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Fault-Tolerant Control of Linear Systems with Unmatched Uncertainties Based on Integral Sliding Mode Technique

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Abstract: This paper proposes a novel fault-tolerant control method based on the integral sliding mode technique for unmatched uncertain linear systems with external perturbations. Differently from the existing works, the uncertainties under consideration have an unmatched norm-bounded form in the system and input matrix. Based on linear matrix inequalities, the existence conditions of the sliding mode surface are presented. The unknown fault information is then estimated by some adaptive laws. On the grounds of that, an integral sliding mode controller is also obtained to guarantee the disturbance attenuation and fault tolerance for linear uncertain systems with unmatched uncertainties and actuator faults from the initial time. Finally, the comparative simulation results verify the effectiveness of our presented scheme.

Keywords: fault-tolerant control; unmatched uncertainties; integral sliding mode control; linear systems

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1. Introduction

A ubiquitous and inevitable factor for practical engineering systems is uncertainty, which is caused by modeling uncertainty [1,2], exogenous disturbance [3], and communication noise [4]. Due to various attractive advantages, such as fast responses, easy implementation, and absolute insensitivity to matched uncertainties and disturbances [5,6], sliding mode control (SMC) is one promising robust control method for handling the matched uncertainty [7,8] for linear systems. When being faced with an unmatched one, traditional SMC may be invalid. In order to get around it, a few authors have combined SMC and other robust techniques [9,10]. Though lots of attractive results have been achieved to address matched and unmatched uncertainty, the above-mentioned robustness to uncertainties is just acquired during the sliding motion. In the presence of a reaching phase, the robustness may be vanishing. An idea of adding an integral term in the sliding manifold was proposed in [11,12]. By doing this, the system trajectories can exist in the manifold from the very beginning. From then on, many authors paid attention to the integral sliding mode control (ISMC) technique to get around uncertainty problems, such as [13,14].

On the other research front, actuator faults may bring about performance degradation, and even instability. Additionally, some fault-tolerant control (FTC) techniques for uncertainty linear systems and corresponding results have been well developed, ranging from passive control [15,16] to active control [17,18]. It should be mentioned that SMC is considered a promising FTC tool to deal with actuator faults these days [19–21]. Additionally, reference [19] dealt with the problems of actuator fault compensation for uncertain linear plants based in sliding mode. Reference [20] considered the design of sliding mode control for uncertain state-delayed systems with partial actuator degradation. Additionally, reference [21] proposed a novel variable structure control law to address a pre-specified subset of actuator failure. To obtain robustness from the very beginning, ISMC has been applied to get around actuator faults problems. However, such an FTC design technique is no longer available for uncertainty linear systems when unmatched uncertainty resides in the state matrix and input matrix simultaneously. The literature [22] dealt with mismatched disturbances for discrete-time systems, and reference [23] purposed SMC for nonlinear systems with mismatched uncertainty, but these failed to consider fault tolerance. We therefore considered that it is timely to address the FTC control issue for uncertain systems with unmatched uncertainties using ISMC controller design techniques.

Under the previous considerations, we came up with an integral sliding mode control method to address the problems of uncertain systems with actuator faults, unmatched uncertainties and external disturbances. Firstly, in a state of an actuator redundancy, a novel integral sliding mode surface is derived by applying the matrix full-rank factorization approach. The gain of the ISM control law can be changed online through several adaptive laws which can compensate for actuator faults, perturbations and unmatched uncertainties in the input matrix. In this paper, the chief technical contributions are as follows:

1. This study is the first attempt to apply the FTC technique based on ISMC to deal with unmatched uncertainties in the input matrix. Existing related works in [14,24] both only consider unmatched uncertainties in the state matrix.
2. Compared with the existing literature addressing liner systems with unmatched uncertainty [22,23,25], actuator faults are considered into the sliding mode stability. In particular, both fault information and unmatched uncertainty are taken into account simultaneously.
3. Compared with [26], we purposed a novel integral sliding surface based on a matrix full-rank factorization approach to guarantee a sliding mode existing throughout the whole system response, and a sufficient condition, including actuator faults information and unmatched uncertainty, is derived through linear matrix inequalities (LMIs).

The structure of this paper is as follows. The description of an unmatched uncertain system and some preliminary results are introduced in Section 2. In Section 3, the brand new integral sliding surface of linear systems with unmatched uncertainties is designed first. Then, the SMC law that compensates for actuator faults, perturbations and unmatched uncertainties is designed. Section 4 demonstrates the effectiveness of our designed method through the a set of comparison simulations. At last, Section 5 ends our paper by summarizing our work.

2. System Description and Problem Statement

We will consider unmatched uncertain system models of the form

$$\dot{x}(t) = [A + \Delta A(t)]x(t) + [B + \Delta B(t)][u(t) + f(t)]. \tag{1}$$

where the system and input matrices $A \in \Re^{n\times n}$; $B \in \Re^{n\times m}$; and $x(t) \in \Re^{n}$, $u(x) \in \Re^{m}$ and $f(x) \in \Re^{m}$ are the state vector, the control input and the disturbance input, respectively. $\Delta A(t)$ and $\Delta B(t)$ are unmatched uncertainties in the system matrix and the input matrix.

In the next section, the faults covering actuator interrupt, stuck and failure are presented. The unified fault model can be built up as follows:

$$u^F(t) = \rho u(t) + \sigma u_s(t). \tag{2}$$

where the matrix ρ is diagonal and semipositive-definite. Additionally, ρ models the effectiveness level of the actuators. ρ can be obtained from the sets

$$\Delta_{\rho^j} = \left\{\rho^j | \rho^j = diag\left\{\rho_1^j, \rho_2^j, \cdots, \rho_m^j\right\}, \rho_i^j \in [\underline{\rho}_i^j, \overline{\rho}_i^j]\right\},$$

and the elements $0 \leqslant \underline{\rho}_i^j \leqslant \rho_i^j \leqslant \overline{\rho}_i^j \leqslant 1$ for $i = 1, \cdots, m, j = 1, \cdots, L$. L indicates the total number of fault categories. i and j represent the ith actuator and the jth fault case, respectively. The diagonal matrix σ is defined as

$$\sigma_i^j = \begin{cases} 0 & 0 < \rho_i^j \leqslant 1 \\ 0 \ \ or \ \ 1 & \rho_i^j = 0. \end{cases} \tag{3}$$

Obviously, in the condition of $\underline{\rho}_i^j = \overline{\rho}_i^j = 1$, the actuator in this case is normal. However, $\underline{\rho}_i^j = \overline{\rho}_i^j = 0$ and $\sigma_i^j = 0$ imply the actuator is interrupted in this failure condition. In the case of $\underline{\rho}_i^j = \overline{\rho}_i^j = 0$ and $\sigma_i^j = 1$, the actuator gets stuck in the condition. If $0 < \underline{\rho}_i^j \leqslant \overline{\rho}_i^j < 1$, it implies that the actuator gets partial failure.

Combining actuator faults (2) with (1), one can rewrite the system as

$$\dot{x}(t) = [A + \Delta A(t)]x(t) + [B + \Delta B(t)][\rho u(t) + \sigma u_s(t) + f(t)]. \tag{4}$$

In order to achieve the purpose of fault tolerance and ensure the stabilization, under state information availability, a few assumptions have to be made as follows:

Assumption 1. *For given known constants ρ_A, ρ_B and a known scalar function ρ_f, the following constraints hold: $\|\Delta A(t)\| \leqslant \rho_A$, $\|\Delta B(t)\| \leqslant \rho_B$, $\|f(t)\| \leqslant \rho_f(x,t)$.*

Assumption 2. *For any $\rho \in \Delta_{\rho^j}$, $\{A, (B + \Delta B)\rho\}$ is always absolutely stabilizable.*

Assumption 3. *$rank((B + \Delta B)\rho) = rank(B + \Delta B)$ for all $\rho \in \Delta_{\rho^j}, j = 1, 2, \cdots, L$.*

Assumption 4. *The actuator stuck fault is unknown but norm-bounded, and then there is an unknown positive constant $\bar{u}_s$ meeting $\|u_s(t)\| \leqslant \bar{u}_s$.*

Remark 1. *Assumption 1 highlights that the uncertainties $\Delta A(t)$ and $\Delta B(t)$, are unmatched in the input and state matrix. In addition, $\Delta A(t)$ and $\Delta B(t)$ are norm-bounded. For the purpose of fault tolerance, Assumption 2 guarantees the realization of fault tolerance [26]. Just like [26], Assumption 3 reveals an actuator redundancy assumption to completely compensate for stuck faults of an actuator. Assumption 4 is greatly common and natural in this kind of robust FTC literature [14].*

Several preliminary results are introduced to accomplish our main works later on.

