k^p) \quad (26)$$

By learning the mapping  $F_2$  from the sample dataset, the proposed DS-NNC approximates the optimal control variables, namely, the front wheel steering angle and the additional yaw moment in this work, to track currently the reference path.

In the next section, we show the process of training the neural network and deploy the trained network on the car to verify the effectiveness.

### 3. Results

First, a closed-loop path, which consists of various scenarios, is designed for the verification of the proposed path tracking controller. As shown in Figure 5, the test car starts from the position (0,0) to the positive direction of the X-axis and tracks this path under the control of the conventional MPC controller. At the end of the path, the MATLAB profiler is expected to record the simulation data which contain the vehicle state (longitudinal speed and yaw rate), control variables (front wheel steering angle and additional yaw moment), and the execution time of each module. The vehicle parameters adopted in simulations are presented in Table 1. All the simulations are performed in MATLAB2022a on a 16 GB RAM desktop PC with Intel i5-12490 CPU.

The neural network comprises a three-layer fully connected architecture, each layer featuring 40 neurons. The input layer receives a  $p$ -dimension prediction deviation sequence vector, where  $p$  denotes the predictive horizon length. The output is the control variable vector, encompassing the normalized front wheel steering angle and additional yaw moment, balanced to the same magnitude scale pre-training to mitigate scale discrepancies.

The dataset sampled from simulation process is divided into two parts for the training process of the NN controller. Specifically, we train the same NN controller on the whole dataset, the first half dataset, and the second half dataset, respectively. This design aims at verification of the guarantee on system safety, especially when the proposed NN controller encounters a new scenario that has never been trained on. When it is implemented in the path tracking on the whole path, the NN controller trained on the partial dataset will face scenarios that it has never experienced. The backpropagation (BP) algorithm is one of the most commonly used training methods for neural networks with excellent fitting precision, and was adopted to train the NN in this work.

**Table 1.** Vehicle parameters.

Parameter	Symbol	Value
Mass	$m$	1830 kg
Yaw inertia moment	$I_z$	3234 kg·m <sup>2</sup>
Front wheel base	$a$	1400 mm
Rear wheel base	$b$	1650 mm
Front axle cornering stiffness	$C_f$	−125,374 N/rad
Rear axle cornering stiffness	$C_r$	−125,374 N/rad

The simulation results are presented in Figures 5 and 6 and Tables 2 and 3. We have two major observations from this result. First, the vehicle under the control of the three NN controller tracks the reference path accurately. More importantly, this advantage in tracking capability does not come at the cost of high computational power consumption. The trained NN controller could generate the control variables at a speed of ten times or more than the MPC (Table 3).

As the baseline controller, the mean tracking error of MPC is 0.2236 m in the whole path. Here the tracking error is calculated by the following:

$$e_{cg} = \sqrt{(x - x_{ref})^2 + (y - y_{ref})^2} \quad (27)$$

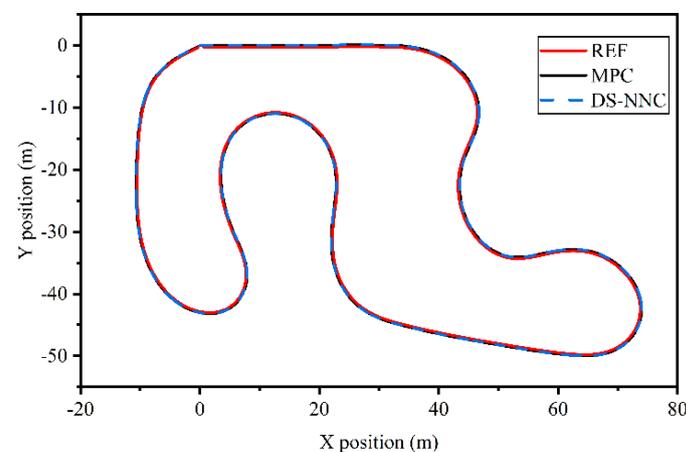
Comparing with the MPC controller, the proposed NN controller reaches a lower level of 0.2234 m. Figures 5 and 6 show an interesting result. Despite the partial train data, the NN controller has a better tracking performance. In the first half of the tracking trajectory, the average tracking error of the NN trained on the second half dataset (NNSH) is 0.05 less than that of the MPC. Similarly, the NN trained on the first half dataset (NNFH) has lower error level in the second half of the tracking trajectory. One underlying reason for this issue is the utilization of root mean square (RMS) as the primary performance metric during the training process. As a result, when encountering input, which it has never seen before, the network is inclined to generate outputs based mainly on its experience

with similar inputs, potentially limiting its effectiveness in dealing with novel situations. The phenomenon raises an issue in that only when the coverage rate of conditions is high enough, can the adaptiveness of NNC be guaranteed. For logical completeness, further discussion is scheduled in the next case.

