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A Mass Appraisal Model Based on Market Segment Parameters

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Abstract: The proposed evaluation scheme is a uni-equation model to evaluate properties of Mass Appraisal (MA) in terms of widespread availability of sample data. It all allows the use of statistical models and in the opposite conditions of the absence of data of comparable properties, the functions of similar market areas are known as well as the ones near to those for which you want to estimate the function. Of course, the accuracy of the evaluation increases with the amount of available data, with other equal conditions and evaluations carried out without data (but in the presence of other market information). It requires extra-statistical appraisal procedures involving a complete knowledge of the real estate market. However, such knowledge is also required in the MA performed by quantitative models with regard to the data sampling and performance monitoring process. The model considers micro-level characteristics of the properties and macro-level parameters of the real estate market segments. The appraisal model defines the prediction function with both the statistical models and estimation procedures. For this purpose, the model considers four specific situations: the construction of a statistical model operating with a sufficiently large sample of market prices; the construction of a prediction function operating with a very few number of market prices samples; in this situation, the appraisal function of market value is defined by using a sample of market prices referred to comparable properties, and these are few for statistical use but perfectly suitable to the appraisal process; the construction of a prediction function operating with only one market price; the construction of a prediction function operating in the absence of real estate data but with similar functions of market areas with other estimated proprieties. The presented model provides a uniform method of estimating the market value of properties (and fees), through the modular functions. The model studied is able to operate also with reduced information, considering the practical circumstances, the boundary conditions, the application precautions and the significance of the results.

Keywords: mass appraisal; real estate; market segment; automatic valuation method

1. Introduction

The Mass Appraisal (MA) regards the evaluation of a set of properties according to processes and methods which are standardized and tested for validity.

This process takes place with models and procedures based on the direct survey of punctual data of individual contracts and other market indications, mainly concerning market segments parameters and statistical and economical indicators [1].

The aim of this work is to suggest an appraisal model of MA which works according to a predefined series, namely: in the availability of a sample of market prices (or a market rents sample) sufficiently numerous for the construction of a statistical model; when it is possible to analyse a sample of market prices or a sample of market rents of comparable properties, not enough to be treated statistically, but perfectly suitable for use in the appraisal process; and in the presence of a single market price (market rental) only of a real transaction; when, finally, there is a lack of real estate data (market prices and market rents), but there are the functions of market areas similar and close to that of the estimate of the function [2].

The appraisal function is the basis of this estimate model. The appraisal function in the main form establishes a bond between cause and effect among the characteristics of the property, the segment parameters and the market price. The appraisal model then uses any kind of real estate information, perfecting it with the market acknowledgement and examining the result according to the purpose of the evaluation.

2. Literature Review

The international scientific and technical literature deals extensively with mass appraisal. The International Valuation Standards [3] in the Guidance note n. 13 on mass appraisal and the Uniform Standards of Professional Appraisal Practice [4] in the Standard 6 on mass appraisal define the phases and indicate the requirements of a mass appraisal process.

Mass appraisal has several evaluation models, many of which are based on the multiple linear regression equation, while others consider multilevel analysis [5,6], artificial neural networks, fuzzy logic, the rough set theory, etc. [7,8]. From a practical point of view, the models based on a geographic information system are particularly interesting. Only recently, research has started being applied to studies of analytical methods based on Geographic Information System. Over the past 15 years, refinement and flexibility of automated valuation models (AVM) have improved the reliability and quality of mass appraisal for cadastral reasons and tax purposes. In this context, the AVM use a system of integrated components and software items that are necessary in order to evaluate a large number of properties using mathematical models, which represent the link between real estate prices and supply and demand factors. Some business websites of private companies conduct evaluations of properties in a georeferenced context.