Lemma 1 ([25])**.** *Given appropriately dimensional matrices X and Y, suppose that $(I + XY)$ is nonsingular; then, $(I + XY)^{-1} = I - X(I + YX)^{-1}Y$.*

Lemma 2 ([25])**.** *Given appropriately dimensional vectors x and y, for any $W > 0$, the following inequalities can hold:*

$$2\sqrt{\|x\|}\sqrt{\|y\|} \leq \|x\| + \|y\|,\ 2x^T y \leq x^T W x + y^T W^{-1} y. \tag{5}$$

Lemma 3 ([25])**.** *For any matrices $\mathcal{A}(t) \in \Re^{n\times n}$, $\mathcal{B}(t) \in \Re^{n\times m}$, $\mathcal{C}(t) \in \Re^{p\times n}$ and $\mathcal{D}(t) \in \Re^{p\times m}$, suppose that $I > \eta^2 \mathcal{D}^T(t)\mathcal{D}(t)$ holds for $\eta \geq 0$, and give this uncertain form of system*

$$\dot{x} = \left[\mathcal{A}(t) + \mathcal{B}(t)[I - \mathcal{E}(t)\mathcal{D}(t)]^{-1}\mathcal{E}(t)\mathcal{C}(t)\right]x, \tag{6}$$

where the unknown matrix $\mathcal{E}(t)$ has a boundary as $\|\mathcal{E}(t)\| \leq \eta$. Assuming that there is a positive-definite matrix P satisfying

$$\begin{bmatrix} P\mathcal{A}(t) + * & \eta P\mathcal{B}(t) & * \\ * & -I & * \\ \mathcal{C}(t) & \eta\mathcal{D}(t) & -I \end{bmatrix} < 0. \tag{7}$$

Then, the above system is stable.

Lemma 4 ([27]). *After full-rank factorization of matrix* (10)*, for any ρ, there is a positive constant μ satisfying*

$$N\rho N^T \geqslant \mu N N^T. \tag{8}$$

Lemma 5 ([28]). *Consider the system:*

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} A_1(t) & A_2(t) \\ 0 & -\varrho I \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}, \tag{9}$$

where $x_1 \in \Re^{n-m}$; $x_2 \in \Re^m$; ϱ is a positive constant; and $A_1(t)$ and $A_2(t)$ have a bounded norm. Then, the previous uncertain system is quadratically stable when the reduced-order system $\dot{x}_1(t) = A_1(t)x(t)$ is stable.

3. Main Results

The diagram of our proposed FTC scheme based on the ISM technique is depicted in Figure 1.

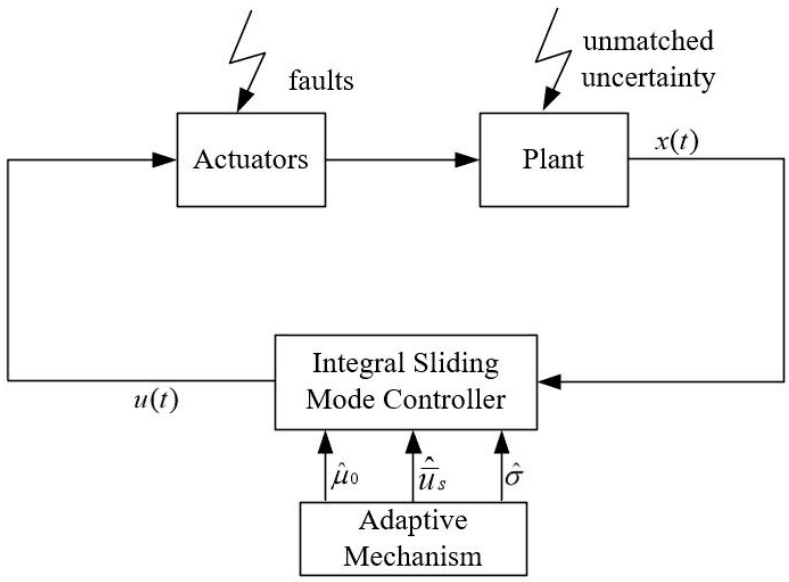

Figure 1. Structure diagram of ISMC fault-tolerant scheme.

3.1. Design of Integral Sliding Mode Surface

Through full-rank factorization, the input matrix B is expressed as:

$$B = B_v N, \tag{10}$$

where $B_v \in \Re^{n\times l}$, $N \in \Re^{l\times m}$ and rank(B_v) = rank(N) = $l \le m$.

Based on (10), the following equation can be obtained:

$$\Delta B(t) = \Delta B_v(t) \cdot N. \tag{11}$$

Now, the switching function is defined by the set $\{x : \alpha(x) = 0\}$, with

$$\alpha(x) = Sx + z,\ S = (B_v^T X^{-1} B_v)^{-1} B_v^T X^{-1}, \tag{12}$$

where $X \in \Re^{n\times n}$ can be designed to make the sliding mode asymptotically stable. z is a an m-order vector given as follows:

$$\dot{z} = \tau S x,\ z(0) = -Sx(0). \tag{13}$$

For convenience, we define that

$$\eta_0 = \rho_{Bv}\|S\|. \tag{14}$$

Theorem 1. *For given matrix B_v, $\tilde{B}_v$ is any basis of the nullspace of B_v^T, and $\lambda_B = \sqrt{\lambda_{min}(B_v^T B_v)}$. Let A, B_v, ρ_A and ρ_{Bv} be given. If there exists a symmetric positive-definite matrix $X > 0$ and scalars $d_0, d_1, d_2 > 0$, $0 < \delta < 1$ and $0 < \omega < 1$ satisfying the following LMIs (15) and (16):*

$$\begin{bmatrix} \tilde{B}_v^T(AX + XA^T + d_0 I)\tilde{B}_v & * & * & * & * \\ \eta\tilde{B}_v & -I & * & 0 & 0 \\ AX\tilde{B}_v & \eta I & -(1-\delta)I & 0 & 0 \\ \rho_A X\tilde{B}_v & 0 & 0 & -d_0 I & 0 \\ \rho_A X\tilde{B}_v & 0 & 0 & 0 & -\delta I \end{bmatrix} < 0, \tag{15}$$

$$\begin{bmatrix} X & * & 0 & 0 \\ I & d_1 I & 0 & 0 \\ 0 & 0 & d_2 I - X & 0 \\ 0 & 0 & 0 & 2\eta\lambda_B - \rho_{Bv}(d_1 + d_2) \end{bmatrix} > 0, \tag{16}$$

then the sliding mode dynamics is stable.

Proof of Theorem 1. We first can see that $\|S\Delta B_v(t)\| < 1$, which is a premise for the existence of the equivalent control [29] and has been proved in [25].

From [25] we can also derive that

$$\|S\Delta B_v(t)\| \leqslant \|S\|\|\Delta B_v(t)\| \leqslant \rho_{Bv}\|S\| = \eta_0 < \eta < 1. \tag{17}$$

Additionally, the LMI characterization of the sliding surface (16) also has been given in [25].

