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Fixed-Time Congestion Control for a Class of Uncertain Multi-Bottleneck TCP/AWM Networks

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Abstract: As network technology continues to advance, network congestion has become an inevitable aspect of network communication. Considering the external interference, unmodeled uncertainty and the interaction between nodes, a multi-bottleneck TCP/AWM network model is established in this paper. A new fixed-time congestion controller was designed by combining a neural network and the backstepping technique. The neural network approximation property is used to eliminate the interference of unmodeled uncertainty and UDP flow in the system. The controller designed in this paper can ensure the stability of the TCP/AWM closed-loop system in a fixed time. Finally, the simulation results demonstrate the effectiveness of the proposed TCP/AWM controller.

Keywords: TCP/AWM system; congestion control; multiple bottleneck; fixed-time stability

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1. Introduction

With the wide application of Internet technology, the network has become an essential component of daily life and work. However, with the surge in the number of users and the sharp increase in the amount of data transmission, the problem of network congestion is becoming increasingly severe, significantly impacting both user experience and network performance. In order to effectively alleviate this problem, Active Queue Management (AQM), as the main method of TCP network congestion control, has been paid attention by researchers [1,2]. In 1998, Braden et al. proposed the concept of AQM, expecting to ensure high throughput while reducing the queuing delay [1]. AQM notifies the sender by actively predicting and discarding partial packets to reduce the transmission rate in advance, thus effectively avoiding network congestion. The earliest AQM algorithm is Random Early Detection (RED): by proactively managing the queue length within routers, RED effectively mitigates the potential for congestion by dropping packets before they can cause significant delays or loss, thereby enhancing the overall user experience and network performance [3]. Subsequently, to mitigate the RED algorithm's heightened sensitivity to design parameters and enhance its robustness, some researchers have proposed some improved RED algorithms [4–6]. However, most of these results are heuristic algorithms, and design parameters remain among the problems that congestion control algorithms need to address.

Building on TCP/AQM network modeling, Misra et al. introduced a nonlinear dynamics model, rooted in fluid flow modeling principles, which laid the foundation for solving the network congestion problem by control theory [7]. Since then, based on modern control theory, many researchers have designed congestion solutions with good results [8–13]. Ref. [10] proposed a new AQM algorithm grounded in a backstepping technique and minimum method, which shortened the signal convergence time to a certain extent. In [11],

Zheng et al. considered the influence of external disturbance and unknown hysteresis, and designed an adaptive finite-time AQM network congestion controller, ensuring that the queue length efficiently aligns with the required queue within a specified finite time. By combining the prescribed performance technology, Chen et al. designed a multi-bottleneck congestion controller that can improve robustness under different numbers of sessions [12]. On this basis, Li et al. viewed the TCP/AQM network as a whole and were the first to consider the influence of dead zones on the establishment of dynamic models. The congestion control algorithm of multi-bottleneck network can optimize the utilization of link resources to their fullest potential between nodes and alleviate network congestion [13].

However, since the network has a transmission time lag, the congestion information such as the sampled queue length reflects the network condition of a previous time. When the congestion information is fed back to the sender, additional delay time is added, so the response of AQM is relatively lagging behind the actual network condition, and because of this lag, the congestion control effect may not meet expectations. In order to solve this problem, Barbera et al. introduced a novel strategy known as "Active Window Management" (AWM) [14] and analyzed the stability of the AWM algorithm [15]. The algorithm focuses more on the behavior control of the sender, and dynamically adjusts the congestion window size according to the real-time condition of the network, which can more directly cause the sending rate of the sender to respond quickly to network congestion. Based on the performance analysis of AWM, Barbera further studied the impact of parameter design on AWM networks and proposed to improve TCP performance by using network bandwidth to regulate TCP transmission rates [16]. Subsequently, Yuan et al. developed an enhanced AWM algorithm, marking its first application to routers. They controlled the sending rate by directly controlling the size of the sender window, so as to more directly control the queue length in the router [17]. Xie et al. presented an improved network model of network congestion control system, and proposed a new AWM congestion control algorithm with the help of specified performance control and an event-triggered mechanism [18]. This study focuses on single-bottleneck networks, while multi-bottleneck networks are more common in practical applications. Currently, congestion control in AWM networks remains an understudied yet challenging area of research.

Stability time is an important performance measure in control algorithms, and finite time stability with good robustness has received much attention [19–22]. In finite time control, settling time is influenced by the system's initial value, yet in practical applications, these initial conditions are often unknown. For the first time, Polyakov formulated a sufficient criterion for achieving stability within a fixed time frame, effectively addressing the limitation where settlement time was previously influenced by the initial state. This innovation rendered the settlement time independent of initial conditions, solely determined by the designated design parameters [23]. For the parameter uncertain system, Wang et al. proposed a novel adaptive fixed time control approach in which the settlement time is exclusively governed by the control design parameters, eliminating any dependence on other factors [24]. A new adaptive fixed-time stability analysis criterion was developed to avoid the singularity problem in the controller design [25]. Meng et al.'s research delves into systems encompassing nonlinear functions with unspecified boundaries. They have devised a fixed-time controller, ensuring that all system states can be adjusted to zero [26]. Combining fixed time control and specified performance control, Shen et al. designed a AQM controller to ensure that the closed-loop system responds within a fixed time [27].

Based on the above discussion, we aimed to design a new multi-bottleneck AWM network congestion control algorithm in this paper. The key contributions can be encapsulated as follows:

(1) We studied the multi-bottleneck TCP/AWM network, regarded it as a whole, considered the influence of unknown factors such as the mutual influence between the network nodes and the unmodeled uncertainties, and established a multi-bottleneck TCP/AWM model which is closer to the real network.

(2) We applied fixed-time stability to multi-bottleneck TCP/AWM network congestion control for the first time. The stability duration of the system is solely determined by the design parameters, divorcing it from any reliance on the initial value state of the network system, which is more suitable for practical applications. In the design of the congestion controller, a new parameter adaptive rate and inequality is designed, which is crucial for analyzing the fixed-time stability of the closed-loop system.

The subsequent sections of this paper are structured as follows: Section 2 introduces the TCP/AWM network model and preliminaries. The adaptive neural controller design and its stability analysis are show in Section 3. In Section 4, the effectiveness of the algorithm is verified by simulation experiments. Finally, Section 5 summarizes the conclusion.