**Table 2.** Comparison of computation time between MPC and NN controller.

Controller	Module	Computation Time (s)
MPC	Longitudinal controller	11.990
	Vehicle dynamic model	5.740
	Predictive state calculate	0.988
	<b>QP solver</b>	9.502
NNC	Longitudinal controller	12.735
	Vehicle dynamic model	5.548
	Predictive state calculate	0.754
	<b>Network</b>	0.333

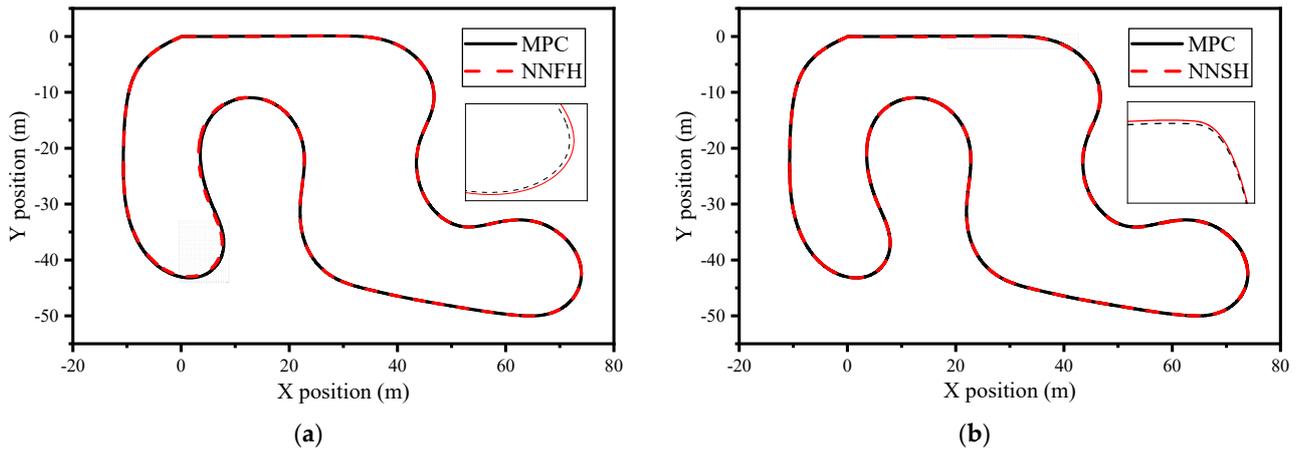
Second, compared to the baseline NN trained on the whole dataset, the NNFH and the NNSH show generalization when encountering a new scenario that has never been trained on. As is shown in Figure 7, Despite a small discrepancy, the NNFH performs better in the first half path than in the second half. Similarly, the NNSH has a greater tracking error when the car is running in the path that has never been trained before. In addition, the control variables and additional yaw moment, could better illustrate this phenomenon. As is shown in Figure 8, the additional yaw moment generated by the NNFH has a slight deviation from the baseline in the second half path tracking process. Although the overall imitation is good, some discrepancy appears in the expected position. Fortunately, this deviation has little impact on the tracking control.



**Figure 5.** NN controller trained on the whole dataset. The initial position is (0,0) and the initial heading angle is 0 rad.

With the purpose of further security verification of the NN controller in the face of unknown scenarios, the path and vehicle model are different in the next two cases. As shown in Figure 9, we design a bigger and more complex closed-loop path and keep the MPC and the NN controller with the same parameters (predictive horizon and control weight) as before. The vehicle longitudinal velocity is set to 18 km/h. The simulation result is shown in Figures 9–11, including the tracking trajectories as well as the tracking error profiles for the lateral deviation and yaw angle error profiles. From the result of the tracking trajectories, an observation is that the MPC controller shows a gradually increasing error when the car runs in the second half of the path, while the NN controller tracks the reference path more currently. Furthermore, the tracking position error and the yaw angle error profiles in Figure 10 show a more detailed trend than in the last case. The tracking