Consider the system (4) with the equivalent control approach of [29] and set $\dot{\alpha} = \alpha = 0$; then, one can derive the following equivalent control:

$$\begin{aligned} u_{eq}(t) = -[N\rho]^+[S(B_v + \Delta B_v(t))]^{-1}S[(A + \Delta A(t)) + \gamma I]x(t) - [N\rho]^+ N\sigma u_s(t) \\ - [N\rho]^+ Nf(t). \end{aligned} \tag{18}$$

Substitute $u_{eq}(t)$ into the system (4). The following $(n-l)$ reduced-order system can be expressed as:

$$\dot{x} = [A + \Delta A(t)]x - [B_v + \Delta B_v(t)][I + S\Delta B_v(t)]^{-1}[S(A + \Delta A(t))x + \tau Sx]. \tag{19}$$

Next, a transformation matrix M and associated vector v are defined:

$$M \triangleq \begin{bmatrix} (\tilde{B}_v^T X\tilde{B}_v)^{-1}\tilde{B}_v^T \\ S \end{bmatrix}, \quad v \triangleq \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = Mx, \tag{20}$$

where $v_1 \in \mathfrak{R}^{n-l}$, $v_2 \in \mathfrak{R}^l$. Then, one can obtain that $M^{-1} = [X\tilde{B}_v, B_v]$. By using the transformation, Equation (19) can be converted as

$$\begin{bmatrix} \dot{v}_1 \\ \dot{v}_2 \end{bmatrix} = \begin{bmatrix} A_1(t) & A_2(t) \\ 0 & -\varrho I \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \tag{21}$$

where

$$A_1(t) = (\tilde{B}_v^T X\tilde{B}_v)^{-1}\tilde{B}_v^T\left[I - \Delta B_v(t)[I + S\Delta B_v(t)]^{-1}S\right][A + \Delta A(t)]X\tilde{B}_v, \tag{22}$$

Applying Lemma 1, $A_1(t)$ can be rewritten as

$$A_1(t) = (\tilde{B}_v^T X \tilde{B}_v)^{-1} \tilde{B}_v^T \Big[I - [I + \Delta B_v(t) S]^{-1} \Delta B_v(t) S \Big] [A + \Delta A(t)] X \tilde{B}_v. \tag{23}$$

We can note that $\varrho > 0$, $\|A_1(t)\| < \infty$ and $\|A_2(t)\| < \infty$. Lemma 5 indicates that the reduced-order transformed system (21) is asymptotically stable when $\dot{v}_1 = A_1(t) v_1$ is stable. Since (17) suggests $\|\Delta B_v(t) S\| \leqslant \eta < 1$, in light of Lemma 3 with $\mathcal{E}(t) = -\Delta B_v(t) S$, it is clear that when there is a positive-definite matrix $P_0 \in \Re^{(n-l)\times(n-l)}$ such that the following LMI holds:

$$\begin{bmatrix} P_0 A_0(t) + * & * & * \\ \eta B_0^T P_0 & -I & * \\ C_0(t) & \eta I & -I \end{bmatrix} < 0; \tag{24}$$

then, the subsystem is stable, where $A_0(t) = (\tilde{B}_v^T X \tilde{B}_v)^{-1} \tilde{B}_v^T [A + \Delta A(t)] X \tilde{B}_v$, $B_0 = (\tilde{B}_v^T X \tilde{B}_v)^{-1} \tilde{B}_v^T$, $C_0(t) = [A + \Delta A(t)] X \tilde{B}_v$. If we define $P_0 = \tilde{B}_v^T X \tilde{B}_v$, where X can be obtained from LMIs (15) and (16), the above matrix inequality (24) can be expressed:

$$\begin{bmatrix} \tilde{B}_v^T [A + \Delta A(t)] X \tilde{B}_v + * & * & * \\ \eta \tilde{B}_v & -I & * \\ [A + \Delta A(t)] X \tilde{B}_v & \eta I & -I \end{bmatrix} < 0. \tag{25}$$

For any nonzero vector $z^T = [z_1^T, z_2^T, z_3^T]$, the previous LMI (25) can be rewritten as:

$$2 z_1^T \tilde{B}_v^T [A + \Delta A(t)] X \tilde{B}_v z_1 + 2 z_3^T [A + \Delta A(t)] X \tilde{B}_v z_1 + 2\eta z_2^T \tilde{B}_v z_1 + 2\eta z_3^T z_2 - z_2^T z_2 - z_3^T z_3 < 0, \tag{26}$$

By Lemma 2 and $\|\Delta A(t)\| \leqslant \rho_A$, the following inequalities satisfy:

$$2 z_1^T \tilde{B}_v^T \Delta A(t) X \tilde{B}_v z_1 \leqslant d_0 z_1^T \tilde{B}_v^T \tilde{B}_v z_1 + \rho_A^2 \frac{1}{d_0} z_1^T \tilde{B}_v^T X^2 \tilde{B}_v z_1, \tag{27}$$

$$2 z_3^T \Delta A(t) X \tilde{B}_v z_1 \leqslant \delta z_3^T z_3 + \rho_A^2 \frac{1}{\delta} z_1^T \tilde{B}_v^T X^2 \tilde{B}_v z_1, \tag{28}$$

Taking (27) and (28) into (26), one can yield that

$$\begin{aligned} & 2 z_1^T \tilde{B}_v^T A X \tilde{B}_v z_1 + d_0 z_1^T \tilde{B}_v^T \tilde{B}_v z_1 + \rho_A^2 \frac{1}{d_0} z_1^T \tilde{B}_v^T X^2 \tilde{B}_v z_1 \\ & \quad + \rho_A^2 \frac{1}{\delta} z_1^T \tilde{B}_v^T X^2 \tilde{B}_v z_1 + 2 z_3^T A X \tilde{B}_v z_1 + 2\eta z_2^T \tilde{B}_v z_1 \\ & \quad + 2\eta z_3^T z_2 + \delta z_3^T z_3 - z_2^T z_2 - z_3^T z_3 < 0. \end{aligned} \tag{29}$$

Consider the Schur complement formula. The inequality (29) is expressed as the LMI (15). Thus, LMI (25) holds with $P_0 = \tilde{B}_v^T X \tilde{B}_v > 0$, and the matrix X is just the solution to the LMIs (15) and (16) which implies the reduced-order equivalent system (21) is asymptotically stable. □

Remark 2. *A novel integral sliding surface* (12) *design scheme based on full-rank factorization of the input matrix is proposed. The stability condition of the sliding mode, including actuator faults information and unmatched uncertainty, are given in terms of LMIs* (15) *and* (16).

Remark 3. *The stability condition of sliding mode is similar to [27]. The difference from the existing literature is that the considered uncertainties have unmatched norm bounded uncertainty in the input matrix.*

3.2. Design of Integral Sliding Mode Fault Tolerant Controller

Now, consider the following ISM control law:

$$u(t) = -\omega(x,t)\hat{\mu}_0 N^T \frac{\alpha(x)}{\|\alpha(x)\|}, \tag{30}$$

where $\hat{\mu}_0$ is an estimated value of the unknown positive parameter $\mu_0 = \frac{1}{\mu}$. $\omega(x,t)$ is a constant coefficient given as:

$$\begin{aligned}\omega(x,t) = \frac{1}{\lambda_1 - \eta_0\hat{\mu}_0\lambda_2}\Big(&\rho_A\|S\|\cdot\|x\| + (1+\eta_0)\|N\|\rho_f(x,t) + \|S(A+\tau I)x\| \\ &+(1+\eta_0)\sum_{i=1}^{m}\|N_i\|\hat{\sigma}_i\hat{\bar{u}}_{si} + \epsilon\Big),\end{aligned} \tag{31}$$

where N_i is the ith column of N. λ_1 and λ_2 are the smallest and the biggest eigenvalues of NN^T, respectively. $\hat{\sigma}$ is an estimate of the actuator failure impact factor σ, and $\hat{\bar{u}}_s$ is an estimate of the stuck fault upper threshold $\bar{u}_s$. ϵ is any positive scalar. $\hat{\mu}_0$ can be obtained through the following adaptive law [27]:

$$\dot{\hat{\mu}}_0(t) = Proj_{[0,\frac{\lambda_1}{\eta_0\lambda_2}]}\{\Gamma\} = \begin{cases} 0 \quad if \quad \hat{\mu}_0 = \frac{\lambda_1}{\eta_0\lambda_2} \; and \; \Gamma <= 0 \\ \qquad\quad or \;\; \hat{\mu}_0 = 0 \; and \; \Gamma >= 0 \\ \Gamma \quad otherwise \end{cases} \tag{32}$$

where $\Gamma = \gamma\omega\lambda_1\|\alpha(x)\|$, and the positive constant γ is the adaptive gain which can be set by the practical applications. $Proj\ \{\cdot\}$ projects $\hat{\mu}_0$ to the range $(0, \frac{\lambda_1}{\eta_0\lambda_2})$ to satisfy $\omega(x,t) > 0$. Aiming to define the control law completely, it is henceforth assumed that when $\alpha(x) = 0$, we define $u(t) = 0$.

Moreover, the adaptive laws

$$\begin{cases} \dot{\hat{\bar{u}}}_{si}(t) = \gamma_{1i}\|\alpha(x)\|\|N_i\|, \hat{\bar{u}}_{si}(0) = \bar{u}_{si0}, \\ \dot{\hat{\sigma}}_i(t) = \gamma_{2i}\|\alpha(x)\|\|N_i\|\hat{\bar{u}}_{si}, \hat{\sigma}_i(0) = \sigma_{i0}, \quad i = 1,\ldots,m. \end{cases} \tag{33}$$

where $\bar{u}_{si0}$ and σ_{i0} are initial conditions which can be given artificially. N_i is defined in (31). γ_{1i} and γ_{2i} can be given same with γ.