2. Model and Preliminaries

Consider the following TCP/AWM model proposed in [14]:

$$\begin{cases} \dot{W}(t) = u(t) \\ \dot{q}(t) = \frac{N(t)W(t)}{R(t)} - C(t) \end{cases} \tag{1}$$

where W is the congestion window, q is the length of the queue for the bottleneck node, C is the link bandwidth, N is the number of sessions, $R(t) = \frac{q(t)}{C} + T_p$ is the round-trip time, and T_p is the propagation delay. In order to make the model of this article more generic, Model (1) will be written as the form of the switching system, namely, $N(t) = N_{\sigma(t)}$, where $\sigma(t) : [0, \infty) \to \mathbb{N} = \{1, \ldots, N_{max}\}$ is the switching signal. We regard $W_\varsigma(t) = N_{\sigma(t)}W(t)$ as the size of the total congestion window. Based on this, given the external unknown disturbance, Equation (1) can be rewritten:

$$\begin{cases} \dot{W}_\varsigma(t) = N_{\sigma(t)}u(t) \\ \dot{q}(t) = \frac{N_{\sigma(t)}W_\varsigma(t)}{R(t)} - C(t) + \omega(t) \end{cases} \tag{2}$$

where $\omega(t)$ is uncertain UDP flows.

The intermediate routing nodes in the network are a two-way transmission. Consider the interaction between upstream and downstream the nodes in the transmission process. The representation of the model pertaining to the i-th node can be formulated as follows.

$$\begin{cases} \dot{W}_{\varsigma,i}(t) = N_{\sigma(t),i}u(t) \\ \dot{q}_i(t) = \frac{W_{\varsigma,i}(t)}{R_i(t)} - C_i(t) + \sum_{j=1}^{n} a_{ij}\frac{W_{\varsigma,j}(t)}{N_{j,\sigma(t)}} + \omega_i(t) \end{cases} \tag{3}$$

Let $x_{1,i} = q_i, x_{2,i} = W_{\varsigma,i}$, Equation (3) can rewritten as

$$\begin{cases} \dot{x}_{1,i}(t) = \frac{C_i x_{2,i}}{x_{1,i} + T_{p,i}C_i} - C_i + \sum_{j=1}^{n} a_{ij}\frac{x_{2,j}}{N_{j,\sigma(t)}} + \omega_i(t) + \Delta_i(x_{1,i}, x_{2,i}) \\ \dot{x}_{2,i}(t) = N_{\sigma(t),i}u(t) \\ y_i = x_{1,i} \end{cases} \tag{4}$$

where $\Delta_i(x_{1,i}, x_{2,i})$ is the unmodeled uncertainty. To streamline the design process of the TCP/AWM controller, the subsequent definitions and lemmas are presented herein.

Remark 1. *Due to the wide existence of model inaccuracy and measurement errors, the modeling of TCP/AWM network is often accompanied by unmodeled dynamic uncertainty. In many studies, the influence of unmodeled dynamic uncertainty is ignored, but the unmodeled uncertainty should be substituted back to the closed-loop system for analysis in stability analysis; otherwise, it may lead to the instability of the system. In this paper, the adaptive fuzzy method is used to approximate the uncertain factors in the system, and the control effect is good.*

Definition 1. *For any initial condition $x(0) \in \mathbb{R}^n$, the system described in (4) is considered practically fixed-time stable if it is practically finite-time stable and the convergence time function $T(x_0)$ is bounded, i.e., there is a $T_{max} \in \mathbb{R}_+$ such that $T(x_0) \leq T_{max}$.*

Lemma 1 ([28]). *For the system (4), if there is a Lyapunov function $V(x)$ as follows:*

$$\dot{V}(x) \leq -\nu_1 V^m(x) - \nu_2 V^n(x) + \zeta \tag{5}$$

for $\nu_1, \nu_2 > 0, m > 1, 0 < n < 1, 0 < \psi < 1$, then it is globally fixed-time stable if the following holds:
(1) The system (4) is globally finite-time stable.
(2) There is a global upper bound stability time function T_{max}, and for all $x_0 \in R^n$, the following inequality holds:

$$T(x_0) \leq T_{max} := \frac{1}{\nu_1(m-1)} + \frac{1}{\nu_2(1-n)}. \tag{6}$$

The solution of System (4) eventually converges to the following set of residuals:

$$x \in \left\{ V(x) \leq min\left\{ \left(\frac{\eta}{(1-\psi)\nu_1} \right)^{\frac{1}{m}}, \left(\frac{\eta}{(1-\psi)\nu_2} \right)^{\frac{1}{n}} \right\} \right\} \tag{7}$$

Lemma 2 ([29]). *For any real m and n, there is an inequality as follows:*

$$|p|^{\phi}|q|^{\varphi} \leq \frac{\phi}{\phi+\varphi}\psi|p|^{\phi+\varphi} + \frac{\varphi}{\phi+\varphi}\psi^{-\frac{\phi}{\varphi}}|q|^{\phi+\varphi} \tag{8}$$

where ϕ, φ and ψ are any given positive constants.

Lemma 3 ([24]). *For $\forall \rho \geq x \geq 0$ and $n > 1$, the following inequality holds:*

$$x(\rho - x)^n \leq \frac{n}{n+1}(\rho^{n+1} - x^{n+1}). \tag{9}$$

Lemma 4 ([24]). *Consider a differential equation as follows:*

$$\dot{\vartheta}(t) = \lambda\vartheta(t) - \tau\vartheta(t) - \gamma\vartheta^p(t) \tag{10}$$

where $\vartheta(t) > 0, \tau > 0, \gamma > 0$ and $p = \frac{2q+l}{2q+1}$. Given the initial condition $\vartheta(t_0) \geq 0$, $\vartheta(t) \geq 0$ is satisfied for $\forall t \geq t_0$.

