

Article

Reliability-Aware Robust Optimization for Multi-Type Sensor Placement Under Sensor Failures

Shenghuan Zeng ¹, Ding Luo ¹, Pujingru Yan ^{2,*}, Naiwei Lu ² , Ke Huang ² and Lei Wang ^{2,*}

¹ Shenzhen Expressway Engineering Testing Co., Ltd., Shenzhen 518000, China; zengshenghuan@szewec.com (S.Z.); luoding@szewec.com (D.L.)

² School of Civil and Environmental Engineering, Changsha University of Science & Technology, Changsha 410114, China; lunaiwei@csust.edu.cn (N.L.); kehuang@csust.edu.cn (K.H.)

* Correspondence: 21002020037@csust.edu.cn (P.Y.); leiwang_csust@163.com (L.W.)

Abstract

In the field of structural health monitoring systems, sensors serve as the fundamental components for assessing infrastructure integrity. The rationality of their spatial configuration significantly influences the precision of structural performance assessment, the efficacy of damage detection algorithms, and the operational reliability of the system throughout its designated lifecycle. A robust optimization methodology for the placement of multi-type sensors is proposed in this study, explicitly formulated to mitigate the negative impact of sensor malfunctions during long-term operation. First, a rigorous evaluation framework for sensor placement schemes is established based on Bayesian inference and the minimization of information entropy, thereby quantifying the uncertainty inherent in parameter identification. Then, a probabilistic model of sensor failure is developed utilizing the Weibull distribution to capture time-dependent reliability characteristics, combined with a modified information entropy calculation method that mathematically assimilates these failure probabilities into the optimization objective. Finally, a heuristic search strategy is employed to achieve the robust optimal placement of multi-type sensors, efficiently navigating the complex combinatorial search space. In contrast to deterministic information entropy (DIE) methodologies, which assume ideal sensor functionality, the robust information entropy (RIE) approach comprehensively accounts for the stochastic nature of sensor failures and their impact on the information content of the monitoring network, thereby significantly augmenting the robustness and redundancy of the sensor configuration. Validations utilizing a numerical frame structure and a finite element bridge model demonstrate that the RIE method effectively integrates the sensor failure probability model to yield robust optimal placement schemes, minimizing the risk of information loss and ensuring reliable structural health monitoring throughout the engineering lifecycle.



Academic Editor: Antonio Caggiano

Received: 4 February 2026

Revised: 27 February 2026

Accepted: 2 March 2026

Published: 5 March 2026

Copyright: © 2026 by the authors.

Licensee MDPI, Basel, Switzerland.

This article is an open access article distributed under the terms and conditions of the [Creative Commons Attribution \(CC BY\)](https://creativecommons.org/licenses/by/4.0/) license.

Keywords: sensor placement; Bayesian inference; Weibull distribution; information entropy; sensor failure; robust optimization

1. Introduction

The systematic acquisition of structural response data via deployed sensory apparatus constitutes a critical phase in the establishment and operation of comprehensive structural health monitoring (SHM) systems [1–6]. The fidelity and completeness of this data are paramount, as the efficacy of critical downstream tasks, encompassing structural damage identification and localization [7–10], finite element model updating and calibration [11–13],

and external load reconstruction [14–16], is heavily dependent on the quality of the sensor placement scheme. In light of stringent fiscal limitations and practical constraints associated with data transmission and bandwidth, optimizing the configuration of a finite number of sensors to efficiently acquire structural state information has emerged as a significant research focus within civil engineering.

Optimal sensor placement is typically realized through the application of mathematical criteria designed to maximize information gain or minimize estimation error. Prevalent optimization criteria include the effective independence method, which maximizes the linear independence of mode shapes [17]; the modal assurance criterion, which ensures the distinctiveness of identified modes [18]; the Fisher information criterion, which maximizes information content relative to model parameters [19–21]; and the information entropy criterion, which minimizes uncertainty in parameter estimation [22]. Addressing the uncertainties inherent in engineering applications, Kim et al. [23] proposed an improved effective independence method by introducing stochastic terms to optimize sensor placement, the efficacy of which was verified on a truss bridge model. Furthermore, Gao et al. [24] constructed an objective function based on the modal assurance criterion and proposed an adaptive gravitational search algorithm, a meta-heuristic approach that significantly enhanced computational efficiency in complex search spaces. Ghosh et al. [25] utilized the Fisher information criterion to maximize the determinant of the Fisher information matrix, thereby maximizing information regarding unknown structural parameters and diminishing posterior uncertainty. Similarly, Yuen et al. [26] proposed a heuristic algorithm utilizing the information entropy of parameter identification as a performance index, achieving optimal placement of multi-type sensors through a sequential approach.

Constructing multi-objective functions based on diverse optimization criteria can effectively enhance the aggregate performance of SHM systems by addressing conflicting design goals simultaneously [27–30]. Yang et al. [31] synthesized the Fisher information matrix with the modal assurance criterion to formulate a comprehensive multi-objective optimization function. In their work, interval analysis was introduced to manage uncertainty, and the Pareto front concept was utilized to optimize sensor placement for a spacecraft subsystem, balancing modal distinctness with parameter sensitivity. Addressing the dual challenge of localization and reconstruction of external loads, Liu et al. [32] utilized the modal shape matrix as a theoretical foundation, integrating the Fisher information matrix, the condition number of the mode shape matrix, and the modal assurance criterion to represent validity, stability, and orthogonality, respectively. By augmenting robustness through interval analysis and employing Pareto front solutions, optimal sensor placement and load reconstruction for a mild steel cantilever beam were achieved. Civera et al. [33] further advanced this domain by constructing a multi-objective optimization function based on the auto-modal and cross-modal assurance criteria, proposing a method that integrates genetic algorithms to ensure high-quality modal identification.

However, traditional sensor placement optimization criteria are primarily designed around specific SHM objectives under the assumption of perfect sensor functionality; the probability of sensor failure is frequently omitted during optimization. During the prolonged operation of an SHM system, which may span decades, sensors are subjected to non-negligible failure risks attributable to harsh environmental erosion, electromagnetic interference, and cumulative service duration [34–38]. When a sensor failure occurs, the topology of the sensor network changes, and the objective function values of the original placement scheme deviate from their design values, subsequently compromising the reliability of the monitoring system. Consequently, incorporating failure probability factors into the optimization process is of paramount significance for ensuring continuous, stable, and reliable health monitoring throughout the structural lifecycle.

The establishment of an accurate and representative sensor failure probability model is essential for quantitatively assessing the robustness of monitoring systems. Currently, prevalent failure probability models in reliability engineering are frequently constructed based on exponential [39–42] or Weibull [43–45] probability density functions. In a comparative analysis concerning wireless sensor networks, Le et al. [46] posited that the Weibull distribution delineates sensor failure characteristics, particularly those related to aging and wear-out, with greater accuracy than the memoryless exponential distribution. Based on extensive experimental data, Qiu [47] verified that the sensor failure probability distribution for typical monitoring nodes conforms to the Weibull distribution utilizing the Kolmogorov–Smirnov test, providing a solid empirical basis for its adoption.

This study proposes a robust placement method for multi-type sensing equipment that explicitly and quantitatively accounts for sensor failure risks. First, an evaluation framework for sensor placement is established based on Bayesian inference and the minimization of information entropy, thereby quantifying the uncertainty inherent in parameter identification. Then, a sensor failure probability model is developed using the Weibull distribution to capture time-dependent reliability characteristics. This model is integrated into a modified information entropy calculation method to evaluate the expected performance of the sensor network. Finally, a heuristic search strategy is employed to determine the optimal sensor configuration, efficiently balancing computational cost with solution optimality. Compared to the DIE method, this approach systematically incorporates the impact of potential sensor failures on posterior information entropy, significantly improving the robustness of the optimization results. This ensures the accuracy and reliability of parameter identification throughout the structural lifecycle, maintaining system effectiveness even in the event of partial sensor degradation.

2. Information Entropy Calculation Based on Bayesian Method

Consider a discrete linear structural system possessing N_d degrees of freedom (DOFs) and the dynamics of which are governed by the following equation:

$$M\ddot{\mathbf{x}}(t) + C\dot{\mathbf{x}}(t) + K\mathbf{x}(t) = \mathbf{f}(t) \quad (1)$$

where $M, C, K \in \mathbb{R}^{N_d \times N_d}$ represent the mass, damping, and stiffness matrices of the structure, respectively, encapsulating the mass, damping, and stiffness properties of the system; $\ddot{\mathbf{x}}(t), \dot{\mathbf{x}}(t), \mathbf{x}(t) \in \mathbb{R}^{N_d}$ denote the acceleration, velocity, and displacement vectors of the structure at time t ; and $\mathbf{f}(t)$ is the external load vector at time t , representing the excitation forces applied to the system.

Consideration is given to the utilization of multi-type sensors, specifically displacement transducers, velocimeters, and accelerometers, to synchronously acquire structural response information across distinct physical dimensions. While accelerometers remain the most widely deployed instruments in practice, modern SHM systems increasingly integrate displacement sensors (e.g., RTK-GNSS, vision-based systems) to accurately capture low-frequency drifts without integration errors, and velocimeters (e.g., geophones) for robust mid-frequency kinetic energy assessment. Fusing these modalities provides a comprehensive broadband representation of the structural state. Denote the sensor sampling interval by Δt . The full-state structural response vector at the j^{th} sampling time step is defined as $\chi_j = [\mathbf{x}(j\Delta t)^T, \dot{\mathbf{x}}(j\Delta t)^T, \ddot{\mathbf{x}}(j\Delta t)^T]^T$, wherein $\chi_j \in \mathbb{R}^{N_\chi}$ and $N_\chi = 3N_d$. Assuming N_0 sensors are employed to monitor the dynamic response, the sensor placement scheme is

represented by the Boolean selection matrix $\Delta \in \mathbb{R}^{N_x}$. The monitoring data $\mathbf{y}_j \in \mathbb{R}^{N_0}$ at the j^{th} time step may be expressed as a linear transformation of the state vector with noise:

$$\mathbf{y}_j = P(\Delta)\mathbf{x}_j + \varepsilon_j, \quad j = 1, \dots, N \quad (2)$$

where $P(\Delta) \in \mathbb{R}^{N_0 \times N_x}$ represents the sensor placement position matrix mapping the full state to the measured DOFs; $\varepsilon_j \in \mathbb{R}^{N_0}$ denotes the measurement noise at the j^{th} time step, typically assumed to be Gaussian white noise; and N is the total quantity of monitoring data points collected. The aggregate dataset $\mathcal{D}$ is expressed as:

$$\mathcal{D} = \{\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_N\} \quad (3)$$

For modal identification, N_m frequency bands containing significant resonant peaks are extracted from the structural response spectrum. The frequency domain response within the i^{th} band is utilized to identify the structural modal parameters $\boldsymbol{\theta}_i$ for the i^{th} mode:

$$\boldsymbol{\theta}_i = \{\lambda_i, \boldsymbol{\phi}_i\}, i = 1, \dots, N_m \quad (4)$$

where λ_i represents the square of the i^{th} natural angular frequency (i.e., the i^{th} eigenvalue of the structural system), and $\boldsymbol{\phi}_i \in \mathbb{R}^{N_0}$ represents the mode shape component vector corresponding to that mode at the measured locations. Parameterizing the identification problem utilizing the eigenvalue λ_i rather than the frequency simplifies the subsequent analytical calculation of the local curvature (Hessian matrix) of the log-likelihood function. Modal parameters are selected as the primary identification targets in this study because they are directly and stably extractable from ambient vibration data. While updating physical parameters (such as element stiffness) is highly valuable for downstream damage assessment, formulating the Bayesian information entropy objective around modal parameters ensures the likelihood evaluation remains computationally tractable across the vast combinatorial search space of sensor placement. In accordance with Bayes' theorem, which provides a probabilistic framework for updating beliefs based on observed data, the posterior probability density function (PDF) of the modal parameters $\boldsymbol{\theta}_i$ can be expressed as:

$$p(\boldsymbol{\theta}_i|\mathcal{D}, \Delta) \propto p(\mathcal{D}|\boldsymbol{\theta}_i, \Delta)p(\boldsymbol{\theta}_i) \quad (5)$$

where $p(\boldsymbol{\theta}_i|\mathcal{D}, \Delta)$ is the posterior PDF of parameter $\boldsymbol{\theta}_i$, representing the updated knowledge of the parameters; $p(\mathcal{D}|\boldsymbol{\theta}_i, \Delta)$ is the likelihood function, representing the probability of observing the data given the parameters; and $p(\boldsymbol{\theta}_i)$ represents the prior PDF, encapsulating prior knowledge. In the absence of specific prior information, $p(\boldsymbol{\theta}_i)$ may be assumed to be a non-informative uniform distribution [48]. In this case, the posterior distribution of $\boldsymbol{\theta}_i$ is directly proportional to the likelihood function. When the measurement data is sufficient and the noise is Gaussian, $p(\mathcal{D}|\boldsymbol{\theta}_i, \Delta)$ is asymptotically approximated as [49]:

$$p(\mathcal{D}|\boldsymbol{\theta}_i, \Delta) = \prod_k p(F_k|\boldsymbol{\theta}_i, \Delta) = \prod_k \pi^{-N_0} |E_k(\boldsymbol{\theta}_i, \Delta)|^{-1} \exp\left[-F_k^* E_k(\boldsymbol{\theta}_i, \Delta)^{-1} F_k\right] \quad (6)$$

where $F_k \in \mathbb{C}^{N_0}$ represents the frequency domain response vector of the measurement data $\mathcal{D}$ at the k^{th} frequency subsequent to the application of the fast Fourier transform (FFT); F_k^* denotes the conjugate transpose of F_k ; and $E_k(\boldsymbol{\theta}_i, \Delta) \in \mathbb{R}^{N_0 \times N_0}$ represents the theoretical power spectral density matrix of the measurement data at the k^{th} frequency, contingent upon the parameters $\boldsymbol{\theta}_i$ and the sensor placement Δ [49].