Denote $\tilde{\mu}_0(t) = \hat{\mu}_0(t) - \mu_0$, $\tilde{\bar{u}}_s(t) = \hat{\bar{u}}_s(t) - \bar{u}_s$ and $\tilde{\sigma}(t) = \hat{\sigma}(t) - \sigma$. Based on the fact that $\mu_0, \bar{u}_{si}, \sigma_i$ are all unknown but constants, and the previous error equations are simplified into: $\dot{\tilde{\mu}}_0(t) = \dot{\hat{\mu}}_0(t)$, $\dot{\tilde{\bar{u}}}_s(t) = \dot{\hat{\bar{u}}}_s(t)$ and $\dot{\tilde{\sigma}}(t) = \dot{\hat{\sigma}}(t)$.

Remark 4. *The designed controller is an active fault-tolerant one whose gain can be adjusted dynamically when actuator failures occur. Additionally, the adaptive technique is applied to estimate the lower bound of fault information and the stuck fault.*

Theorem 2. *Suppose that Assumptions 1–4 hold and LMIs (15) and (16) are feasible. Let the ISMC law be given in (30) and some adaptive laws be given in (32) and (33). Consider the system (4) with an actuator fault model, unmatched uncertainties and disturbance; then, a sliding mode is stable from the very beginning.*

Proof of Theorem 2. To demonstrate that robustness starts from the very beginning, it is sufficient that the value of $\alpha(t)$ is initially set to $\alpha(0) = 0$ and the standard η-reachability condition $\alpha^T\dot{\alpha} < -\epsilon\|\alpha\|$ is satisfied for all $\alpha(t) \neq 0$.

Let us define the Lyapunov function as

$$V = \frac{1}{2}\alpha^T\alpha + \frac{1}{2}\gamma^{-1}\mu\tilde{\mu}_0^2 + \frac{1+\eta_0}{2}\left(\sum_{i=1}^{m}\frac{\sigma_i\tilde{\bar{u}}_{si}^2}{\gamma_{1i}} + \sum_{i=1}^{m}\frac{\tilde{\sigma}_i^2}{\gamma_{2i}}\right).$$

The time derivative of V along the (4)–(13) meets

$$\begin{aligned}\dot{V} =&\alpha^T\dot{\alpha}+\gamma^{-1}\mu\tilde{\mu}_0\dot{\hat{\mu}}_0+(1+\eta_0)\left(\sum_{i=1}^{m}\frac{\sigma_i\tilde{\bar{u}}_{si}\dot{\hat{\bar{u}}}_{si}}{\gamma_{1i}}+\sum_{i=1}^{m}\frac{\tilde{\sigma}_i\dot{\hat{\sigma}}_i}{\gamma_{2i}}\right)\\ =&\alpha^T[S(A+\Delta A(t))x+(I+S\Delta B_v(t)Nf(t))+\tau Sx]\\ &+\alpha^T N\rho u(t)+\alpha^T S\Delta B_v(t)N\rho u(t)+\alpha^T N\sigma u_s(t)+\alpha^T S\Delta B_v(t)N\sigma u_s(t)\\ &+\gamma^{-1}\mu\tilde{\mu}_0\dot{\hat{\mu}}_0+(1+\eta_0)\left(\sum_{i=1}^{m}\frac{\sigma_i\tilde{\bar{u}}_{si}\dot{\hat{\bar{u}}}_{si}}{\gamma_{1i}}+\sum_{i=1}^{m}\frac{\tilde{\sigma}_i\dot{\hat{\sigma}}_i}{\gamma_{2i}}\right).\end{aligned} \tag{34}$$

Then, considering the control law $u(t)$ in (30), Assumption 1 and Lemma 4, one can secure that

$$\begin{aligned}\alpha^T N\rho u(t)&=-\alpha^T\omega\hat{\mu}_0 N\rho N^T\frac{\alpha(x)}{\|\alpha(x)\|}\\ &\leqslant -\omega\hat{\mu}_0\mu\lambda_1\|\alpha\|\\ &=-\omega\lambda_1\tilde{\mu}_0\mu\|\alpha\|-\omega\lambda_1\|\alpha\|,\end{aligned} \tag{35}$$

$$\begin{aligned}\alpha^T N\sigma u_s(t)+\alpha^T S\Delta B_v(t)N\sigma u_s(t)&\leqslant(1+\eta_0)\sum_{i=1}^{m}\|\alpha\|\|N_i\|\sigma_i\bar{u}_{si}\\ &=(1+\eta_0)\sum_{i=1}^{m}\|\alpha\|\|N_i\|(\hat{\sigma}_i\hat{\bar{u}}_{si}-\tilde{\sigma}_i\hat{\bar{u}}_{si}-\sigma_i\tilde{\bar{u}}_{si}).\end{aligned} \tag{36}$$

Notice that NN^T and $N\rho N^T$ are both positive matrices. $\hat{\mu}_0$ and ω are both positive scalars in (31). Since the inequality $\rho\leqslant I$ holds, it follows that

$$\begin{aligned}\alpha^T S\Delta B_v N\rho u(t)&=-\alpha^T S\Delta B_v N\rho\omega\hat{\mu}_0 N^T\frac{\alpha}{\|\alpha\|}\\ &\leqslant\|\alpha\|\cdot\|S\Delta B_v\|\cdot\omega\hat{\mu}_0\cdot\left\|N\rho N^T\right\|\\ &\leqslant\|\alpha\|\eta_0\omega\hat{\mu}_0\lambda_2,\end{aligned} \tag{37}$$

where λ_2 is defined in (31); η_0 is defined in (14).

Further, substituting (35)–(37) into (34), the inequality can be converted into

$$\begin{aligned}\dot{V}\leqslant&\|\alpha\|[\|S(A+\tau I)x\|+\rho_A\|S\|\cdot\|x\|+(1+\eta_0)\|N\|\rho_f(x,t)]\\ &-\omega\lambda_1\tilde{\mu}_0\mu\|\alpha\|-\omega\lambda_1\|\alpha\|+\|\alpha\|\eta_0\omega\hat{\mu}_0\lambda_2\\ &+(1+\eta_0)\sum_{i=1}^{m}\|\alpha\|\|N_i\|(\hat{\sigma}_i\hat{\bar{u}}_{si}-\tilde{\sigma}_i\hat{\bar{u}}_{si}-\sigma_i\tilde{\bar{u}}_{si})\\ &+\gamma^{-1}\mu\tilde{\mu}_0\dot{\hat{\mu}}_0+(1+\eta_0)\left(\sum_{i=1}^{m}\frac{\sigma_i\tilde{\bar{u}}_{si}\dot{\hat{\bar{u}}}_{si}}{\gamma_{1i}}+\sum_{i=1}^{m}\frac{\tilde{\sigma}_i\dot{\hat{\sigma}}_i}{\gamma_{2i}}\right).\end{aligned} \tag{38}$$

Considering the adaptive laws in (32) and (33), $V(t)$ can be reduced to:

$$\begin{aligned}\dot{V}\leqslant&\|\alpha\|[\|S(A+\tau I)x\|+\rho_A\|S\|\cdot\|x\|+(1+\eta_0)\|N\|\rho_f(x,t)]\\ &-\omega\lambda_1\|\alpha\|+\|\alpha\|\eta_0\omega\hat{\mu}_0\lambda_2\\ &+(1+\eta_0)\sum_{i=1}^{m}\|\alpha\|\|N_i\|\hat{\sigma}_i\hat{\bar{u}}_{si}.\end{aligned} \tag{39}$$

Taking the (31) into (39), and simplifying terms, yields:

$$\dot{V}\leqslant-\epsilon\|\alpha\|<0. \tag{40}$$

The above inequality (40) conforms to the η-reachability condition, and indicates that the sliding motion exists throughout all the time. □

The whole proposed FTC algorithm is shown in Figure 2.