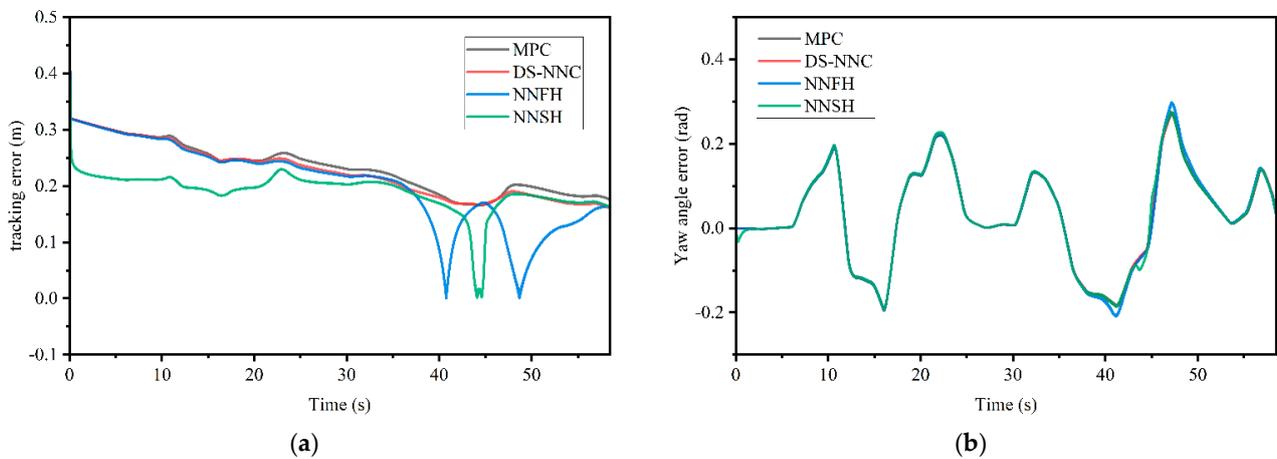
position error of the NN controller is around 1 m while that of MPC gradually rises to 14 m at the end of the path. In the different path, the tracking position error of the MPC is more significant. The increasing error is attributed to the unsuitable parameters. For example, an excessive predictive horizon can lead to increased sensitivity to model errors and less reliable control actions. As is mentioned in Section 2, the DS-NNC removes the control weight from the MPC architecture and reduces the impact of predictive horizon on control variable generation. Consequently, the DS-NNC can be implemented in various scenarios without redesigned parameters.



**Figure 6.** Comparison of tracking trajectory in case 1. The initial position is (0,0) and the initial heading angle is 0 rad. (a) NN controller trained on the first half dataset; (b) NN controller trained on the second half dataset. The boxes are enlarged view and show detail.

**Table 3.** Path tracking error of MPC and NN controller.

Deviation	Module	Mean Tracking Error
Position	MPC	0.2236 m
	NN	0.2234 m
	NNFH	0.2052 m
	NNSH	0.1912 m
Yaw angle	MPC	0.0299 rad
	NN	0.0299 rad
	NNFH	0.0300 rad
	NNSH	0.0299 rad



**Figure 7.** Comparison of vehicle state in case 1. The additional yaw moment generated by controller reflects the approximate performance. (a) Tracking position error; (b) tracking error of yaw angle.

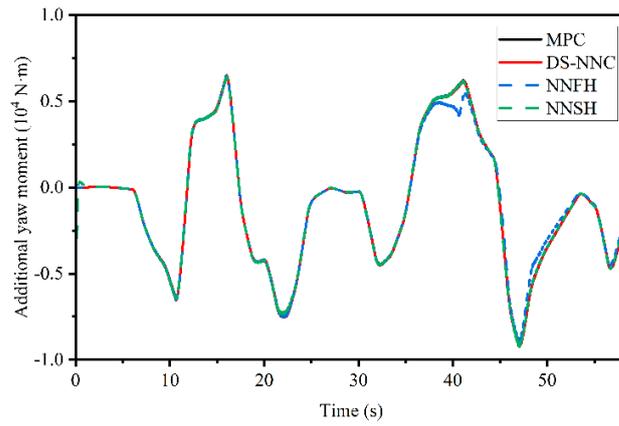


Figure 8. Comparison of additional yaw moment in case 1.