The optimal estimate $\hat{\boldsymbol{\theta}}_i$ of the modal parameters, which corresponds to the mode of the posterior distribution, may be obtained by solving the following maximum likelihood optimization problem:

$$\hat{\boldsymbol{\theta}}_i = \underset{\boldsymbol{\theta}_i}{\operatorname{argmin}} [-\ln p(\mathcal{D}|\boldsymbol{\theta}_i, \Delta)] \quad (7)$$

Predicated on the optimal value $\hat{\boldsymbol{\theta}}_i$, the local curvature of the log-likelihood function is analyzed to quantify the posterior uncertainty. The Hessian matrix $\mathbf{H}_{\boldsymbol{\theta}_i}$ is defined as the second-order partial derivative of the negative log-likelihood function with respect to the modal parameters. Given the complexity of the theoretical power spectral density matrix $E_k(\boldsymbol{\theta}_i, \Delta)$, which incorporates the distinct characteristics of multi-type sensors (displacement, velocity, and acceleration) via the selection matrix $P(\Delta)$, an analytical derivation is computationally intractable. Consequently, the Finite Difference Method is utilized to approximate the Hessian matrix elements [50]. By perturbing the parameter vector $\boldsymbol{\theta}_i$ near the optimal solution $\hat{\boldsymbol{\theta}}_i$, the curvature is estimated numerically, capturing the sensitivity of the specific sensor mix defined in Δ . This matrix $\mathbf{H}_{\boldsymbol{\theta}_i}$ essentially quantifies the sharpness of the likelihood peak; a larger curvature implies higher information content and lower uncertainty. Finally, by integrating the Hessian matrices associated with the first N_m modal parameters, the global uncertainty quantification matrix $\mathbf{H}_{\boldsymbol{\theta}}$ for the overall modal parameter estimation is obtained.

Utilizing the Laplace approximation for the integral of the posterior distribution, the information entropy, which serves as a scalar measure of uncertainty, corresponding to the sensor placement scheme Δ may be approximated as:

$$H(\Delta) \propto \frac{1}{2} N_m (N_0 + 1) [\ln(2\pi) + 1] + \frac{1}{2} \ln |\mathbf{H}_{\boldsymbol{\theta}}^{-1}| \quad (8)$$

The initial term on the right side represents the entropy contribution derived from the dimensionality of the parameter space, where $N_m(N_0 + 1)$ corresponds to the total number of modal parameters (frequencies and mode shapes) being estimated. It serves as a baseline offset determined by the multivariate Gaussian assumption. Whilst this term is constant for a fixed number of sensors, the second term specifically reflects the impact of the placement scheme on the volume of the uncertainty ellipsoid of the parameters. A diminished value of $H(\Delta)$ indicates lower information entropy, implying a greater magnitude of information acquired by the corresponding placement and thus a more precise estimation of the structural parameters.

3. Sensor Optimization Driven by Heuristic Search Strategy

The optimal sensor placement problem is mathematically formulated as a discrete optimization problem, often hindered by combinatorial explosion. As the number of candidate positions and sensors increases, the search space expands factorially, rendering exhaustive search methods computationally intractable. To overcome this barrier, a heuristic search strategy, specifically the sequential sensor placement algorithm, is adopted to enhance optimization efficiency by reducing the number of schemes evaluated at each step.

Consider the fundamental case of placing a single sensor. There exist N_χ possible schemes, constituting the initial placement set $\mathcal{L}^{(1)} = \{\Delta_i^{(1)} = \mathbf{e}_i, i = 1, \dots, N_\chi\}$, where $\mathbf{e}_i \in \mathbb{R}^{N_\chi}$ is a unit vector with a single non-zero entry corresponding to the sensor location. Subsequent to the calculation of the information entropy for each scheme in this initial set, a pruning strategy is employed. N_w schemes possessing the lowest entropy are selected to form the candidate set $\mathcal{W}$ and N_b schemes possessing the absolute lowest entropy are selected to form the current optimal set $\mathcal{B}^{(1)}$. The sizes of the candidate set N_w and the retained optimal set N_b act as the beam width for the heuristic search, directly dictating the

trade-off between computational speed and global optimality. A minimal value (e.g., $N_b = 1$) reduces the process to a greedy forward search, which is highly efficient but susceptible to local optima. Conversely, a large N_b approaches an exhaustive search, increasing the probability of finding the true global optimum at the cost of significant computational time. The results are sensitive to this assumption at lower bounds; however, beyond a certain threshold, further increasing N_b yields diminishing returns. Therefore, these values are determined empirically to balance breadth of search with computational speed [26].

Predicated on the initial set $\mathcal{B}^{(1)}$, the sensor placement configuration is expanded iteratively. In the q^{th} step (where $q \geq 2$), the algorithm seeks to add an additional sensor to the existing best configurations. The schemes in the precedent optimal set $\mathcal{B}^{(q-1)}$ are combined with the single-sensor schemes in the candidate set $\mathcal{W}$ to generate a new expanded candidate set $\mathcal{L}^{(q)}$:

$$\mathcal{L}^{(q)} = \left\{ \Delta^{(q)} \mid \Delta^{(q)} = \Delta_i^{(q-1)} + \delta_j, \forall \Delta_i^{(q-1)} \in \mathcal{B}^{(q-1)}, \forall \delta_j \in \mathcal{W} \right\} \quad (9)$$

where $\Delta_i^{(q-1)}$ represents the i^{th} optimized configuration of $q - 1$ sensors, and δ_j represents the j^{th} candidate sensor position from set $\mathcal{W}$. The information entropy of all placements in the generated set $\mathcal{L}^{(q)}$ is calculated, and the N_b schemes with the minimum entropy are selected to form the new optimal set $\mathcal{B}^{(q)}$. These steps are reiterated sequentially until the number of sensors reaches the target N_0 . Finally, the placement scheme with the global minimum information entropy is selected from the final set $\mathcal{B}^{(N_0)}$ as the optimal solution Δ_{opt} .

However, the traditional information entropy criterion assumes all sensors remain fully operational throughout the monitoring period [26], thereby ignoring the stochastic risk of failure. Sensors may malfunction due to environmental erosion, moisture ingress, or aging. Neglecting this factor diminishes the robustness of the monitoring system; a system optimized for a specific configuration may suffer catastrophic information loss if a critical sensor fails. Incorporating a failure probability model into the optimization criterion facilitates the design of placement schemes with inherent redundancy, thereby improving operational reliability and maintaining data quality even in the event of partial sensor degradation.

4. Robust Optimization of Multi-Type Sensor Placement Considering Sensor Failure

To address the limitations of deterministic optimization, this study advances a robust optimization framework. Within the iterative procedure described above, for each candidate placement in the set $\mathcal{L}^{(q)}$ ($q > 1$), consideration is explicitly given to the scenario where at most one sensor in the system fails. By introducing a modified information entropy metric to evaluate the robustness of each placement scheme, a method capable of accommodating and mitigating the impact of sensor failure is constructed.

To mathematically characterize the failure behavior of sensors during long-term service, the Weibull distribution is adopted to establish the failure probability model. The Weibull distribution is widely recognized in reliability engineering for its versatility in modeling various stages of a component's bathtub curve. Crucially, it is capable of capturing both progressive aging and sudden, random failures. Accidental damage (e.g., physical impact or environmental severing) can be modeled as a constant failure rate by setting the shape parameter $\beta = 1$, which reduces the model to an exponential distribution. Conversely,

setting $\beta > 1$ models an increasing failure rate, characteristic of the wear-out phase. The probability of failure for a sensor within a projected service life T is expressed as:

$$Q(T) = 1 - \exp \left[- \left(\frac{T}{\eta} \right)^\beta \right] \quad (10)$$

where the scale parameter η (characteristic life) assumes values η_d, η_v, η_a corresponding to displacement, velocity, and acceleration sensors, respectively, reflecting the inherent durability of different sensor technologies. The shape parameter β ($\beta_d, \beta_v, \beta_a$) delineates the distribution characteristics of failure probability evolution over time; a value of $\beta > 1$ indicates an increasing failure rate, consistent with aging components. Figure 1 illustrates the cumulative failure probability curves. It is important to note that the specific values of β and η depicted in this figure (and utilized in subsequent case studies) are based on representative engineering assumptions designed to demonstrate the algorithm's capability to handle heterogeneous sensor networks. They are not derived from specific empirical datasheets; rather, they reflect the relative inherent durability of the technologies. As depicted, failure probability increases non-linearly with service duration. Sensors with higher β values, such as accelerometers, demonstrate pronounced wear-out characteristics with a rapid operational decline (modeling the potential fatigue of complex MEMS or piezoelectric components), whereas displacement sensors exhibit a more gradual degradation, reflecting their comparative mechanical robustness (modeling simpler devices like LVDTs).

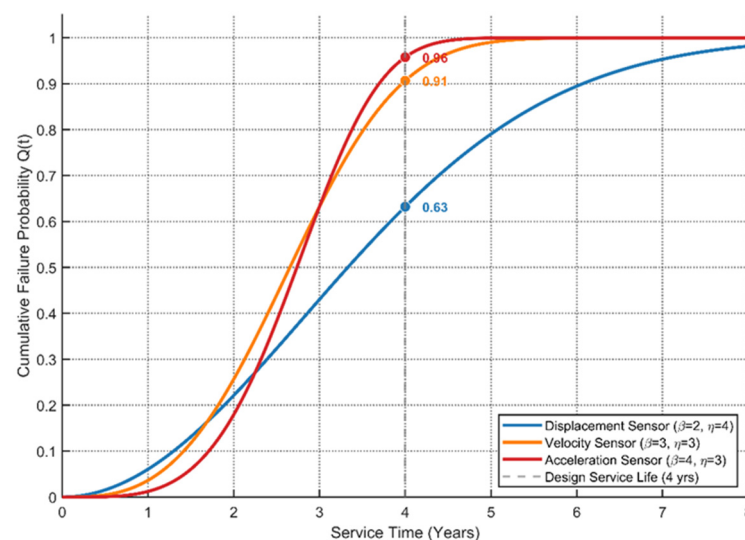


Figure 1. Cumulative failure probability curves for displacement, velocity, and acceleration sensors over service time.