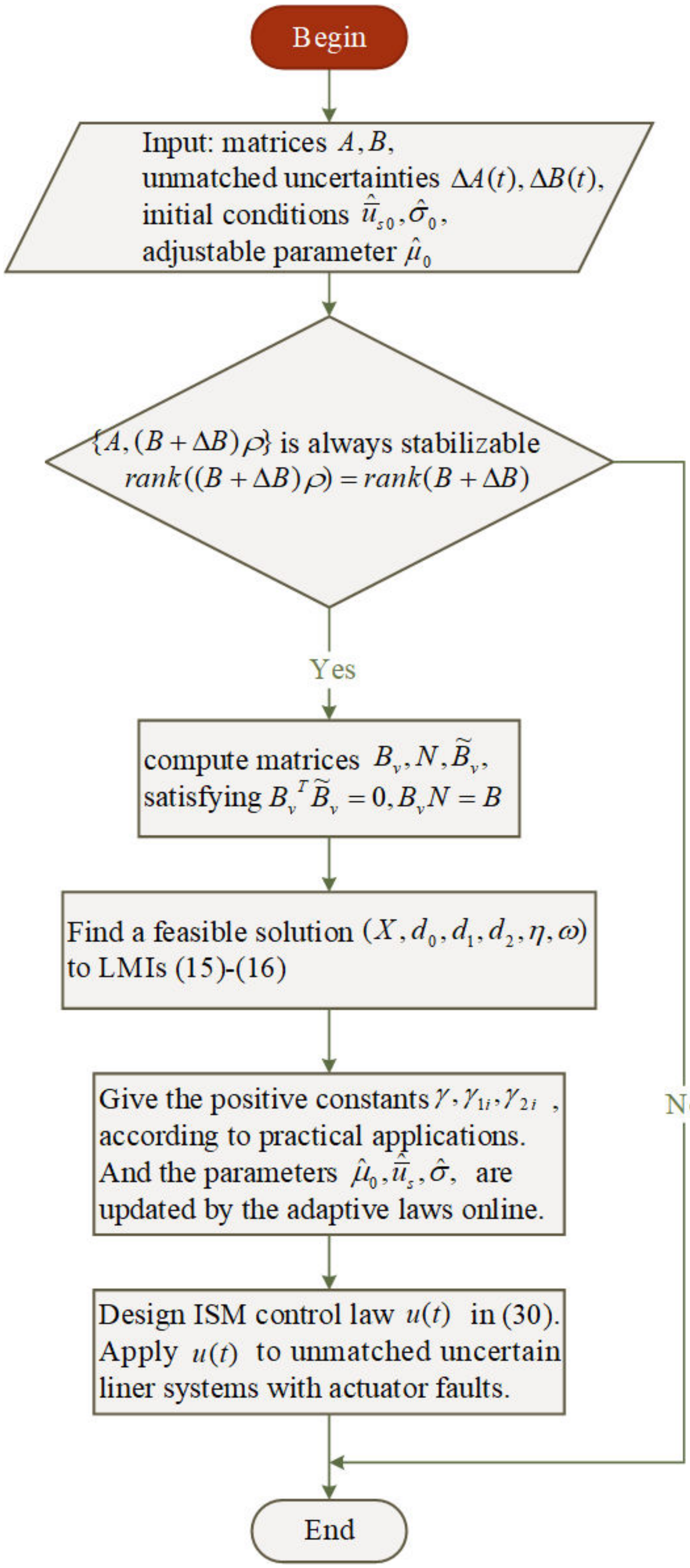

Figure 2. Flowchart of fault-tolerant control based on the ISMC algorithm.

4. Simulation Results

In this section, to demonstrate the effectiveness of the proposed fault-tolerant control scheme, two comparative simulation results are shown with a numerical example from [25] and a single-mode fairing model from [30], which also prove the necessity of considering the unmatched uncertainty.

4.1. Numerical Example

Consider the unmatched uncertain liner system of (4) with the system matrices [25].

$$A = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0.5 & 0.8 & 1 \end{bmatrix}, B_v = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, N = \begin{bmatrix} 0.5 & 0.8 & 1 \end{bmatrix}. \tag{41}$$

where B_v and N are obtained by the factorization of control input matrix B. Additionally, $\Delta A(t)$ and $\Delta B(t)$ are chosen to be

$$\Delta A(t) = \begin{bmatrix} 0 & 0.2cos(t) & 0 \\ 0.2cos(t) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \Delta B_v(t) = \begin{bmatrix} 0 \\ 0.1sin(t) \\ 0 \end{bmatrix}, \Delta B(t) = \Delta B_v(t) N. \tag{42}$$

According to Assumption 1, we can have $\rho_A = 0.2, \rho_{Bv} = 0.1, \rho_f(x,t) = 1$. By applying MATLAB LMI toolbox with the data (41) and (42), the solution to LMIs (15) and (16) can be yielded by

$$X = \begin{bmatrix} 0.8697 & 0.1310 & -0.0419 \\ 0.1310 & 0.3086 & -0.5913 \\ -0.0419 & -0.5913 & 4.4696 \end{bmatrix},$$

$$d_0 = 0.2995,\ d_1 = 5.5313,\ d_2 = 6.1998,\ \delta = 0.2464,\ \eta = 0.6482.$$

The existing sliding mode fault-tolerant scheme proposed in [26] fails to handle unmatched, uncertain system with uncertainty in the input matrix. By contrast, the proposed ISM fault-tolerant control technique of this paper can work well.

To prove the effectiveness of our designed control strategy, related parameters and initial conditions are set by: $x(0) = [1.5; 0.6; 0.1]$, $\hat{u}_{s1}(0) = \hat{u}_{s2}(0) = \hat{u}_{s3}(0) = 1$, $\hat{\mu}_0(0) = 1$, $\hat{\sigma}_1(0) = \hat{\sigma}_2(0) = \hat{\sigma}_3(0) = 0$, $\gamma = 0.1$, $\gamma_{11} = \gamma_{12} = \gamma_{13} = 0.1$, $\gamma_{21} = \gamma_{22} = \gamma_{23} = 0.1$, $\tau = 1$.

In order to weaken the discontinuity in (30), the nonlinear control law is smoothed by employing the continuous approximation

$$u(t) = -\omega \hat{\mu}_0 N^T \frac{\alpha(x)}{\|\alpha(x)\| + 0.001}. \tag{43}$$

where $\omega > 0$ is a constant coefficient given in (31), which can be figured out by some estimate based on adaptive laws and some known values. $\hat{\mu}_0$ belongs to the range $(0, \frac{\lambda_1}{\eta_0 \lambda_2})$, and it can be obtained by the projection in (32). N is yielded by the full rank factorization of B in (10).

To show the ability of our designed fault-tolerant controller, simulations with different faulty cases were sorted out such that

1. The actuator does not have any faults until t = 15 s.
2. There is 50% actuator failure in the first actuator; the second one gets stuck at $u_{s2}(t) = 1 + 1 * sin(t)$ in the meantime.

Additionally, the perturbations $f(t) = [\ 0.6cos(t)\ 0.6sin(t)\ 0.6cos(t)\ ]^T$, $15 < t < 20$.

Additionally, for the sake of demonstrating the ability of the proposed method to cope with the problems related to unmatched uncertainties in the input matrix, the results in this paper are compared with those without taking the influence of the input matrix uncertainties into consideration.

Figures 3–6 illustrate the comparison results in the above fault case. In Figure 3, the states tend to zero under the control initially. After 15 s, actuator faults and disturbance input occur. The states go to zero again under the ISMC law and then stay stable. By contrast, it can be shown that the response curve of the system states adopting the designed scheme (blue solid line) is more stable than the one adopting the existing scheme (red dotted-dashed line) when actuator faults and perturbations are added to the system. We can see the comparison results of sliding surface response curves in Figure 4. Applying the proposed scheme (blue solid line), as the actuator fault happens at 19s, the stability of sliding control is better than the existing control (red dotted-dashed line). Figure 5 presents the estimates of unknown parameter μ_0. Figure 6 shows the curves of adaptive laws $\dot{\hat{u}}_{si}(t)$, $\dot{\hat{\sigma}}_i(t)$ in (33). We can notice that the estimates are convergent; only then can the system be stable. By comparing the simulation results in Figures 3 and 4, it is obvious that the amplitude without considering the unmatched uncertainty is larger. If the uncertainty is large enough, divergence may be caused.