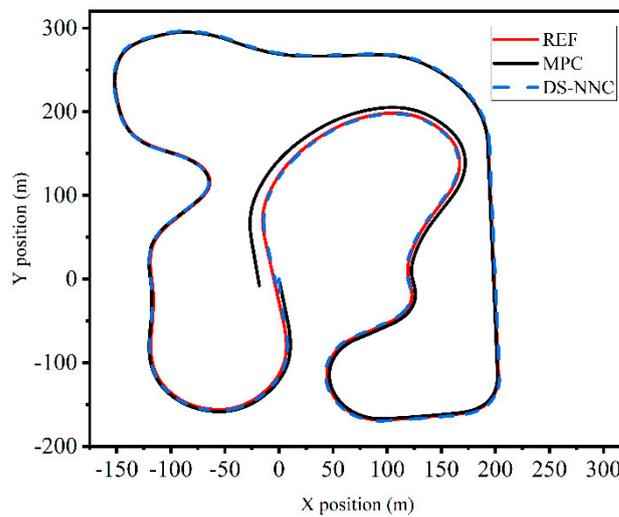


Figure 9. Comparison of tracking trajectory in case 2. The initial position is (0,0) and the initial heading angle is  $-1.4$  rad.

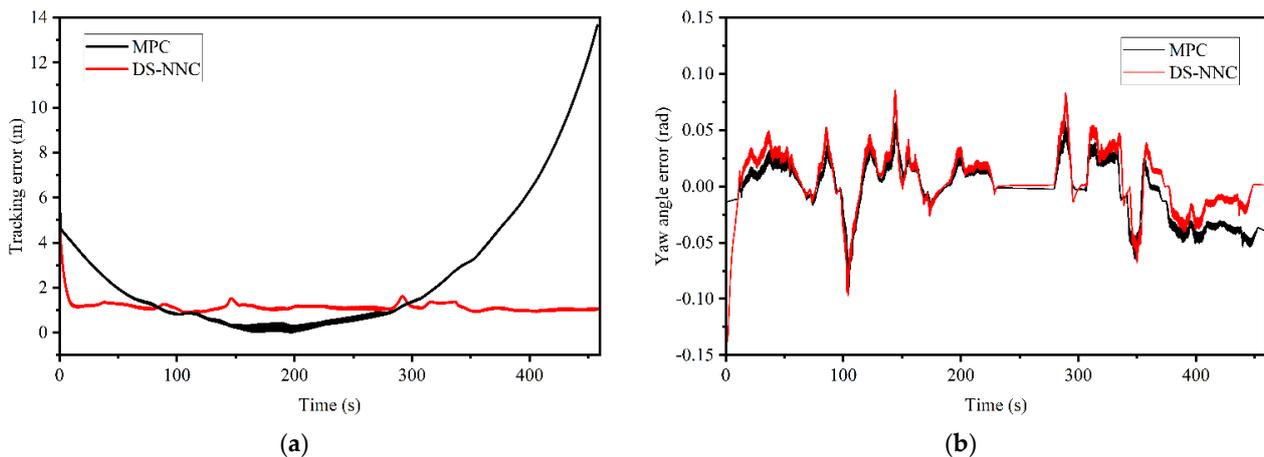


Figure 10. Comparison of vehicle state in case 2. The additional yaw moment generated by controller reflects the approximate performance. (a) Tracking position error; (b) tracking error of yaw angle.

Next, we further explore the adaptiveness of the proposed NN controller for different vehicle models. Because the calculation of the input of the NN controller, i.e., predictive deviation sequence  $E$ , is independent of the network, the changes of the vehicle parameters can be considered as prior knowledge. In this case, the car is significantly lighter and

shorter to ensure differentiation of the system model. The vehicle parameters are shown in Table 4. The vehicle mass is reduced by approximately 40% and the length is reduced by approximately 30%. Remarkably, we also use the same MPC controller and NN controller as before. The simulation result is shown in Figures 12 and 13, including the tracking trajectories as well as the tracking profiles. Similarly, the MPC cannot still track the reference path currently as a result of the control parameters. Although the MPC is redesigned based on the new vehicle dynamic model, the inappropriate control parameters limit its accuracy. As a comparison, the proposed NN controller has a better tracking performance. At the end of the tracking trajectory, the tracking position error keeps below 0.5 m. The three cases show the tracking performance of the proposed DS-NNC. Although NNC can also approximate the original controller, DS-NNC has more generalization. A brief example in Figure 14 indicates that advantage.