In this paper, RBFNNs are applied to estimate the unknown nonlinear functions. The RBFNNs can be written as

$$F_{nn}(\varsigma) = \Phi^T\xi(\varsigma),$$

where $\varsigma \in \Omega_\varsigma \subset \mathscr{R}_q$ is the input vector, $\Phi = (\Phi_1, \Phi_2, \ldots, \Phi_\aleph)^T \in \mathscr{R}_\aleph$ is the weight vector, $\aleph > 1$ represents the number of RBFNN nodes, and $\xi(\varsigma) = [\xi_1(\varsigma), \xi_2(\varsigma), \ldots, xi_\aleph(\varsigma)]^T \in \mathscr{R}_\aleph$ denotes the basis function vector. Select the Gaussian basis function $\xi_i(\varsigma)$ as follows:

$$\xi_i(\varsigma) = exp\left[-\frac{(\varsigma - \kappa_i)^T(\varsigma - \kappa_i)}{\chi^2} \right], i = 1, 2, \ldots, \aleph \tag{11}$$

where $\kappa_i = [\kappa_{i1}, \ldots, \kappa_{iq}]$ is the center of the receptive field, and χ denotes the width of the Gaussian function.

Lemma 5 ([30]). *Let $F(\varsigma)$ be a continuous function defined on a compact set Ω. Then, for $\forall \varepsilon > 0$, the following holds:*

$$F(\varsigma) = \Phi^{*T}\xi(\varsigma) + \epsilon(\varsigma). \tag{12}$$

where Φ^* is the ideal constant weight vector defined as

$$\Phi = arg\min_{\Phi}[\sup_{\varsigma\in\Omega}\left|F(\bar{x}) - \Phi^T\xi(\varsigma)\right|], \epsilon(\varsigma) \leq \varepsilon. \tag{13}$$

3. Adaptive Controller Design and Stability Analysis

First, based on the backstepping method design, the transfer error of the network nodes is expressed as follows:

$$\begin{cases} z_{1,i} = \sum_{j=1}^{n} a_{ij}(y_i - y_j) + (y_i - y_{ref}) \\ z_{2,i} = x_{2,i} - \alpha_{1,i} \end{cases} \tag{14}$$

where $\alpha_{1,i}$ is the virtual control law, and y_{ref} is the tracking objective function.

According to (14), one has

$$\begin{aligned} \dot{z}_{1,i} = & \sum_{j=1,j\neq i}^{n} a_{ij}(y_i - y_j) + (y_i - y_{ref}) \\ = & \sum_{j=1,j\neq i}^{n} a_{ij}\left[\left(\frac{C_i x_{2,i}}{x_{1,i} + T_p C_i} - C_i + \sum_{j=1}^{n} a_{ij}\frac{x_{2,j}}{N_{j,\sigma}} + \omega_i + \Delta_i\right)\right. \\ & \left. -\left(\frac{C_j x_{2,j}}{x_{1,j} + T_p C_j} - C_j + \sum_{j=1}^{n} a_{jk}\frac{x_{2,k}}{N_{k,\sigma}} + \omega_j + \Delta_j\right)\right] \\ & + \left(\frac{C_i x_{2,i}}{x_{1,i} + T_p C_i} - C_i + \sum_{j=1}^{n} a_{ij}\frac{x_{2,j}}{N_{j,\sigma}} + \omega_i + \Delta_i\right) - \dot{q}_{ref} \end{aligned} \tag{15}$$

Step 1. Consider the following Lyapunov function:

$$V_{1,i} = \frac{1}{2}z_{1,i}^2 + \frac{1}{2}\tilde{\theta}_{1,i}^2 \tag{16}$$

where $\theta_{k,i} = \|\Phi_{k,i}^*\|^2, k = 1, 2, \cdots, n$, where $\hat{\theta}_{k,i}$ represents the estimation of $\theta_{k,i}$, with the estimation error denoted as $\tilde{\theta}_{k,i} = \theta_{k,i} - \hat{\theta}_{k,i}$. Then, the derivative of 1 $V_{1,i}$ is obtained:

$$\begin{aligned} \dot{V}_{1,i} = & z_{1,i}\dot{z}_{1,i} - \tilde{\theta}_{1,i}\dot{\hat{\theta}}_{1,i} \\ = & z_{1,i}\left[\sum_{j=1,j\neq i}^{n} a_{ij}\left(\frac{C_i x_{2,i}}{x_{1,i} + T_p C_i} - C_i + \sum_{j=1}^{n} a_{ij}\frac{x_{2,j}}{N_{j,\sigma}} + \omega_i + \Delta_i\right.\right. \\ & \left. -\frac{C_j x_{2,j}}{x_{1,j} + T_p C_j} + C_j - \sum_{j=1}^{n} a_{jk}\frac{x_{2,k}}{N_{k,\sigma}} - \omega_j - \Delta_j\right) \\ & \left. + \frac{C_i x_{2,i}}{x_{1,i} + T_p C_i} - C_i + \sum_{j=1}^{n} a_{ij}\frac{x_{2,j}}{N_{j,\sigma}} + \omega_i + \Delta_i - \dot{q}_{ref}\right] \\ & - \tilde{\theta}_{1,i}\dot{\hat{\theta}}_{1,i} \end{aligned} \tag{17}$$

Let $\sum_{j=1}^{N} a_{ij} + 1 = \varrho_i$. Then (17) can be expressed as

$$\begin{aligned}\dot{V}_{1,i} =& z_{1,i}\left[\varrho_i\left(\frac{C_i x_{2,i}}{x_{1,i}+T_pC_i} - C_i + \sum_{j=1}^{n} a_{ij}\frac{x_{2,j}}{N_{j,\sigma}} + \omega_i + \Delta_i\right)\right.\\ &\left. - \sum_{j=1,j\neq i}^{n} a_{ij}\left(\frac{C_j x_{2,j}}{x_{1,j}+T_pC_j} - C_j + \sum_{j=1}^{n} a_{jk}\frac{x_{2,k}}{N_{k,\sigma}} + \omega_j + \Delta_j\right) - \dot{q}_{ref}\right]\\ &- \tilde{\theta}_{1,i}\dot{\hat{\theta}}_{1,i}\end{aligned} \tag{18}$$

The RBFNNs effectively approximates the subsequent nonlinear functions, and based on Lemma 5, there is a neural network $\Phi_{1,i}^T\xi_{1,i}(X)$ that meets the requirements for this approximation.

$$\begin{aligned}F_{1,i}(X) =& \sum_{j=1,j\neq i}^{n} a_{jk}\left(\frac{C_j x_{2,j}}{x_{1,j}+T_pC_j} - C_j + \sum_{k=1}^{n} a_{jk}\frac{x_{2,k}}{N_{k,\sigma}} + \omega_j + \Delta_j\right)\\ &+ \varrho_i\omega_i + \varrho_i\Delta_i + d_{2,1}z_{1,i}^{2n-1} + \frac{1}{2}z_{1,i}^2\\ =& \Phi_{1,i}^T\xi_{1,i}(X) + \epsilon_{1,i}(X), \epsilon_{1,i}(X) \leq \varepsilon_{1,i}\end{aligned}$$