Shenkel and Eidson [9] presented a method for a comparable sales retrieval system. Dilmore [10] created a simple mass appraisal model by "matching property attributes". More recently, Todora and Whiterell [11] have produced an automated sales comparison method for validating results from regression-based mass assessment models. This model uses the Minkowski metric to select comparable properties and then uses regression model coefficients to make standard, adjustment-grid-type adjustments to the comparables. Borst and McCluskey [12] review and strengthen the relationship between multiple regression models, locally weighted regression—specifically the geographically weighted regression (GWR) models (Fotheringham, Brunsdon, and Charlton [13]) and the sales comparison approach. They have developed a comparable sales method (CSM) and compared it to an OLS model, a sub marketed model, and a GWR model. Their results show that the CSM outperforms all other models in both prediction errors and spatial autocorrelation measures. Regression models with geographically specific dummy variables and distance coefficients have been considered by several authors [14–16]. To improve the valuation accuracy other authors, consider spatial information in pricing models using the direct spatial modelling with Cartesian coordinates [17,18], geostatistical models [19], or response surfaces [20–22]. Other research [23–26] has focused on submarkets in which the marginal price contributions of independent variables are more likely to be similar. Quintos in 2013 [27] used Spatial Lag Models to create location based base prices and location adjustment factors. Conway et al. [28] developed a spatial lag hedonic model to capture price effects of urban green space. Following the non-agency mortgage crisis, numerous contributions have been offered in order to improve the efficiency and quality of an automated valuation methodology (AVM) dealing with

emerging problems and different contexts. Spatial issues [29,30], evolution of AVM standards [31–33], multilevel models [34], fuzzy and rough set applications [35] and quantitative methods to define comparables are just some of the topics discussed.

3. Base of the Valuation

The appraisal model has, as a basis of evaluation, the market value defined by the current valuation standards (International Valuation Standards Committee, 2017) [36]. The main references of the evaluations are generally two economic variables, market prices and rents, as concrete displays of the real estate market.

Generally, the MA, with numerous sample data and using statistical and mathematical analysis items, are applied to appraise the market value, the market rent and the net income of the residential properties. The mass appraisal may also be applied to properties with destinations other than residential, in other words special uses or special property (industrial, agricultural and commercial real estate, etc.) with a limited market and which often show the structure, shape and size.

The appraisal model is to provide a uniform assessment procedure of real estate also for special destination.

4. Market Area

The real estate market is divided into segments, as a result of inequality, atypical nature and complexity of the properties. This diversity is measured by economic estimation parameters. In order to value similar units, similar characteristics occur and the data must be in the same market segment. Through segment parameters, it is possible to measure a series of market indicators and ratios.

The market segment is typically a small unit of analysis (neighbourhoods, blocks, parts of the building, etc.) but with a parameters definition, it is possible to combine similar or comparable segments in a larger unit [37].

According to studies, the main application unit of MA is a market area, which delimits, by a continuous line, a set of market segments for construction purposes and application of model functions.

The boundary of the market area varies on different parameters taken into account. According to studies, the parameters that define the market area are: the type of contract, the destination, the building and property typology. The market areas thus defined include segments in which there are only market prices obtained from a survey of purchase agreements.

This survey follows principles and guidelines set forth by the valuation standards. The market area can be represented in the map by following the perimeter lines that physically correspond to the boundary lines between buildings and areas.

According to the study and knowledge of the market and its segmentation process, the delimitation of its area can be done by aggregation of similar properties with an inductive procedure. Considering that the greater the extent of market area, with an increased perimeter, the greater is the variability introduced by the properties.

5. Appraisal Function

The appraisal model allows for building a market value function with reference to the market prices survey, the market area, the real estate characteristics and segment parameters. The coefficients of the prediction function are represented by the hedonic prices of the real estate characteristics and parameters. The marginal price of the real estate characteristics is the variation in the market value by varying the characteristics. The marginal price of the parameter segment expresses the variation in the market value by varying the parameter.

The function used to appraise the market value is presented in the deterministic form.

According to the same procedures, it is possible to define and apply the model to market rent, with reference to market rents survey, to the rents market area, the property characteristics and segment parameters.

The appraisal model defines the prediction function with the statistical models and estimation procedures.

For this purpose, the model considers four specific situations:

- the first situation, construction of a statistical model operating with a sufficiently large sample of market prices;
- the second situation, construction of a prediction function operating with a very few number of market prices samples;
- the third situation, construction of a prediction function operating with only one market price;
- the fourth situation, construction of a prediction function operating in the absence of real estate data but with similar functions of market areas with other estimated proprieties [2].

The appraisal model analyses the macro-level characteristics related to the market area and the micro-level characteristics, demonstrating that is able to exploit any type of real estate information, and perfecting it with market knowledge and considering the result according to the purpose of the valuation.