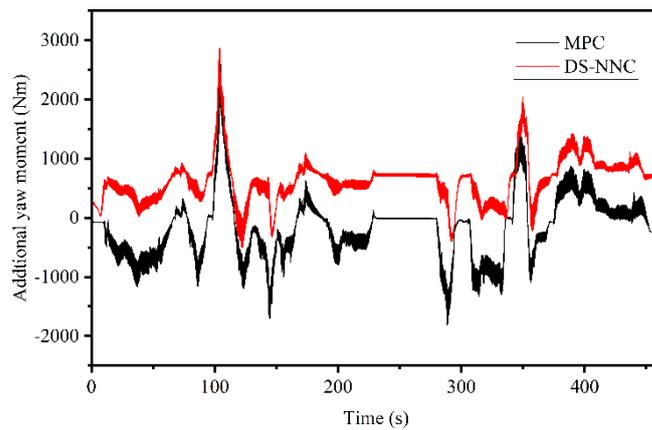


Figure 11. Comparison of additional yaw moment in case 2.

Table 4. Vehicle parameters.

Parameter	Symbol	Value
Mass	$m$	1140 kg
Yaw inertia moment	$I_z$	1020 kg·m <sup>2</sup>
Front wheel base	$a$	1165 mm
Rear wheel base	$b$	1165 mm
Front axle cornering stiffness	$C_f$	−29,517 N/rad
Rear axle cornering stiffness	$C_r$	−29,517 N/rad

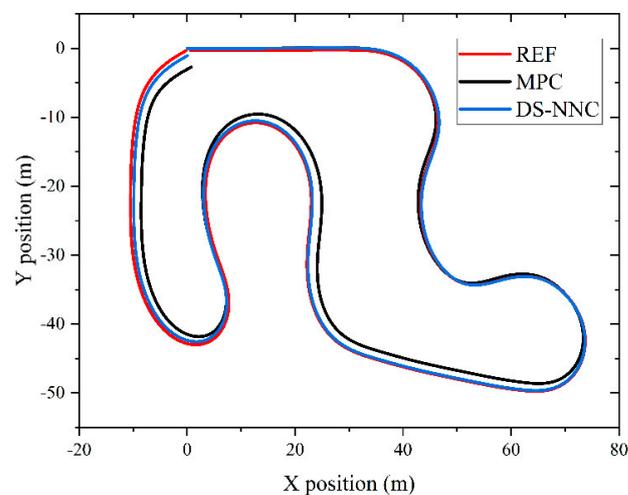
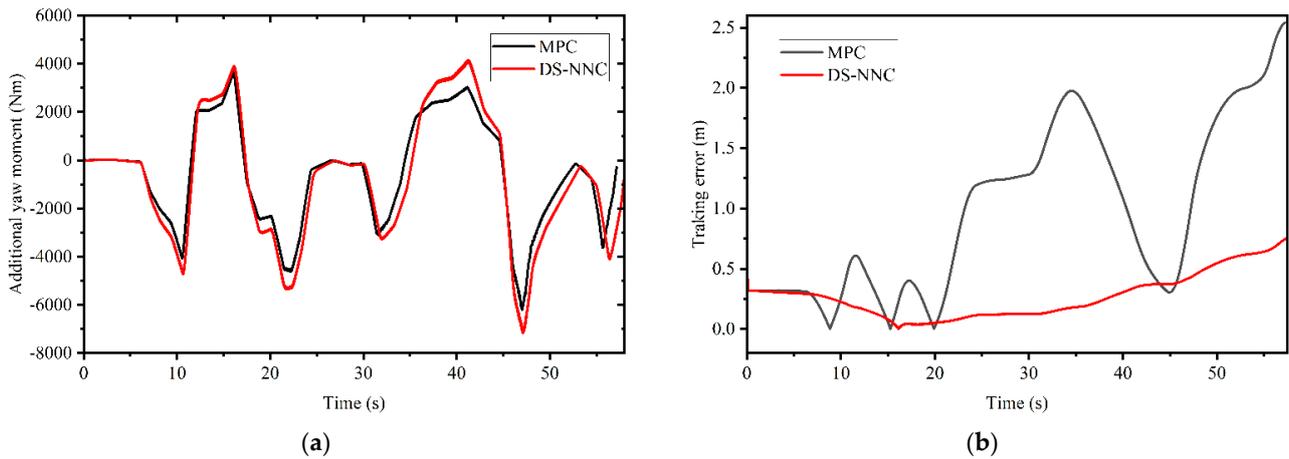
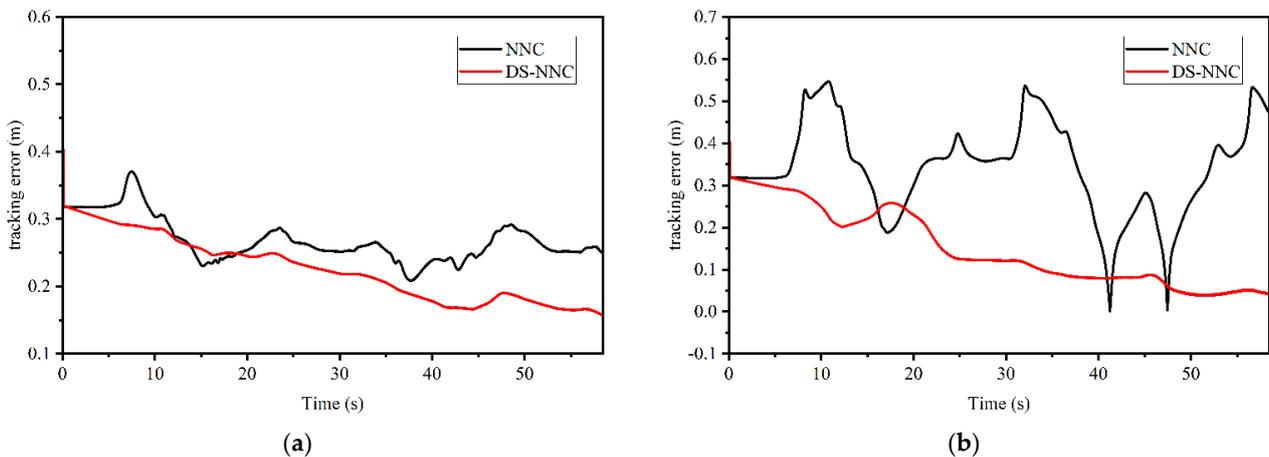