Then, Equation (18) is recalculated as

$$\begin{aligned}\dot{V}_{1,i} =& z_{1,i}\left[\varrho_i\left(\frac{C_i x_{2,i}}{x_{1,i}+T_pC_i} - C_i + \sum_{j=1}^{n} a_{ij}\frac{x_{2,j}}{N_{j,\sigma}}\right) + \Phi_{1,i}^T\xi_{1,i}(X)\right.\\ &\left. + \epsilon_{1,i}(X) - \dot{q}_{ref} - d_{2,1}z_{1,i}^{2n-1} - \frac{1}{2}z_{1,i}^2\right] - \tilde{\theta}_{1,i}\dot{\hat{\theta}}_{1,i}\end{aligned} \tag{19}$$

From Young's inequality, the following inequality holds:

$$\begin{aligned}z_{1,i}\left(\Phi_{1,i}^T\xi_{1,i} + \epsilon_{1,i}\right) &= z_{1,i}\Phi_{1,i}^T\xi_{1,i} + z_{1,i}\epsilon_{1,i}\\ &\leq z_{1,i}\frac{\Phi_{1,i}^T}{\|\Phi_{1,i}\|}\|\Phi_{1,i}\|\xi_{1,i} + z_{1,i}\epsilon_{1,i}\\ &\leq \frac{1}{2a_1^2}z_{1,i}^2\theta_{1,i}\xi_{1,i}^T\xi_{1,i} + \frac{1}{2}a_1^2 + \frac{1}{2}z_{1,i}^2 + \frac{1}{2}\varepsilon_{1,i}^2\end{aligned} \tag{20}$$

Due to $z_{2,i} = x_{2,i} - \alpha_{1,i} \Rightarrow x_{2,i} = z_{2,i} + \alpha_{1,i}$, substituting (20) into (19), we can ascertain the calculation of $V_{1,i}$ as

$$\begin{aligned}\dot{V}_{1,i} =& z_{1,i}\varrho_i\left(\frac{C_i(z_{2,i}+\alpha_{1,i})}{x_{1,i}+T_pC_i} - C_i + \sum_{j=1}^{n} a_{ij}\frac{x_{2,j}}{N_{j,\sigma}}\right) + \frac{1}{2a_1^2}z_{1,i}^2\theta_{1,i}\xi_{1,i}^T\xi_{1,i}\\ &+ \frac{1}{2}a_1^2 + \frac{1}{2}\varepsilon_{1,i}^2 - \dot{q}_{ref} - d_{2,1}z_{1,i}^{2n-1} - \tilde{\theta}_{1,i}\dot{\hat{\theta}}_{1,i}\end{aligned} \tag{21}$$

The virtual control law and adaptive law can be chosen as follows:

$$\begin{aligned}\alpha_{1,i} =& -\frac{x_{1,i}+T_pC_i}{\varrho_iC_i}\left(\varrho_i\sum_{j=1}^{n} a_{ij}\frac{x_{2,j}}{N_{j,\sigma}} - \varrho_iC_i - \dot{q}_{ref}\right.\\ &\left. - \frac{1}{2a_1^2}z_{1,i}\hat{\theta}_{1,i}\xi_{1,i}^T\xi_{1,i} + d_{1,1}z_{1,i}^{2m-1}\right)\end{aligned} \tag{22}$$

$$\dot{\hat{\theta}}_{1,i} = \frac{1}{2a_1^2} z_{1,i}^2 \xi_{1,i}^T \xi_{1,i} - \tau_{1,i} \hat{\theta}_{1,i} - \gamma \hat{\theta}_{2,i}^{2n-1} \tag{23}$$

where $\tau_{1,i} > 0$ is a design constant.

Substituting (22) and (23) into (21), the calculation of $V_{1,i}$ can be determined as

$$\begin{aligned} \dot{V}_{1,i} \leq & - d_{1,1} z_{1,i}^{2m} - d_{2,1} z_{1,i}^{2n} + \frac{\varrho_i C_i}{x_{1,i} + T_p C_i} z_{1,i} z_{2,i} \\ & + \frac{1}{2} a_1^2 + \frac{1}{2} \varepsilon_{1,i}^2 + \tau_{1,i} \tilde{\theta}_{1,i} \hat{\theta}_{1,i} + \gamma \tilde{\theta}_{1,i} \hat{\theta}_{2,i}^{2n-1} \end{aligned} \tag{24}$$

Step 2. Consider the following Lyapunov function:

$$V_{2,i} = V_{1,i} + \frac{1}{2} z_{2,i}^2 + \frac{1}{2} \tilde{\theta}_{2,i}^2 \tag{25}$$

From (14), we have

$$\dot{z}_{2,i} = \dot{x}_{2,i} - \dot{\alpha}_{1,i} = N_{i,\sigma} u(t) - \dot{\alpha}_{1,i} \tag{26}$$

The derivative of $V_{2,i}$ is formulated as follows:

$$\begin{aligned} \dot{V}_{2,i} = & \dot{V}_{1,i} + z_{2,i} \dot{z}_{2,i} - \tilde{\theta}_{2,i} \dot{\hat{\theta}}_{2,i} \\ = & \dot{V}_{1,i} + z_{2,i} N_{i,\sigma} u(t) - z_{2,i} \dot{\alpha}_{1,i} - \tilde{\theta}_{2,i} \dot{\hat{\theta}}_{2,i} \end{aligned} \tag{27}$$

As in Step 1, the RBFNN is used to approximate the following unknown function in the controller:

$$\begin{aligned} F_{2,i}(X) = & \Phi_{2,i}^T \xi_{2,i}(X) + \epsilon_{2,i}(X) \\ = & \frac{\varrho_i C_i}{x_{1,i} + T_p C_i} z_{1,i} - \dot{\alpha}_{1,i} + \frac{1}{2} z_{2,i} \end{aligned}$$