In the stepwise placement process, the set $\mathcal{L}^{(q)}$ ($q > 1$) contains $N_b N_w$ schemes, each consisting of q sensors. For the i^{th} placement scheme $\Delta_i^{(q)}$ ($i = 1, \dots, N_b N_w$), in accordance with the independent failure assumptions and the Weibull model in Equation (10), the reliability function, denoting the probability that all sensors function normally within service life T , is given by:

$$R_{0,i}^{(q)}(T) = \prod_{j=1}^q \left(1 - Q_{j,i}^{(q)}(T) \right) \quad (11)$$

where $Q_{j,i}^{(q)}(T)$ represents the failure probability of the j^{th} individual sensor in the configuration $\Delta_i^{(q)}$ within T years. If the specific case is considered where the p^{th} sensor fails whilst

the remaining $q - 1$ sensors function normally, the joint probability of this specific failure state is given by:

$$R_{p,i}^{(q)}(T) = Q_{p,i}^{(q)}(T) \prod_{j=1, j \neq p}^q (1 - Q_{j,i}^{(q)}(T)) \quad (12)$$

Let $H_{0,i}^{(q)}$ denote the standard information entropy when all sensors are operational, and $H_{p,i}^{(q)}$ denote the information entropy of the reduced sensor set when the p^{th} sensor fails. The modified information entropy, which serves as the robust objective function for placement $\Delta_i^{(q)}$, can be expressed as the expected value of the entropy over the considered failure scenarios:

$$\tilde{H}_i^{(q)}(T) = \sum_{p=0}^q \kappa_{p,i}^{(q)}(T) H_{p,i}^{(q)} \quad (13)$$

where $\tilde{H}_i^{(q)}(T)$ is the modified information entropy considering failure risk; and $\kappa_{p,i}^{(q)}(T) = R_{p,i}^{(q)}(T) / \sum_{k=0}^q R_{k,i}^{(q)}(T)$ represents the normalized weight of the scenario where the p^{th} sensor fails (with $p = 0$ indicating the scenario of no failure). This formulation effectively penalizes configurations that rely heavily on sensors with high failure probabilities or configurations where the loss of a single sensor leads to a drastic increase in information entropy, namely loss of observability.

Note that the robust objective in Equation (13) evaluates a truncated scenario set consisting of the nominal state (no failure) and independent single-sensor failure states. In operational environments, it is highly realistic that multiple sensors, particularly those in close spatial proximity or of identical hardware types, may experience correlated or simultaneous failures due to localized environmental hazards or systemic aging. To rigorously include such scenarios, the independent probability model (Equation (12)) would need to be replaced by spatially correlated joint probability models (e.g., Copulas), and the objective function would need to aggregate higher-order failure combinations ($p \geq 2$). However, multi-sensor and correlated failure scenarios are excluded from the current scope to maintain computational tractability within the heuristic search algorithm. Evaluating all combinatorial permutations of simultaneous failures would cause the computational cost to grow exponentially at each sequential step. Nevertheless, penalizing independent single-sensor failures serves as a highly effective first-order robustness measure. It actively prevents the algorithm from selecting brittle configurations that rely excessively on a single, failure-prone critical node to maintain system observability, effectively eliminating single points of failure without incurring prohibitive computational costs. The probability mass of multi-sensor failures is neglected in the current objective, representing a complex extension reserved for future research.

Predicated on the modified information entropy and the heuristic search strategy described in Section 3, the robust optimal sensor placement Δ is finally obtained. The specific procedure involves replacing the standard entropy calculation in the sequential sensor placement algorithm with this modified entropy metric. By quantifying the impact of distinct failure scenarios on the magnitude of monitoring information, this method significantly improves the reliability of the placement scheme, ensuring that the SHM system remains effective throughout its design life.

The detailed algorithmic procedure for the RIE method is delineated as follows:

Step 1: Initialization and Preliminary Measurement. Define the discrete set of potential sensor locations $\mathcal{S}$, distinguishing between sensor types (displacement, velocity, acceleration). For large-scale civil infrastructure, defining $\mathcal{S}$ requires spatial down-sampling of the finite element model to maintain computational tractability (e.g., discretizing a continuous deck into 15 m or 20 m intervals). Furthermore, practical engineering constraints

must be applied to exclude physically inaccessible nodes or boundary supports with negligible modal participation. While the optimal solution is bounded by this discrete set, the spatial smoothness of lower-order mode shapes ensures that the macroscopic sensor distribution remains relatively insensitive to minor variations in the grid discretization. Instead of relying solely on numerical simulations, conduct a preliminary measurement campaign (e.g., utilizing a temporary dense sensor array) to acquire the actual baseline structural responses. Construct a preliminary numerical Finite Element Model (FEM) of the structure to identify the reference modal parameters (frequencies λ_i and mode shapes ϕ_i) that will serve as the ground truth for the optimization process. While optimal sensor placement is typically an a priori numerical procedure, if resources permit, an initial measurement campaign utilizing a small set of temporary, roving sensors can be optionally conducted to calibrate this initial FEM and acquire highly accurate baseline responses. Establish the Weibull reliability parameters (η , β) for each sensor type. In practical engineering applications, these parameters are not arbitrary; they should be rigorously estimated by fitting historical maintenance logs from similar SHM deployments or by utilizing Accelerated Life Testing (ALT) and Mean Time Between Failures (MTBF) data provided by hardware manufacturers, typically employing Maximum Likelihood Estimation (MLE) to derive the shape and scale variables.

It is important to note the sensitivity of the optimization results to the assumed Weibull parameters. The algorithm acts as a dynamic scale, balancing a sensor's modal sensitivity against its failure probability. Consequently, the final configuration is highly sensitive to the inputs β and η ; if the characteristic life (η) of the accelerometers were improved by hardware advancements to match that of the displacement sensors, the proposed robust configuration would naturally converge back toward the accelerometer-dominated traditional layout. Therefore, the accuracy of the robust design is fundamentally tied to the quality of the empirical reliability data used to define the failure model.

Define the target service duration T . Compute the baseline failure probability $Q(T)$ for each sensor type using Equation (10) to serve as static constants for the remainder of the algorithm.

Step 2: Initial Screening ($q = 1$). Compute the modified information entropy $\tilde{H}^{(1)}$ for every available single-sensor location, incorporating the specific failure probability of that sensor type. Identify the subset of optimal configurations $\mathcal{B}^{(1)}$ (of size N_b) and the candidate pool $\mathcal{W}$ (of size N_w) by minimizing the entropy metric.

Step 3: Iterative Augmentation Process. For each sequential step q from 2 to the target sensor quantity N_0 ,

- **a. Configuration Expansion.** Construct the expanded search space $\mathcal{L}^{(q)}$ by systematically appending sensor locations from the candidate pool $\mathcal{W}$ to the existing optimal configurations in $\mathcal{B}^{(q-1)}$.
- **b. Reliability Assessment.** For each candidate configuration within $\mathcal{L}^{(q)}$, retrieve the individual failure probability $Q(T)$ for every constituent sensor from the pre-calculated values established in Step 1.
- **c. Scenario Probability Calculation.** Compute the joint probabilities for the nominal (non-failure) state R_0 and all independent single-sensor failure states R_p utilizing Equations (11) and (12).
- **d. Robust Objective Evaluation.** Calculate the modified information entropy for the configuration according to Equation (13), thereby aggregating the weighted entropies of all considered operational scenarios. From a computational implementation perspective, the most expensive operation is calculating the standard entropy $H(\Delta)$ via the Hessian matrix. If the user has previously executed a DIE optimization, the intermediate standard entropy values can be cached in a lookup table. The robust

evaluation can then reuse these cached values for the required subsets $H_{0,i}^{(q)}$ and $H_{p,i}^{(q)}$, avoiding redundant Hessian calculations and significantly accelerating the algorithm, even though the heuristic search path itself must be re-executed due to the modified objective.

- **e. Optimal Selection.** Update the optimal set $\mathcal{B}^{(q)}$ by retaining the N_b configurations that yield the lowest $\tilde{H}^{(q)}(T)$ values.

Step 4: Final Determination. Upon conclusion of the iterative process ($q = N_0$), designate the configuration within the final set $\mathcal{B}^{(N_0)}$ that possesses the global minimum modified information entropy as the definitive robust optimal sensor placement scheme Δ_{opt}

The flowchart of the algorithm is given in Figure 2.

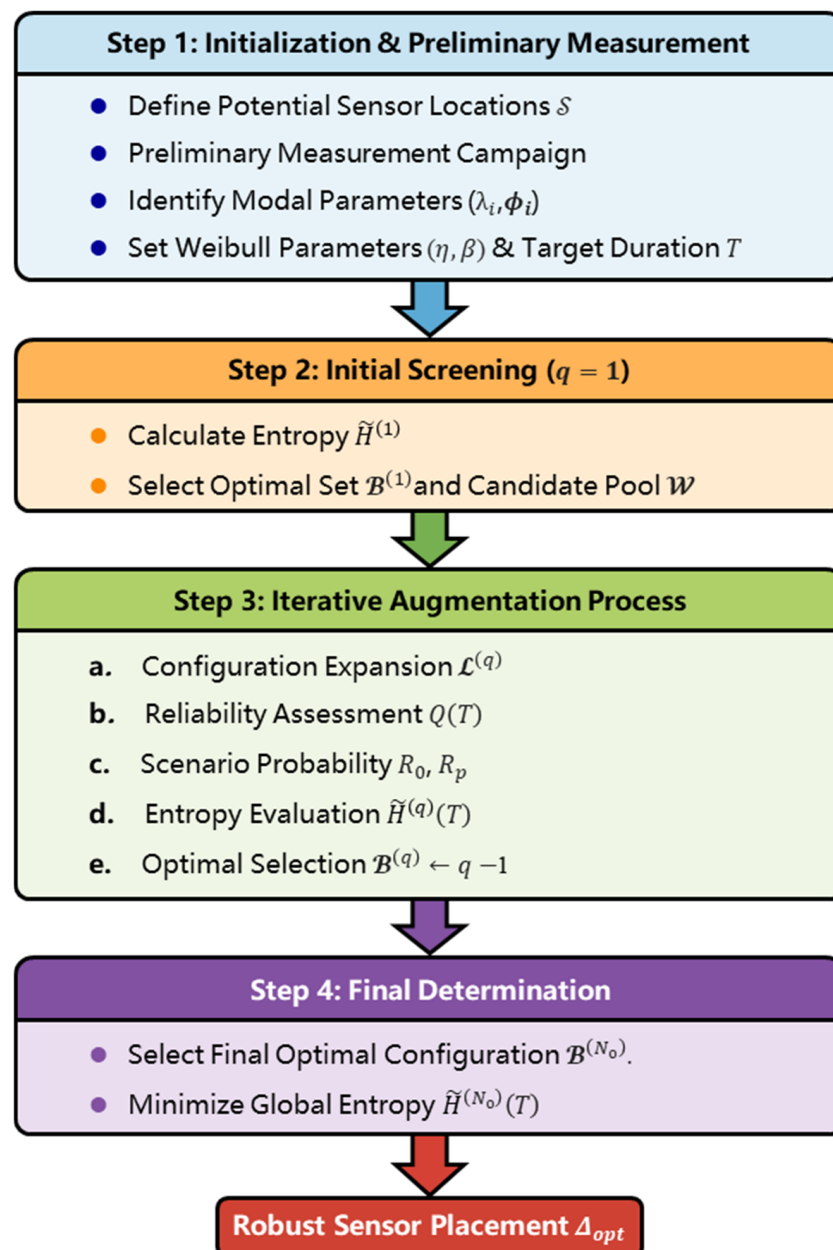


Figure 2. Flowchart of the RIE sensor placement optimization algorithm.

5. Validations

5.1. Multi-Story Frame

A ten-story shear frame structure was analyzed. The stiffness parameters are $k_i = 1500 \text{ N/m}$ and mass parameters are $m_i = 2 \text{ kg}$ for each story $i = 1, \dots, 10$. To model energy dissipation, the Rayleigh damping model is utilized, with damping ratios for the first two modes established at $\zeta_1 = \zeta_2 = 0.01$. The external load is modeled as seismic excitation, simulated as Gaussian white noise with a power spectral density of $3.6 \times 10^{-5} \text{ m}^2/\text{s}^3$ applied to the ground. Regarding sensor configuration, a heterogeneous network comprising both displacement and acceleration sensors is considered. The failure model parameters are set to reflect the higher reliability of displacement sensors compared to accelerometers, namely for displacement sensors, $\beta_d = 2, \eta_d = 4$; and for acceleration sensors, $\beta_a = 4, \eta_a = 3$. It should be noted that these specific values are selected as representative examples to simulate distinct reliability profiles for numerical validation. In practical engineering applications, these parameters should be fitted from historical maintenance data or manufacturer reliability testing results (e.g., Mean Time Between Failures) specific to the hardware deployed. The root mean square (RMS) of measurement noise for all sensors is set to 10% of the actual structural response RMS to simulate realistic signal-to-noise ratios. Specifically, for the zero-mean Gaussian white noise term ε_j in Equation (2), this equates to a standard deviation of $\sigma_{\varepsilon,i} = 0.1 \times \text{RMS}(y_{\text{clean},i})$ for the i -th sensor channel, where $y_{\text{clean},i}$ is the simulated noise-free dynamic response at that location. The data sampling duration is 400 s with a sampling frequency of 100 Hz, and the design service life of the sensors is assumed to be 4 years. It should be clarified that 400 s represents the duration of a single, typical data acquisition window (a snapshot) utilized for one instance of modal parameter identification. In a long-term SHM paradigm, such snapshots are collected periodically over the multi-year service life. The robust optimization algorithm evaluates the expected information entropy of such a 400 s snapshot over the 4-year horizon, factoring in the probability that sensors will have failed during the prolonged operational lifespan, rather than assuming failure occurs within a single short sampling window.