The statistical appraisal function of the market price of the generic real estate characteristic j (with j = 1, 2, ..., m) is the following formula:

$$y_j = b_0 + \sum_{f=1}^{n} b_f \cdot x_{jf} + \sum_{g=1}^{m} B_g \cdot X_{jg} + e_j, \quad (1)$$

where b_0 is the constant term (euro); b_f is the coefficient of the generic real estate characteristic f (with f = 1, 2, ..., n); B_g is the coefficient of the generic market segment parameter g (with g = 1, 2, ..., m); x_{jf} is the generic real estate characteristic; and X_{jg} is the generic market segment parameter with e_j the stochastic error.

In the appraisal model, the general form of the function used to appraise the market price can be proposed as follows in the following formula:

$$V = L_0 + \sum_{f=1}^{n} p_f \cdot x_f + \sum_{g=1}^{m} q_g \cdot X_g. \quad (2)$$

V is the market value, with L_0 the constant term; p_f is the marginal price of the generic real estate characteristic; q_g is the marginal price of the generic market segment parameter; x_f is the generic real estate characteristic and X_g is the generic market segment parameter.

There is complete identity between the appraisal Formula (1) and the statistical Formula (2) in the constant component (L_0 and b_0) and in marginal prices of real estate characteristics (p_f and b_f) and in marginal prices of the parameters (q_g and B_g).

The construction of the appraisal function of market value is analysed according to four situations linked to market data availability.

In the first situation, the appraisal function of market value of a property (V_0), is estimated by using numerous market data, sufficient for the construction of a statistical model. Once the prices related to the market area, the real estate characteristics and the market segment parameters are known, the multiple linear regression equation Formula (2) is interpolated to the market value:

$$V_0 = b_0 + \sum_{f=1}^{n} b_f \cdot x_{0f} + \sum_{g=1}^{m} B_g \cdot X_{0g}, \quad (3)$$

where x_{0f} is the generic real estate characteristic of the property assessed and X_{0g} is the generic segment parameter of the property assessed. The appraisal function is able to estimate individually by interpolation all the properties of the market area.

In the second situation, the appraisal function of market value is defined by using a sample of market prices referred to comparable properties. These are limited for statistical use but perfectly suitable to the appraisal process. Once we know the market price P_j (with $j = 1, 2, \ldots, k$) of a market area, the real estate characteristics and the market segment parameters, the appraisal function is developed as a compound k system in which each one of the equations is based on the appraisal function of the Formula (2):

$$\begin{cases} V_0 = P_1 - \sum_{f=1}^{n} p_f \cdot x_{1f} - \sum_{g=1}^{m} q_g \cdot X_{1g} + \sum_{f=1}^{n} p_f \cdot x_{0f} + \sum_{g=1}^{m} q_g \cdot X_{0g} \\ V_0 = P_2 - \sum_{f=1}^{n} p_f \cdot x_{2f} - \sum_{g=1}^{m} q_g \cdot X_{2g} + \sum_{f=1}^{n} p_f \cdot x_{0f} + \sum_{g=1}^{m} q_g \cdot X_{0g} \\ \ldots = \ldots \\ V_0 = P_k - \sum_{f=1}^{n} p_f \cdot x_{kf} - \sum_{g=1}^{m} q_g \cdot X_{kg} + \sum_{f=1}^{n} p_f \cdot x_{0f} + \sum_{g=1}^{m} q_g \cdot X_{0g} \end{cases}. \quad (4)$$

In order to consider the possible solution of the equations system (4), it can be presented as follows:

$$\begin{cases} V_0 + \sum_{f=1}^{n} p_f \cdot \left(x_{1f} - x_{0f}\right) + \sum_{g=1}^{m} q_g \cdot \left(X_{1g} - X_{0g}\right) = P_1 \\ V_0 + \sum_{f=1}^{n} p_f \cdot \left(x_{2f} - x_{0f}\right) + \sum_{g=1}^{m} q_g \cdot \left(X_{2g} - X_{0g}\right) = P_2 \\ \ldots = \ldots \\ V_0 + \sum_{f=1}^{n} p_f \cdot \left(x_{kf} - x_{0f}\right) + \sum_{g=1}^{m} q_g \cdot \left(X_{kg} - X_{0g}\right) = P_k \end{cases}. \quad (5)$$

The market value of the property being appraised, the marginal prices of the characteristics and real estate segment parameters represent the unknowns of the equations system. The recorded market prices are the known terms of the system.