Then (27) can be rewritten:

$$\begin{aligned} \dot{V}_{2,i} = & \dot{V}_{1,i} + z_{2,i} \Bigg(N_{i,\sigma} u(t) + \Phi_{2,i}^T \xi_{2,i}(X) + \epsilon_{2,i}(X) - \frac{1}{2} z_{2,i} \\ & - \frac{\varrho_i C_i}{x_{1,i} + T_p C_i} z_{1,i} \Bigg) - \tilde{\theta}_{2,i} \dot{\hat{\theta}}_{2,i} \end{aligned} \tag{28}$$

From Young's inequality, the following inequality holds:

$$\begin{aligned} z_{2,i} \left(\Phi_{2,i}^T \xi_{2,i} + \epsilon_{2,i} \right) &= z_{2,i} \Phi_{2,i}^T \xi_{2,i} + z_{2,i} \epsilon_{2,i} \\ &\leq z_{2,i} \frac{\Phi_{2,i}^T}{\| \Phi_{2,i} \|} \| \Phi_{1,i} \| \xi_{2,i} + z_{2,i} \epsilon_{2,i} \\ &\leq \frac{1}{2a_2^2} z_{2,i}^2 \theta_{2,i} \xi_{2,i}^T \xi_{2,i} + \frac{1}{2} a_2^2 + \frac{1}{2} z_{2,i}^2 + \frac{1}{2} \varepsilon_{2,i}^2 \end{aligned} \tag{29}$$

Substituting (29) into (28),

$$\begin{aligned}\dot{V}_{2,i} \leq & \dot{V}_{1,i} + z_{2,i}N_{i,\sigma}u(t) - \frac{1}{2a_2^2}z_{2,i}^2\theta_{2,i}\xi_{2,i}^T\xi_{2,i} + \frac{1}{2}a_2^2 - \tilde{\theta}_{2,i}\dot{\hat{\theta}}_{2,i} \\ & - \frac{\varrho_i C_i}{x_{1,i} + T_p C_i}z_{1,i} - \tilde{\theta}_{2,i}\dot{\hat{\theta}}_{2,i} \\ \leq & -d_{1,1}z_{1,i}^{2m} - d_{2,1}z_{1,i}^{2n} + \frac{1}{2}a_1^2 + \frac{1}{2}\varepsilon_{1,i}^2 + \tau_{1,i}\tilde{\theta}_{1,i}\hat{\theta}_{1,i} + \gamma\tilde{\theta}_{1,i}\hat{\theta}_{2,i}^{2n-1} \\ & + z_{2,i}N_{i,\sigma}u(t) + \frac{1}{2a_2^2}z_{2,i}^2\theta_{2,i}\xi_{2,i}^T\xi_{2,i} + \frac{1}{2}a_2^2 + \frac{1}{2}\varepsilon_{2,i}^2 - \tilde{\theta}_{2,i}\dot{\hat{\theta}}_{2,i}\end{aligned} \tag{30}$$

Define the following TCP/AWM congestion control law and the adaptive law as follows:

$$u_i = -N_{i,\sigma(t)}^{-1}\left(d_{1,2}z_{2,i}^{2m-1} + d_{2,2}z_{2,i}^{2n-1} + \tfrac{1}{2a_2^2}z_{2,i}\hat{\theta}_{2,i}\xi_{2,i}^T\xi\right) \tag{31}$$

$$\dot{\hat{\theta}}_{2,i} = \frac{1}{2a_2^2}z_{2,i}^2\xi_{2,i}^T\xi_{2,i} - \tau_{2,i}\hat{\theta}_{2,i} - \gamma\hat{\theta}_{2,i}^{2n-1} \tag{32}$$

where $\tau_{2,i} > 0, \gamma > 0$ are design constants, and $\hat{\theta}(0) \geq 0$.

Substituting (31) and (32) into (30),

$$\begin{aligned}\dot{V}_{2,i} \leq & -d_{1,1}z_{1,i}^{2m} - d_{2,1}z_{1,i}^{2n} + \frac{1}{2}a_1^2 + \frac{1}{2}\varepsilon_{1,i}^2 + \tau_{1,i}\tilde{\theta}_{1,i}\hat{\theta}_{1,i} + \gamma\tilde{\theta}_{1,i}\hat{\theta}_{2,i}^{2n-1} \\ & - d_{1,2}z_{2,i}^{2m} - d_{2,2}z_{2,i}^{2n} + \frac{1}{2}a_2^2 + \frac{1}{2}\varepsilon_{2,i}^2 + \tau_{2,i}\tilde{\theta}_{2,i}\hat{\theta}_{2,i} + \gamma\tilde{\theta}_{2,i}\hat{\theta}_{2,i}^{2n-1}\end{aligned} \tag{33}$$

Noting $V_i = V_{2,i}$, we can derive

$$\begin{aligned}\dot{V}_i \leq & -\sum_{k=1}^{2} d_{1,k}z_{k,i}^{2m} - \sum_{k=1}^{2} d_{2,k}z_{k,i}^{2n} + \sum_{k=1}^{2}\frac{1}{2}a_k^2 + \sum_{k=1}^{2}\frac{1}{2}\varepsilon_{k,i}^2 \\ & + \sum_{k=1}^{2}\tau_{k,i}\tilde{\theta}_{k,i}\hat{\theta}_{k,i} + \sum_{k=1}^{2}\gamma\tilde{\theta}_{k,i}\hat{\theta}_{k,i}^{2n-1}\end{aligned} \tag{34}$$

To ensure fixed-time stability of the system, we will apply Lemma 3 to address the relevant terms $\tilde{\theta}_{k,i}\hat{\theta}_{k,i}^{2n-1}$. When solving terms $\tilde{\theta}_{k,i}\hat{\theta}_{k,i}^{2n-1}$, Lemma 3 states that, under (31), for $\forall t \geq t_0$, $\hat{\theta}_{k,i}(0) \geq 0$ must be satisfied. Furthermore, by Lemma 3, the following is obtained:

$$\tilde{\theta}_{k,i}\hat{\theta}_{k,i}^{2n-1} = \tilde{\theta}_{k,i}(\theta_{k,i} - \tilde{\theta}_{k,i})^{2n-1} \leq \frac{2n-1}{2n}(\theta_{k,i}^{2n} - \tilde{\theta}_{k,i}^{2n}). \tag{35}$$