Figure 3 presents the optimal sensor placements obtained using the DIE method [26] for different target-mode sets (i.e., simultaneously targeting the combined first 3, 4, 5, and 6 structural modes) and varying numbers of sensors. In the figure, accelerometers are denoted by red squares and displacement sensors by blue triangles. However, no displacement sensors are selected in these configurations, indicating that the traditional objective consistently favors accelerometers under the no-failure assumption. This preference is consistent with fundamental physical principles. The acceleration response scales with the square of frequency, leading to higher sensitivity to modal information than displacement, and the acceleration frequency response typically exhibits steeper resonance features that can enhance parameter identifiability under ideal conditions. Moreover, an accelerometer is consistently placed at the first story very early in the sequential selection process (specifically, within the first three allocated sensors across all target mode sets), and this location remains permanently occupied as the sensor network expands. This choice is reasonable because the first story stiffness strongly influences the transmission of shear from the ground into the structure and thus plays a critical role in global system identification. Placing a high-sensitivity accelerometer at this location can therefore improve the observability of key structural parameters.

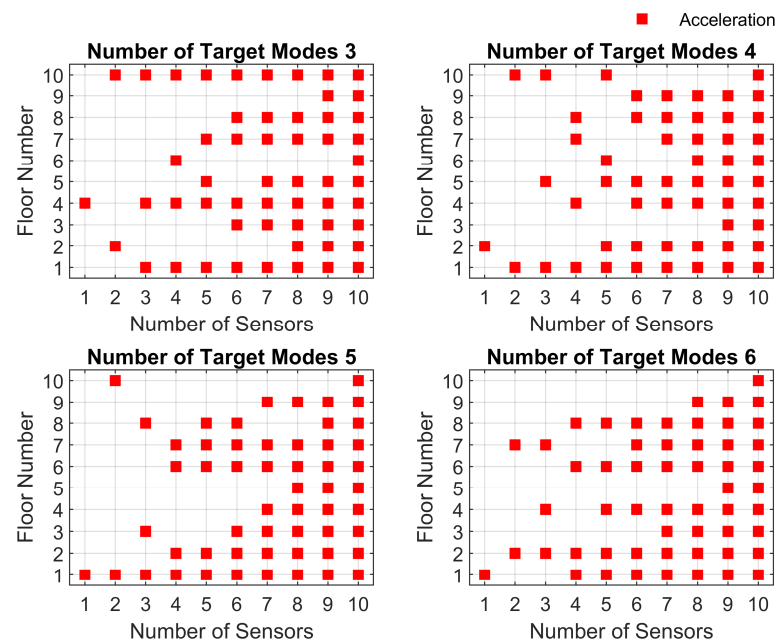


Figure 3. Optimal sensor placements using the DIE method for different target-mode sets.

Figure 4 illustrates the optimal sensor placement configurations obtained using the RIE method. Compared with the DIE results in Figure 3, the robust design produces a clear reconfiguration by systematically introducing displacement sensors, typically placed at the first story and occasionally at the second story, while still retaining accelerometers as the dominant sensor type as the sensor count increases. This change results from incorporating long-term sensor reliability into the objective. According to the Weibull failure model in Equation (10), the cumulative failure probability over a 4-year service period is 63% for displacement sensors and 95% for accelerometers, which encourages the robust method to include more reliable displacement measurements as an anchoring component of the sensing portfolio, while leveraging accelerometers for their strong modal sensitivity and resonance features. While the specific Weibull parameters utilized here are representative, the resulting magnitude of these cumulative failure probabilities (e.g., reaching significant percentages over a 4-year lifespan) aligns with empirical realities in civil Structural Health Monitoring. Sensors deployed on actual infrastructure frequently experience high attrition rates within the first few years of operation due to harsh environmental exposures, including moisture ingress, extreme temperature cycling, lightning-induced power surges, and physical cable damage. Consequently, designing against such high probability failure scenarios is a practical necessity.

The RIE method incorporates failure-probability weights into the information-entropy optimization framework, thereby balancing information maximization with long-term sensing reliability. When the higher failure risk of accelerometers is accounted for in the objective, their exclusive dominance observed under the DIE method is reduced. As a result, the robust designs systematically include displacement sensors, typically at the first story, while retaining accelerometers as the primary modality to exploit their strong modal sensitivity. This yields a rational trade-off between instantaneous modal information and hardware durability, improving the expected information acquisition capability and supporting continuous operation over the service life.

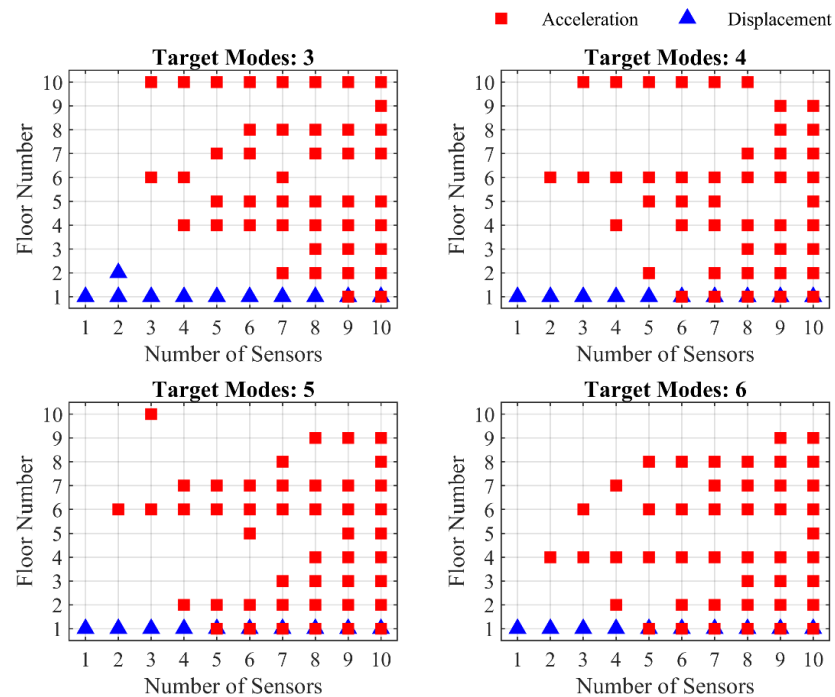


Figure 4. Optimal sensor placement obtained using the RIE method.

Figure 5 compares the expected information entropy of the sensor placement schemes obtained by the DIE method and the RIE method. For most schemes, the RIE method exhibits slightly higher expected entropy than the DIE method, while the gap diminishes as the number of sensors increases. This behavior reflects a key trade-off in reliability-aware design. The DIE method minimizes nominal uncertainty by favoring high-sensitivity accelerometers under the assumption of perfect sensor availability. In contrast, the RIE method prioritizes more reliable sensing choices, accepting a modest increase in entropy to improve long-term monitoring continuity and robustness over the service life.

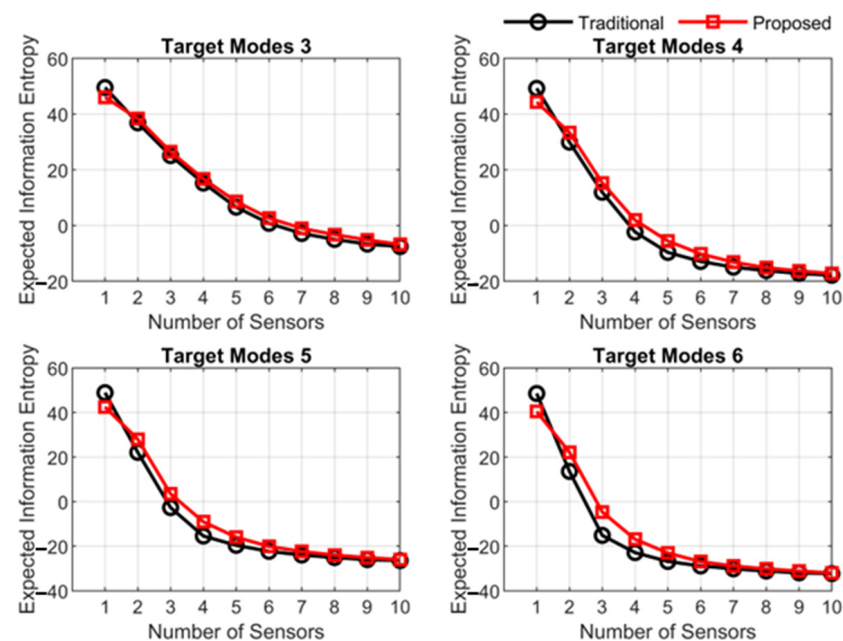


Figure 5. Information entropy comparisons of the optimal sensor placement schemes.

Figures 6–9 present the modal frequency identification results obtained using the RIE sensor placement method, targeting the first 3, 4, 5, and 6 modes, respectively. Each

figure depicts the convergence of the identified natural frequencies using the optimal sensor placement configurations presented in Figure 4. The solid blue lines represent the estimated posterior mean values, while the red boundaries delineate the 95% confidence intervals, providing a quantitative measure of identification uncertainty. A consistent trend across all scenarios is the monotonic reduction in uncertainty boundaries as the sensor network expands, confirming that the accumulation of monitoring data progressively constrains the structural parameter space. Specifically, for lower-order modes, modes 1 and 2, the confidence intervals narrow rapidly, achieving high-precision identification with as few as 3 to 4 sensors. This rapid convergence is attributed to the high energy contribution and global observability of fundamental modes. Conversely, higher-order modes, e.g., Modes 5 and 6 in Figure 9, exhibit wider initial uncertainty bands, necessitating a denser sensor array to achieve comparable precision. This disparity highlights the inherent difficulty in capturing local high-frequency behaviors, which are less sensitive to global modal parameters. Furthermore, by comparing the subplots across Figures 6–9, it is evident that expanding the target mode set effectively broadens the bandwidth of reliable identification. While targeting fewer modes, e.g., Figure 6 yields rapid convergence for the fundamental frequencies, it fails to provide constraints for higher modes. In contrast, the configurations optimized for 6 modes in Figure 9 successfully balance the information gain across a wider spectral range, ensuring that all targeted frequencies are identified with bounded uncertainty within the constraints of the available sensor quantity. It is crucial to note that the results depicted in Figure 6, specifically, the perfect convergence of the estimated values to the actual values and the near-zero uncertainty at high sensor counts—are theoretical artifacts of the idealized numerical simulation. In this study, the measurement error was modeled as spatially independent, zero-mean white noise, and the structural responses were generated without epistemic model error. Under these idealized Bayesian conditions, adding sensors continuously contributes independent information, causing the theoretical uncertainty to asymptotically approach zero. In realistic structural health monitoring applications, however, perfect measurement equipment does not exist. Measurement errors are frequently spatially correlated due to environmental factors (e.g., cabling noise, temperature gradients, or ambient wind excitation affecting the entire structure). Furthermore, inherent discrepancies between the mathematical model and the physical structure (model error) dictate that a non-zero lower bound of uncertainty will always remain, regardless of sensor density. Therefore, Figure 6 should be interpreted as demonstrating the relative comparative efficiency of the robust optimization scheme rather than predicting absolute field uncertainties.

To quantify identification precision, Figure 10 shows the posterior coefficient of variation (COV) of the identified modal frequencies versus the number of sensors for different target mode sets. The COV provides a normalized measure of posterior uncertainty, with smaller values indicating higher confidence. Across all cases, the COV exhibits a clear decreasing trend as the sensor network expands, demonstrating that additional measurements reduce uncertainty. For the fundamental modes, e.g., 1 to 3 modes, all target sets converge rapidly to very low COV levels with only a few sensors. For mode 4, configurations optimized with larger target sets, e.g., 4 to 6 modes, generally yield faster reduction and lower COV than 3 modes, indicating that explicitly including higher modes in the optimization improves uncertainty control for those modes. Overall, the RIE optimization maintains low COV across modes while accommodating different target sets under limited sensor resources.