The marginal price of the real estate characteristics and the marginal price of the segment parameter require an external valuation model. The application of the market comparison method allows for analysing the hedonic prices. With a good amount of data and in the same conditions, not considering the marginal price which is an exception, it is possible to apply the paired data analysis. In the case of transparent market and availability of data, it is possible to determine the percentage changes and apply them to the marginal price calculation according to the considered parameter.

It is possible to estimate the market value and the marginal prices of both characteristics and parameters by solving the equations system of Formula (5). The system admits a unique solution when all the unknown coefficients of the Formula (5), called the coefficient matrix, are all grouped in a single matrix.

In cases in which the variables outnumber the equations used for the comparison, the market value of each property may be arithmetically calculated:

$$V_0 = \frac{1}{k} \cdot \left(\sum_{j=1}^{k} P_j - \sum_{f=1}^{n} p_f \cdot \sum_{j=1}^{k} x_{jf} - \sum_{g=1}^{m} q_g \cdot \sum_{j=1}^{k} X_{jg} \right) + \sum_{f=1}^{n} p_f \cdot x_{0f} + \sum_{g=1}^{m} q_g \cdot X_{0g}. \quad (6)$$

In the appraisal function of market value, the constant term is mainly related to the location of properties and the effect of other characteristics and segment parameters different from those reported in the same function.

The constant term L_0 according to the Formula (6) is equal to:

$$L_0 = \frac{1}{k} \cdot \left(\sum_{j=1}^{k} P_j - \sum_{f=1}^{n} p_f \cdot \sum_{j=1}^{k} x_{jf} - \sum_{g=1}^{m} q_g \cdot \sum_{j=1}^{k} X_{jg} \right). \quad (7)$$

In the third situation, the market value is defined using the market price of a single purchase. Thus, to value individual property according to the general Formula (2), knowing the market price P_j, the real estate characteristics and the market segment parameters of the contracted property, the appraisal function is:

$$V_0 = L_0 + \sum_{f=1}^{n} p_f \cdot x_{0f} + \sum_{g=1}^{m} q_g \cdot X_{0g}. \tag{8}$$

The appraisal function is able to determine single estimates by the interpolation of all properties of that market area.

In this situation, the constant term is also related to the location and the other reference characteristics and segment parameters. The constant term is calculated by developing an evaluation equation related to the comparison of all the properties being valued and the generic comparable property:

$$V_0 - P_j = \sum_{f=1}^{n} p_f \cdot x_{0f} + \sum_{g=1}^{m} q_g \cdot X_{0g} - \sum_{f=1}^{n} p_f \cdot x_{jf} - \sum_{g=1}^{m} q_g \cdot X_{jg}. \tag{9}$$

As a result, the market value of each property is equal to:

$$V_0 = \left(P_j - \sum_{f=1}^{n} p_f \cdot x_{jf} - \sum_{g=1}^{m} q_g \cdot X_{jg} \right) + \sum_{f=1}^{n} p_f \cdot x_{0f} + \sum_{g=1}^{m} q_g \cdot X_{0g}, \tag{10}$$

where the constant term is:

$$L_0 = P_j - \sum_{f=1}^{n} p_f \cdot x_{jf} - \sum_{g=1}^{m} q_g \cdot X_{jg}. \tag{11}$$

In the fourth situation, with no comparable market data, the market value is defined by using the appraisal function of market areas next to the one where the property to estimate conveys.