By applying Young's inequality, we have

$$\tilde{\theta}_{k,i}\hat{\theta}_{k,i} = -\tilde{\theta}_{k,i}^2 + \tilde{\theta}_{k,i}\theta_{k,i} \leq -\frac{1}{2}\tilde{\theta}_{k,i}^2 + \frac{1}{2}\theta_{k,i}^2. \tag{36}$$

According to Lemma 2, let $p = \sum_{k=1}^{2}\frac{\tilde{\theta}_i^2}{2}, q = 1, \phi = m, \varphi = 1-m$ and $\psi = \frac{1}{m}$, the following inequality can be obtained:

$$\left(\sum_{k=1}^{2}\frac{\tilde{\theta}_{k,i}^2}{2}\right)^m \leq \sum_{k=1}^{2}\frac{\tilde{\theta}_{k,i}^2}{2} + (1-m)m^{\frac{m}{1-m}}. \tag{37}$$

Substituting (35)–(37) into (34), we have

$$\begin{aligned}\dot{V}_i \leq & -\sum_{k=1}^{2} d_{1,k} z_{k,i}^{2m} - \sum_{k=1}^{2} d_{2,k} z_{k,i}^{2n} + \sum_{k=1}^{2} \frac{1}{2} a_k^2 + \sum_{k=1}^{2} \frac{1}{2} \varepsilon_{k,i}^2 - \tau_{k,i} \left(\sum_{k=1}^{2} \frac{\tilde{\theta}_{k,i}^2}{2} \right)^m \\ & - 2^{2n} \gamma \frac{2n-1}{2n} \left(\sum_{k=1}^{2} \frac{\tilde{\theta}_{k,i}^2}{2} \right)^n + \tau_{k,i} \sum_{k=1}^{2} \frac{\theta_{k,i}^2}{2} + \frac{2n-1}{2n} \gamma \sum_{k=1}^{2} \theta_{k,i}^{2n} \\ & + \tau_{k,i} (1-m) m^{\frac{m}{1-m}} \\ \leq & -\sum_{k=1}^{2} d_{1,k} z_{k,i}^{2m} - \sum_{k=1}^{2} d_{2,k} z_{k,i}^{2n} - \tau_{k,i} \left(\sum_{k=1}^{2} \frac{\theta_{k,i}^2}{2} \right)^m - 2\gamma \left(\sum_{k=1}^{2} \frac{\tilde{\theta}_{k,i}^2}{2} \right)^n + \eta \end{aligned} \tag{38}$$

where $\eta = \sum_{k=1}^{2} \frac{1}{2} a_k^2 + \sum_{k=1}^{2} \frac{1}{2} \varepsilon_{k,i}^2 + \tau_{k,i} \sum_{k=1}^{2} \frac{\theta_i^2}{2} + \frac{2n-1}{2n} \gamma \sum_{k=1}^{2} \theta_{k,i}^{2n} + \tau_{k,i} (1-m) m^{\frac{m}{1-m}}$.

The block diagram shows the design procedure of the controller in Figure 1. Next, we give the main results and proofs in Theorem 1.

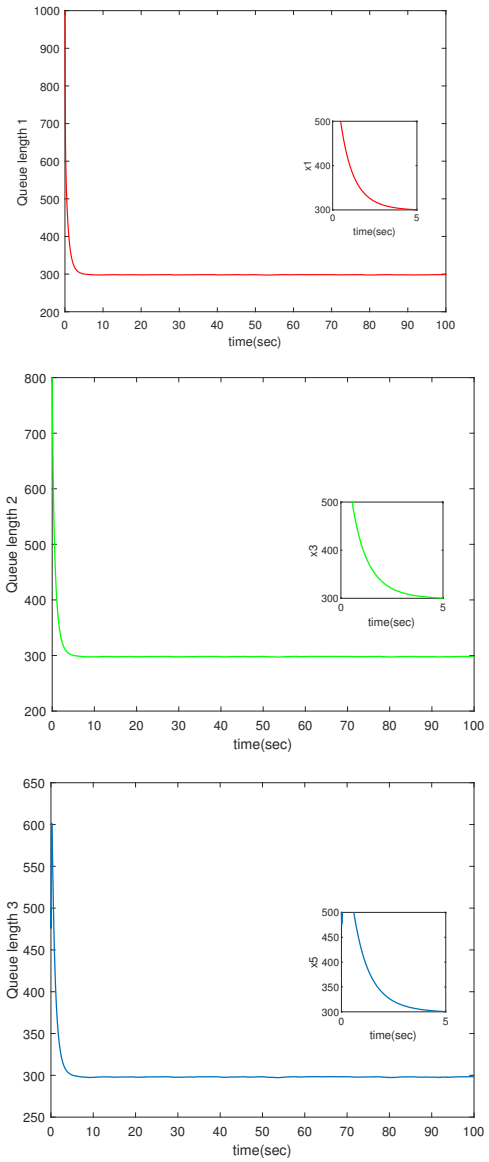
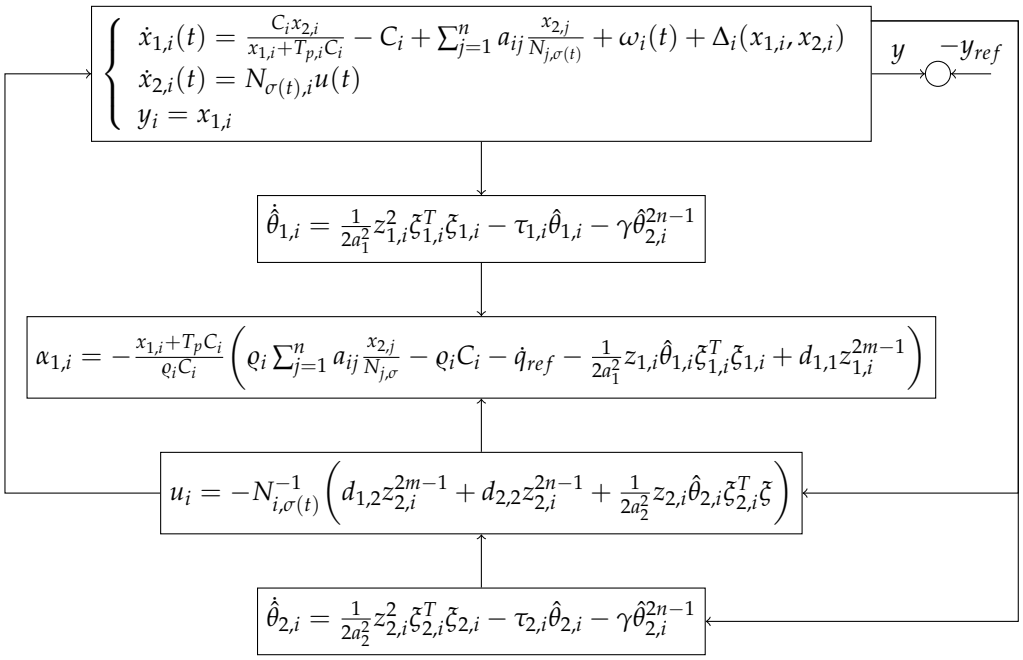


Figure 1. The queue length of three nodes.