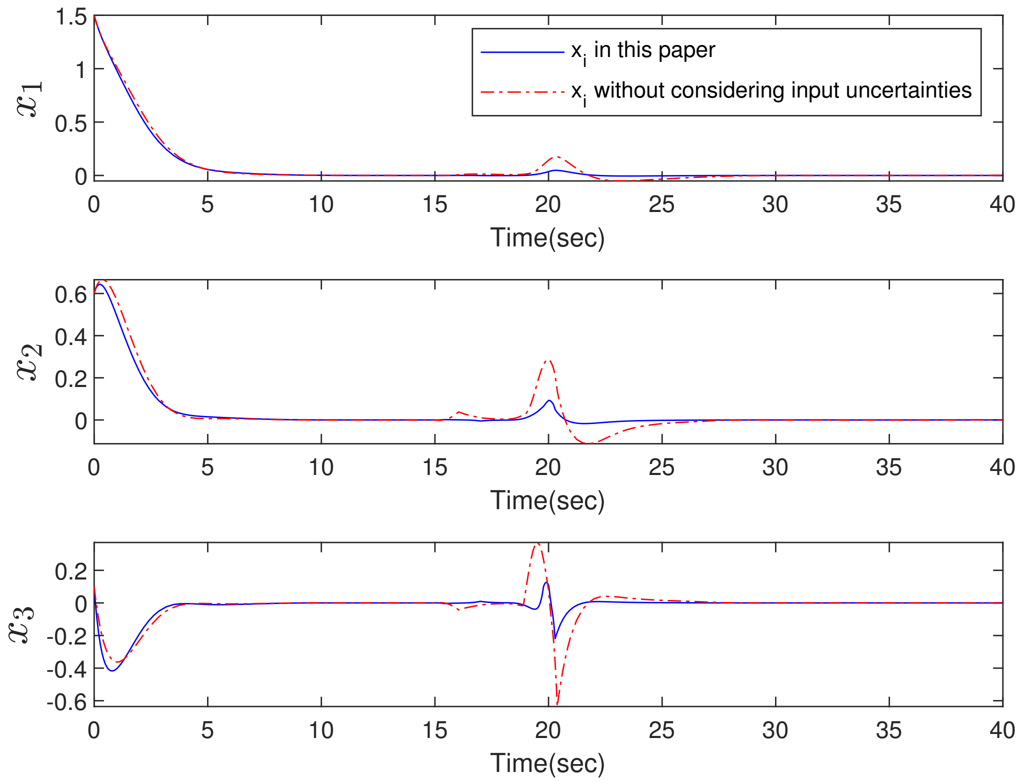

Figure 3. The comparison result of the responses curves of the system states.

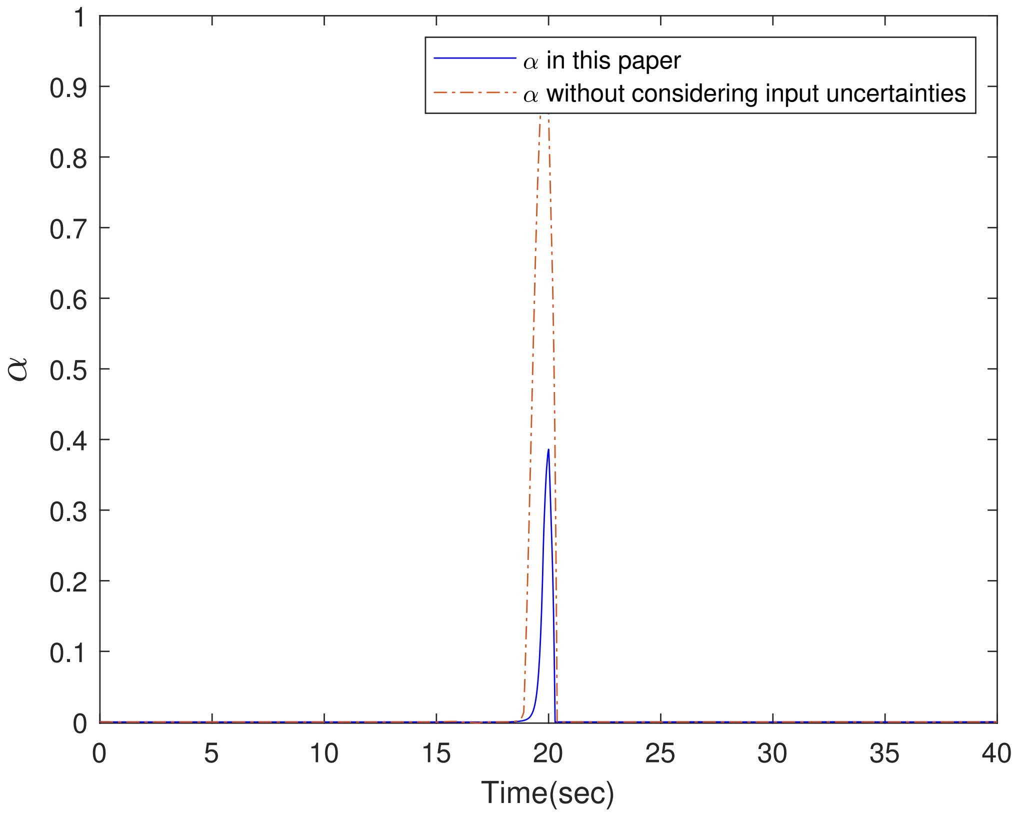

Figure 4. The comparison figure of the sliding surface response curves.

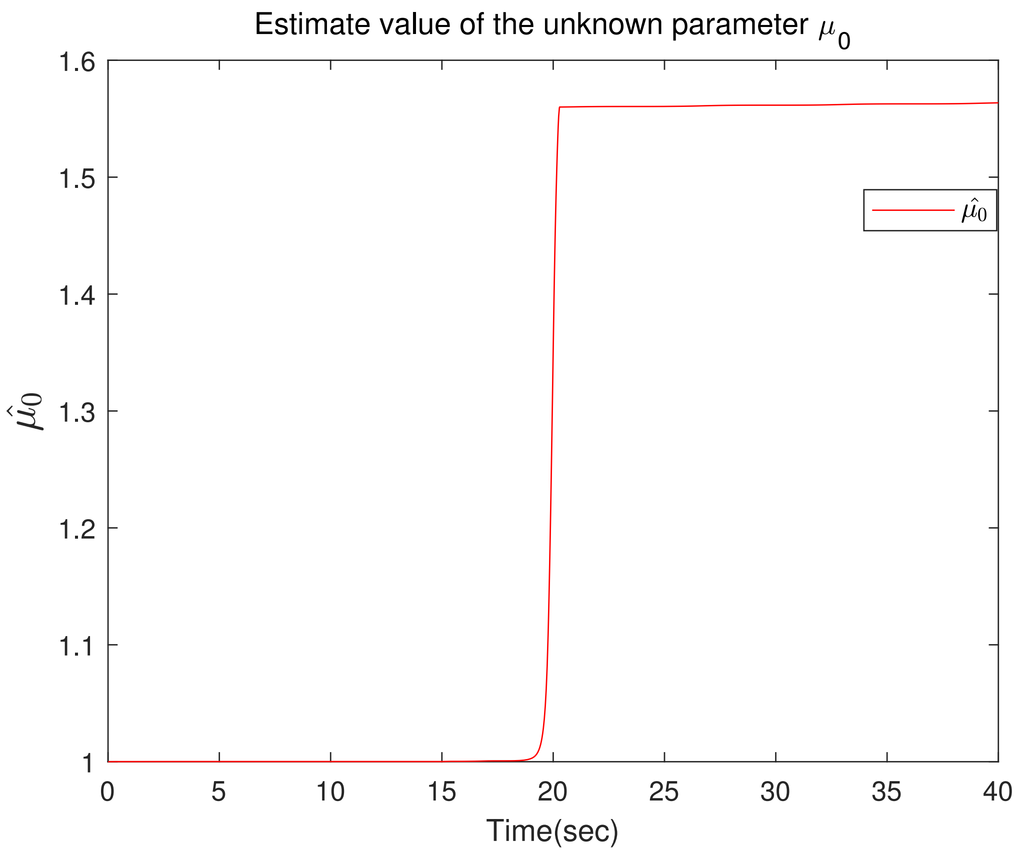

Figure 5. The estimate value of the unknown parameter μ_0.