As an example, provided two market areas A and B, the appraisal function comes from the equations of the two areas according to the Formula (2) as follows:

$$\begin{cases} V_A = L_{0A} + \sum\limits_{f=1}^{n} p_{fA} \cdot x_f + \sum\limits_{g=1}^{m} q_{gA} \cdot X_g \\ V_B = L_{0B} + \sum\limits_{f=1}^{n} p_{fB} \cdot x_f + \sum\limits_{g=1}^{m} q_{gB} \cdot X_g \end{cases}. \tag{12}$$

The locational factor L_{0A} of market area A and the locational factor L_{0B} of market area B, are:

$$\begin{array}{l} L_{0A} = P_{jA} - \sum\limits_{f=1}^{n} p_f \cdot x_{jfA} - \sum\limits_{g=1}^{m} q_g \cdot X_{jgA} \\ L_{0B} = P_{jB} - \sum\limits_{f=1}^{n} p_f \cdot x_{jfB} - \sum\limits_{g=1}^{m} q_g \cdot X_{jgB} \end{array}, \tag{13}$$

In the fourth situation, the market value of the individual property being appraised is:

$$V_0 = \left(\frac{L_{0A} + L_{0B}}{2} \right) + \frac{p_{fA} + p_{fB}}{2} \cdot \sum_{f=1}^{n} x_{0f} + \frac{q_{gA} + q_{gB}}{2} \cdot \sum_{g=1}^{m} X_{0g}. \tag{14}$$

6. Prototypes

The prototypes relate to the situations in which it is possible to determine the market value V of the subject S using the four situations analysed in the previous section.

In the first situation, the appraisal function of market value of a property (V_0), is estimated by using numerous market data, sufficient for the construction of a statistical model.

In the numerical grid (Table 1), there are summarized prices, real estate characteristics and the segment parameters of the comparables, and, in the numerical grid (Table 2), there are summarized prices, real estate characteristics and the segment parameters of the subject.

Table 1. Numerical grid—Comparables.

Sales Price (Euro) PRS	Size (Square Feet) SIZ	Balconies (Square Feet) BAL	Floor Level (Floor Number) LEV	Typology (0–1) TYP
90,000.00	93.00	6.00	4.00	0.00
45,000.00	60.00	6.00	1.00	0.00
60,000.00	60.00	8.00	1.00	0.00
65,000.00	65.00	6.00	1.00	0.00
242,000.00	120.00	19.00	1.00	1.00
150,000.00	80.90	2.30	1.00	1.00
155,000.00	88.40	3.40	3.00	0.00
160,000.00	95.10	4.90	1.00	1.00
71,000.00	75.00	22.00	1.00	0.00
200,000.00	130.40	16.00	1.00	1.00
250,000.00	130.00	23.00	2.00	1.00
160,000.00	150.00	30.00	2.00	1.00
202,000.00	103.00	12.00	2.00	1.00
250,000.00	116.40	20.18	1.00	1.00
245,000.00	115.30	10.58	2.00	1.00
250,000.00	115.00	10.00	1.00	1.00
250,000.00	140.00	7.40	2.00	1.00
290,000.00	104.00	18.83	2.00	1.00
270,000.00	150.00	11.10	2.00	1.00
335,000.00	126.00	12.50	2.00	1.00
215,000.00	135.00	12.00	2.00	1.00
270,000.00	102.00	12.75	2.00	1.00
320,000.00	123.00	15.00	2.00	1.00
75,000.00	65.00	3.00	1.00	1.00
300,000.00	105.00	6.00	2.00	1.00
250,000.00	120.00	23.00	2.00	1.00
254,000.00	121.00	16.00	2.00	1.00
240,000.00	136.00	9.70	2.00	1.00
285,000.00	90.00	8.50	2.00	1.00
85,000.00	75.00	8.00	2.00	1.00
217,000.00	123.00	12.00	2.00	1.00
276,000.00	123.00	10.00	2.00	1.00
278,000.00	129.00	19.00	2.00	1.00
274,000.00	130.00	14.00	1.00	1.00
200,000.00	105.00	9.00	1.00	1.00
265,000.00	116.00	10.00	2.00	1.00
280,000.00	100.00	12.00	1.00	1.00
290,000.00	128.00	21.00	2.00	1.00
210,000.00	130.00	7.00	1.00	1.00
235,000.00	120.00	11.50	1.00	1.00
250,000.00	132.00	7.00	2.00	1.00
185,000.00	110.00	15.00	2.00	1.00
200,000.00	100.00	12.00	2.00	1.00
215,000.00	120.00	6.00	1.00	1.00
140,000.00	100.60	19.34	1.00	1.00
110,000.00	84.22	5.78	1.00	1.00

Table 2. Numerical grid—Subject S.