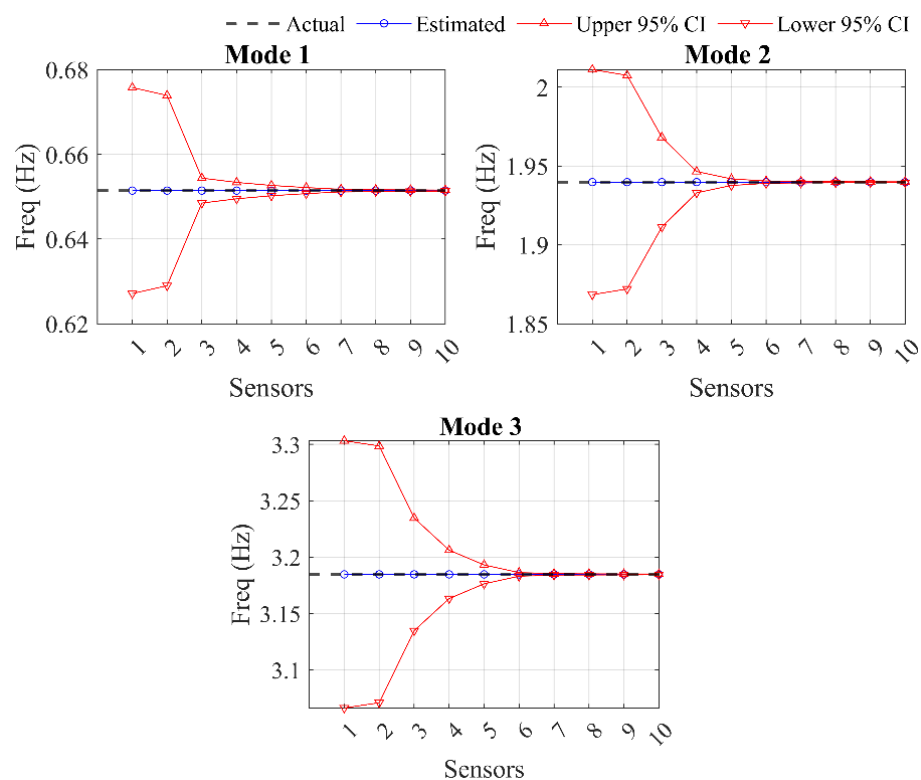


Figure 6. Modal frequency identification results showing the actual values and estimated uncertainty bounds for optimal sensor configurations targeting the first three modes (Example 1).

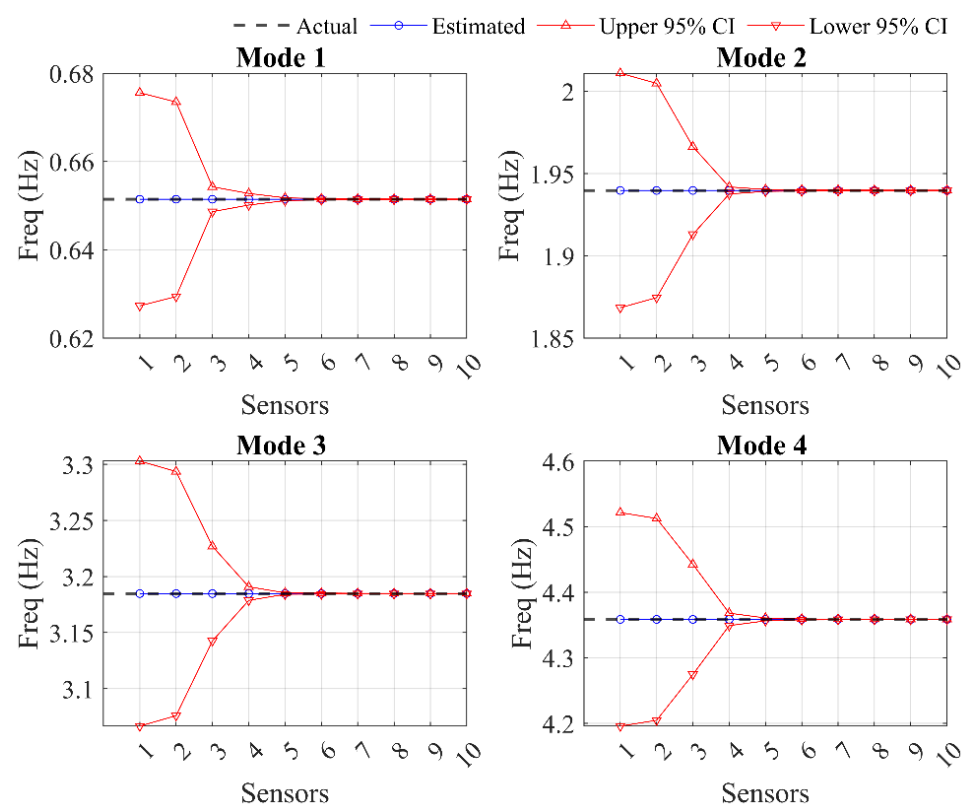


Figure 7. Modal frequency identification results showing the actual values and estimated uncertainty bounds for optimal sensor configurations targeting the first four modes (Example 1).

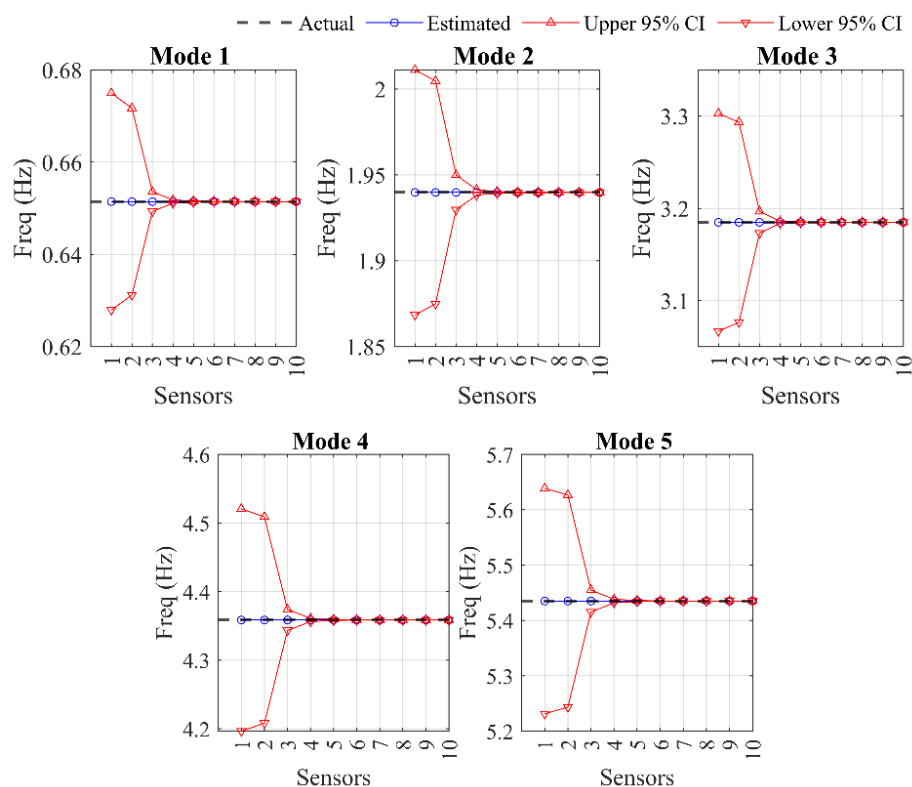


Figure 8. Modal frequency identification results showing the actual values and estimated uncertainty bounds for optimal sensor configurations targeting the first five modes (Example 1).

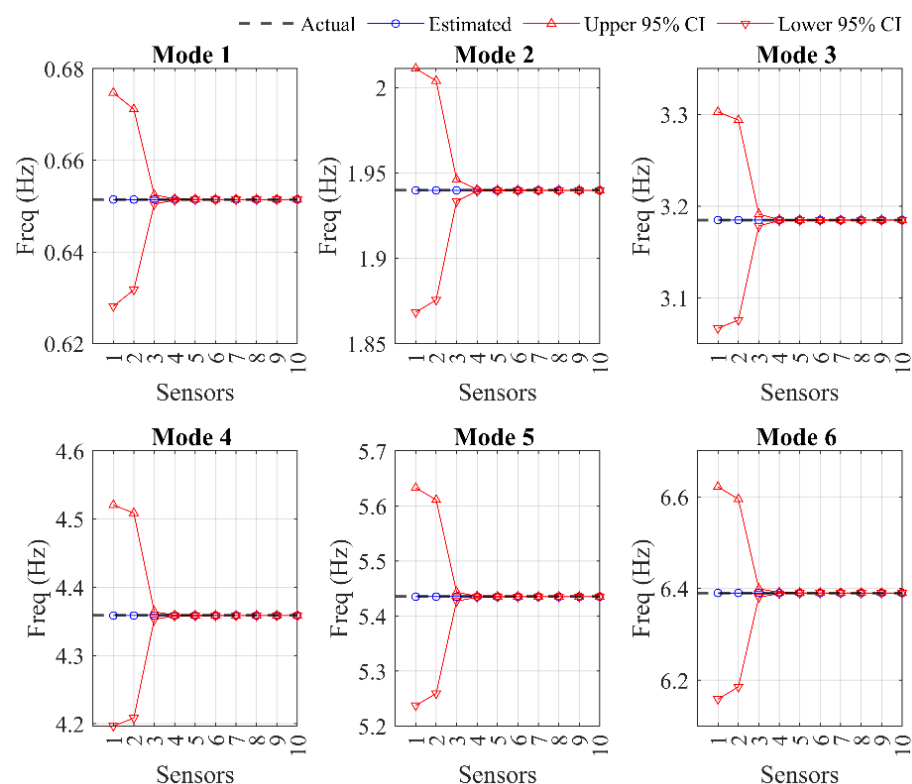


Figure 9. Modal frequency identification results showing the actual values and estimated uncertainty bounds for optimal sensor configurations targeting the first six modes (Example 1).

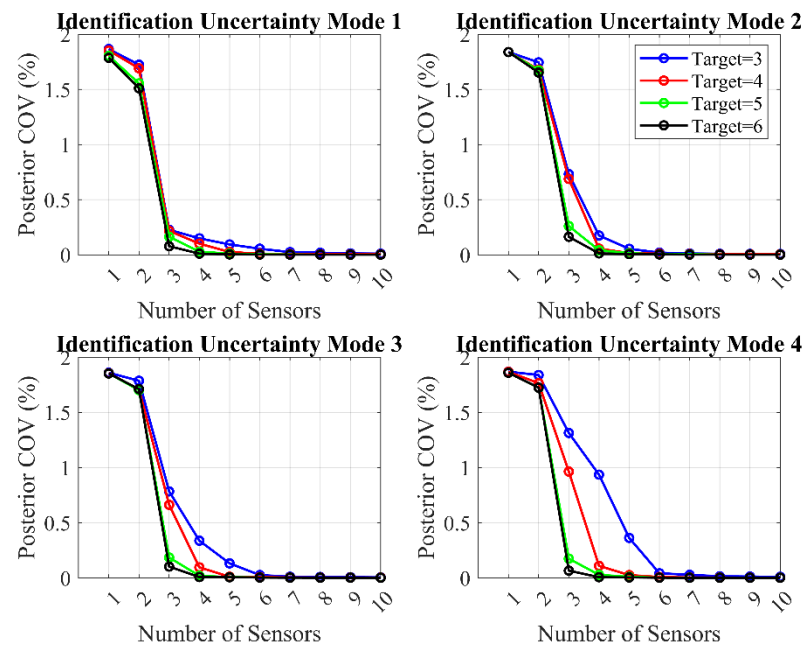


Figure 10. Posterior coefficient of variation in the identified modal frequencies versus the number of sensors for optimization schemes with different target mode sets.

It should be explicitly noted that while the COV curve in Figure 10 visually appears to converge to zero, this is a graphical artifact resulting from the linear scaling of the vertical axis. The initial uncertainty values are large enough that the subsequent, highly optimized configurations produce COV values that are orders of magnitude smaller (e.g., reaching a final minimum COV of 0.002). Thus, a small but strictly non-zero baseline of uncertainty is maintained, accurately reflecting that perfect parameter estimation is practically impossible.