Figure 12. Comparison of tracking trajectory in case 3. The initial position is (0,0) and the initial heading angle is 0 rad.



**Figure 13.** Comparison of tracking performance in case 3. (a) Additional yaw moment; (b) tracking position error.



**Figure 14.** Comparison of tracking performance. Here the NNC is trained by using a general structure, as shown in Equation (1). (a) Vehicle dynamic model in Case1; (b) vehicle dynamic model in case 3.

#### 4. Conclusions

This paper presents a deviation sequence neural network control (DS-NNC) for path tracking of AVs. The algorithm is based on offline sampling and rich data set, and the real-time computation time is reduced by 96%. The two significant parts are feature engineering and the improved structure of NNC, which allow more driving scenarios and provide more generalization. It can be summarized as follows:

- (1) Introducing the deviation sequence into the input structure of neural network control improves the generalization and reduces the model complexity and the training burden. As is shown in the theory analysis, it contains more driving scenarios and better future motion tendency and thus can represent multiple states.
- (2) The proposed structure separates the vehicle dynamic model from the approximation process and adds a computation module for the predictive state, making full use of the real-time vehicle dynamic model. Compared to directly approximating the mapping of states to control inputs, this structure reduces the complexity of the neural network training because it does not need to consider the dynamic model during the approximation process. Additionally, when the dynamic model is changed, an NN trained offline approximates an out-of-date dynamic model and results in an incremental tracking error. This error could be avoided.

In this paper, simulation experiments are conducted in two environments with different complexity levels in Matlab/Simulink. The simulation results indicate that the proposed

path tracking controller possesses adaptability and learning capabilities, enabling it to generate optimal control variables within a shorter computation time and handle variations in vehicle models and driving scenarios.

In summary, the path tracking controller based on the proposed DS-NNC can improve the speed and adaptiveness. However, although most driving scenarios are covered, it is possible for a real-time controller to reach an unavailable state. Ideally, one would want an NNC that is a drop-in replacement for the original controller but runs faster and preserves all of its desirable features. For controllers that can already run in real time, hot starting has been proved to be a general and effective method with strict guarantee, where a conventional solver is still being used at every control iteration [34]. By themselves, the outputs of the network have no guarantees but because all primal variables are predicted, a simple algebraic check can be performed to assess the feasibility and suboptimality of the solution. Projecting onto feasible sets is another method [35]. This coercion preserves the recursive feasibility guarantees of MPC but requires significant overhead to perform the projection, and computation of the maximal control invariant set which is only feasible for some problems. These methods increase the computational burden to some extent, which runs counter to the purpose of neural network control. Additionally, the proposed DS-NNC loses some information on the weight matrix. In our simulations, general control parameters are chosen. This prevents the controller from improving to better performance. A promising approach would be the combination of weight matrix and deviation sequence. That may improve the performance in a large maneuver.

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