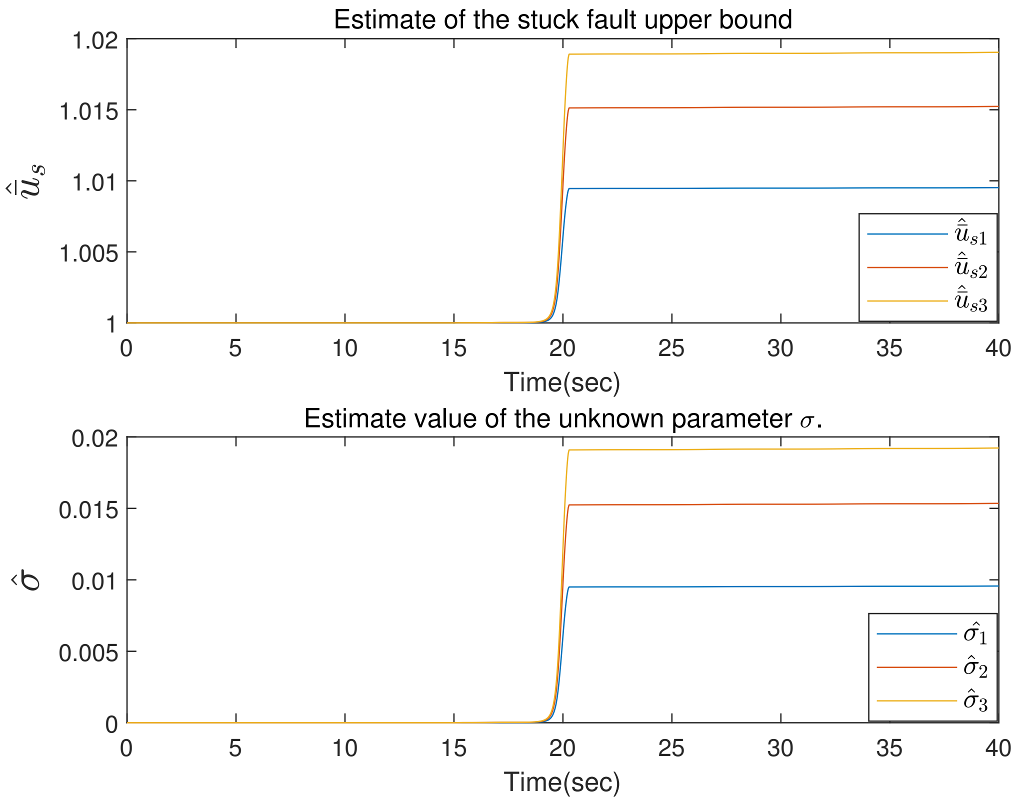

Figure 6. The estimate of the stuck fault upper bound $\bar{u}_s$ and the unknown parameter σ.

4.2. Single-Mode Fairing Model

Apply our fault-tolerant control strategy to a single-mode fairing model. The single-mode fairing model from [30] added with unmatched uncertainties and actuator faults can be presented as follows:

$$A = \begin{bmatrix} 0 & 1 & 0.0802 & 1.0415 \\ -0.1980 & -0.115 & -0.0318 & 0.3 \\ -3.0500 & 1.1880 & -0.4650 & 0.9 \\ 0 & 0.0805 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 1.55 & 0.75 \\ 0.975 & 0.8 & 0.85 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{44}$$

The matrix B can be factorized as

$$B_v = \begin{bmatrix} 1 & 0.975 & 0 & 0 \\ 1.55 & 0.8 & 0 & 0 \end{bmatrix}^T, N = \begin{bmatrix} 1 & 0 & 1.0088 \\ 0 & 1 & -0.167 \end{bmatrix}. \tag{45}$$

The unmatched uncertainties are given as

$$\Delta A(t) = \begin{bmatrix} 0 & 0.25cos(t) & 0 & 0 \\ 0.25cos(t) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \Delta B(t) = \begin{bmatrix} 0 & 0.02sin(t) \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}. \tag{46}$$

it can be defined that $\rho_A = 0.25$, $\rho_{Bv} = 0.02$ and $\rho_f(x,t) = 1.03$.

Solving LMIs (15) and (16) with known parameters, one can obtain that

$$X = \begin{bmatrix} 1.4702 & -0.3230 & 0.1230 & 0.0631 \\ -0.3230 & 1.2445 & 0.1597 & -0.2668 \\ 0.1230 & 0.1597 & 0.4389 & -0.2980 \\ 0.0631 & -0.2668 & -0.2980 & 0.4052 \end{bmatrix}.$$

To present the advantages of our fault-tolerant control strategy, related initial conditions and designed parameters are selected: $x(0) = [1.5; 0.6; 0.1; -0.2]$, $\hat{u}_{s1}(0) = \hat{u}_{s2}(0) = \hat{u}_{s3}(0) = 0.02$, $\hat{\mu}_0(0) = 1$, $\hat{\sigma}_1(0) = \hat{\sigma}_2(0) = \hat{\sigma}_3(0) = 0$, $\gamma = 1$, $\gamma_{11} = \gamma_{12} = \gamma_{13} = 0.01$, $\gamma_{21} = \gamma_{22} = \gamma_{23} = 0.01$, $\tau = 5$.

Similarly to the previous simulation, the ISM fault-tolerant control law can be designed as

$$u(t) = -\omega\hat{\mu}_0 \begin{bmatrix} 1 & 0 & 1.0088 \\ 0 & 1 & -0.167 \end{bmatrix}^T \frac{\alpha(x)}{\|\alpha(x)\| + 0.001}. \tag{47}$$

Additionally, the disturbance input $f(t) = [\ 0.6cos(t)\ 0.6sin(t)\ 0.6cos(t)\ ]^T, 15 < t < 20$. The following cases were taken into consideration in this simulation.

1. The actuators being normal until $t = 15$ s.
2. There is 50% actuator failure in the first actuator, the second one is normal and the third one gets stuck at $u_{s2}(t) = 0.02 + 0.02sin(t)$ simultaneously.

Figures 7–10 demonstrate the comparison simulation results in the above fault case. It can be noted that the response curves of the system states and the sliding surface adopting the designed scheme are more stable. This shows that it is necessary to consider the unmatched uncertainty.

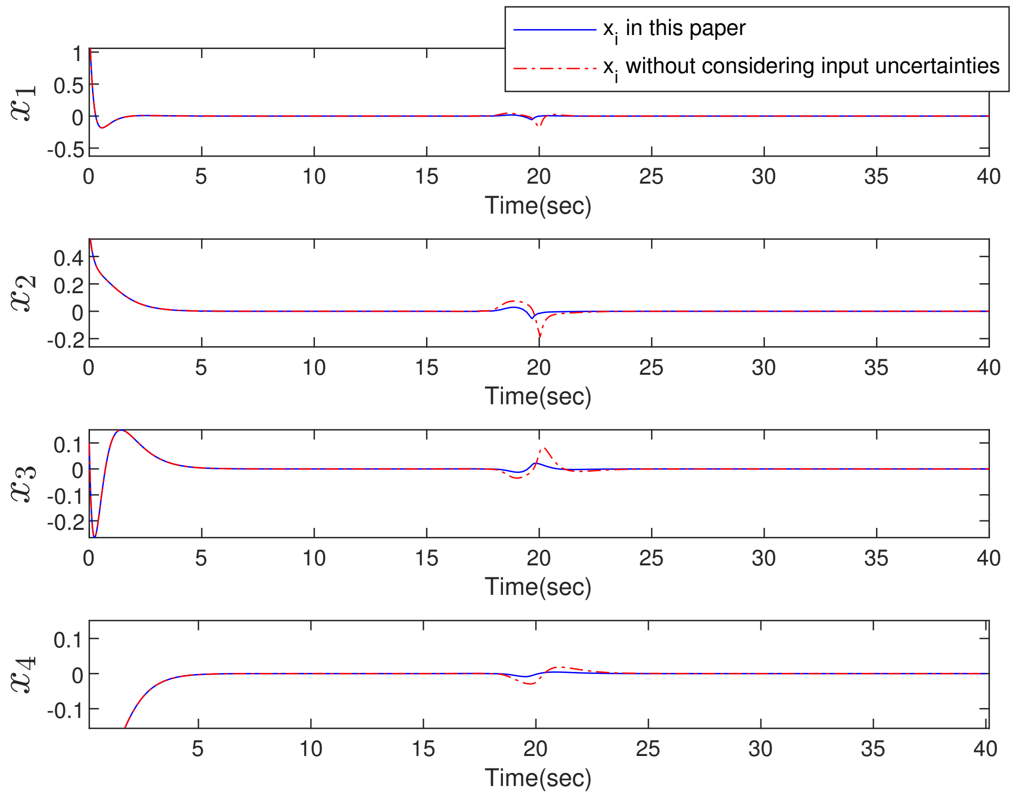

Figure 7. The comparison result of the responses curves of the system states.