Sales Price (Euro) PRSs	Size (Square Feet) SIZs	Balconies (Square Feet) BALs	Floor Level (Floor Number) LEVs	Typology (0–1) TYPs
?	150.00	9.00	1.00	1.00

Generally, the appraisal of market value is based on the main surface of the property, on the secondary surfaces and on a set of other characteristics different from the surface. In the approach, we don't consider the characteristics ceteris paribus. For the analysis, we considered the market prices referred to comparable, expressed in euro; the real estate characteristics such as the size (SIZ) of greater importance apartment rooms, measured in square feet; the balconies (BAL) measured in square feet and the floor level (LEV) measured as the number of floors above street level. The market area considered is the typology of building (TYP), the apartments in buildings are measured with 0 and apartments in multistory buildings with 1. These features have been chosen because they are representative of the sample analysed.

Knowing market price, the real estate characteristics and the parameters of the market segment, it is possible determine the market value searched by the multiple linear regression equation Formula (15):

$$V_0 = b_0 + b_1 \times SIZ_S + b_2 \times BAL_S + b_3 \times LEV_S + b_4 \times TYP_S, \quad (15)$$

where the coefficients $b_0, b_1, \ldots, b_4$, of the regression model are the constant term and the marginal prices of the characteristics and parameters:

$$\begin{aligned} V_0 \; = \; & -19,759.10 \; + \; 597.56 \times 150.00 \; + \; 318.02 \times 9.00 \; + \\ & +29,494.85 \times 1.00 \; + \; 133,618.04 \times 1.00 \; = \; 235,850.58 \text{ euro} \end{aligned} \quad (16)$$

In the second situation, the appraisal function of market value is defined by using a sample of prices referred to comparable properties, and these are few for a statistic use but perfectly suitable to the appraisal process.

In the numerical grid (Table 3), there are summarized prices, real estate characteristics and the segment parameters of the comparables, and, in the numerical grid (Table 4), there are summarized price, real estate characteristics and the segment parameters of the *subject*.

Table 3. Numerical grid—Comparables.

Sales Price (Euro) PRS	Size (Square Feet) SIZ	Balconies (Square Feet) BAL	Bathrooms (Number) BAT	Typology (0–1) TYP
90,000.00	93.00	6.00	1.00	1.00
120,000.00	100.00	10.00	2.00	1.00
150,000.00	85.00	8.00	1.00	0.00
150,000.00	70.00	8.00	1.00	0.00

Table 4. Numerical grid—Subject S.

Sales Price (Euro) PRS_S	Size (Square Feet) SIZ_S	Balconies (Square Feet) BAL_S	Bathrooms (Number) BAT_S	Typology (0–1) TYP_S
?	80.00	7.00	1.00	1.00

Considered for the analysis are the market prices referred to as comparables, expressed in euro; the real estate characteristics such as the size (SIZ) of greater importance apartment rooms, measured in square feet; the balconies (BAL) measured in square feet and the bathrooms (BAT) measured as the number. The market area considered is the typology of building (TYP), the apartments in buildings are measured with 0 and apartments in multistory buildings are measured with 1. These features have been chosen because they are representative of the sample analysed.

In order to consider the possible solution of the equations system (5), the recorded market prices are the known terms of the system, the marginal prices of the real estate characteristics and the marginal prices of the segment parameter, which are estimated by the market comparison method.

In the numerical grid (Table 5), the marginal prices calculation is summarized:

Table 5. Marginal prices.

Characteristics	Formula	Calculation
Size SIZ (euro/sqf)	$p_{SIZ} = min\overline{p}_{SIZ} = min\frac{PRS_i}{SIZ_i + \pi_{BAL} \times BAL_i}$	949.36
Balconies BAL (euro/sqf)	$p_{BAL} = \pi_{BAL} \times p_{SIZ}$	284.81
Bathrooms BAT (euro)	p_{BAT} = detected by market surveys	7500.00
Typology TYP (euro)	q_{TYP} = detected by market surveys	20,000.00