5.2. Bridge

To further validate the RIE method on a complex civil infrastructure, a three-span continuous bridge, as depicted in Figure 11, is analyzed. The structure features a span arrangement of 60 m + 80 m + 60 m with pier heights of 80 m. The bridge is discretized utilizing the finite element method. In particular, the element length for side spans is 15 m, and for the middle span and pier regions is 20 m. This relatively coarse element length was deliberately selected to balance dynamic fidelity with computational tractability. While a coarser mesh cannot resolve high-frequency local modes, it accurately captures the spatially smooth, lower-order global modes targeted in this study. Furthermore, the element size dictates the spatial resolution of the candidate sensor grid $\mathcal{S}$. Because lower-order mode shapes are macroscopic and continuous, the objective function landscape is generally insensitive to minor spatial refinements; thus, refining the mesh would drastically increase the combinatorial search cost without significantly altering the macroscopic optimal sensor distribution. The principal mechanical parameters are listed in Table 1. A Rayleigh damping model is adopted with damping ratios for the first two modes set to $\zeta_1 = \zeta_2 = 0.01$. Ambient excitation is simulated as Gaussian white noise with a power spectral density of $1 \times 10^8 \text{ N}^2/\text{Hz}$, applied independently to each degree of freedom. Three types of sensors, including displacement, velocity, and acceleration, are evaluated, with failure probabilities modeled by the Weibull distribution. To capture distinct reliability characteristics typical of hardware components, the specific parameters are defined as follows, displacement sensors ($\beta_d = 1.5, \eta_d = 10$), representing high-reliability hardware with a long service life; velocity sensors ($\beta_v = 3, \eta_v = 6$), representing balanced performance; and acceleration sensors ($\beta_a = 5, \eta_a = 3$), representing high-precision but higher-risk components prone to

earlier failure. Consistent with the methodology detailed in Section 4, these parameters are representative engineering assumptions intended to model the relative durability profiles of a heterogeneous sensor network (e.g., the higher relative fatigue susceptibility of accelerometers compared to displacement transducers). As noted previously, the optimal spatial configuration is highly sensitive to these values, as the algorithm dynamically trades off modal observability against regional failure probability; therefore, in physical deployments, these parameters must be rigorously calibrated using manufacturer Mean Time Between Failures (MTBF) data or empirical maintenance logs. To provide a clearer, more practically meaningful quantification of this uncertainty, this assumed measurement noise propagates to a baseline identification error in the extracted dynamic features. Specifically, it translates to an initial Coefficient of Variation (COV) of approximately 1% for the extracted natural frequencies (λ_i) and an expected relative error variance yielding a Modal Assurance Criterion (MAC) uncertainty of approximately 2% for the mode shape vectors (ϕ_i). Defining the error strictly in the modal domain ensures that the optimization's baseline uncertainty is clearly understood in standard structural dynamics terms. The data sampling duration is configured to 1000 s with a sampling frequency of 200 Hz. Consistent with a long-term permanent monitoring paradigm, this 1000 s represents a single periodic data acquisition window. The sensors themselves are deployed for a design service life of 4 years, necessitating a robust layout that maintains acceptable identification accuracy for these periodic snapshots even as individual hardware nodes degrade over the multi-year lifespan.

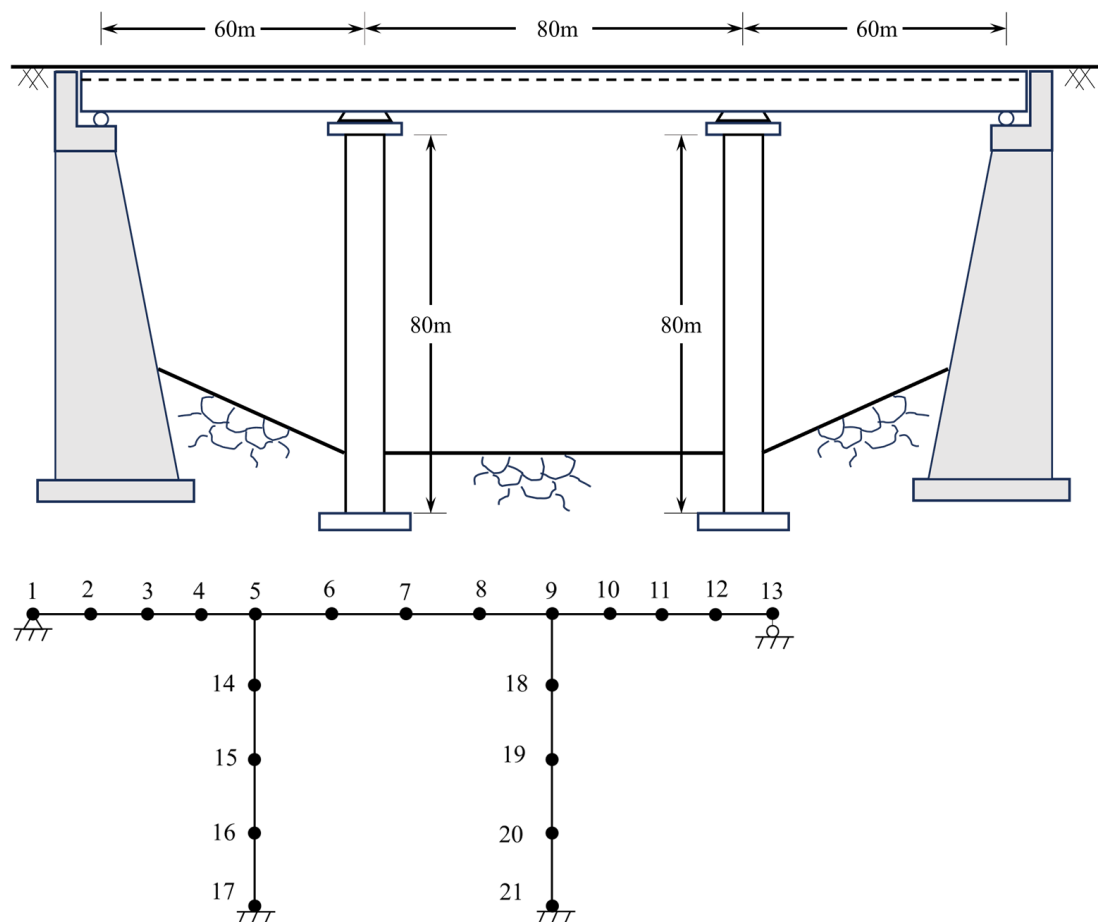


Figure 11. A three-span continuous bridge.

Table 1. Mechanical Parameters of Bridge.

Component	Cross-Sectional Area (m ²)	Moment of Inertia (m ⁴)	Elastic Modulus (GPa)	Density (kg/m ³)
Main Girder	9.8	10.7	34.5	2700
Pier	18.5	55.5	32.0	2600

To simulate a challenging yet realistic engineering scenario, the optimization targets the identification of the first ten structural modes using a network limited to 15 sensors. Unlike simpler building frames, complex bridge structures exhibit closely spaced frequencies and rely on higher-order modes to capture critical coupled behaviors (e.g., lateral-torsional coupling). Furthermore, higher-order modes are significantly more sensitive to localized stiffness degradation; therefore, ensuring the sensor network can accurately observe a broad bandwidth of up to 10 modes is a practical necessity to support downstream structural health monitoring tasks such as damage detection. Constraining the candidate set to these specific discrete nodes, rather than allowing complete spatial freedom across a highly refined mesh, reflects the practical realities of field deployments. In actual infrastructure, sensor placement is strictly governed by physical accessibility, cabling logistics, and the necessity to mount equipment on robust primary structural members. Furthermore, because the target global mode shapes are spatially smooth, minor spatial adjustments (e.g., sub-meter shifts) offer negligible improvements in modal observability while exponentially increasing the combinatorial computational burden. Thus, the constrained grid depicted in Figure 11 represents a necessary and realistic balance between installation feasibility and optimization efficiency. Unlike the previous case study, no artificial constraints are imposed on the number of specific sensor types. The algorithms are granted full freedom to select the optimal mix based solely on the trade-off between information gain and reliability.

Figures 12 and 13 compare the evolution of the optimal sensor placement obtained using the DIE method and the RIE reliability-aware method, respectively. The deterministic solution is dominated by accelerometers across almost all sensor budgets (i.e., the maximum discrete number of sensors permitted for deployment due to financial, logistical, and data-acquisition constraints), with only a limited introduction of velocity sensors at higher sensor counts, reflecting optimization under the assumption of fully functional sensors. In contrast, the RIE method selects predominantly velocity sensors, while introducing a small number of displacement sensors at higher sensor counts. This selection is driven by the specific reliability parameters defined for this case study, where velocity sensors ($\eta_v = 6$) offer a sweet spot between performance and durability. While accelerometers possess superior theoretical sensitivity, their high failure rate ($\eta_a = 3$) renders them a liability for the robust objective function. Velocity sensors, therefore, emerge as the optimal compromise: they retain sufficient dynamic sensitivity to capture the requisite modal information while possessing a survival probability high enough to ensure data continuity over the 4-year design life. This shift indicates that incorporating failure probability in the objective leads to a different sensor-type composition that prioritizes long-term information sustainability.

Figure 14 compares the lifecycle evolution of the expected entropy for sensor networks designed by the DIE method and the RIE method. At $t = 0$ years (where t denotes the elapsed operational time, representing the moment of initial deployment prior to any hardware degradation), the DIE design achieves lower nominal entropy under the assumption of fully functional sensors. With increasing service time, its expected entropy rises sharply, especially near the 4-year design life, reflecting the rapid degradation of accelerometers. In contrast, the RIE design maintains consistently lower expected entropy

over the service period and exhibits a slower increase after the design-life point, indicating improved tolerance to sensor failures and more stable long-term information acquisition.

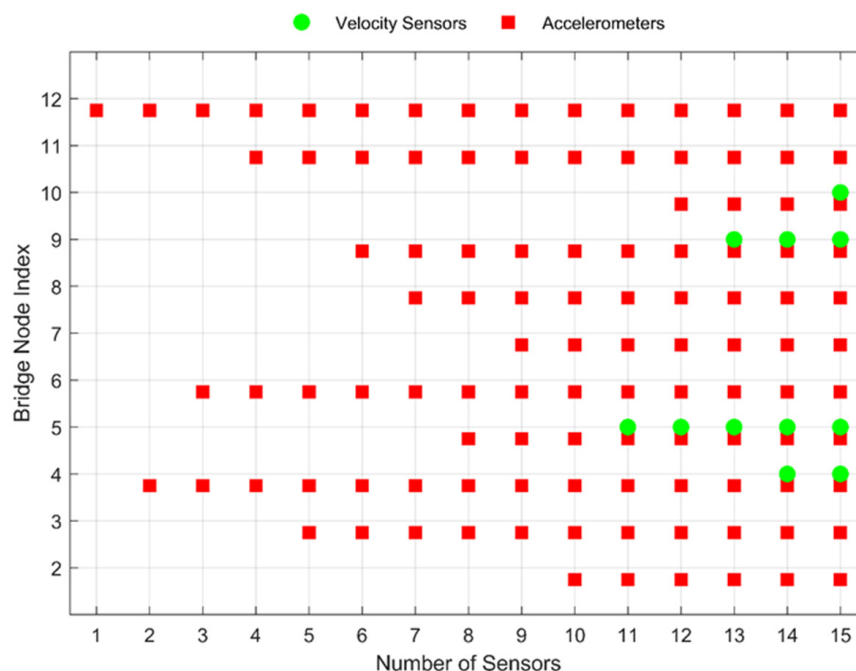


Figure 12. Optimal sensor placement configurations for the bridge model using the DIE method (excluding sensor failure). The deployed sensor types are indicated as follows: Accelerometers (red squares), and Velocity Sensors (green circles).

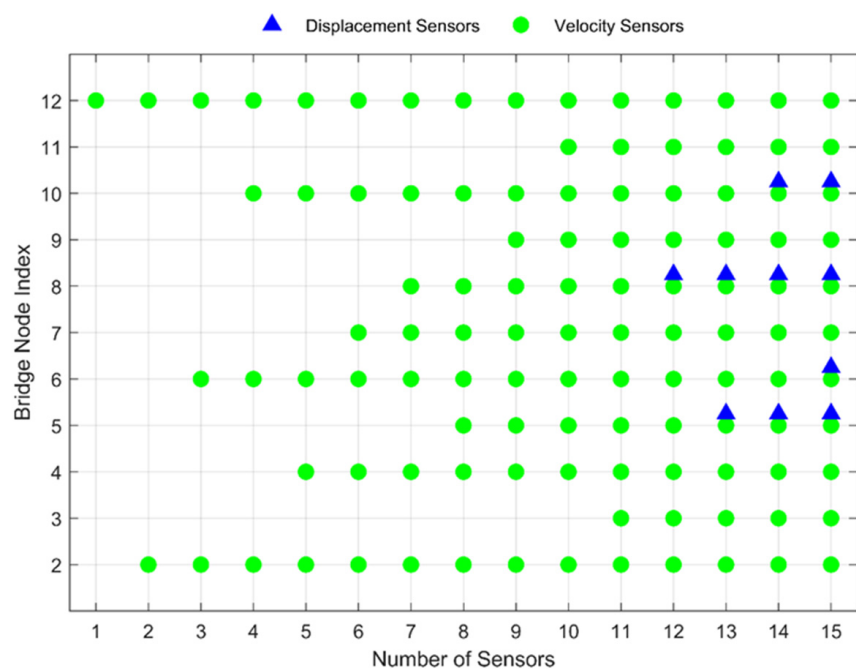


Figure 13. Optimal sensor placement configurations based on the RIE method (accounting for sensor failure). The deployed sensor types are indicated as follows: Velocity Sensors (green circles), and Displacement Sensors (blue triangles).