Theorem 1. *For the TCP/AWM system* (4)*, considering the virtual control law* (22)*, the actual control input* (31)*, and the adaptive laws* (23) *and* (32)*, all signals in the TCP/AWM closed-loop system remain bounded, and the tracking error can achieve stability within a fixed time.*



Proof. Obviously, from $\dot{V}_i$, the following is obtained:

$$\begin{aligned}\dot{V}_i \leq & -\sum_{k=1}^{2} d_{1,k}z_{k,i}^{2m} - \sum_{k=1}^{2} d_{2,k}z_{k,i}^{2n} - \tau_{k,i}\left(\sum_{k=1}^{2}\frac{\theta_{k,i}^2}{2}\right)^m - 2\gamma\left(\sum_{k=1}^{2}\frac{\tilde{\theta}_{k,i}^2}{2}\right)^n + \eta \\ \leq & -\sum_{k=1}^{2} 2^m d_{1,k}\left(\frac{z_{k,i}^2}{2}\right)^m - \tau_{k,i}\left(\sum_{k=1}^{2}\frac{\theta_{k,i}^2}{2}\right)^m - \sum_{k=1}^{2} 2^n d_{2,k}\left(\frac{z_{k,i}^2}{2}\right)^n \\ & - 2\gamma\left(\sum_{k=1}^{2}\frac{\tilde{\theta}_{k,i}^2}{2}\right)^n + \eta \\ \leq & -\alpha V^m - \beta V^n + \eta\end{aligned} \tag{39}$$

where $\alpha = min\{2^m d_{1,k}, \tau_{k,i}, k = 1, 2\}, \beta = min\{2^n d_{2,k}, 2\gamma, k = 1, 2\}$. □

It is evident from Equation (39) that V is bounded. Thus, the proof of Theorem 1 is completed.

4. Simulation Result

Based on the controller design, this section examines a TCP network with three bottlenecks and evaluates the effectiveness of the congestion control algorithm using Matlab R2021b.

The initial system parameters and the controller design parameters can be selected as follows:

$$\begin{aligned}&\tau_{1,1} = 1, \tau_{1,2} = 1, \tau_{1,3} = 1, \tau_{2,1} = 1, \tau_{2,1} = 1, \tau_{2,1} = 1, \\ &\varrho_1 = 10, \varrho_2 = 20, \varrho_3 = 30, a_1 = 0.5, a_2 = 0.5, a_3 = 0.5, \\ &d_{1,1} = 10, d_{1,2} = 20, d_{2,1} = 20, d_{2,2} = 30, T_p = 0.1, q_{ref} = 300, \\ &N_1 = 1000, N2 = 800, N3 = 500, C_1 = 3500, C_2 = 3500, C_3 = 3500, \\ &m = 1.07, n = 0.5, \omega = cos(t)\end{aligned}$$

The detailed simulation results are presented in Figures 1–5. Figure 1 displays the queue length of the three bottleneck nodes, which tends to be stable despite the uncertainties and external disturbances. Figure 2 shows the window size at the three bottleneck nodes. Obviously, the congestion controller stabilizes the window at a certain value, thus

maintaining the transmission rate at the TCP source at a stable value and reducing the generation of network congestion.

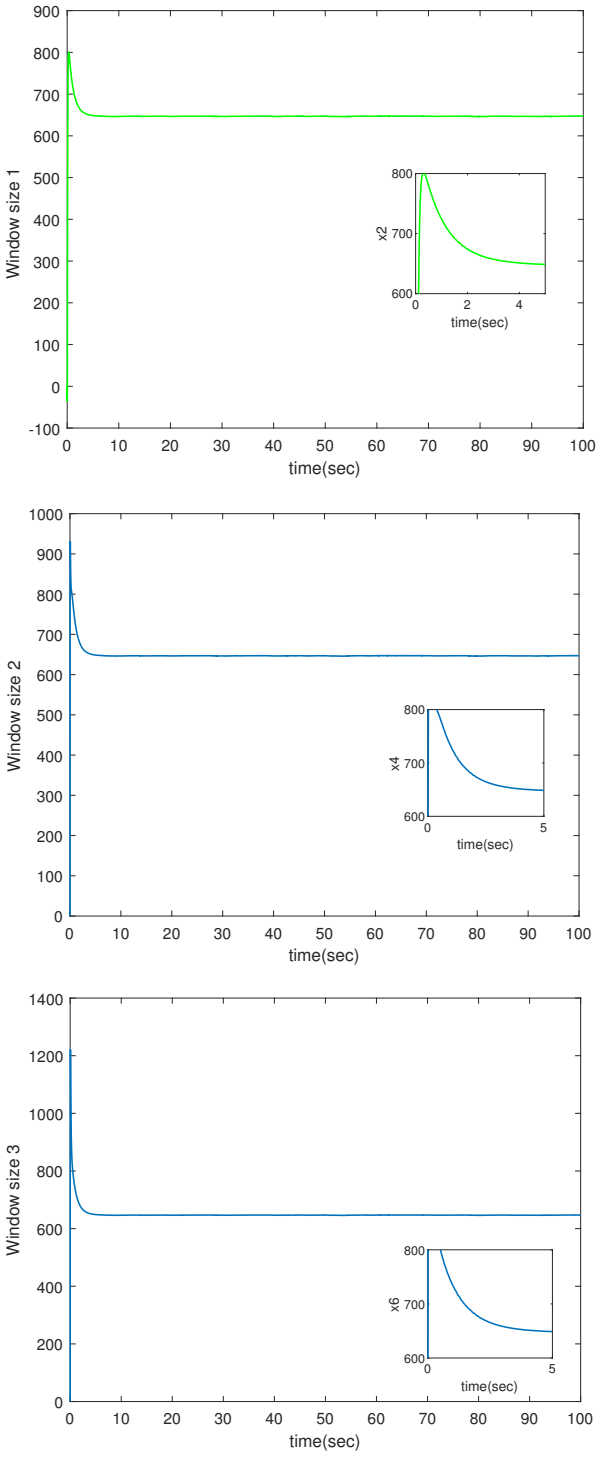


Figure 2. The window size of three nodes.