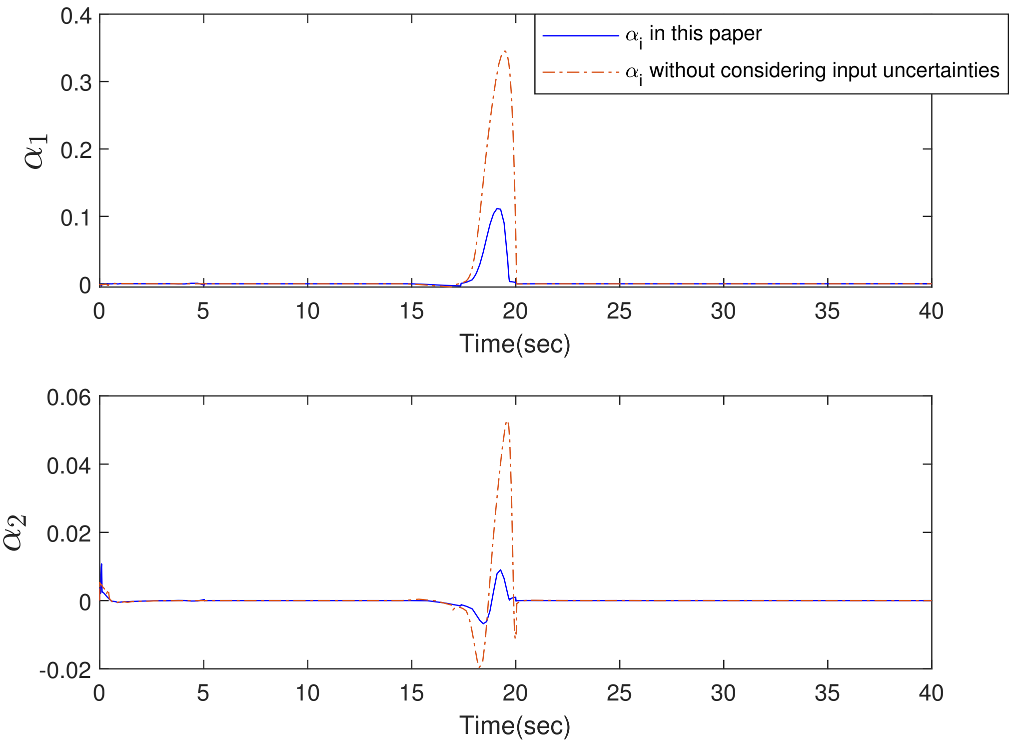

Figure 8. The comparison figure of the sliding surface response curves.

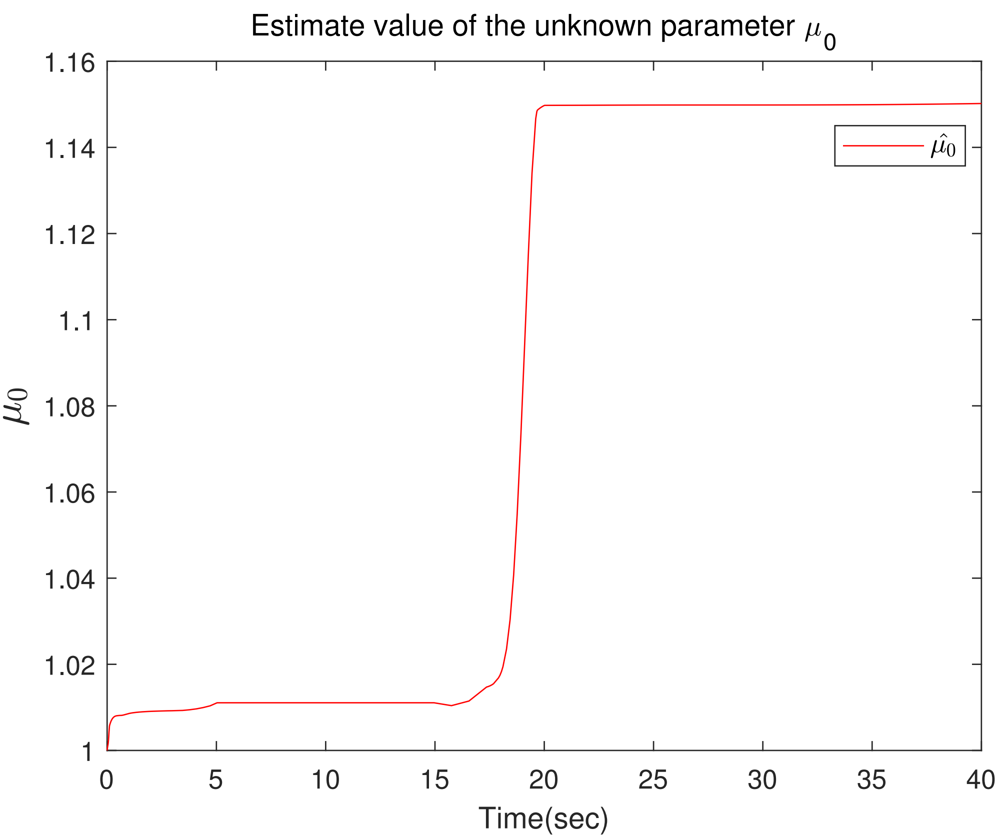

Figure 9. The estimate value of the unknown parameter μ_0.

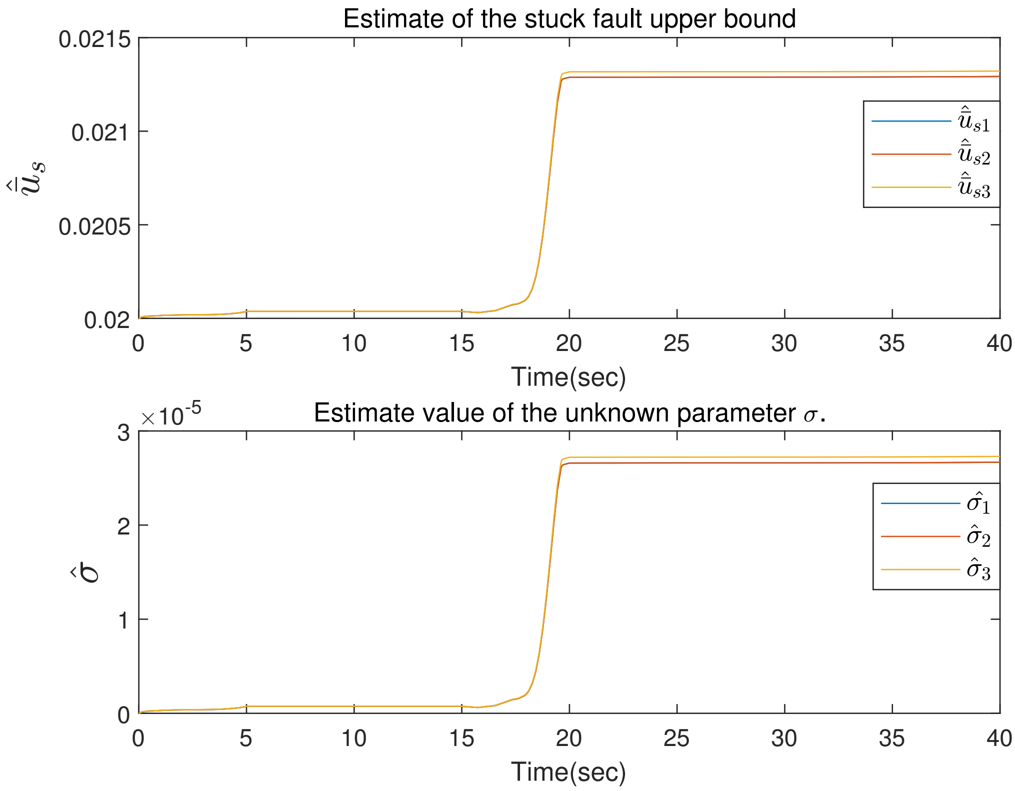

Figure 10. The estimate of the stuck fault upper bound $\bar{u}_s$ and the unknown parameter σ.

5. Conclusions

In this paper, a novel integral sliding mode scheme has been designed to get around fault-tolerant problems of liner systems with unmatched uncertainty in the input matrix. Based on two LMIs, the existence conditions of the sliding mode surface have been solved. Through adaptive laws estimating unknown fault information online, the design of the adaptive fault-tolerant controller can deal with liner systems with actuator faults and unmatched uncertainty. Finally, two sets of comparison simulation results have testified to the efficiency and advantages of the proposed control method. In future work, nonlinear systems with mismatched uncertainty and actuator faults should be investigated.

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