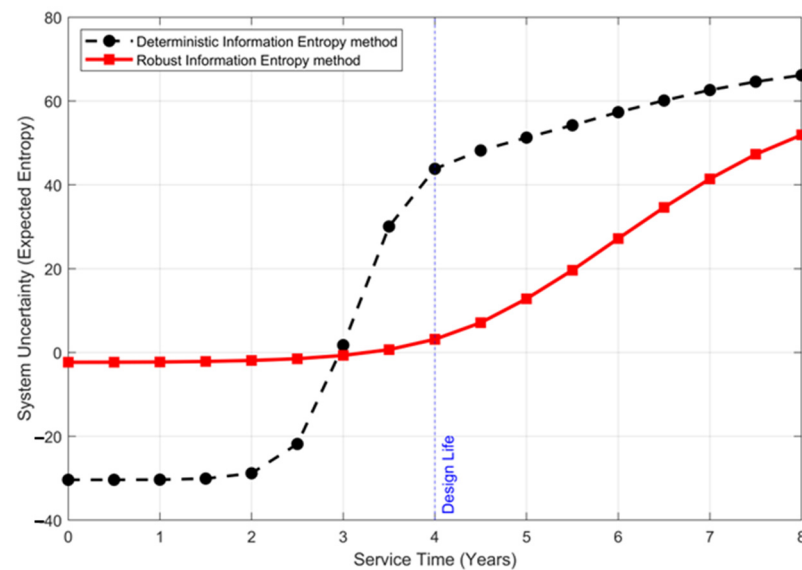


Figure 14. Lifecycle performance comparison of optimal sensor placement schemes.

Furthermore, the identification accuracy of the optimized sensor configurations is evaluated through the frequency convergence results in Figure 15 and the posterior COV analysis in Figure 16. As shown in Figure 15, the RIE sensor placement achieves progressively tighter 95% confidence intervals for all ten target modes as the sensor budget increases, indicating stable and well-conditioned modal identification across both low- and high-frequency ranges. Figure 16 further confirms this observation, where the posterior COV values obtained using the robust configuration decrease consistently with the number of sensors and reach levels comparable to those of the DIE design. These results demonstrate that explicitly accounting for sensor reliability in the placement optimization does not compromise identification precision. Instead, the RIE method preserves high-quality modal estimation while enabling a sensor network structure that is better suited for long-term operation. As previously noted in the discussion of the earlier case study, it is important to clarify that while the COV curves visually appear to converge to absolute zero, this is a graphical artifact of the linear axis scaling. The initial uncertainties are large enough that the highly optimized, multi-sensor configurations produce COV values that are orders of magnitude smaller. A strict, non-zero baseline of uncertainty is always maintained, reflecting the practical realities of parameter estimation.

In summary, the three-span continuous bridge case study highlights the importance of incorporating sensor failure probabilities into the placement optimization. By moving from a deterministic design based on nominal sensitivity to a reliability-aware objective, the RIE method explicitly balances instantaneous information gain against lifecycle uncertainty. The resulting configuration adopts a diversified mix of sensor types, which reduces reliance on high-risk components and improves long-term information sustainability. This advantage is reflected in the lifecycle analysis, where the proposed design exhibits a slower increase in expected entropy over time. Meanwhile, the frequency-convergence and COV results confirm that the robust configuration achieves competitive identification accuracy across all targeted modes as the sensor budget increases. Overall, the proposed framework provides a practical basis for designing sustainable monitoring systems for complex civil infrastructure.

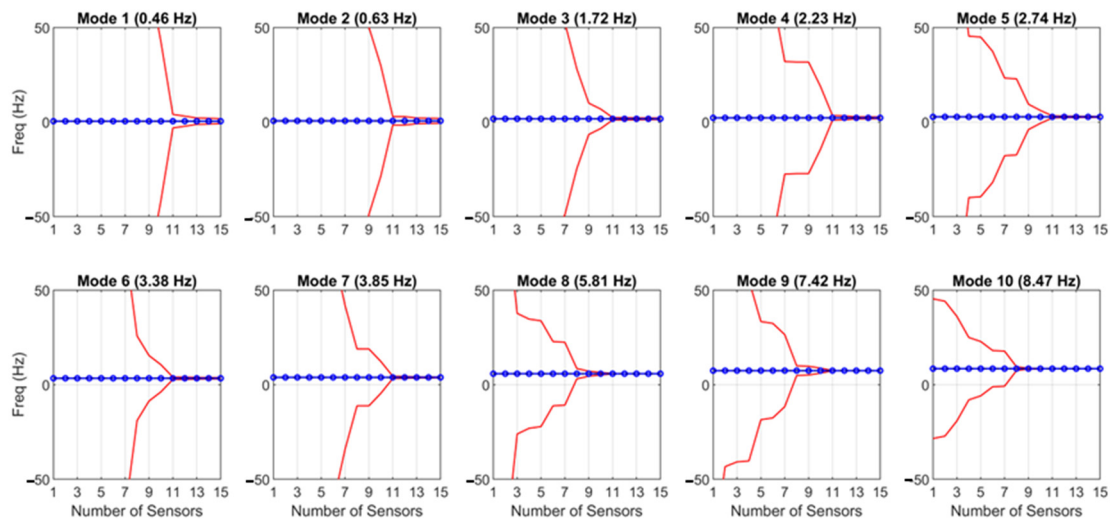


Figure 15. Identified natural frequencies versus the number of sensors. The blue line represents the identification results, the red line indicates the 95% confidence interval, and the blue circles denote the actual values.

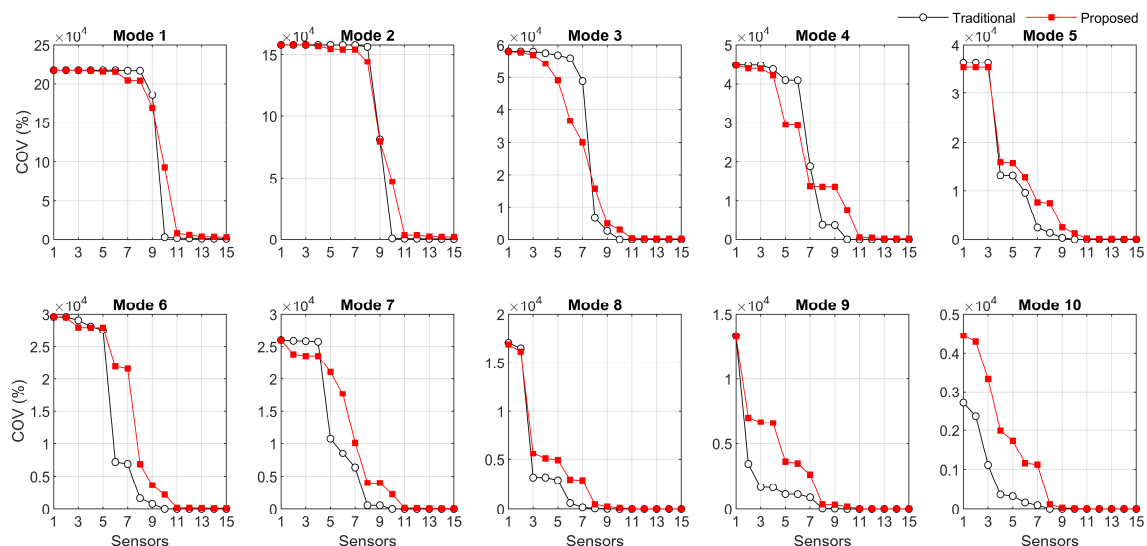


Figure 16. Posterior coefficient of variation in the identified modal frequencies versus the number of sensors for optimal sensor placement schemes obtained by DIE and RIE methods.

6. Conclusions

A robust optimization methodology for multi-type sensor placement, explicitly accounting for sensor failure, is proposed and validated in this study. First, an information entropy evaluation framework was established based on Bayesian inference to quantify parameter identification uncertainty. Subsequently, the Weibull distribution was utilized to construct a sensor failure probability model, capturing time-dependent hardware reliability characteristics. By integrating these failure probabilities into a modified information entropy metric and employing a heuristic search strategy, an optimal sensor configuration was derived. Numerical validations on a frame structure and a bridge model demonstrate that, under identical sensor quantity constraints, the RIE method yields lower expected information entropy compared to DIE approaches. This effectively reduces identification uncertainty over the lifecycle, providing a robust monitoring solution that ensures data availability despite hardware degradation.

A modified information entropy index with modal parameters as the identification target is constructed in this paper; however, the RIE method may be extended to other structural parameter identification scenarios. Based on the identical theoretical framework, the optimization objective can be replaced with structural physical parameters. In terms of implementation, this would require redefining the parameter vector θ to represent element stiffness or damping coefficients and updating the forward model in the likelihood function (Equation (6)) to compute the theoretical power spectral density directly from the system matrices (M , C , K). By applying the finite difference sensitivity analysis to these physical parameters, an optimal sensor placement scheme that balances information content and robustness may similarly be obtained, demonstrating the method's favorable generalization capability.

It is acknowledged that the current optimization loop considers only single-sensor failure scenarios to manage computational tractability. However, as service duration extends, the probability of simultaneous multi-sensor failures rises significantly. Analysis indicates that to fully address life-cycle monitoring requirements, the optimization model must eventually accommodate higher-order failure combinations, despite the associated increase in computational complexity. Consequently, the development of efficient algorithms capable of navigating the combinatorial search space of multi-sensor failures remains a critical direction for future research.

While the proposed Robust Information Entropy (RIE) method effectively accounts for the stochastic degradation and failure of the sensor hardware, it currently assumes that the underlying structure's baseline dynamic properties remain constant. In reality, as civil infrastructure ages and degrades (e.g., through fatigue cracking or corrosion), the localized loss of stiffness alters the structural mode shapes. Consequently, a sensor layout optimized exclusively for a pristine baseline may exhibit reduced observability if severe, unanticipated structural damage significantly shifts the dynamic response. To design a network capable of optimally tracking deep degradation, future research must extend this methodology to account for structural epistemic uncertainty. This would involve optimizing the sensor layout across a probabilistically defined ensemble of simulated damage scenarios, effectively coupling hardware reliability with structural damage robustness.

Furthermore, while the current RIE method successfully balances modal observability against hardware failure probabilities, it represents a purely technical optimization. In practical SHM deployments, particularly when utilizing a heterogeneous network of varying sensor types, the financial costs of the hardware and the logistics of replacement are decisive factors. A cheaper sensor with a shorter design life might be economically preferable if maintenance access is facile, whereas an expensive, highly durable sensor is required for inaccessible locations. To holistically address this, future iterations of this methodology should be expanded into a multi-objective optimization framework. By defining an Expected Life-Cycle Cost (LCC) function—incorporating initial capital expenditures and the probabilistic costs of future replacement interventions derived from the sensor failure models—the algorithm could generate a Pareto front. This would allow practitioners to explicitly trade-off robust information entropy against total financial cost.

Author Contributions: S.Z.: Conceptualization, Methodology, Software, Writing—original draft; D.L.: Data curation, Visualization, Investigation; P.Y.: Conceptualization, Methodology, Software, Writing—original draft; N.L.: Formal analysis, Validation; K.H.: Writing—original draft, Writing—review & editing; L.W.: Funding acquisition, Writing—review & editing. All authors have read and agreed to the published version of the manuscript.

Funding: This study is funded by the Shenzhen Science and Technology Planning Project (CJGJZD20220517141800001).

Data Availability Statement: Data will be made available on request.