Knowing market price, the real estate characteristics and the parameters of the market segment, it is possible to build the appraisal function by first calculating the constant term L_0 related to the location and other characteristics and parameters. The constant term is calculated as from (17):

$$L_0 = \frac{1}{k} \times \left(\sum_{j=1}^{k} PRS \; - \; p_{SIZ} \times \sum_{j=1}^{k} SIZ \; - \; p_{BAL} \times \sum_{j=1}^{k} BAL \; - \; p_{BAT} \times \sum_{j=1}^{k} BAT \; - \; q_{TYP} \times \sum_{J=1}^{k} TYP \right) \quad (17)$$

$$L_0 = \frac{1}{4} \times \left(\begin{array}{l} (90,000.00 + 120,000.00 + 150,000.00 + 150,000.00) + \\ -949.36 \times (93.00 + 100.00 + 85.00 + 70.00) - 284.81 \times (6.00 + 10.00 + 8.00 + 8.00) + \\ -7,500.00 \times (1.00 + 2.00 + 1.00 + 1.00) - 20,000.00 \times (1.00 + 1.00 + 0.00 + 0.00) \end{array} \right) \quad (18)$$

As a result, the market value of the individual property being appraised with Formula (6) is equal to:

$$V_0 = L_0 + p_{SIZ} \times SIZ_S + p_{BAL} \times BAL_S + p_{BAT} \times BAT_S + q_{TYP} \times TYP_S, \quad (19)$$

$$\begin{array}{l} V_0 = 23,251.58 \; + \; (949.36 \times 80 + 284.81 \times 7.00 + \\ +7,500.00 \times 1.00 + 20,000.00 \times 1.00) \; = \; 145,949.14 \text{euro}. \end{array} \quad (20)$$

In the third situation, the market value is defined using the market price of a single purchase.

In the numerical grid (Table 6), there are summarized prices, real estate characteristics and the segment parameters of the *subject* and the comparable.

Table 6. Numerical grid.

Sale Price Element of Comparison	Comparable A	Subject S
Sales price PRS (euro)	21,000,0,00	-
Size SIZ (square feet)	15,000	14,000
Balconies BAL (square feet)	900	600
Bathrooms BAT (number)	100	200
Typology TYP (0–1)	000	100

Table 7 reports the marginal prices of real estate properties and the parameters of the market segment, endogenously estimated (as in the second situation):

Table 7. The analysis of the adjustments.

Marginal Price Element of Comparison	Adjustments
p_{SIZ} (euro/sqf)	1098.25
p_{BAL} (euro/sqf)	362.42
p_{BAT} (euro/n)	7500.00
p_{TYP} (euro)	10,000.00

Knowing market price, the real estate characteristics and the parameters of the market segment of property contract A, it is possible to build the appraisal function by first calculating the constant term

L_0 related to the location and other characteristics and parameters. The constant term is calculated by setting a comparison equation between the subject and the comparable property, as from [21]:

$$L_0 = PRS_A - (p_{SIZ} \times SIZ_A + p_{BAL} \times BAL_A + p_{BAT} \times BAT_A + p_{TYP} \times TYP_A), \quad (21)$$

$$L_0 = 210,000.00 - \left(\begin{array}{c} 1,098.25 \times 150.00 + 362.42 \times 9.00 + \\ +7,500.00 \times 1.00 + 10,000.00 \times 0.00 \end{array} \right) = 34,500.00 \text{ euro}. \quad (22)$$

Consequently, the market value of subject according to Formula (10) is equal to:

$$V_0 = L_0 + (p_{SIZ} \times SIZ_S + p_{BAL} \times BAL_S + p_{BAT} \times BAT_S + p_{TYP} \times TYP_S). \quad (23)$$

Therefore, the market value from Formula (23) is:

$$V_0 = 34,500.00 + \left(\begin{array}{c} 1,098.25 \times 140.00 + 362.42 \times 6.00 + \\ +7,500.00 \times 2.00 + 10,000.00 \times 1.00 \end{array} \right) = 215,430.18 \text{ euro}. \quad (24)$$

Table 8 shows the incidence of characteristics and parameters on the appraisal value.

Table 8. The analysis of the price components.

Features	Component of Elements (Euro)	Incidence of Elements (%)
Constant (L_0)	34,500.00	16.01%
Size (SIZ)	153,755.64	71.37%
Balconies (BAL)	2174.54	1.01%
Bathrooms (BAT)	15,000.00	6.96%
Typology (TYP)	10,000.00	4.64%
Total price	215,430.18	100.0%

In the fourth situation, with no comparable market data, the market value is defined by using the appraisal function of market areas next to the one for the property to estimate conveys.