The control input signal is shown in Figure 3. As evident from the figure, u asymptotically converges to zero at 0.37 s, 0.28 s, and 0.43 s, respectively, signifying the commendable convergence performance of the congestion controller devised in this paper.

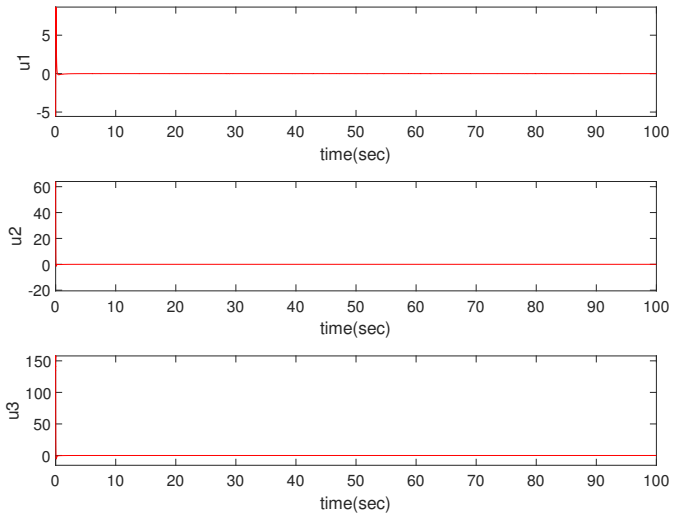


Figure 3. Control input.

Figure 4 vividly illustrates the role of the adaptive law, showing that the significant approximation characteristics of RBFNNs play a good compensation effect, effectively reducing the influence of unknown uncertainties in the system.

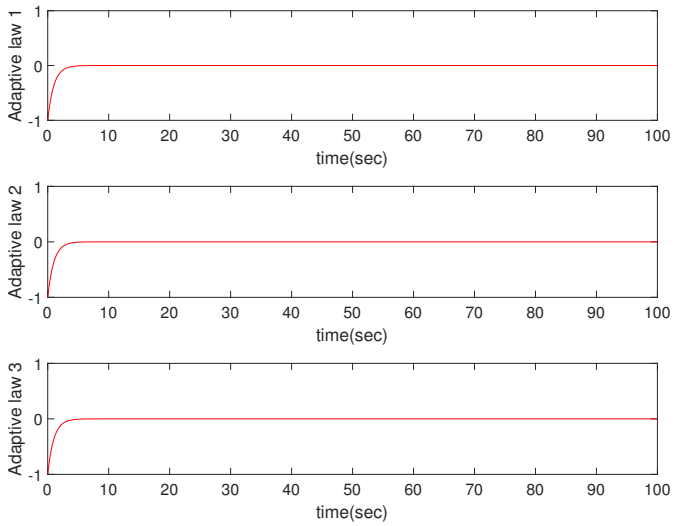


Figure 4. Adaptive law.

Figure 5 shows the queue tracking error of three nodes and the comparison with the EO-PID [31] control algorithm and the RED [32] algorithm. The equilibrium optimization algorithm (EO) algorithm is an optimization algorithm inspired by the physics of controlled volume mass balance proposed by Faramarzi et al. in 2020 [33]. The optimization characteristic of the intelligent algorithm can more effectively realize the control scheme. Although the compared algorithms can remain within the prescribed range, the congestion control algorithm proposed in this paper shows improved steady-state performance and robustness.

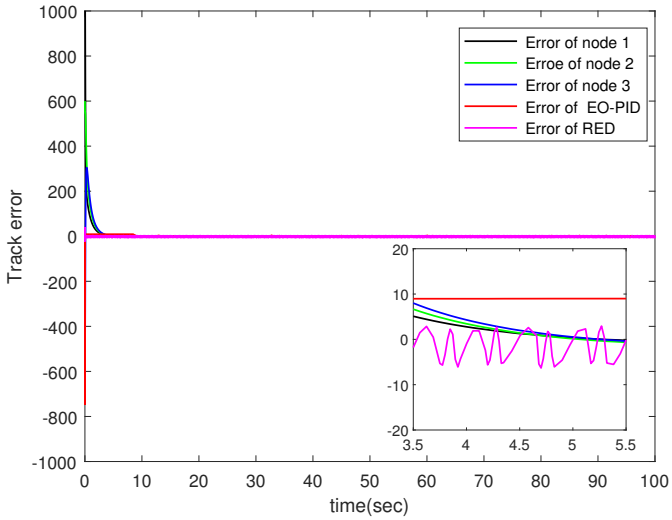


Figure 5. Comparison of the tracking error.

5. Conclusions

A fixed time congestion control of the TCP/AWM network system was deeply studied. A new adaptive fixed-time TCP/AWM congestion controller was constructed for the TCP/AWM network model with UDP flow and unmodeled uncertainty under multiple bottlenecks, taking into account the interaction of intermediate nodes within the multi-bottleneck network. Neural network approximation was utilized to handle unmodeled uncertainties and external disturbances in the system. The designed TCP/AWM congestion controller ensures tracking performance and closed-loop system stability within a fixed time frame. Comparative analysis and simulation results demonstrate the feasibility of the proposed controller.

The model used in the design of the congestion controller described in this paper is a continuous network nonlinear model, and the network system is discretized in actual operation, and the designed controller should also be discretized before it can be configured and implemented in the router. The discretization of the continuous controller will inevitably have a certain impact on the effectiveness of the controller. Therefore, in follow-up work, it is necessary to develop the network congestion controller according to the discrete network model. In multi-bottleneck networks, different bottleneck nodes are affected by different degrees of impact. Different operating conditions, mixed wired transmission nodes and wireless transmission nodes, and other problems encountered in congestion controller design need more in-depth discussion and further research.

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