Conflicts of Interest: Authors Shenghuan Zeng and Ding Luo were employed by Shenzhen Expressway Engineering Testing Co., Ltd. The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

References

1. Wang, Y.; Chen, Y.; Yao, Y.H.; Ou, J.P. Advancements in optimal sensor placement for enhanced structural health monitoring: Current insights and future prospects. *Buildings* **2023**, *13*, 3129. [\[CrossRef\]](#)
2. Tan, Y.; Zhang, L.M. Computational methodologies for optimal sensor placement in structural health monitoring: A review. *Struct. Health Monit.* **2020**, *19*, 1287–1308. [\[CrossRef\]](#)
3. Wang, K.; Er, G.K.; Zhu, Z.H. 3D nonlinear dynamic analysis of cable-moored offshore structures. *Ocean Eng.* **2020**, *213*, 107759. [\[CrossRef\]](#)
4. Wang, K.; Er, G.K.; Iu, V.P. Nonlinear random vibrations of 3D cable-moored floating structures under seismic and wave excitations. *J. Sound Vib.* **2019**, *452*, 58–81. [\[CrossRef\]](#)
5. Nasr, D.; Dahr, R.E.; Assaad, J.; Khatib, J. Comparative analysis between genetic algorithm and simulated annealing-based frameworks for optimal sensor placement and structural health monitoring purposes. *Buildings* **2022**, *12*, 1383. [\[CrossRef\]](#)
6. Chen, G.; Shi, W.X.; Yu, L.; Huang, J.Z.; Wei, J.G.; Wang, J. Wireless sensor placement optimization for bridge health monitoring: A critical review. *Buildings* **2024**, *14*, 856. [\[CrossRef\]](#)
7. Lu, N.W.; Xiao, X.Y.; Cui, J.; Liu, Y.R.; Huang, K.; Yuen, K.V. An unsupervised cross-domain method for bridge damage detection based on multichannel symmetric dot pattern feature alignment. *Comput. Aided Civ. Infrastruct. Eng.* **2025**, *40*, 5698–5718. [\[CrossRef\]](#)
8. Lu, N.W.; Wang, K.; Wang, H.H.; Liu, X.F.; Luo, Y. Real-time fatigue crack detection and prediction in steel structures based on an automated digital twin-driven framework. *Thin-Walled Struct.* **2026**, *218*, 114099. [\[CrossRef\]](#)
9. Chen, D.H.; Zhang, Y.S.; Xu, S.Z.; Li, Z.; Yang, C.X. Experimental study on bridge structural damage identification based on quasi-static displacement effects and wavelet packet decomposition. *Buildings* **2023**, *13*, 2974. [\[CrossRef\]](#)
10. Wang, Y.; Shi, Y.; Yang, T.Y.; Wang, W.N.; Zhang, Y.Q.; Xi, W. Damage identification of gas station double layer grid structure based on time domain response sensitivity analysis. *Buildings* **2025**, *15*, 3959. [\[CrossRef\]](#)
11. Abrahamsson, T.J.S.; Kammer, D.C. Finite element model calibration using frequency responses with damping equalization. *Mech. Syst. Signal Process.* **2015**, *62*, 218–234. [\[CrossRef\]](#)
12. Girardi, M.; Padovani, C.; Pellegrini, D.; Robol, L. A finite element model updating method based on global optimization. *Mech. Syst. Signal Process.* **2021**, *152*, 107372. [\[CrossRef\]](#)
13. Raviolo, D.; Civera, M.; Fragonara, L.Z. A comparative analysis of optimization algorithms for finite element model updating on numerical and experimental benchmarks. *Buildings* **2023**, *13*, 3010. [\[CrossRef\]](#)
14. Tondo, G.R.; Kavrakov, I.; Morgenthal, G. Efficient dynamic modal load reconstruction using physics-informed Gaussian processes based on frequency-sparse Fourier basis functions. *Mech. Syst. Signal Process.* **2025**, *225*, 112295. [\[CrossRef\]](#)
15. Liu, Y.R.; Wang, L.; Qiu, Z.P.; Chen, X. A dynamic force reconstruction method based on modified Kalman filter using acceleration responses under multi-source uncertain samples. *Mech. Syst. Signal Process.* **2021**, *159*, 107761. [\[CrossRef\]](#)
16. Liu, D.X.; Liu, B.; Li, X.G.; Shi, K. Theoretical and numerical examination of a novel method for identifying bridge moving force using an instrumented vehicle. *Buildings* **2023**, *13*, 1481. [\[CrossRef\]](#)
17. Yang, C.; Lu, Z.X. An interval effective independence method for optimal sensor placement based on non-probabilistic approach. *Sci. China Technol. Sci.* **2017**, *60*, 186–198. [\[CrossRef\]](#)
18. Yi, T.H.; Zhou, G.D.; Li, H.N.; Wang, C.W. Optimal placement of triaxial sensors for modal identification using hierarchic wolf algorithm. *Struct. Control Health Monit.* **2017**, *24*, e1958. [\[CrossRef\]](#)
19. Castro-Triguero, R.; Murugan, S.; Gallego, R.; Friswell, M.I. Robustness of optimal sensor placement under parametric uncertainty. *Mech. Syst. Signal Process.* **2013**, *41*, 268–287. [\[CrossRef\]](#)
20. Mandler, A.; Döhler, M.; Ventura, C.E. Sensor placement with optimal damage detectability for statistical damage detection. *Mech. Syst. Signal Process.* **2022**, *170*, 108767. [\[CrossRef\]](#)
21. Dogancay, K. Optimal geometries for AOA localization in the Bayesian sense. *Sensors* **2022**, *22*, 9802. [\[CrossRef\]](#)
22. Yin, T.; Yuen, K.V.; Lam, H.F.; Zhu, H.P. Entropy-based optimal sensor placement for model identification of periodic structures endowed with bolted joints. *Comput. Aided Civ. Infrastruct. Eng.* **2017**, *32*, 1007–1024. [\[CrossRef\]](#)

23. Kim, T.; Youn, B.D.; Oh, H. Development of a stochastic effective independence (SEFI) method for optimal sensor placement under uncertainty. *Mech. Syst. Signal Process.* **2018**, *111*, 615–627. [[CrossRef](#)]
24. Gao, B.; Bai, Z.H.; Song, Y.B. Optimal placement of sensors in bridge monitoring based on an adaptive gravity search algorithm. *J. Vib. Shock* **2021**, *40*, 86–92+189. [[CrossRef](#)]
25. Ghosh, D.; Mukhopadhyay, S. Fisher information-based optimal sensor locations for output-only structural identification under base excitation. *Struct. Control Health Monit.* **2022**, *29*, e3021. [[CrossRef](#)]
26. Yuen, K.V.; Kuok, S.C. Efficient Bayesian sensor placement algorithm for structural identification: A general approach for multi-type sensory systems. *Earthq. Eng. Struct. Dyn.* **2015**, *44*, 757–774. [[CrossRef](#)]
27. Xu, Z.Y.; Guo, Y.J.; Saleh, J.H. Multi-objective optimization for sensor placement: An integrated combinatorial approach with reduced order model and Gaussian process. *Measurement* **2022**, *187*, 110370. [[CrossRef](#)]
28. Zhou, G.D.; Yi, T.H.; Xie, M.X.; Li, H.N.; Xu, J.H. Optimal wireless sensor placement in structural health monitoring emphasizing information effectiveness and network performance. *J. Aerosp. Eng.* **2021**, *34*, 04020112. [[CrossRef](#)]
29. Pei, X.Y.; Hou, Y.; Huang, H.B.; Zhang, J.X. A multi-objective sensor placement method considering modal identification uncertainty and damage detection sensitivity. *Buildings* **2025**, *15*, 821. [[CrossRef](#)]
30. Nong, S.X.; Yang, D.H.; Yi, T.H. Pareto-based bi-objective optimization method of sensor placement in structural health monitoring. *Buildings* **2021**, *11*, 549. [[CrossRef](#)]
31. Yang, C.; Xia, Y.Q. Interval Pareto front-based multi-objective robust optimization for sensor placement in structural modal identification. *Reliab. Eng. Syst. Saf.* **2024**, *242*, 109703. [[CrossRef](#)]
32. Liu, Y.R.; Wang, L.; Ng, B.F. Load-independent multi-objective sensor placement method for localization and reconstruction of external excitations under interval uncertainties. *Comput. Methods Appl. Mech. Eng.* **2023**, *416*, 116344. [[CrossRef](#)]
33. Civera, M.; Pecorelli, M.L.; Ceravolo, R.; Surace, C.; Fragonara, L.Z. A multi-objective genetic algorithm strategy for robust optimal sensor placement. *Comput. Aided Civ. Infrastruct. Eng.* **2021**, *36*, 1185–1202. [[CrossRef](#)]
34. Liu, B.; Xu, Q.; Chen, J.Y.; Li, J.; Wang, M.M. A new framework for isolating sensor failures and structural damage in noisy environments based on stacked gated recurrent unit neural networks. *Buildings* **2022**, *12*, 1286. [[CrossRef](#)]
35. Nong, X.Z.; Luo, X.; Lin, S.; Ruan, Y.M.; Ye, X.J. Multimodal deep neural network-based sensor data anomaly diagnosis method for structural health monitoring. *Buildings* **2023**, *13*, 1976. [[CrossRef](#)]
36. Xu, X.L.; Li, F.C.; Hu, Y.; Zhu, Y.P. A joint sparse representation-based approach for anomaly rectification in extended-duration SHM systems with sensor networks. *Mech. Syst. Signal Process.* **2025**, *231*, 112666. [[CrossRef](#)]
37. Wu, W.; Cantero-Chinchilla, S.; Prescott, D.; Remenyte-Prescott, R.; Chiachío, M. A general approach to assessing SHM reliability considering sensor failures based on information theory. *Reliab. Eng. Syst. Saf.* **2024**, *250*, 110267. [[CrossRef](#)]
38. Taylor, S.G.; Park, G.; Farinholt, K.M.; Todd, M.D. Diagnostics for piezoelectric transducers under cyclic loads deployed for structural health monitoring applications. *Smart Mater. Struct.* **2013**, *22*, 025024. [[CrossRef](#)]
39. Zuo, L.; Sun, H.K.; He, Y.G.; Yin, B.Q.; Hu, X.M. Reliability analysis of MEMS sensors with zero-failure data. *J. Electron. Meas. Instrum.* **2019**, *33*, 69–75. [[CrossRef](#)]
40. Wang, Y.W.; Lv, S.F.; Zhuang, Z.Q.; Albalawi, O.; Alshanbari, H.M. A novel probabilistic model: Simulation and modeling the time duration in musical education and engineering. *Alex. Eng. J.* **2024**, *106*, 392–402. [[CrossRef](#)]
41. Zhao, X.F.; Li, B.T.; Mizutani, S.; Nakagawa, T. A revisit of age-based replacement models with exponential failure distributions. *IEEE Trans. Reliab.* **2022**, *71*, 1477–1487. [[CrossRef](#)]
42. Yin, M.L.; Angus, J.E.; Trivedi, K.S. Optimal preventive maintenance rate for best availability with hypo-exponential failure distribution. *IEEE Trans. Rel.* **2013**, *62*, 351–361. [[CrossRef](#)]
43. Sun, X.; Wu, J.E.; Liang, X.F.; Lu, Y.; Xu, Y.W.; Liang, L.W.; Wang, S.Z.; He, X.; Liu, X.; Ju, D.F.; et al. Statistical reliability analysis and lifetime prediction for magnetoelectric sensors. *IEEE Trans. Instrum. Meas.* **2025**, *74*, 9525109. [[CrossRef](#)]
44. Sun, W.W.; Shao, H.; Li, J.L.; Wu, T.; Fainman, E.Z. Multi-type traffic sensor location problem for origin-destination estimation considering spatiotemporal correlation and sensor failure. *Transp. Res. Part C Emerg. Technol.* **2025**, *179*, 105288. [[CrossRef](#)]
45. Dubek, K.; Schneidhofer, C.; Dörr, N.; Rojacz, H.; Plank, B.; Schmid, U. Accelerated lifetime estimation and failure analysis of micromachined humidity sensors under high temperature and high humidity conditions. *Microelectron. Reliab.* **2025**, *174*, 115885. [[CrossRef](#)]
46. Le, Z.Y.; Becker, E.; Konstantinides, D.G.; Ding, C.; Makedon, F. Modeling reliability for wireless sensor node coverage in assistive testbeds. In Proceedings of the 3rd International Conference on Pervasive Technologies Related to Assistive Environment, New York, NY, USA, 23 June 2010.
47. Qiu, L.J. Research on the Wireless Sensor Network Reliability Model. Master's Thesis, Jilin University, Changchun, China, 2011.
48. Li, Z.Y.; Wang, Y.H.; Ding, J.M.; Yang, G.L. Research on deformation of foundation based on maximum entropy principle. *J. Railw. Sci. Eng.* **2008**, *5*, 46–51.

49. Au, S.K. Fast Bayesian ambient modal identification in the frequency domain, Part I: Posterior most probable value. *Mech. Syst. Signal Process.* **2012**, *26*, 60–75. [\[CrossRef\]](#)
50. Yuen, K.V. *Bayesian Methods for Structural Dynamics and Civil Engineering*; John Wiley & Sons: Singapore, 2010.

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.