In the numerical grid (Table 9) there are summarized real estate properties and the segment parameters of the *subject*:

Table 9. Numerical grid—Subject S.

Sales Price (Euro) PRS_S	Size (Square Feet) SIZ_S	Balconies (Square Feet) BAL_S	Bathrooms (Number) BAT_S	Typology (0–1) TYP_S
?	100.00	9.00	1.00	1.00

rea tt:it 3 being appraised isrecedenti (seconda e terza situazione) The two market areas considered in previous applications (the second and third situations) can define the locational factor L_{0A} of market area A (second situation) and locational factor L_{0B} of market area B (third situation), equal to:

$$\begin{array}{rcl} L_{0A} & = & 23,251.58 \\ L_{0B} & = & 34,500.00 \end{array} \quad (25)$$

The market value of subject according to Formula (14) is equal to:

$$V_0 = \left(\frac{L_{0A} + L_{0B}}{2} \right) + \frac{p_{ASIZ} + p_{BSIZ}}{2} \times SIZ_S + \frac{p_{ABAL} + p_{BBAL}}{2} \times BAL_S + \\ + \frac{p_{ABAT} + p_{BBAT}}{2} \times BAT_S + \frac{q_{ATYP} + q_{BTYP}}{2} \times TYP_S \quad . \quad (26)$$

By substituting the numbers to the symbols:

$$V_0 = \left(\frac{25{,}251.58+34{,}500.00}{2}\right) + \frac{949.36+1{,}098.25}{2} \times 100.00 + \frac{284.81+362.42}{2} \times 9.00 + \\ + \frac{7{,}500.00+7{,}500.00}{2} \times 1.00 + \frac{20{,}000.00+10{,}000.00}{2} \times 1.00 = 156{,}669.18 \text{ euro} \quad (27)$$

7. Conclusions

The mass appraisal model is based on a multiple linear regression equation that considers as independent variables the real estate characteristics and market segment parameters and the market price as a dependent variable. The construction of the function is linked to the availability of market data: in the presence of a large sample of market prices, the function applies statistical models, whereas in the presence of a small sample, or of only one, or of no given data, the function proposes real estate evaluation.

The choice of the linear form is required by estimating the coefficients of the function that takes place on the basis of marginal prices with extra-statistical procedures.

The model indicates the basis of the evaluation in the market value and can be extended to the estimate of the market rent. The main unit of application is the market area consists of one or more market segments.

For market data samples sufficiently numerous to allow the use of statistical models, the appraisal model is expressed by a multiple linear regression equation. For not many data samples or one data sample, the coefficients of the real estate characteristics and segment parameters are estimated based on market evidence in the form of adjustments to market prices of real estate or property of comparison according to the market comparison approach. The constant term of the function is interpreted as a monetary effect on the location of the building and the independent variables in the same conditions not explicitly considered in the function. Though this effect is not expected in the adjustments of estimation procedures, the calculation of the constant term is proposed independently by a formula derived from the analysis of the adjustments. In the absence of market data, the constant term of the function is interpolated between the segment components of the market area.

The model presents uniformity of application regarding the estimation of market values taking place for all properties exclusively and evenly with the appraisal functions. Since the model operates also with reduced data, particularly if it extracts in narrow market segments, it gives rise to the estimate of market value properties in special destinations.

In order to show the easiness of application of the estimated model, in exemplification terms, there is reference to a situation of estimate of market value, where you have a single transaction for the market area. The example shows the simplicity of the calculations and the immediate use of the appraisal function, as well as to clarify the analysis of the market value components highlighting the impact of each element (characteristic and segment's parameter) on the estimated price.

Ultimately, the model presented provides a consistent estimation procedure through the modular functions, which form a system of interrelationships among market areas, including data and market information, between statistical and estimation procedures. The model based on the estimation functions is offered as a simple and economic model for the estimation of the market value properties (and the rent), and it represents a concrete possibility to define a mass appraisal item that is able to operate also with reduced information, considering practical circumstances, boundary conditions, application cautions and significant results.

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